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Technical report 07-026

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If you want to cite this report, please use the following reference instead:
M. van den Berg, A. Hegyi, B. De Schutter, and J. Hellendoorn, "Integrated traffic control for mixed urban and freeway networks: A model predictive control approach," *European Journal of Transport and Infrastructure Research*, vol. 7, no. 3, pp. 223–250, Sept. 2007. doi:10.18757/ejtir.2007.7.3.3390

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Integrated traffic control for mixed urban and freeway networks:
A model predictive control approach

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Abstract: We develop a control method for networks containing both urban roads and freeways. These two road types are closely connected: congestion on the freeway often causes spill-back leading to urban queues, slowing down the urban traffic, and vice versa. As a consequence, control measures taken in one of the two areas can have a significant influence on the other area. We first develop a model that describes the evolution of the traffic flows in mixed networks. Next, we propose the control method that is used for the integrated control. This approach is based on model predictive control, in which the optimal control inputs are determined on-line using numerical optimization and a prediction model in combination with a receding horizon approach. We also compare our newly developed control method with systems that are similar to existing dynamic traffic control systems like SCOOT and UTOPIA/SPOT, in a qualitative as well as in a quantitative way via a simple case study. The results illustrate the potential benefits of the proposed approach and motivate further development and improvement of the proposed control method.

Keywords: coordinated control, integrated traffic control, mixed urban and freeway networks, model predictive control, traffic management, traffic signal control.

1 Introduction

The need for mobility is increasing, as can be seen from the growing number of road users as well as from the increasing number of movements per user (AVV, 2006). This leads to an increase in the frequency of traffic jams and growing lengths of the queues in the traffic network. These traffic jams cause large delays, resulting in higher travel costs and they also have a negative impact on the environment due to e.g. noise and pollution. Due to these disadvantages dealing with traffic jams has become an important issue.

To tackle the above congestion problems there exist different methods: construction of new roads, levying tolls, promoting public transport, or making more efficient use of the existing infrastructure. In this paper we consider the last approach, implemented using dynamic traffic management or control, because this approach is effective on the short term, and inexpensive compared to constructing new infrastructure.

Current traffic control approaches usually focus on either urban traffic or freeway traffic. In urban areas traffic signals are the most frequently used control measures. Traditionally, they are controlled locally using fixed time settings, or they are vehicle-actuated, meaning that they react on the prevailing traffic situation. Nowadays sophisticated, dynamic systems are also making progress. They coordinate different available control measures to improve the total performance. Systems such as SCOOT (Robertson and Bretherton, 1991), SCATS (Wolson and Taylor, 1999), Toptrac (TPA, n.d.), TUC (Diakaki et al., 2000), Mitrop (Gartner et al., 1976), Motion (Busch and Kruse, 2001), and UTOPIA/SPOT (Peek Traffic, 2002) use a coordinated control method to improve the urban traffic, e.g. by constructing green waves, or to improve the traffic circulation. Control on freeways is done using different traffic control measures. Ramp metering is applied on on-ramps, using systems like ALINEA (Papageorgiou et al., 1991). Overviews of ramp metering methods and results are given in (Taale and Middelham, 2000; Papageorgiou and Kotsialos, 2002). The use of variable speed limits on freeways is described in (Alessandri et al., 1999; Lenz et al., 1999; Mulders, 1990; Hegyi et al., 2005), and the use of route guidance e.g. in (Deflorio, 2003; Diakaki et al., 2000; Karimi et al., 2004). Several authors have described methods for coordinated control for freeways using different traffic control measures (Kotsialos, Papageorgiou, Mangeas and Haj-Salem, 2002; Bellemans, 2003; Hegyi, 2004; Kotsialos and Papageorgiou, 2004).
Several authors have also investigated corridor control (Karimi et al., 2004; Wu and Chang, 1999; Diakaki et al., 2000), where arterials are controlled together with freeways. In this paper we describe the coordinated and integrated control of networks that contain both freeways and urban roads, since the traffic flows on freeways are often influenced by traffic flows on urban roads, and vice versa. Freeway control measures like ramp metering or speed limits allow a better flow, higher speeds, and larger throughput but lead to longer queues on on-ramps. These queues may spill back and block urban roads. On the other hand, urban traffic management policies often try to get vehicles on the freeway network as soon as possible, displacing the congestion toward neighboring freeways. The problems between the two road types are often increased by the fact that in many countries urban roads and freeways are managed by different management bodies, each with their own policies and objectives.

By considering a coordinated control approach the performance of the overall network can be improved significantly. Therefore we develop a control approach for coordinated control of mixed urban and freeway networks that makes an appropriate trade-off between the performance of the urban and freeway traffic operations, and that prevents a shift of problems between the two. This paper improves and extends the conference papers (van den Berg et al., 2003) and (van den Berg et al., 2004). The new contributions of these papers and the current paper with respect to the state-of-the-art are a macroscopic model that describes networks that contain both urban roads and freeways, and an integrated control method that takes the traffic flows on both types of road into account. In addition, this paper contains a case study in which different control

methods are compared in a qualitative and a quantitative way.
As control method we propose a model predictive control (MPC) approach (Camacho and Bordons, 1995; Maciejowski, 2002). MPC is an on-line model-based predictive control approach that has already been applied successfully to coordinated control of freeway networks (Hegyi, 2004; Bellemans, 2003; Kotsialos, Papageorgiou, Mangeas and Haj-Salem, 2002). MPC optimizes the settings of the control measures over a certain prediction horizon. Using a receding horizon approach, only the first step of the computed control signal is applied, and next the optimization is started again with the prediction horizon shifted one time step further.

As MPC requires a model to predict the behavior of the traffic, we will first develop a traffic model for networks that contain both urban roads and freeways. Traffic flow models can be distinguished according to the level of detail they use to describe the traffic. In this paper we use a macroscopic model. Macroscopic models are suited for on-line control since these models give a balanced trade-off between accurate predictions and computational efforts. The computation time for a macroscopic model does not depend on the number of vehicles in the network, making the model more suited for on-line control, where the prediction should run on-line in an optimization setting, which requires that the model should run several times faster than real-time, and where the results should always be available within a specified amount of time. Examples of macroscopic models are the LWR model (Lighthill and Whitham, 1955; Richards, 1956), the models of Helbing (Helbing et al., 2002) and Hoogendoorn (Hoogendoorn and Bovy, 2001a), and METANET (Messmer and Papageorgiou, 1990). An overview of existing models is given in (Hoogendoorn and Bovy, 2001b).

In particular, we use an extended version of the METANET traffic flow model to describe the freeway traffic, and a modified and extended model based on a queue length model developed by Kashani (Kashani and Saridis, 1983) for the urban traffic. We also discuss how the freeway and the urban model have to be coupled. This results in a macroscopic model for mixed networks with urban roads and freeways, especially suited for the MPC-based traffic control approach developed in this paper.
In a case study we illustrate how the developed MPC control method performs with respect to existing control systems. A simple benchmark network is simulated, and simple implementations of existing control systems are applied. The performance of these existing systems is compared with our theoretical MPC method. The results of this case study motivate the further development of the MPC method.

The remainder of the paper is organized as follows. We first describe the model for mixed urban and freeway networks in Section 2. Next we develop the MPC-based traffic control method in Section 3, and in Section 4 we compare the developed method with existing methods like SCOOT (Robertson and Bretherton, 1991) and UTOPIA/SPOT (Peek Traffic, 2002).

2 Model development

As indicated above the model for mixed networks containing both urban roads and freeways that we develop is based on the METANET model (Messmer and Papageorgiou, 1990) for the freeway part, and on a queue length model based on a model developed by Kashani (Kashani and Saridis, 1983) for the urban part.
Note that we will explicitly make a difference between the simulation time step T_{f} for the freeway part of the network, the simulation time step T_{u} for the urban part of the network, and the controller sample time T_{c}. We will also use three different counters: k_{f} for the freeway part, k_{u} for the urban part, and k_{c} for the controller. For simplicity, we assume that T_{u} is an integer divisor of T_{f}, and that T_{f} is an integer divisor of T_{c}:

$$T_{\mathrm{f}} = TT_{\mathrm{u}}, \ \ T_{\mathrm{c}} = KT_{\mathrm{f}} = KTT_{\mathrm{u}},$$

with T and K integers. The value for T_{f} must be selected in such a way that no vehicle can cross a freeway segment in one time step, which results in a typical value of 10 s for segments of 0.5 km. The value of T_{u} is selected small enough to obtain an accurate description of the traffic, typically between 1 and 5 s, depending on the length of the roads. In our case study we will select T_{c} to be 120 s, because for an on-line controller T_{c} should be long enough to determine the new control signal, which depends on the required computation time, and short enough to deal with changing traffic conditions.

2.1 Freeway traffic model

In order to model traffic flows in the freeway part of the network we use the destination-independent version of the METANET model, developed by Papageorgiou and Messmer (Messmer and Papageorgiou, 1990). This model is also used in earlier work for the coordinated control of freeways, (Kotsialos, Papageorgiou, Mangeas and Haj-Salem, 2002; Hegyi, 2004; Bellemans, 2003; Kotsialos and Papageorgiou, 2004). In this paper we add an extension to the model to obtain a better modeling of the outflow toward off-ramps when blocking phenomena on the off-ramp occur. For completeness we will first describe the original METANET model based on (Messmer and Papageorgiou, 1990), and next present the extension.

2.1.1 Basic METANET model

In the METANET model the freeway network is divided into links. Each link m is further divided in segments, as illustrated in Figure 1. All the segments in a link have the same characteristics, e.g. number of lanes, capacity, length, etc.
The traffic state in each segment i of link m at time $t = k_{\mathrm{f}}T_{\mathrm{f}}$ is described with the macroscopic variables average density $\rho_{m,i}(k_{\mathrm{f}})$ in veh/km/lane, space mean speed $v_{m,i}(k_{\mathrm{f}})$ in km/h, and average flow $q_{m,i}(k_{\mathrm{f}})$ in veh/h.
The outflow of segment i of link m at time step k_{f} is given by:

$$q_{m,i}(k_{\mathrm{f}}) = \rho_{m,i}(k_{\mathrm{f}})v_{m,i}(k_{\mathrm{f}})n_m$$

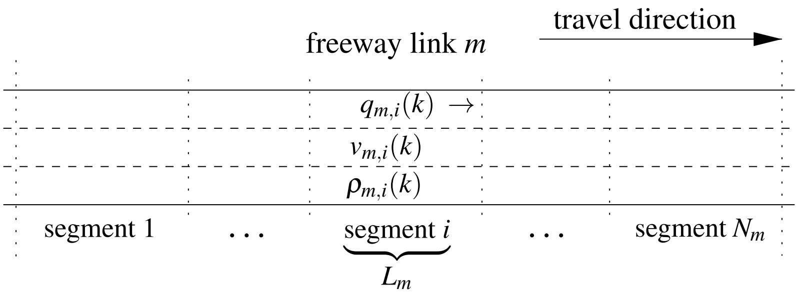


Figure 1: A freeway link in the METANET model divided in segments

where n_m denotes the number of lanes of link m. The density in each segment evolves as follows:

$$\rho_{m,i}(k_\mathrm{f}+1) = \rho_{m,i}(k_\mathrm{f}) + \frac{T_\mathrm{f}}{L_m n_m}\big(q_{m,i-1}(k_\mathrm{f}) - q_{m,i}(k_\mathrm{f})\big)$$

where L_m denotes the length of the segments in link m. This equation represents the law of conservation of vehicles: no vehicles appear or disappear within a link.
The equation for the evolution of the speed contains three main terms. The relaxation term expresses that the drivers try to achieve a desired speed $V(\rho)$ for the current density ρ. The convection term expresses that the speed changes due to the inflow of vehicles with a different speed, and the anticipation term expresses that drivers change their speed when the downstream density changes. The updated speed is then computed with:

$$v_{m,i}(k_\mathrm{f}+1) = v_{m,i}(k_\mathrm{f}) + \frac{T_\mathrm{f}}{\tau}\Big(V\big(\rho_{m,i}(k_\mathrm{f})\big) - v_{m,i}(k_\mathrm{f})\Big) + \frac{T_\mathrm{f}}{L_m} v_{m,i}(k_\mathrm{f})\big(v_{m,i-1}(k_\mathrm{f}) - v_{m,i}(k_\mathrm{f})\big)$$
$$- \frac{\nu T_\mathrm{f}}{\tau L_m}\frac{\rho_{m,i+1}(k_\mathrm{f}) - \rho_{m,i}(k_\mathrm{f})}{\rho_{m,i}(k_\mathrm{f}) + \kappa} \tag{1}$$

where τ, ν and κ are model parameters. They can be identified from data as described in (Kotsialos, Papageorgiou, Diakaki, Pavlis and Middelham, 2002). The desired speed $V(\rho_{m,i}(k_\mathrm{f}))$ is given by:

$$V\big(\rho_{m,i}(k_\mathrm{f})\big) = v_{\mathrm{free},m} \exp\left[-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k_\mathrm{f})}{\rho_{\mathrm{crit},m}}\right)^{a_m}\right] \tag{2}$$

where $v_{\mathrm{free},m}$ is the free flow speed on link m, $\rho_{\mathrm{crit},m}$ the critical density on this link, and a_m a model parameter.
Mainstream origins are modeled with a queue model:

$$w_o(k_\mathrm{f}+1) = w_o(k_\mathrm{f}) + T_\mathrm{f}\big(d_o(k_\mathrm{f}) - q_{m,o}(k_\mathrm{f})\big) \tag{3}$$

where w_o is the queue length at origin o connected to link m, d_o the demand at the origin and $q_{m,o}$ the flow leaving the origin o toward link m, which is determined by the number of available vehicles, the capacity of the freeway and the traffic conditions on the freeway:

$$q_{m,o}(k_\mathrm{f}) = \min\left(d_o(k_\mathrm{f}) + \frac{w_o(k_\mathrm{f})}{T_\mathrm{f}}, Q_{\mathrm{cap},m}\frac{\rho_{\max,m} - \rho_{m,1}(k_\mathrm{f})}{\rho_{\max,m} - \rho_{\mathrm{crit},m}}\right) \tag{4}$$

where $Q_{\text{cap},m}$ is the capacity of link m and $\rho_{\text{max},m}$ is the maximum possible density on the freeway link.

Freeway links are coupled via nodes, e.g. on-ramps, off-ramps, or intersections. Flows that enter a node p are distributed over the leaving nodes. They are first distributed according to the turning rates[1]:

$$Q_{\text{tot},p}(k_\text{f}) = \sum_{\mu \in I_p} q_{\mu,n_{\text{last},\mu}}(k_\text{f})$$

$$q_{m,0}(k_\text{f}) = \beta_{p,m}(k_\text{f}) Q_{\text{tot},p}(k_\text{f}) \text{ for each } m \in O_p$$

where $Q_{\text{tot},p}$ is the total flow entering node p, I_p is the set of all freeway links entering node p, $n_{\text{last},\mu}$ is the last segment of link μ, $\beta_{p,m}$ is the turning rate from node p to leaving link m, and O_p the set of leaving links of node p.

When a node p has more than one leaving link, the virtual downstream density $\rho_{\mu,n_{\text{last},\mu}+1}(k_\text{f})$ of the link μ that enters the node is approximated with:

$$\rho_{\mu,n_{\text{last},\mu}+1}(k_\text{f}) = \frac{\sum\limits_{m \in O_p} \rho_{m,1}^2(k_\text{f})}{\sum\limits_{m \in O_p} \rho_{m,1}(k_\text{f})} \ .$$

The virtual downstream density is used in the speed update equation (1) for the last segment $n_{\text{last},\mu}$ of link μ.

When a node p has more than one entering link, the virtual entering speed $v_{m,0}(k_\text{f})$ of leaving link m is given by:

$$v_{m,0}(k_\text{f}) = \frac{\sum\limits_{\mu \in I_p} v_{\mu,n_{\text{last},\mu}}(k_\text{f}) q_{\mu,n_{\text{last},\mu}}(k_\text{f})}{\sum\limits_{\mu \in I_p} q_{\mu,n_{\text{last},\mu}}(k_\text{f})} \ .$$

The virtual entering speed is used in the speed update equation (1) to compute the speed of the traffic that enters the first segment of link m.

In a link or segment where weaving and/or merging effects are taking place extra terms are added to improve the description of these effects, as described in (Kotsialos, Papageorgiou, Mangeas and Haj-Salem, 2002; Messmer and Papageorgiou, 1990).

2.1.2 Extension for off-ramp links

When the urban network is congested, it often happens that a nearby off-ramp is also blocked. This blockage will spill back onto the freeway. We propose an extension to the METANET model that more accurately models the behavior of off-ramp flows.

Consider an off-ramp r connected to a freeway link m as shown in Figure 2. The available space on off-ramp r limits the maximum flow that can enter it. This maximum flow $q_{r,1}^{\text{max}}(k_\text{f})$ is seen

[1]The index 0 in $q_{m,0}(k_\text{f})$ corresponds to a virtual segment that is located upstream of the first segment of link m. This virtual segment is used to describe the traffic that will enter link m.

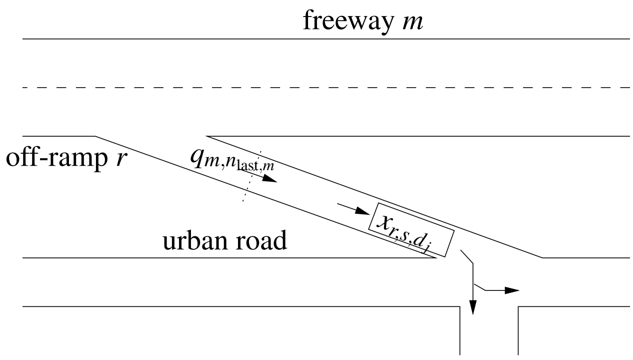


Figure 2: Layout of an off-ramp

as a boundary condition for the flow that leaves the freeway link $q_{m,n_{\text{last},m}}(k_\text{f})$ connected to the off-ramp:

$$q_{m,n_{\text{last},m}}(k_\text{f}) = \min\left(q^{\text{normal}}_{m,n_{\text{last},m}}(k_\text{f}), q^{\max}_{r,1}(k_\text{f})\right) \tag{5}$$

where $q^{\text{normal}}_{m,n_{\text{last},m}}(k_\text{f})$ is the flow that would have entered the freeway when the off-ramp was not blocked. When the flow is indeed limited to $q^{\max}_{r,1}(k_\text{f})$, the speed of the last segment of the freeway must be recalculated as follows:

$$v_{m,n_{\text{last},m}}(k_\text{f}) = \begin{cases} v^{\text{normal}}_{m,n_{\text{last},m}}(k_\text{f}) & \text{if } q^{\text{normal}}_{m,n_{\text{last},m}}(k_\text{f}) < q^{\max}_{r,1}(k_\text{f}) \\ v^{\text{normal}}_{m,n_{\text{last},m}}(k_\text{f}) \dfrac{q^{\max}_{r,1}(k_\text{f})}{q^{\text{normal}}_{m,n_{\text{last},m}}(k_\text{f})} & \text{otherwise} \end{cases}$$

where $v^{\text{normal}}_{m,n_{\text{last},m}}(k_\text{f})$ denotes the speed in the segment when no spill-back occurs, i.e. the speed computed with equation (1).
Further extensions describing e.g. dynamic speed limits, and mainstream metering are given in (Hegyi, 2004; Hegyi et al., 2005). The effects of control measures such as ramp metering and variable speed limits will be described in Section 3.2.

The external inputs for a simulation of the freeway model are the initial state of the links and the origin queues, and the signals that describe the evolution over the entire simulation period of the turning rates[2] $\beta_{p,m}(k_\text{f})$, the demands $d_o(k_\text{f})$, the boundary conditions $q^{\max}_{r,1}(k_\text{f})$, and the control signals such as the ramp metering rates and the variable speed limits.

2.2 Urban traffic model

Several authors have developed models to describe traffic in urban areas (Kashani and Saridis, 1983; Diakaki et al., 2002; Yin et al., 2002; Peek Traffic, 2002). Due to the fact that we want

[2]These turning rates can be given externally or they can be determined using a (dynamic) traffic assignment model (see e.g. (Daganzo and Sheffi, 1977; Sheffi and Powell, 1982; Chabibi and He, 2000)).

to model and control mixed networks under all conditions, the model we use should satisfy the following requirements:

1. It should be able to describe both light and congested traffic;

2. It should contain horizontal queues because queues often become long compared with buffer capacities, which can lead to blockage of intersections. When an intersection is blocked, no vehicles should be able to cross it.

There are many macroscopic urban traffic models that meet one or more of these requirements, such as the Kashani model (Kashani and Saridis, 1983) and the IN-TUC model (Diakaki et al., 2000, 2002). We will base our model on the Kashani model because it has the first of the above features, and because the model can easily be extended.

2.2.1 Extended urban model

Our model is based on the model developed by Kashani (Kashani and Saridis, 1983), but to fulfill all the requirements given above we make the following extensions:

1. Horizontal, turning-direction-dependent queues,

2. Blocking effects, represented by maximal queue lengths and a flow constraint on flows that want to enter the blocked link, so no vehicle will be able to cross a blocked intersection,

3. A shorter time step[3], to get a more accurate description of the traffic flows.

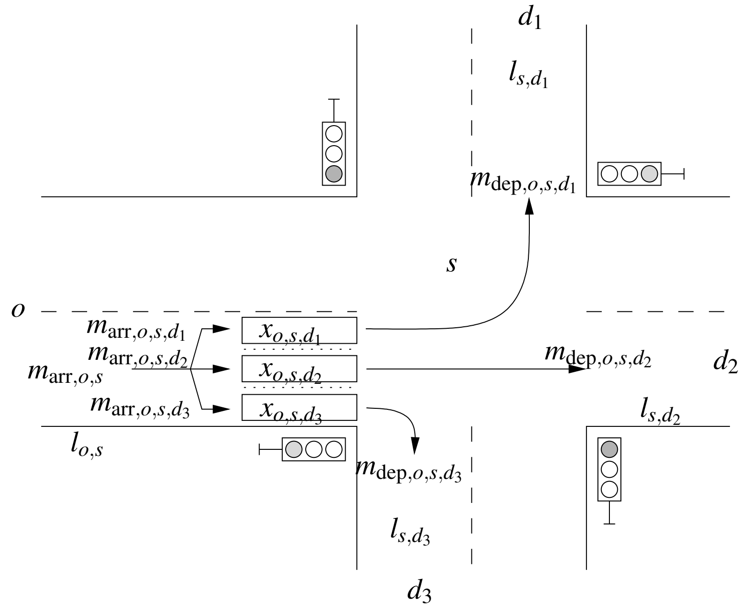
The main variables used in the urban model are shown in Figures 3(a) and 3(b). The most important variables are the queue length x expressed in number of vehicles, the number of arriving vehicles m_{arr}, and the number of departing vehicles m_{dep}. Using these variables, the model is formulated as follows.
The number of vehicles that intend to leave the link $l_{o_i,s}$, connecting origin o_i and intersection s, toward destination d_j at time $t = k_{\mathrm{u}}T_{\mathrm{u}}$ is given by:

$$m_{\mathrm{dep,int},o_i,s,d_j}(k_{\mathrm{u}}) = \begin{cases} 0 & \text{if } g_{o_i,s,d_j}(k_{\mathrm{u}}) = 0, \\ \min\big(x_{o_i,s,d_j}(k_{\mathrm{u}}) + m_{\mathrm{arr},o_i,s,d_j}(k_{\mathrm{u}}), & \\ \qquad S_{s,d_j}(k_{\mathrm{u}}), T_{\mathrm{u}}Q_{\mathrm{cap},o_i,s,d_j}\big) & \text{if } g_{o_i,s,d_j}(k_{\mathrm{u}}) = 1, \end{cases} \tag{6}$$

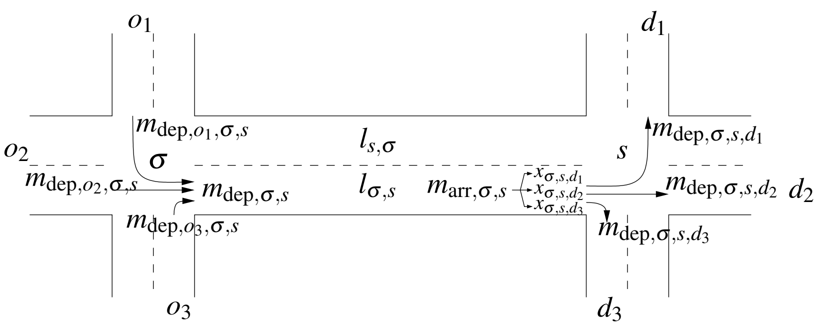
where T_{u} is the urban step with k_{u} as counter, $x_{o_i,s,d_j}(k_{\mathrm{u}})$ is the queue length consisting of vehicles coming from origin o_i and going to destination d_j at intersection s, $m_{\mathrm{arr},o_i,s,d_j}(k_{\mathrm{u}})$ is the number of vehicles arriving at the end of this queue, $S_{s,d_j}(k_{\mathrm{u}})$ is the free space in the downstream link expressed in number of cars, $Q_{\mathrm{cap},o_i,s,d_j}$ is the saturation flow[4], and $g_{o_i,s,d_j}(k_{\mathrm{u}})$ a binary signal that

[3] Kashani uses the cycle time as time step, which restricts the model to effects that take longer than the cycle time. For MPC-based traffic control the other effects can also be interesting, and one might also want to control the cycle times as part of the control measures.

[4] The saturation flow is the maximum flow that can cross the intersection under free flow conditions.



(a) Variables for an urban intersection



(b) Variables for an urban link

Figure 3: Overview of urban network variables

is 1 when the specified traffic direction has green, and zero otherwise. This means that $g_{o_i,s,d_j} = 0$ corresponds to a red traffic signal, and $g_{o_i,s,d_j} = 1$ to a green one.[5]

The free space $S_{\sigma,s}$ in a link $l_{\sigma,s}$ expresses the maximum number of vehicles that can enter the link. It can never be larger than the length $L_{\sigma,s}$ of the link expressed in number vehicles, and is computed as follows:

$$S_{\sigma,s}(k_{\mathrm{u}}+1) = S_{\sigma,s}(k_{\mathrm{u}}) - m_{\mathrm{dep},\sigma,s}(k_{\mathrm{u}}) + \sum_{d_j \in D_s} m_{\mathrm{dep},\sigma,s,d_j}(k_{\mathrm{u}})$$

where $m_{\mathrm{dep},\sigma,s}(k_{\mathrm{u}})$ is the number of vehicles departing from intersection σ towards link $l_{\sigma,s}$, and D_s is the set of destinations connected to intersection s.

[5]The computed green time is the effective green time. The exact signal timing including the amber time can easily be derived from this effective green time.

The number of vehicles departing from intersection s towards link l_{s,d_j} can be computed as

$$m_{\text{dep},s,d_j}(k_\text{u}) = \sum_{o_i \in O_s} m_{\text{dep},o_i,s,d_j}(k_\text{u}).$$

These vehicles drive from the beginning of the link l_{s,d_j} toward the tail of the queue waiting on the link. This gives a time delay $\delta_{s,d_j}(k_\text{u})$ which is approximated as:

$$\delta_{s,d_j}(k_\text{u}) = \text{ceil}\left(\frac{S_{s,d_j}(k_\text{u})\,L_{\text{av,veh}}}{v_{\text{av},s,d_j}}\right) \tag{7}$$

where $L_{\text{av,veh}}$ is the average length of a vehicle, and v_{av,s,d_j} the average speed on link l_{s,d_j}.
The time the vehicle enters the link and its delay on the link result in the time the vehicle will arrive at the end of the queue. It can happen that vehicles that have entered the link at different instants reach the end of the queue during the same time step. To take this into account the variable $m_{\text{arr},s,d_j}(k_\text{u})$ that describes the vehicles arriving at the end of the queue is updated accumulatively every time step. This results in:

$$m_{\text{arr},s,d_j}(k_\text{u}+\delta_{s,d_j}(k_\text{u}))_\text{new} = m_{\text{arr},s,d_j}(k_\text{u}+\delta_{s,d_j}(k_\text{u}))_\text{old} + m_{\text{dep},s,d_j}(k_\text{u})$$

where $m_{\text{arr},s,d_j}(k_\text{u}+\delta_{s,d_j}(k_\text{u}))$ is the number of vehicles arriving at the end of the queue at time $k_\text{u}+\delta_{s,d_j}(k_\text{u})$, and $m_{\text{dep},s,d_j}(k_\text{u})$ the number of vehicles entering link l_{s,d_j}.
The traffic flow reaching the tail of the queue in link l_{s,d_j} divides itself over the subqueues according to the turning rates $\beta_{o_i,s,d_j}(k_\text{u})$:

$$m_{\text{arr},o_i,s,d_j}(k_\text{u}) = \beta_{o_i,s,d_j}(k_\text{u}) m_{\text{arr},o_i,s}(k_\text{u}).$$

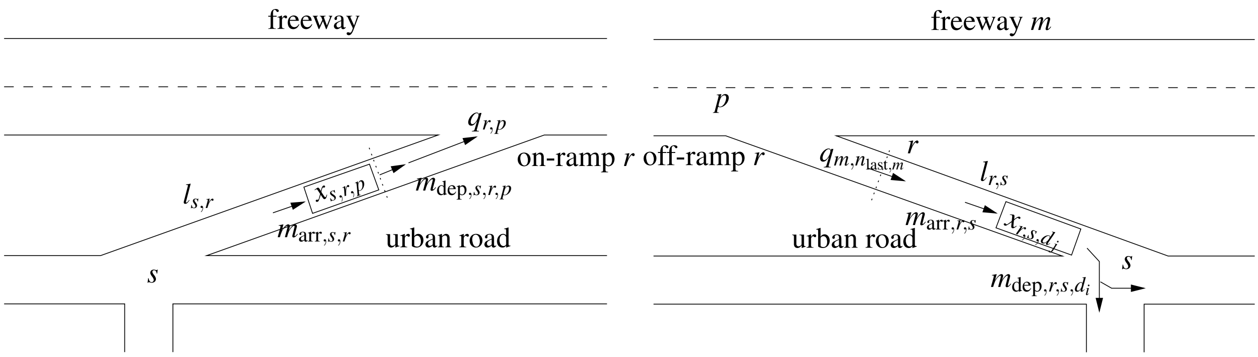
The subqueues are then updated as follows:

$$x_{o_i,s,d_j}(k_\text{u}+1) = x_{o_i,s,d_j}(k_\text{u}) + m_{\text{arr},o_i,s,d_j}(k_\text{u}) - m_{\text{dep},o_i,s,d_j}(k_\text{u}).$$

The total flow entering a destination link consists of several flows from different origins. The available space in the destination link should be divided over the entering flows, since the total number of vehicles entering the link may not exceed the available space. We divide this available space equally over the different entering flows. When one flow does not fill its part of the space, the remainder is proportionally divided over the rest of the flows. To illustrate how the effective values of $m_{\text{dep},o_i,s,d_j}(k_\text{u})$ can be computed let us assume that there are two origins, and so two queues from which vehicles want to drive into the same link. Let $m_{\text{dep,int},1}(k_\text{u})$ and $m_{\text{dep,int},2}(k_\text{u})$ denote the number of vehicles that intend to enter the link l_{s,d_j} from respectively origin 1 and origin 2. If we assume without loss of generality that $m_{\text{dep,int},1}(k_\text{u}) \leq m_{\text{dep,int},2}(k_\text{u})$, then the effective values for $m_{\text{dep},1}(k_\text{u})$ and $m_{\text{dep},2}(k_\text{u})$ can be computed as follows:

- if $m_{\text{dep,int},1}(k_\text{u}) + m_{\text{dep,int},2}(k_\text{u}) \leq S_{s,d_j}(k_\text{u})$, then

$$m_{\text{dep},1}(k_\text{u}) = m_{\text{dep,int},1}(k_\text{u}) \;\text{ and }\; m_{\text{dep},2}(k_\text{u}) = m_{\text{dep,int},2}(k_\text{u}) \;\;,$$



(a) Variables for an on-ramp (b) Variables for an off-ramp

Figure 4: Overview of variables on on-ramps and off-ramps

- if $m_{\text{dep,int},1}(k_\text{u}) + m_{\text{dep,int},2}(k_\text{u}) \geq S_{s,d_j}(k_\text{u})$, then

$$\begin{cases} m_{\text{dep},1}(k_\text{u}) = m_{\text{dep,int},1}(k_\text{u}) \\ m_{\text{dep},2}(k_\text{u}) = S_{s,d_j}(k_\text{u}) - m_{\text{dep,int},1}(k_\text{u}) \end{cases} \quad \text{if } m_{\text{dep,int},1}(k_\text{u}) < \tfrac{1}{2} S_{s,d_j}(k_\text{u}),$$
$$m_{\text{dep},1}(k_\text{u}) = m_{\text{dep},2}(k_\text{u}) = \tfrac{1}{2} S_{s,d_j}(k_\text{u}) \quad \text{if } m_{\text{dep,int},1}(k_\text{u}) \geq \tfrac{1}{2} S_{s,d_j}(k_\text{u}).$$

The extension to a situation with more upstream queues is straightforward.

The external inputs for a simulation of the urban model are the initial state of queues, arriving vehicles, and free space, and the signals that describe the evolution over the entire simulation period of the turning rates $\beta_{o_i,s,d_j}(k_\text{u})$ and of the green/red indicators $g_{o_i,s,d_j}(k_\text{u})$.

2.3 Interface between the models

The urban part and the freeway part are coupled via on-ramps and off-ramps. In this section we present the formulas that describe the evolution of the traffic flows on these on-ramps and off-ramps. The main problems are the different simulation time steps T_f and T_u and the boundary conditions that the models create for each other. We assume that the time steps are selected such that $T_\text{f} v_{\text{free},m} < L_m$.

2.3.1 On-ramps

Consider an on-ramp r that connects intersection s of the urban network to node p of the freeway network, as shown in Figure 4(a). The number of vehicles that enter the on-ramp from the urban network is given by $m_{\text{arr},s,r}(k_\text{u})$. These vehicles have a delay $\delta_{s,r}(k_\text{u})$ similar to equation (7). The evolution of the queue length is first described with the urban model. At the end of each freeway time step, the queue length as described in the urban model is translated to the queue length for the freeway model as explained below.

Now consider the freeway time step k_f corresponding to the urban time step $k_\mathrm{u} = Tk_\mathrm{f}$. In order to get a consistent execution of the urban and freeway models the computations should be done in the following order:

1. Determine the on-ramp departure flow $q_{r,p}(k_\mathrm{f})$ during the period $[k_\mathrm{f}T_\mathrm{f}, (k_\mathrm{f}+1)T_\mathrm{f})$ using (4).

2. Assume that these departures spread out evenly over the equivalent urban simulation period $k_\mathrm{u}T_\mathrm{u}, \ldots, (k_\mathrm{u}+T)T_\mathrm{u}$. Compute the departures for each urban time step in this period using $m_{\mathrm{dep},s,r,p}(k) = \dfrac{q_{r,p}(k_\mathrm{f})T_\mathrm{f}}{T}$ for $k = k_\mathrm{u}, \ldots, k_\mathrm{u}+T-1$ (note that $T_\mathrm{u} = T_\mathrm{f}/T$).

3. The number of arriving vehicles, the free space, and the queue length $x_{s,r,p}$ at link $l_{s,r}$ can now be computed using the equations for the urban traffic model given in Section 2.2.

4. When the queue length $x_{s,r,p}(k_\mathrm{u}+T)$ is computed, we set $w_o(k_\mathrm{f}+1) = x_{s,r,p}(k_\mathrm{u}+T)$. It is easy to verify that this is equivalent to (3).

2.3.2 Off-ramps

The evolution of the traffic flows on an off-ramp r is computed for the same time steps as for the on-ramp, starting at time step $k_\mathrm{u} = Tk_\mathrm{f}$. The variables are shown in Figure 4(b). The following steps are required to simulate the evolution of the traffic flows, in order to get a consistent execution of the urban and freeway models:

1. Determine the number of departing vehicles from link $l_{r,s}$ at intersection s during the period $[k_\mathrm{u}T_\mathrm{u}, (k_\mathrm{u}+T)T_\mathrm{u})$ using the urban traffic flow model.

2. Compute the maximal allowed flow $q_{r,1}^{\max}(k_\mathrm{f})$ that can enter the off-ramp in the period $[k_\mathrm{f}T_\mathrm{f}, (k_\mathrm{f}+1)T_\mathrm{f})$ based on the available storage space in the link $l_{r,s}$ at the end of the period. We have
$$q_{r,1}^{\max}(k_\mathrm{f}) = \frac{1}{T_\mathrm{f}}S_{r,s}(k_\mathrm{u}) + \sum_{k=k_\mathrm{u}}^{k_\mathrm{u}+T-1} \sum_{d_j \in D_s} m_{\mathrm{dep},r,s,d_j}(k).$$
The effective outflow $q_{m,n_{\mathrm{last},m}}(k_\mathrm{f})$ of freeway link m between node p and off-ramp r is then given by (5).

3. Now the METANET model can be updated for simulation step $k_\mathrm{f}+1$.

4. We assume that the outflow of the off-ramp is distributed evenly over the period $[k_\mathrm{f}T_\mathrm{f}, (k_\mathrm{f}+1)T_\mathrm{f})$ such that
$$m_{\mathrm{arr},r,s}(k+\delta_{r,s}) = \frac{q_{m,n_{\mathrm{last},m}}(k_\mathrm{f})T_\mathrm{f}}{T} \quad \text{for } k = k_\mathrm{u}, \ldots, k_\mathrm{u}+T-1.$$
The corresponding urban queue lengths $x_{r,s,d_j}(k)$ for $k = k_\mathrm{u}+1, \ldots, k_\mathrm{u}+T$ can be updated using the urban traffic flow model.

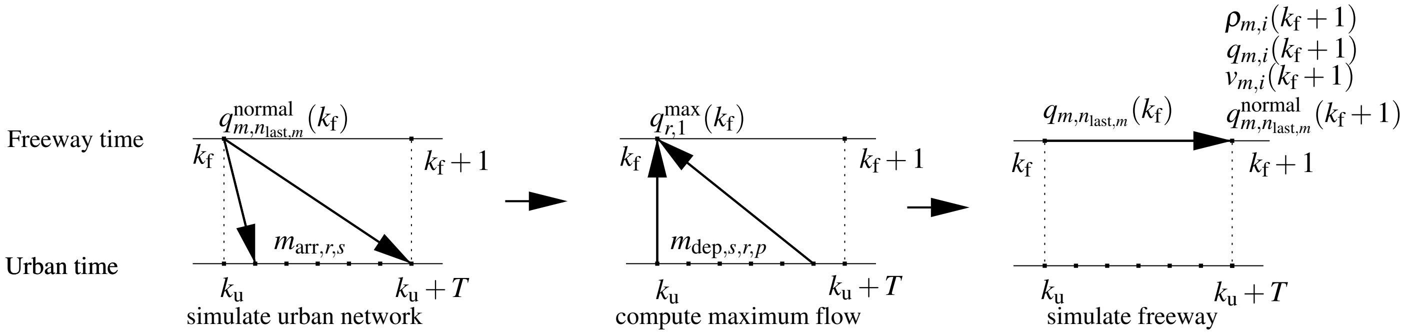


Figure 5: Order of computations during a simulation

In summary, the model for the off-ramp as well as the model for the on-ramp require a special order in which the computations are done. For simulating the whole network this means the computations should be done in the order shown in Figure 5. At the bottom each subfigure shows the urban time steps, at the top the freeway time steps. The first subfigure shows that with the flow at time k_f the number of arriving vehicles in the urban network can be computed for time $k_\mathrm{u}+1,\ldots,k_\mathrm{u}+T$. Next, as shown in the second subfigure, the urban variables at time steps $k_\mathrm{u},\ldots,k_\mathrm{u}+T-1$ are used to adapt the flows at time k_f. Last, the freeway variables on time $k_\mathrm{f}+1$ are computed with the variables at time k_f.

3 Coordinated control for mixed networks

In the previous section we have developed a model that describes traffic networks that contain both urban roads and freeways. This model forms the basis for our model predictive control-based method. In this section we first give a general description of model predictive control (MPC). Next we formulate the traffic controller for mixed urban and freeway networks, which is based on MPC.
We have selected MPC because it has the following features and advantages:

1. It can easily handle multi-input multi-output systems,
2. Only a few parameters have to be tuned,
3. It can handle constraints on inputs and outputs in a systematic way.

One of the first applications of MPC for traffic control is described in (Gartner, 1984). Other publications that deal with MPC or MPC-like approaches for traffic control are (Peek Traffic, 2002; Kotsialos, Papageorgiou, Mangeas and Haj-Salem, 2002; Diakaki et al., 2002). As described in (Bellemans, 2003; Hegyi, 2004) MPC can be extended to coordinated control of freeway networks.

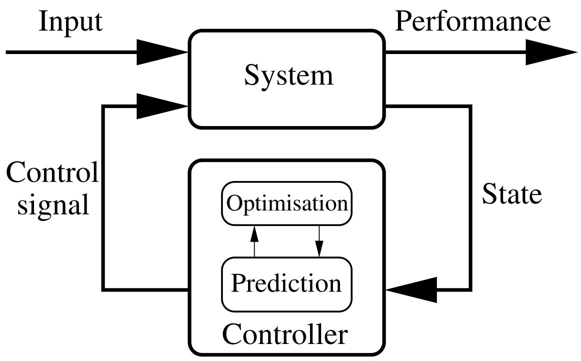


Figure 6: Working principle of MPC

3.1 Model Predictive Control

Model predictive control (MPC) (Camacho and Bordons, 1995; Maciejowski, 2002) is a control method that is applied in the process industry, where it is widely accepted for its ability to deal effectively with increasing productivity demands, environmental regulations and tighter product quality specifications. MPC is also suited for traffic control because it can easily handle changes in demands and in external conditions.

3.1.1 MPC approach

The goal of MPC is to minimize a cost function over a given prediction period $T = N_\mathrm{p}T_\mathrm{c}$, where N_p is called the prediction horizon. This cost function should give an indication for the performance of the system.

Figure 6 gives an overview of the operation of MPC. Assume that we are at time $t = k_\mathrm{c}T_\mathrm{c} = k_\mathrm{f}T_\mathrm{f} = k_\mathrm{u}T_\mathrm{u}$ where T_c is the controller time step. The current state of the system is measured, and fed into the controller. Now the current state and a prediction model are used to predict the behavior of the traffic during the period $[k_\mathrm{c}T_\mathrm{c},(k_\mathrm{c}+N_\mathrm{p})T_\mathrm{c})$. Note that in principle any traffic model can be used, but in this paper we use the model described in Section 2 because it provides a good trade-off between accuracy and efficiency. With the obtained prediction the value $J(k_\mathrm{c})$ of the cost function for this period is computed.

The cost function should be minimized by selecting the optimal control signal sequence $c^*(k_\mathrm{c})$, $c^*(k_\mathrm{c}+1),\ldots,c^*(k_\mathrm{c}+N_\mathrm{p}-1)$. In order to reduce the number of optimization variables (and thus the computational complexity) a control horizon N_c (with $N_\mathrm{c} \leq N_\mathrm{p}$) is introduced and the control sequence is only allowed to vary over the period $[k_\mathrm{c}T_\mathrm{c},(k_\mathrm{c}+N_\mathrm{c})T_\mathrm{c})$ and is set constant afterwards, i.e. we have $c^*(k_\mathrm{c}+k) = c^*(k_\mathrm{c}+N_\mathrm{c}-1)$ for $k = N_\mathrm{c}, N_\mathrm{c}+1,\ldots,N_\mathrm{p}-1$.

From the optimal control signal sequence only the first sample $c^*(k_\mathrm{c})$ is applied to the real system. The next control time step, a new optimization is performed with the prediction horizon N_p that is shifted one control time step further. Of the resulting control signal again only the first sample is applied, and so on. This is called the receding horizon approach. This system allows for updating the state from measurements every iteration, and also for adaptive control by regularly updating the model parameters using system identification. Together with the measurement of the current state it introduces a feedback mechanism.

3.1.2 Control signal, constraints, cost function, and prediction model

The MPC method requires defining the control signal c, the cost function J, and the constraints. Further, a suitable prediction model should be selected. Below we describe these elements for a general setting. In Section 3.2 they will be made specific for traffic control for mixed networks.
The control signal contains the signals that are able to influence the system, e.g. the setting for the control measures.
The constraints can contain upper and lower bounds on the control signal, but also linear or non-linear equality and inequality constraints on the states of the system. The constraints are used e.g. to keep the system working within safety limits, or to avoid unwanted situations.
The cost function J represents the performance of the network. Different performance criteria are possible. In practice, cost functions are often a combination of the different performance indicators:

$$J_{\text{total}}(k_c) = \alpha_1 J_1(k_c) + \alpha_2 J_2(k_c) + \alpha_3 J_3(k_c) + \ \ldots$$

where the weights α_i of each term can be determined by the user of the controller. Note that for more complex functions or different domains selecting a non-linear weighting function can be useful.
MPC uses a model of the system to make predictions. MPC is an on-line control approach, and thus requires models that give a balanced trade-off between accurate predictions and computational efforts. The model should be able to run several times faster than real-time, to ensure that the optimization algorithm can have results available within a specified amount of time.

3.1.3 Optimization algorithms

At each control step MPC computes an optimal control sequence over a given prediction horizon. In general, this optimal control sequence is the solution of a non-linear, non-convex optimization problem in which the cost function is minimized subject to the model equations and the constraints. To solve this optimization problem different numerical optimization techniques can be applied, such as multi-start sequential quadratic program (SQP) (see e.g. (Pardalos and Resende, 2002)) or pattern search (see e.g. (Pierre, 1986)) for real-valued problems, and genetic algorithms (Davis, 1991), tabu search (Glover and Laguna, 1997), or simulated annealing (Eglese, 1990) for mixed integer problems arising when discrete measures are included.

3.2 MPC-based traffic control for mixed urban and freeway networks

The principle of MPC is explained above. In this section we describe how MPC can be used to design a traffic controller for mixed networks. Note that the elements of an MPC controller (model, control signal, cost function, constraints, optimization algorithm) can be selected separately, and that the elements that we select over here are just an example of a possible implementation.

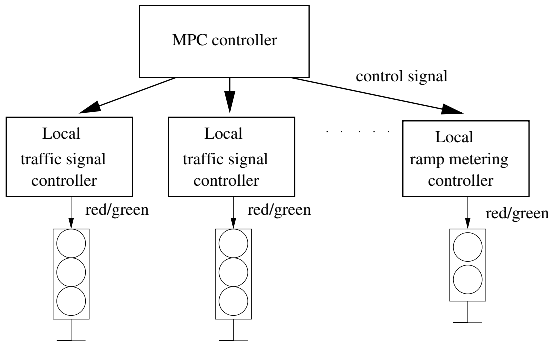


Figure 7: MPC-based control implemented with local controllers

The model requirements for MPC lead to the selection of a macroscopic traffic flow model to predict the behavior of the traffic. Macroscopic models are suited since the computation time is relatively low and does not depend on the number of vehicles in the network, and since they offer a good trade-off between accuracy and computational efforts. In Section 2 we have developed such a model, which we now include in our controller. Note that to be able to make a prediction of the traffic flows, the current state of the network should be known. This current state can be obtained via direct measurements or by using a state estimator, e.g. based on Kalman filtering (Jazwinski, 1970).

The control signal c can contain e.g. traffic signals for urban networks, presenting the green times and off-sets for each intersection. For freeway networks it can contain e.g. ramp metering rates, variable speed limits, or lane closure settings. The MPC controller is often used as a higher level controller. In this case the control signal contains control signals and set-points for the local controllers. The low level local controllers translate these signals and set-points in the red/green signals for the real traffic control measures, as illustrated in Figure 7.

The traffic signals work as given in (6). During the green time (i.e. when $g_{o_i,s,d_j} = 1$) the saturation flow $Q_{\mathrm{cap},o_i,s,d_j}$ is present, during red the flow is zero. The green time is included in the global control signal via a cycle time T_{cyc}, the green offset $o_{\mathrm{green},o_i,s,d_j}$ (expressed as a percentage of the cycle time), and the green time $\pi_{\mathrm{green},o_i,s,d_j}$ (also expressed as a percentage of the cycle time). These percentage are translated into the binary green/red signal g_{o_i,s,d_j} as follows:
Suppose that we have to compute the control signals over the period $[t^0, t^{\mathrm{end}}]$ with $t^0 = k^0 T_{\mathrm{u}}$ and $t^{\mathrm{end}} = k^{\mathrm{end}} T_{\mathrm{u}}$. In this period the number of cycles is equal to $N_{\mathrm{cyc}} = \mathrm{ceil}\left(\frac{t^{\mathrm{end}} - t^0}{T_{\mathrm{cyc}}}\right)$. For each cycle $\ell = 0, 1, \ldots, N_{\mathrm{cyc}} - 1$ the vehicles coming from origin o_i and going to destination d_j at intersection s have green from time instant $t^0 + \ell T_{\mathrm{cyc}} + o_{\mathrm{green},o_i,s,d_j}(\ell) T_{\mathrm{cyc}}$ up to time instant[6]

[6]Note that in fact time instants beyond t^{end} do not have to be considered.

$t^0 + \ell T_{\text{cyc}} + o_{\text{green},o_i,s,d_j}(\ell) T_{\text{cyc}} + \pi_{\text{green},o_i,s,d_j}(\ell) T_{\text{cyc}}$. So we have

$$g_{o_i,s,d_j}(k) = \begin{cases} 1 & \text{if } kT_{\text{u}} \in \bigcup_{\ell=1}^{N_{\text{cyc}}-1} [t^0 + \ell T_{\text{cyc}} + o_{\text{green},o_i,s,d_j}(\ell) T_{\text{cyc}}, \\ & \qquad t^0 + \ell T_{\text{cyc}} + o_{\text{green},o_i,s,d_j}(\ell) T_{\text{cyc}} + \pi_{\text{green},o_i,s,d_j}(\ell) T_{\text{cyc}}] \\ 0 & \text{otherwise} \end{cases}$$

for $k = k^0, k^0+1, \ldots, k^{\text{end}}$.
This implies that the actual urban control inputs computed by the MPC controller consist of the cycle times T_{cyc}, the offset percentages $o_{\text{green},o_i,s,d_j}$, and the green time percentages $\pi_{\text{green},o_i,s,d_j}$ for each traffic cycle in the given prediction period.

Ramp metering installations limit the flow that leaves the on-ramp:

$$q_{r,p}^{\text{onramp,metering}}(k_{\text{f}}) = \min(q_{r,p}^{\text{onramp,no metering}}(k_{\text{f}}), b(k_{\text{f}}) Q_{\text{cap},r})$$

where $q_{r,p}^{\text{onramp,no metering}}(k_{\text{f}})$ is the flow on the on-ramp when no metering is applied (cf. equation (4)), $Q_{\text{cap},r}$ is the capacity of the on-ramp, and $b(k_{\text{f}})$ the metering rate of the controller.
The last part of the control signal contains the freeway speed limits. Speed limits influence the speed of the drivers by changing their desired speed (Hegyi, 2004):

$$v_{m,i}^{\text{desired,limits}}(k_{\text{f}}) = \min(v_{m,i}^{\text{desired,no limits}}(k_{\text{f}}), (1+\alpha_l) v_{m,i}^{\text{limit}}(k_{\text{f}})) \tag{8}$$

where $v_{m,i}^{\text{desired,no limits}}(k_{\text{f}})$ is the desired speed of the drivers when there are no speed limits applied (cf. equation (2)), $v_{m,i}^{\text{limit}}(k_{\text{f}})$ is the value of the applied speed limits, and α_l is a parameter which represents the fact that drivers will freely interpret and adhere to the speed limits. When enforcement is used α_l will typically be around -0.1, but without enforcement drivers will tend to drive to fast, and α_l can be around 0.1.
The subsequent values of the ramp metering rates and the variable speed limits over the prediction period make up the freeway part of the control signal.

Furthermore, we can impose constraints for the controller. For traffic control such constraints can consist of e.g. maximum queue lengths at intersections, on-ramps or off-ramps, minimum and maximum green times or speed limits, maximum flows on roads, constraints that the traffic signal plans should be conflict-free, etc. These constraints could be prescribed by regulations, or they could express a policy selected by the traffic management authorities.

The cost function can be determined by the traffic management authorities of the traffic network, to represent their traffic management policies. The cost function can contain e.g. the total time that the vehicles spend in the network, the average queue length, the number of stops, the total delay, the throughput, vehicle loss hours, variation in the travel times, the total fuel consumption, the emission levels, the noise production, etc., or a combination of them. The cost functions for the urban and freeway parts of the network are often computed separately, to allow a trade-off between the two:

$$J_{\text{total}}(k_{\text{c}}) = J_{\text{freeway}}(k_{\text{c}}) + \alpha J_{\text{urban}}(k_{\text{c}})$$

where α is a weight factor to determine the relative influence of the urban traffic.

A cost function that is often used in literature (see e.g. (Kotsialos, Papageorgiou, Mangeas and Haj-Salem, 2002; Bellemans, 2003; Hegyi, 2004; Kotsialos and Papageorgiou, 2004)) is the total time spent (TTS) by all vehicles in the network. We will also use this objective function for our case study in Section 4. Therefore, we will now expand somewhat on this particular objective function. To compute the TTS for the urban part of the network the number of vehicles in each urban link $n_{\text{vehicles},l_{\sigma,s}}$ is required:

$$n_{\text{vehicles},l_{\sigma,s}}(k_{\text{u}}) = L_{\sigma,s} - S_{\sigma,s}(k_{\text{u}})$$

where $L_{\sigma,s}$ is the maximum number of vehicles that the link can contain. Using this equation the number of vehicles for all urban links, on-ramps and off-ramps can be computed.
The TTS will be computed over the period $[k_{\text{c}}^0 T_{\text{c}}, (k_{\text{c}}^0 + N_{\text{p}})T_{\text{c}})$ when we are at time $t = k_{\text{c}}^0 T_{\text{c}}$. Now define k_{u}^0 and k_{f}^0 such that $k_{\text{c}}^0 T_{\text{c}} = k_{\text{u}}^0 T_{\text{u}} = k_{\text{f}}^0 T_{\text{f}}$ and $k_{\text{u}}^{0,\text{end}}$ and $k_{\text{f}}^{0,\text{end}}$ such that $(k_{\text{c}}^0 + N_{\text{p}})T_{\text{c}} = (k_{\text{u}}^{0,\text{end}} + 1)T_{\text{u}} = (k_{\text{f}}^{0,\text{end}} + 1)T_{\text{f}}$. The total time spent in the urban part of the network during the period $[k_{\text{c}}^0 T_{\text{c}}, (k_{\text{c}}^0 + N_{\text{p}})T_{\text{c}})$ is then given by:

$$\begin{aligned}
\text{TTS}_{\text{urban}}(k_{\text{c}}^0) = \; & T_{\text{u}} \sum_{k=k_{\text{u}}^0}^{k_{\text{u}}^{0,\text{end}}} \Big(\sum_{l_{o_i,s} \in I} n_{\text{vehicles},l_{o_i,s}}(k) + \sum_{l_{s,r} \in R_{\text{on}}} n_{\text{vehicles},l_{s,r}}(k) + \sum_{o \in O_{\text{urban}}} n_{\text{vehicles},o}(k) \Big) \\
& + T_{\text{f}} \sum_{k=k_{\text{f}}^0}^{k_{\text{f}}^{0,\text{end}}} \sum_{l_{r,s} \in R_{\text{off}}} n_{\text{vehicles},l_{r,s}}(k)
\end{aligned}$$

where $\text{TTS}_{\text{urban}}(k_{\text{c}}^0)$ denotes the total time spent in the urban part of the network during the period $[k_{\text{c}}^0 T_{\text{c}}, (k_{\text{c}}^0 + N_{\text{p}})T_{\text{c}})$, I the set of all urban links, O_{urban} the set of all urban origins o, R_{on} the set of links $l_{r,s}$ connected to the on-ramps, and R_{off} the set of links $l_{s,r}$ connected to the off-ramps.
The TTS in the freeway part of the network is computed using the density on the segments:

$$\text{TTS}_{\text{freeway}}(k_{\text{c}}^0) = \sum_{k=k_{\text{f}}^0}^{k_{\text{f}}^{0,\text{end}}} \sum_{m \in M} \Big(L_m n_m \sum_{i \in I_m} \rho_{m,i}(k) + \sum_{o \in O_{\text{freeway}}} n_{\text{vehicles},o}(k) \Big)$$

where $\text{TTS}_{\text{freeway}}(k_{\text{c}}^0)$ is the total time spent in the freeway part of the network during the period $[k_{\text{c}}^0 T_{\text{c}}, (k_{\text{c}}^0 + N_{\text{p}})T_{\text{c}})$, M the set of all freeway links m, I_m the set of all segments i in link m, and O_{freeway} the set of all freeway origins.
The total cost function is given by the weighted sum of the urban and freeway cost functions:

$$\text{TTS}(k_{\text{c}}^0) = \text{TTS}_{\text{freeway}}(k_{\text{c}}^0) + \alpha \text{TTS}_{\text{urban}}(k_{\text{c}}^0) \;.$$

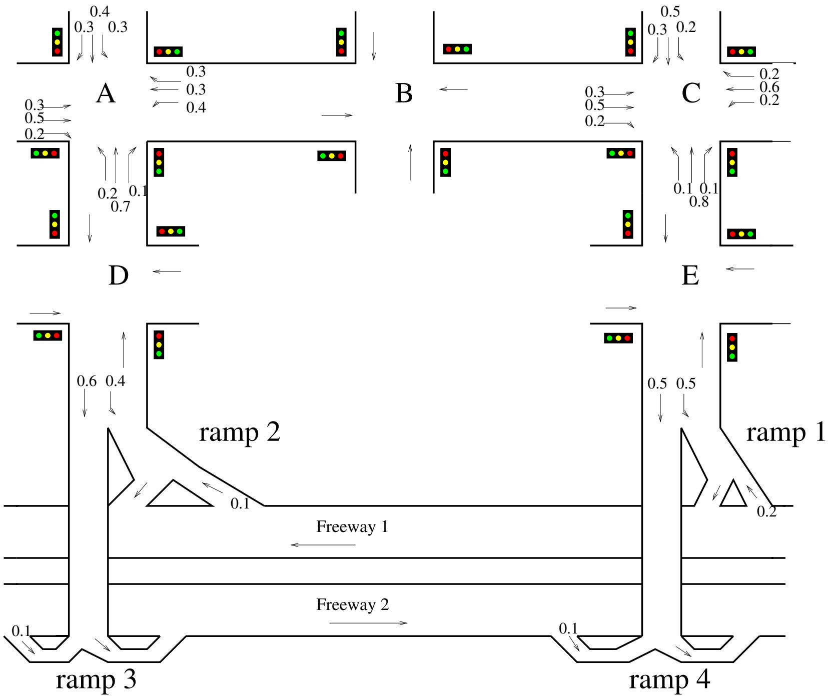


Figure 8: Network used in the case study

4 Case study

To illustrate the performance of the MPC method we will present a simple case study. The case study concentrates on urban control but in a network that also contains a freeway. We have done this to be able to make a comparison with existing dynamic control systems, which have mainly been developed for urban control measures.

4.1 Set-up of the case study

For the case study a simple network is used, as shown in Figure 8. The network consists of two freeways (freeway 1 and 2) each with two on-ramps and two off-ramps (ramp 1 to 4). Furthermore, there are two urban intersections (A and C), which are connected to the freeway and to each other. Between these intersections and the freeways there are some crossing roads (B, D and E), where there is only crossing traffic that does not turn into other directions, e.g. pedestrian traffic, bicycles, etc. We have selected this network because it contains most essential elements from mixed networks. There are freeways with on-ramps and off-ramps and controlled intersections not too far away from the freeways resulting in a strong relation between the traffic on the two road types. The network is small enough to use intuition to analyze and interpret the results, but large enough to make the relevant effects visible.
The performance of the control systems will be shown for different traffic scenarios. Three of them are scenarios with different traffic situations, while the fourth is a control-related scenario. We have selected these scenarios because they clearly show the influence of the urban traffic on

the freeway traffic and vice versa, because this influence occurs frequently, and because some properties of the control methods will become clearly visible. The 'basic' scenario has a demand of 3600 veh/h for freeway origins and 1000 veh/h for urban origins, and turning rates as shown in Figure 8. Each of the scenarios is a variation on this 'basic' scenario, with one variable or parameter changed or a constraint added. The total simulated time is 30 minutes. These are the four scenarios:

Scenario 1: congestion on freeway A congestion exists on the downstream end of freeway 1. This congestion grows into the upstream direction and blocks the on-ramps, causing a spill-back leading to urban queues. The congestion is started by creating a downstream density of 65 veh/km/lane for the last segment of the freeway.

Scenario 2: blockage of an urban intersection On intersection D an incident has occurred, and the whole intersection is blocked. The queues spill back to neighboring intersections, and also block the off-ramps of the freeways. The incident is simulated by setting the saturation flow of all links leaving the intersection to zero.

Scenario 3: rush hour In this scenario the demand at the origins becomes larger during a short period, for example during a rush-hour. We selected a flow of 500 veh/h with a peak of 2000 veh/h for the urban origins, and a flow of 2000 veh/h with a peak of 4000 veh/h for freeway origins. The duration of the peak is 10 minutes.

Scenario 4: maximum queue length Here, the queue on the link from intersection A toward intersection B may not become longer than 20 vehicles. This can be a management policy, e.g. when the link is in a residential area.

4.2 Simulation set-up

For all control systems the implementation of the simulations and the controller is completely done in the mathematical computation environment Matlab. We use the model described in Section 2 both as real world model and as prediction model. With this set-up we can give a proof-of-concept of the developed control method, without introducing unnecessary side-effects. In our case study the MPC optimization problem is a non-convex, non-linear problem with real-valued optimization variables. To solve this problem we have selected multi-start SQP (Pardalos and Resende, 2002) as optimization algorithm. This algorithm is implemented in the `fmincon` function of the Matlab optimization toolbox (The MathWorks, 2007).

As cost function we select the total time spent (TTS). The model parameters are selected as follows. The parameters of the METANET model are selected according to (Kotsialos et al., 1999): $v_{\text{free},m} = 106$ km/h, $\rho_{\text{crit},m} = 33.5$ veh/km/lane, $\rho_{\max,m} = 180$ veh/km/lane, $Q_{\text{cap},m} = 4000$ veh/h, $\tau = 18$ s, $\nu = 65$, $\kappa = 40$, and $a_m = 1.867$. The parameters of the urban model are $Q_{\text{cap},o,s,d} = 1000$ veh/h, $L_{\text{av,veh}} = 6$ m, and $v_{\text{av},l_{s,d}} = 50$ km/h.
We have selected the following time steps: $T_{\text{c}} = 120$ s, $T_{\text{f}} = 10$ s, and $T_{\text{u}} = 1$ s. A small value is selected for the urban time step to obtain detailed information. The freeway time step of 10 s forms a trade-off between computational effort and accuracy.

There are three parameters that can be tuned for the MPC controller. We have selected $N_\mathrm{p} = 8$ and $N_\mathrm{c} = 3$ as horizons, and $\alpha = 1$ as trade-off between urban and freeway performance in the cost function.

4.3 Alternative control methods

Many dynamic traffic control systems are implemented in the real world. Some of these systems are SCATS (Wolson and Taylor, 1999), Toptrac (TPA, n.d.), SCOOT (Robertson and Bretherton, 1991), UTOPIA/SPOT (Peek Traffic, 2002), MOTION (Busch and Kruse, 2001), IN-TUC (Diakaki et al., 2000), etc. Here we will use SCOOT and UTOPIA/SPOT to make a comparison between the developed MPC control method and some existing systems. We have selected these methods because they are good representatives of this kind of dynamic traffic control systems. Note however that these systems are commercial systems, meaning that real specifications are not publicly available. This means that we can only approximate their functioning as follows:

An UTOPIA/SPOT-like system UTOPIA/SPOT (Peek Traffic, 2002) has been developed in Turin, Italy. It is a hierarchical system with a local controller at each intersection, and a central controller. The central controller computes an optimal control signal, using a prediction of the traffic in the whole urban network over a period of 15 minutes. This optimal control signal is sent to the local controllers. Each of these local controllers communicates with its neighbors to obtain their measurements and expected control scheme. With this information the local controllers compute a locally optimal control signal, using predictions of the traffic only on the local intersection during the next cycle, including the arriving traffic. In the cost function used by the local controllers a penalty is added for deviations from the signal computed by the central controller. In this way the central controller can influence the local controllers. A queue length model is used to obtain the predictions.

A SCOOT-like system SCOOT (Robertson and Bretherton, 1991) has a controller on each intersection. These controllers estimate the arriving traffic flows using a cyclic flow profile, which is updated via measurements taken at the beginning of each link. Every control time step the cycle time is updated. This is done according to the ratio between the current queue length and the maximum allowed queue length. When more than 90% of the maximum queue length is reached, the cycle time is increased. The time differences between the beginning of the green times of different intersections are called the offsets. At the beginning of each cycle the offsets are optimized. A prediction of the traffic flows for the next cycle is used to determine the optimal values for each intersection separately, using predictions obtained from neighboring intersections during the previous time step. The green times are updated every urban simulation time step. A prediction of the traffic during the next cycle is made to determine whether it is useful to increase or decrease the green times with 4 s. The model used for the predictions is a simple queue length model. It describes the number of vehicles arriving at the beginning of the link, the delay due to the travel time on the link, the length of the queue, and the number of vehicles leaving the link.

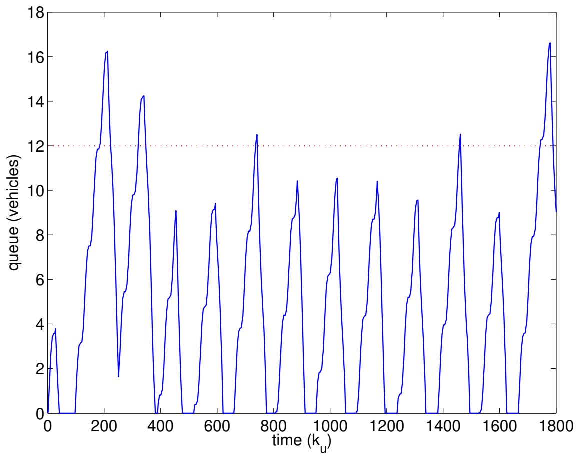
In both systems constraints like maximum queue lengths are introduced by adding a penalty term to the cost function. This penalty term must become relatively large when the maximum queue length is reached. This results in a very high value of the cost when the maximum queue length is violated. While the purpose of the control is to minimize the cost function, a trade-off will be made between minimizing the original cost and violating the queue length constraint.
Both systems target the urban traffic, and they optimize intersections independently of the neighboring freeway.

4.4 Qualitative comparison

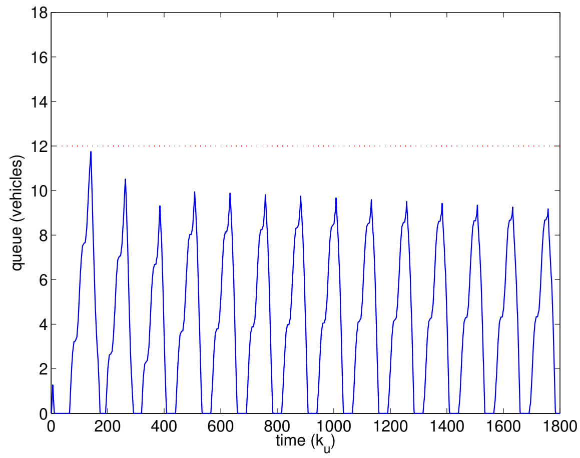
The main difference between the MPC-based system proposed in this paper and the existing systems is that the new system takes the influences and interactions between the urban and freeway parts of the network into account. By simulating the effect of one measure on both kinds of roads, control settings can be found that provide a trade-off between improving traffic conditions on the freeway and delaying traffic on the urban roads and vice versa.
Furthermore, the MPC-based system we have developed can handle hard constraints. All systems can handle constraints that are directly linked to the control signals, e.g. maximum and minimum green times and cycle time constraints. But the MPC-based system can also handle more indirect constraints such as maximum queue lengths, maximum delays, etc. These constraints are included as hard constraints in the MPC optimization problem, which is subsequently solved using a constrained optimization algorithm (e.g. SQP). Figure 9 shows a queue on the link from A to B, where a maximum queue length of 12 is set as constraint. In the other systems such a constraint is implemented by adding a penalty term that penalizes the constraint violation to the performance function. This can lead to either satisfying the constraints with a degraded performance, or violating the constraints and obtaining a better performance. Which of the two occurs depends on the weight that is given to the penalty term.
The three control methods are also characterized by different communication requirements. SCOOT is based on local controllers, each with their own detectors and control algorithm. UTOPIA/SPOT uses different levels: local controllers that communicate with their neighbors, and a centralized control computer that communicates with each local controller, mainly sending set-points for the local control algorithms. The MPC method is in principle a central method in which the control algorithm runs on a central computer, and only the results of the optimization are communicated toward the low-level controllers. In this way an optimum for the total network is found, possibly at the cost of large computation times in the case of large networks (in Section 4.6 we will sketch some ways to address this issue).

4.5 Quantitative comparison

We have applied the three different control methods to the case study network. The results are shown in Table 1. Each traffic scenario is simulated with each control method. The table shows the total time spent for the freeway part of the network, for the urban part, and for the whole network. The last column of the table shows the improvement of the MPC method compared to SCOOT (first number) and to UTOPIA/SPOT (second number). This makes it possible to



(a) Queue length at the link from A to B, for MPC without queue length constraint



(b) Queue length at the link from A to B, for MPC with a queue length constraint of 12 vehicles

Figure 9: The effect of a queue length constraint

determine in which part of the network most improvements are obtained. For the fourth scenario the largest attained queue length is also shown.

The first two scenarios show that the MPC method can improve the performance for the urban as well as for the freeway part of the network when a problem arises on one of the two. The immediate negative effects of such a problem are decreased just as the negative influence on the rest of the network.

Large variations in traffic demands are difficult to handle for some control methods. The third scenario shows that the MPC method can control the traffic slightly better when a large peak in the demand occurs. In this scenario the trade-off between the freeway and urban parts of the network can clearly be seen. A decrease of the performance on the urban network can lead to an improvement of the performance on the freeway network, and vice versa. This can be used to obtain a better performance for the total network.

The maximum queue length constraint is implemented in SCOOT and UTOPIA by adding an extra penalty term in the cost function. This term has a relative weight that allows a trade-off between the performance of the network and the importance of the maximum queue length constraints. When the weight is high the queue length constraint is satisfied but the performance is low, as shown in the first simulations done for the fourth scenario. In the second set of simulations the weighting term for the queue constraint is low, resulting in a better performance but the maximum queue length is exceeded. The values for MPC are the same for both simulation sets because the queue length constraint is implemented as a hard constraint for the optimization algorithm[7].

[7]The MPC-based method violates the constraint with 1 vehicle at the start of the simulation. This is due to infeasibility problems during the optimization, related to the initial state of the network at the start of the simulation. This issue can be solved by increasing the horizons N_p and N_c.

Table 1: Results of the case study: total time spent for the freeway part of the network, for the urban part, and for the total network; and also the improvement of the MPC-based method compared to SCOOT and UTOPIA/SPOT respectively

Scenario 1: congestion on freeway

	SCOOT	**UTOPIA/SPOT**	**MPC**	**improvement**
freeway	595.4	565.1	563.9	5.3 / 0.3%
urban	313.6	335.7	305.7	3.0 / 9.0%
total	909.0	900.8	869.6	4.4 / 3.5%

Scenario 2: blockage of an urban intersection

	SCOOT	**UTOPIA/SPOT**	**MPC**	**improvement**
freeway	498.0	526.2	495.0	0.7 / 6.0%
urban	665.9	672.3	620.3	6.9 / 7.8%
total	1163.9	1198.5	1115.3	4.2 / 7.0%

Scenario 3: rush hour

	SCOOT	**UTOPIA/SPOT**	**MPC**	**improvement**
freeway	244.6	280.1	253.3	-3.5 / 9.6%
urban	409.0	383.5	386.8	5.5 / -1.6%
total	653.6	663.6	640.1	2.1 / 3.5%

Scenario 4: maximum queue length of 20 vehicles with large weight

	SCOOT	**UTOPIA/SPOT**	**MPC**	**improvement**
freeway	367.2	510.3	373.9	-1.8 / 26.8%
urban	309.7	435.4	264.4	15.7 / 39.3%
max. queue	19	19	21	
total	676.9	945.7	638.3	6.8 / 32.6%

Scenario 4: maximum queue length of 20 vehicles with small weight

	SCOOT	**UTOPIA/SPOT**	**MPC**	**improvement**
freeway	367.1	428.1	373.9	-1.8 / 13.7%
urban	303.0	360.5	264.5	13.8 / 26.7%
max. queue	93	43	21	
total	670.1	788.6	638.3	5.8 / 19.1%

4.6 Discussion

Although the MPC-based method gives good results, some parts of it have to be investigated more extensively.
The most important problem at the moment is the required computational effort. The run time for the MPC-based method is larger than for the other methods. This is due to the use of one central computer and to the fact that a larger network is optimized at once. This can be solved by using faster computers, by using the method in a distributed setting, or by using better special, dedicated solvers implemented in object code[8].
The optimization technique also forms an important factor in relation to the computation time and the computed optimal control signal. Different optimization algorithms can have different run times, and find different optimal solutions. To select the best algorithm extensive simulations should be done for a wide range of set-ups and scenarios to compare the various algorithms.
When hard constraints are implemented, it is possible that the optimization problem becomes infeasible. When this occurs, one or more constraints have to be relaxed (see (Camacho and Bordons, 1995; Maciejowski, 2002) for more details). This can in reality mean that the constraints are violated for a short period.
The effects of selecting different cost functions should also be investigated, just as the influence of the weighting parameter α, which determines the trade-off between the urban and freeway costs.

5 Conclusion

Congestion on urban roads and congestion on freeways cannot be seen as separate problems. The traffic on urban roads influences the traffic on freeways and vice versa. As a result, control measures taken on one of the two types of roads have influence on both road types. We have developed a control system that takes this influence into account when the control signals are determined. The system is suitable for integrated control, and makes a balanced trade-off between the urban and the freeway parts of the network.
We first have developed a model that describes the evolution of traffic flows on mixed networks. For the freeway part the METANET model is used, and for the urban roads a queue length model based on Kashani's model is developed. We have made the connection between the urban and freeway parts of the network by modeling on-ramps and off-ramps.
The mixed network model is used to develop a coordinated control method using MPC. In MPC the evolution of the traffic flow is predicted over a certain period, and this prediction is then used to optimize the signal settings, using numerical algorithms. MPC uses a receding horizon approach: only the first step of the optimized signal settings is applied, and then the procedure is started all over again. This makes that the controlled system can also cope with changes in the traffic demand.
We have performed a case study to compare the MPC method with existing control methods. We have selected methods that are an approximation of SCOOT and UTOPIA/SPOT. Different

[8]The current simulations are programmed in Matlab, which is basically an interpreted language.

traffic scenarios are simulated and the result of the three systems are compared qualitatively and quantitatively. The MPC method performs between 2% and 7% better than the other two systems, and can guarantee bounds on the queue lengths without a large decrease in performance.
The results of the simulation are promising: they can be seen as a proof-of-concept for the proposed approach, they show the potential benefits, and encourage further research. This research could include the following steps. First, additional case studies, with several different traffic scenarios and set-ups including larger networks should be performed. Next, case studies should be done where different models are used to model the 'real world' traffic flows (for the prediction model we would keep on using the macroscopic model proposed in this paper). Then, the efficiency of the algorithm should be improved. Some attention should be payed to the robustness and sensitivity of the control method. Last, a real-life test should be done. Other topics that should be investigated are the validation and calibration of the model. Furthermore, for the simulation of larger networks, it is useful to investigate MPC for distributed control in which different adjacent network regions are defined and optimized separately (but with some coordination to avoid negative influences of the control actions of one region on the other regions).

Acknowledgments

Research supported by the NWO-CONNEKT project 014-34-523 "Advanced multi-agent control and information for integrated multi-class traffic networks (AMICI)", by the "Transport Research Centre Delft", and by the BSIK projects "Transition Sustainable Mobility (TRANSUMO)" and "Next Generation Infrastructures (NGI)".

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