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Influencing Long-Term Route Choice by Traffic Control Measures – A Model Study ⋆

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Abstract: Currently used traffic control measures, such as traffic signals, variable speed limits, ramp metering installations etc., are often not designed to influence the route choice of drivers. However, traffic control measures do influence the travel times that are experienced in the network. Since route choice is, at least for a part, based on experienced travel times, the control measures thus also influence the long-term route choice. This influence can be seen as a side-effect of the control measures, but in this paper we will investigate the possibilities to explicitly and actively use the influence of the traffic control measures to change the long-term route choice. Using basic traffic flow and route choice models we investigate how outflow and speed limit control can affect the final equilibrium turning fractions. As an example we consider a case study for a simple network with two routes and use a simple linear outflow controller, which makes the analytical investigation of the effects of the controller possible, but the results can be extended to more sophisticated control methods.

1. INTRODUCTION

In road networks locations are often connected by more than one route, which means that drivers who want to go from an origin to a destination have to choose which route to take. When all drivers make such a route choice this results in a traffic assignment that is preferred according to the preferences of the drivers. A traffic assignment gives the number of drivers that have selected each route. Such an assignment resulting from the preferred route choice of the drivers may lead to large traffic flows on narrow or dangerous roads, to socially undesired situations (e.g. too many vehicles in residential areas or near primary schools), or to too large flows near national parks causing pollution and noise. To prevent negative effects of large traffic flows, road administrators can try to influence the route choice of the drivers, to reach a traffic assignment with less traffic on some roads.

Different methods are available to influence the route choice. At this moment, much attention is payed to providing information, either pre-trip, en-route, or post-trip. Furthermore, some experiments with traffic control measures have shown that these measures can influence the route choice (Haj-Salem and Papageorgiou, 1995; Taale and van Zuylen, 1999). This has led to the theoretical development of methods to incorporate the effect of existing traffic control measures on route choice (Bellemans et al., 2003; Karimi et al., 2004; Wang and Papageorgiou, 2002). In this paper however, we investigate how we can change the settings of the traffic control measures so as to steer the traffic flows towards different, desired equilibria. Route choice is in equilibrium when no driver can change its route without increasing its costs, as described by Wardrop (1952). This means that all routes which are used have equal costs, while the costs of unused routes are larger.

In this paper we consider control methods that can be used for the upper layer controller of a multi-layer control system. We describe an upper layer controller which tries to achieve a desired traffic assignment. This will result in desired settings for the mean outflow capacity of the links in the network, and desired mean speeds on these links. We assume that the lower level controllers use these desired settings as targets, and optimize the local situation in such a way that these desired capacities and speeds are obtained. In our approach the upper level controller uses a basic static route choice model. This allows for analytical descriptions of the behavior of traffic flows, and for formulating intuitive explanations. We assume that the experienced travel time is the most important factor in route choice, which is also argued for by Bogers et al. (2005). We also assume that we can influence the travel time by changing the outflow of the links, and the speed on these links.

To describe the properties of traffic that are useful for influencing route choice, two models are required: a route choice model and a travel time model. Since the main goal of this paper is to gain insight in the mechanisms regarding control of route choices, we select simple models and controllers, to be able to provide analytical descriptions of the behavior of the traffic flows, and to formulate intuitive explanations.

This paper is organized as follows. In the next section we first introduce the small network with two routes that we use for our exposition and for illustrating how traffic control can affect long-term route choice. Note however, that the models introduced in this paper can easily be extended to more complex situations with multiple, possibly overlapping routes. In Section 3 we present a model that describes the experienced travel times, and a model that relates these experienced travel times to route choices. In Section 4 we look at the equilibrium turning fractions that are the result of the selected model, network, and traffic demands when no control is applied. Next, we

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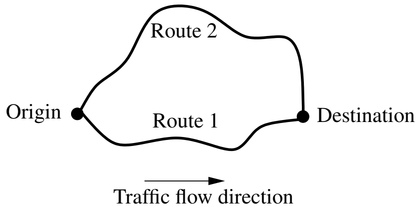


Fig. 1. Network with two routes

investigate the possibilities of outflow control and speed limit control in Section 5. In Section 6 we illustrate the proposed approach using a simple linear outflow controller. At last, some conclusions about the possibilities of route choice control with existing control measures are drawn in Section 7.

2. A BASIC ROUTE CHOICE NETWORK

To obtain simple and intuitive results, we require a simple network. When working with route choice, a network with two routes between an origin and a destination is the most simple, as shown in Figure 1. Such a network contains all features that are required for route choice, but is small enough to make intuitive understanding possible. We will use this network during the remainder of the paper. We assume that drivers enter this network at the origin and make their route choice immediately. Then they experience a travel time during their trip through the network and leave the network at the destination.

We will look at the day-to-day evolution of the traffic flows in particular period of the day, e.g. the morning peak. The length of the period is denoted by T (h). During this period we assume that the traffic demand Q_{tot} (veh/h) in the network is constant. The demand is distributed over the two routes according to the turning fraction $\beta(d)$, which gives the fraction of the vehicles that select route 1 on day d. The turning fraction is computed with the route choice model described in Section 3.2. Each route r ($r \in \{1,2\}$) is characterized by its length l_r (km) and its outflow capacity C_r (veh/h). Another characteristic of a route is the free-flow travel time, which describes the time that a vehicle needs to travel a route when there is no delay (i.e., no congestion). The free-flow travel time on route r is given by:

$$\theta_r^{\text{free}} = \frac{l_r}{v_r^{\max}} \tag{1}$$

where $v_r^{\max}$ denotes the maximum speed (km/h) on route r.

3. BASIC ROUTE CHOICE MODEL

In this section we describe how the travel time experienced on a given route on a particular day affects the route choice on the following day(s). More specifically, we will first formulate the travel time model, and next describe the resulting route choice model.

3.1 Travel time model

Recall that we assume that travel time is the most important factor that influences route choice (Bogers et al., 2005). So to describe the route choice, we thus need a model that describes the experienced travel time when a driver selects a given route. Travel times can be computed in two different ways: either by simulation or analytically. Traffic flow models that can be used for the first approach are described in (Quadstone, 2002; Barceló and Ferrer, 1997; Messmer and Papageorgiou, 1990; Daganzo, 1994). The second approach uses travel time models which are described in e.g. (Fisk, 1980), and of which an overview is given in (Carey and Ge, 2003). We will determine the travel times analytically based on the number of vehicles in the link and C_r.

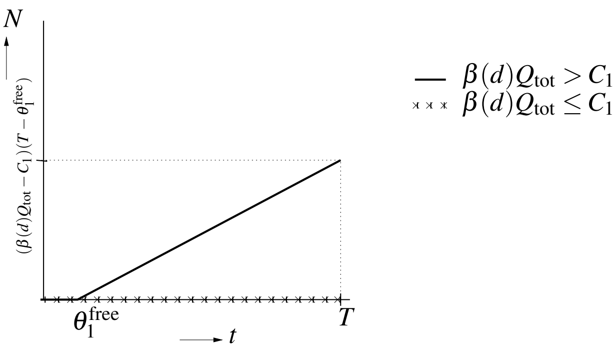


Fig. 2. Queue length on a link during one day

In our model, the queues are assumed to be vertical [1] . This means that the vehicles drive the whole route without delay, experiencing the free-flow travel time. At the end of the route, the vehicles enter the vertical queue and wait in this queue until they can leave the route. As a consequence, the travel time on given route consists of two components: the free-flow travel time and the time spent in the queue (which is taken to be 0 if no queue is present):

$$\theta_r(d) = \theta_r^{\text{free}} + \theta_r^{\text{queue}}(d) \ . \tag{2}$$

The free-flow travel time θ_r^{free} is defined in (1). The time in the queue depends on the number of vehicles in the queue. Let us first consider route $r = 1$. During one peak period, the queue at the end of route 1 grows as shown in Figure 2. Recall that the length of the peak period is denoted by T and that $\beta(d)Q_{\text{tot}}$ gives the flow on route 1. Let $N(t)$ be the number of vehicles in the queue at time t. When the free-flow travel time has passed, the first vehicles reach the end of the route and the queue starts to build up if the demand exceeds the capacity of the route. In Figure 2 the queue length is plotted for different demands. When the demand on route 1 is less than the capacity of route 1, i.e., $\beta(d)Q_{\text{tot}} \leq C_1$, no queue appears. When the demand is larger than the outflow limit a queue starts to grow, with rate $\beta(d)Q_{\text{tot}} - C_1$.

The route choice model that we will describe in Section 3.2 requires the mean travel time for each day. To obtain this mean travel time, we first have to compute the average number of vehicles $N_1^{\text{mean}}(d)$, if any, in the queue for day d. The average number of vehicles is given by the area below the graph of Figure 2 divided by the period in which the queue exists:

$$N_1^{\text{mean}}(d) = \frac{(\beta(d)Q_{\text{tot}} - C_1)(T - \theta_1^{\text{free}})}{2} \quad \text{if } \beta(d)Q_{\text{tot}} \geq C_1 \ ,$$

and is equal to 0 otherwise.

The travel time in the queue is given by the time that a vehicle needs to reach the downstream end of the queue starting from the time instant that the vehicle enters at the upstream end of the queue. Since C_1 vehicles are leaving the queue each hour, this time is given by:

$$\theta_1^{\text{queue}}(d) = \frac{N_1^{\text{mean}}(d)}{C_1} = \frac{(\beta(d)Q_{\text{tot}} - C_1)(T - \theta_1^{\text{free}})}{2C_1}$$

[1] A vertical queue is a virtual queue that has no physical length but stores the vehicles just in front of the bottleneck.

if $\beta(d)Q_{\text{tot}} \geq C_1$. Since $\theta_1^{\text{queue}}(d)$ equals 0 if there is no queue (i.e., if $\beta(d)Q_{\text{tot}} < C_1$) and since we may assume without loss of generality that $\theta_1^{\text{free}} \leq T$, we can combine both situations:

$$\theta_1^{\text{queue}}(d) = \max\left(0, \frac{(\beta(d)Q_{\text{tot}} - C_1)(T - \theta_1^{\text{free}})}{2C_1}\right) \quad . \qquad (3)$$

Similar computations can be done for route 2. Note however that the traffic that enters route 2 is given by $(1-\beta(d))Q_{\text{tot}}$, which results in

$$\theta_2^{\text{queue}}(d) = \max\left(0, \frac{((1-\beta(d))Q_{\text{tot}} - C_2)(T - \theta_2^{\text{free}})}{2C_2}\right). \quad (4)$$

3.2 Route choice model

There exist different models for describing how many drivers select a route. Traffic assignment models (Daganzo and Sheffi, 1977; Bliemer, 2000; Peeta and Mahmassani, 1995) compute an equilibrium traffic assignment for the whole network. Route choice models (Bogers et al., 2005; Mahmassani et al., 2003; Ben-Akiva et al., 1991) describe the route choice of drivers at locations where a route must be selected. Below we develop a route choice model that requires little computational effort and that is very intuitive.

We assume that the route choice only depends on previously experienced travel times (Bogers et al., 2005). The mean experienced travel times are assumed to be known by all drivers, which leads to the assumption that all the drivers are completely informed, meaning that they know the travel times on both routes (e.g., via travel information services), independent of the route they have selected.

We use the following model to update the turning fractions from one day to the next based on the difference in travel times on the two routes:

$$\beta(d+1) = \beta(d) + \kappa(\theta_2(d) - \theta_1(d)) \quad .$$

The factor κ expresses the number of drivers that will change their route based on the travel time difference, combined with a learning factor that describes how fast the drivers will change their behavior.

Now we combine this route choice model with the model for the travel time described in (3) and (4). The total route choice model is then given by:

$$\begin{aligned}\beta(d+1) = {} & \beta(d) + \qquad (5)\\ & \kappa\Big(\max(0, \frac{((1-\beta(d))Q_{\text{tot}} - C_2)(T - \theta_2^{\text{free}})}{2C_2})\\ & - \max(0, \frac{(\beta(d)Q_{\text{tot}} - C_1)(T - \theta_1^{\text{free}})}{2C_1}) + \theta_2^{\text{free}} - \theta_1^{\text{free}}\Big).\end{aligned}$$

4. EQUILIBRIUM TURNING FRACTIONS

In this section we describe the long-term evolution of the turning fractions when no control is applied. Depending on the particular demand Q_{tot}, driver route choice rate κ, the initial turning fraction $\beta(0)$, and capacities C_1 and C_2 of the routes, either oscillations could occur, or the turning fractions could converge to an equilibrium. Oscillations can happen when, e.g., drivers react too strongly on travel time differences or when the capacity of one of the roads is relatively low. In the remainder we will now focus on the equilibrium case.

To compute the equilibrium turning fractions we set $\beta(d+1) = \beta(d)$. The value of the equilibrium turning fraction depends on the free-flow times θ_1^{free} and θ_2^{free} of the two routes, and on the traffic demand Q_{tot}. Note that we use a fixed demand, which together with the selected travel time model results in a static traffic assignment when the equilibrium is reached. This allows for the use of basic analytical methods to describe the resulting equilibria. Note that in general the equilibrium turning fraction does not depend on the parameter κ. This means that the time constant (or learning rate) does not influence the equilibrium that is reached.

In the following subsections, we investigate situations where route 1 has the lowest free flow travel time, where route 2 has the lowest free flow travel time, and where both routes take equally long. For each of these situations, we examine a low demand that leads to no queues at all, a demand that leads to a queue on one route, and a demand that leads to a queue on both routes.

4.1 Route 1 has lowest free flow travel time

When $\theta_1^{\text{free}} < \theta_2^{\text{free}}$ route 1 will in general be the most desired route. When the total demand is lower than the capacity of route 1, all the traffic will take it: $\beta^{\text{eq}} = 1$ if $Q_{\text{tot}} \leq C_1$, where β^{eq} is the turning fraction during the equilibrium.

When the demand is larger than the capacity of route 1, but is smaller than the total capacity $C_1 + C_2$, most of the traffic will take route 1, which will result in a queue on this route. When the travel time on route 1 exceeds the free-flow travel time on route 2, a part of the traffic will use route 2. In this case, (5) can be reduced to:

$$\begin{aligned}\beta(d+1) = {} & \beta(d) + \qquad (6)\\ & \kappa\Big(-\frac{(\beta(d)Q_{\text{tot}} - C_1)(T - \theta_1^{\text{free}})}{2C_1} + \theta_2^{\text{free}} - \theta_1^{\text{free}}\Big)\end{aligned}$$

since there is only a queue on route 1 and not on route 2. Setting $\beta(d+1) = \beta(d) = \beta^{\text{eq}}$ leads to the following equilibrium turning fraction:

$$\beta^{\text{eq}} = \frac{C_1}{Q_{\text{tot}}} + \frac{(\theta_2^{\text{free}} - \theta_1^{\text{free}})2C_1}{Q_{\text{tot}}(T - \theta_1^{\text{free}})} \quad \text{if } Q_{\text{tot}} \geq C_1 \text{ and } Q_{\text{tot}} \leq C_1 + C_2.$$

When the demand is larger than the total capacity, the traffic is divided over the two routes. On both routes a queue forms. Equation (5) can then be rewritten as:

$$\begin{aligned}\beta(d+1) = {} & \beta(d) + \kappa\Big(\frac{((1-\beta(d))Q_{\text{tot}} - C_2)(T - \theta_2^{\text{free}})}{2C_2}\\ & - \frac{(\beta(d)Q_{\text{tot}} - C_1)(T - \theta_1^{\text{free}})}{2C_1} + \theta_2^{\text{free}} - \theta_1^{\text{free}}\Big)\end{aligned}$$

which at equilibrium, with $\beta(d+1) = \beta(d) = \beta^{\text{eq}}$ leads to

$$\beta^{\text{eq}} = \frac{C_1 Q_{\text{tot}}(T - \theta_2^{\text{free}}) + 3C_1C_2(\theta_2^{\text{free}} - \theta_1^{\text{free}})}{Q_{\text{tot}}(C_1(T - \theta_2^{\text{free}}) + C_2(T - \theta_1^{\text{free}}))} \quad \text{if } Q_{\text{tot}} > C_1 + C_2 \quad . \qquad (7)$$

4.2 Route 2 has lowest free flow travel time

In this case $\theta_1^{\text{free}} > \theta_2^{\text{free}}$, and thus route 2 is most desired route for the drivers. The possible equilibria are computed using similar equations as in Section 4.1. In particular, when the demand is larger than the capacity of the second route, this gives:

$$\beta^{\text{eq}} = \frac{Q_{\text{tot}} - C_2}{Q_{\text{tot}}} + \frac{2C_2(\theta_2^{\text{free}} - \theta_1^{\text{free}})}{Q_{\text{tot}}(T - \theta_2^{\text{free}})} \quad \text{if } Q_{\text{tot}} \geq C_2 \text{ and } Q_{\text{tot}} \leq C_1 + C_2 \ . \tag{8}$$

The other equations can be obtained in a similar way from the equations in Section 4.1.

4.3 The routes are equally long: $\theta_1^{\text{free}} = \theta_2^{\text{free}}$

When both routes have the same free-flow travel time, the equilibrium turning fractions can depend on the initial turning fraction, β^0. This is because drivers in this case will only change their route when a queue is formed.

A queue on a route is formed when the initial turning fraction results in a flow larger than the capacity on this route: $\beta^0 Q_{\text{tot}} > C_1$ for route 1 or $(1-\beta^0)Q_{\text{tot}} > C_2$ for route 2. In these cases the equilibrium turning fraction is such that the flow on a route equals the capacity flow of this route:

$$\beta^{\text{eq}} = \frac{C_1}{Q_{\text{tot}}} \quad \text{if } \beta^0 \geq \frac{C_1}{Q_{\text{tot}}} \text{ and } Q_{\text{tot}} > C_1 \text{ and } Q_{\text{tot}} \leq C_1 + C_2$$

when a queue is formed on route 1, and

$$\beta^{\text{eq}} = \frac{Q_{\text{tot}} - C_2}{Q_{\text{tot}}} \quad \text{if } \beta^0 \leq 1 - \frac{C_2}{Q_{\text{tot}}} \text{ and } Q_{\text{tot}} \geq C_2 \text{ and } Q_{\text{tot}} \leq C_1 + C_2$$

when a queue is formed on route 2.

When no queue is formed, so when $\beta^0 Q_{\text{tot}} \leq C_1$ and $(1-\beta^0)Q_{\text{tot}} \leq C_2$ with $Q_{\text{tot}} \leq C_1 + C_2$, the equilibrium turning fraction lies in the interval $\left[1 - \frac{C_2}{Q_{\text{tot}}}, \frac{C_1}{Q_{\text{tot}}}\right]$:

$$\beta^{\text{eq}} = \beta^0 \quad \text{if } \beta^0 \in \left[1 - \frac{C_2}{Q_{\text{tot}}}, \frac{C_1}{Q_{\text{tot}}}\right] \text{ and } Q_{\text{tot}} \leq C_1 + C_2 \ .$$

This shows that when no queue is formed there is no need for the drivers to change their route choice, because without queue the travel times on the routes are equal. When the demand is so low that no queue can be formed at all, i.e. $Q_{\text{tot}} < C_1$ and $Q_{\text{tot}} < C_2$, the equilibrium turning fraction is equal to the initial turning fraction β^0, and can have all values in the interval $[0, 1]$.

A queue will be formed when the demand is larger than the total capacity. In this case the route choice depends on the ratio between the capacities of the two routes, since the free-flow travel times are equal and thus the capacities determine the queue lengths:

$$\beta^{\text{eq}} = \frac{C_1}{C_1 + C_2} \quad \text{if } Q_{\text{tot}} > C_1 + C_2 \ .$$

5. INFLUENCING LONG-TERM ROUTE CHOICE VIA TRAFFIC CONTROL

In this section we will consider two methods to influence the long-term route choice via traffic control measures. Traffic can be controlled in different ways, and the traffic control system can have many different goals, e.g. improve throughput, reduce travel time, decrease the delay, reduce queue lengths, etc. Note that the long-term route choice that is reached without intervention may not always correspond to the desired situation. Goals directly related to route choice are e.g. reducing pollution in specified areas, creating traffic diversions around accidents or maintenance works, or reducing traffic flows in densely populated areas.

We consider outflow control and speed control. With outflow control the flow on a given route is limited to a certain maximal value. Such outflow control can be effectuated using e.g. mainstream metering installations or traffic signals. For speed control the speed limit on a given route is lowered to a certain value and communicated to the drivers via variable speed limit signs.

We assume that the outflow limit and speed limit can change each day and for route r it is characterized by the following notations:

$Q_r(d)$	applied outflow limit at day d (veh/h)
$v_r(d)$	applied speed limit at day d (km/h)
Q_r^{min}	minimum outflow limit (veh/h)
Q_r^{max}	maximum outflow limit (veh/h)
v_r^{min}	minimum speed limit (km/h)
v_r^{max}	maximum (static) speed limit (km/h)

The outflow limit $Q_r(d)$ gives the maximum number of vehicles per hour that is allowed to leave the route. This outflow limit is bounded from above by a maximal value [2] Q_r^{max}. The minimum value Q_r^{min} can be selected to prevent total closure of the road when outflow control is applied. The speed limit $v_r(d)$ gives the maximum speed that is allowed. We assume that this speed limit is enforced, so all drivers have a speed that is lower than or equal to the speed limit. The speed limit is bounded from below by v_r^{min} and from above by v_r^{max}, where v_r^{min} is used to prevent total closure of the road (which will happen when the speed limit equals zero), and where v_r^{max} is selected based on safety considerations.

If we apply outflow or speed control the travel time model of Section 3.1 has to be adapted as follows.

Outflow control With outflow control we reduce the effective capacity of a given route. As a consequence, the capacities C_1 and C_2 in (3) and (4) for the time spent in the queue and in (5) for the evolution of the turning fraction have to be replaced by the effective or controlled capacities C_1^{ctrl} and C_2^{ctrl} respectively, with

$$C_r^{\text{ctrl}}(d) = \min(Q_r(d), C_r) \ ,$$

which in case $Q_r^{\text{max}} \leq C_r$ reduces to $C_r^{\text{ctrl}} = Q_r(d)$.

Speed control Speed control changes the free-flow travel time for the routes, and in this way it changes the route choice. As a consequence the free-flow travel time θ_r^{free} in equations (2), (3) and (4) has to be replaced by the effective or controlled free-flow travel time $\theta_r^{\text{free,ctrl}}(d)$ with

$$\theta_r^{\text{free,ctrl}}(d) = \frac{l_r}{\min(v_r(d), v_r^{\text{max}})}$$

which reduces to $\theta_r^{\text{free,ctrl}}(d) = \frac{l_r}{v_r(d)}$ since $v_r \leq v_r^{\text{max}}$.

6. ANALYSIS FOR A SIMPLE LINEAR CONTROLLER FOR OUTFLOW CONTROL

In this section we illustrate for the two-route network how outflow control can be used to reach a desired flow Q_1^{desired}. Note that speed limit control can also be used, but due to the limited amount of space we will only present outflow control. To keep the analysis simple, we select a basic controller which

[2] Note that we can assume without loss of generality that $Q_r^{\text{max}} \leq C_r$.

can only change the outflow limit $Q_1(d)$ of route 1. The outflow limit for route 2 is kept constant, i.e. $Q_2(d) = C_2$. Our controller aims at steering the flow on route 1 to a desired flow Q_1^{desired}. The control law is as follows:

$$Q_1(d+1) = \max\Big(Q_1^{\min}, \min\big(Q_1^{\max}, Q_1(d) + P(Q_1^{\text{desired}} - \beta(d)Q_{\text{tot}})\big)\Big) \quad (9)$$

where $P > 0$ is the integration gain.

When a controller is applied, the uncontrolled equilibrium traffic assignments described in the previous section can change. We again look at the different combinations of free-flow times and demands, and determine the equilibrium turning fractions β^{eq} and the corresponding values for the outflow limit $Q_1(d)$.

6.1 Route 1 has the lowest free flow travel time

When the demand is lower than the desired flow, $Q_{\text{tot}} \leq Q_1^{\text{desired}}$, the outflow limit $Q_1(d)$ has no influence on the route choice, and so the controller also has no influence. This means that all the traffic enters route 1, so $\beta^{\text{eq}} = 1$. Since the desired flow is not reached the controller keeps increasing the outflow limit, which ends up at its maximum value:

$$Q_1^{\text{eq}} = Q_1^{\max} = C_1 \quad \text{if } Q_{\text{tot}} \leq Q_1^{\text{desired}} \quad . \qquad (10)$$

When the demand is larger than the desired flow but smaller than $Q_1^{\text{desired}} + C_2$, the capacity is changed until the desired flow on route 1 is reached. This equilibrium value is reached when $Q_1(d+1) = Q_1(d) = Q_1^{\text{eq}}$. Together with (9) this gives

$$P(Q_1^{\text{desired}} - \beta(d)Q_{\text{tot}}) = 0, \quad \beta(d)Q_{\text{tot}} = Q_1^{\text{desired}},$$

which can be substituted in Equation 6, and then results in:

$$Q_1^{\text{eq}} = \frac{Q_1^{\text{desired}}(T - \theta_1^{\text{free}})}{(T - \theta_1^{\text{free}}) + 2(\theta_2^{\text{free}} - \theta_1^{\text{free}})} \qquad (11)$$

$$\text{if } Q_{\text{tot}} > Q_1^{\text{desired}} \text{ and } Q_{\text{tot}} \leq Q_1^{\text{desired}} + C_2 \quad . \qquad (12)$$

The resulting flow on route 1 is exactly equal to Q_1^{desired} which results the following equilibrium turning fraction:

$$\beta^{\text{eq}} = \frac{Q_1^{\text{desired}}}{Q_{\text{tot}}} \quad \text{if } Q_{\text{tot}} > Q_1^{\text{desired}} \text{ and } Q_{\text{tot}} \leq Q_1^{\text{desired}} + C_2 \quad .$$

When the demand exceeds the sum of the desired flow and C_2, the controller decreases $Q_1(d)$ in such a way that a long queue appears at route 1, which results in most drivers taking route 2, and so the desired flow on route 1 can still be attained. This leads to the desired equilibrium turning fraction $\beta^{\text{eq}} = Q_1^{\text{desired}}/Q_{\text{tot}}$, so $Q_1^{\text{desired}} = \beta^{\text{eq}}Q_{\text{tot}}$, which can be substituted in (7). This gives the corresponding value for $Q_1(d)$:

$$Q_1^{\text{eq}} = \frac{Q_1^{\text{desired}}(T - \theta_1^{\text{free}})}{\frac{(Q_{\text{tot}} - Q_1^{\text{desired}})(T - \theta_2^{\text{free}})}{C_2} + 3(\theta_2^{\text{free}} - \theta_1^{\text{free}})} \qquad (13)$$

$$\text{if } Q_{\text{tot}} > Q_1^{\text{desired}} + C_2 \quad .$$

6.2 Route 2 has the lowest free flow travel time

Here route 1 is only used when $Q_{\text{tot}} \geq C_2$. When the demand is lower than C_2, $\beta^{\text{eq}} = 0$ and $Q_1^{\text{eq}} = Q_1^{\max}$ as in (10).

When the demand exceeds C_2 but not the total capacity $Q_1^{\text{desired}} + C_2$, the controller has also no influence, and the same turning fraction of (8) is reached. The desired flow is still not reached, and the controller increases the outflow limit on route 1 to its maximum value.

A demand that is larger than $Q_1^{\text{desired}} + C_2$ leads to a turning fraction of $\beta^{\text{eq}} = Q_1^{\text{desired}}/Q_{\text{tot}}$, with an outflow limit as in (13).

6.3 The routes are equally long: $\theta_1^{\text{free}} = \theta_2^{\text{free}}$

If the routes are equally long the drivers will not change their route choice until the flow on a route reaches the outflow limit of this route. This results in two different situations: the demand is higher than the sum of the desired flow and the capacity of route 2 ($Q_{\text{tot}} \geq Q_1^{\text{desired}} + C_2$), or the demand is lower than this sum.

When $Q_{\text{tot}} \geq Q_1^{\text{desired}} + C_2$, the fact that the routes are equally long has no influence. The turning fraction is given by $\beta^{\text{eq}} = Q_1^{\text{desired}}/Q_{\text{tot}}$, and the corresponding outflow limit by (13).

When $Q_{\text{tot}} \leq Q_1^{\text{desired}} + C_2$ the initial flow on route 1 is important. The initial flow can correspond to an equilibrium traffic assignment or not.

An equilibrium traffic assignment is present when

$(1-\beta^0)Q_{\text{tot}} \leq C_2$ and $\beta^0 Q_{\text{tot}} < Q_1^{\text{desired}}$ and $Q_{\text{tot}} \leq Q_1^{\text{desired}} + C_2$

In this case the controller has no influence because the initial flow on route 1 is lower than the desired flow. And because no queue is formed on route 2, the flows on the two routes will not change. In this case, the turning fraction does not change, and the controller reaches the maximum value $\beta^{\text{eq}} = \beta^0$ and $Q_1^{\text{eq}} = Q_1^{\max}$, with $\beta^0 = \beta^{\text{eq}} \in [\max(0, 1 - \frac{C_2}{Q_{\text{tot}}}), \min(1, \frac{Q_1(d)}{Q_{\text{tot}}})]$.

The initial flow does not correspond to an equilibrium traffic assignment in two cases:

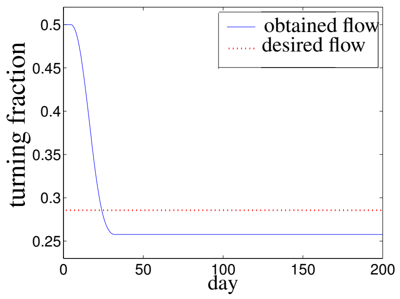
- $\beta^0 > \frac{Q_1^{\text{desired}}}{Q_{\text{tot}}}$ and $Q_{\text{tot}} > Q_1^{\text{desired}}$ and $Q_{\text{tot}} \leq Q_1^{\text{desired}} + C_2$
- $\beta^0 \leq 1 - \frac{C_2}{Q_{\text{tot}}}$ and $Q_{\text{tot}} > C_2$ and $Q_{\text{tot}} \leq Q_1^{\text{desired}} + C_2$.

In these cases the controller tries to reach the equilibrium turning fraction $\beta^{\text{eq}} = Q_1^{\text{desired}}/Q_{\text{tot}}$, but a specific property of the controller results in a deviation of this value.

When $\beta^0 > \frac{Q_1^{\text{desired}}}{Q_{\text{tot}}}$ the controller decreases the outflow limit of route 1, until the flow on this route is lower than the desired flow. However, the controller can have undershoot. This means that, depending on the value of P, the turning fraction may become lower than the desired turning fraction.

The controller can only lower the flow on route 1, but not increase it, and thus cannot correct for the undershoot, and the flow stays too low. This means that the equilibrium turning fraction will also be lower than $Q_1^{\text{desired}}/Q_{\text{tot}}$. This is illustrated in Figure 3 which shows the desired turning fractions corresponding to Q_1^{desired} and the obtained equilibrium flow Q_1^{eq}. The value of the outflow limit, shown in the second graph in Figure 3, first decreases until the undershoot is at its minimum. At this moment the controller will try to increase the flow on route 1 again, and so the outflow limit starts to increase. Since the turning fraction does not change as a reaction on this increase, the controller keeps increasing the outflow limit until the maximum value $Q_1^{\max}$ is reached.

When $\beta^0 \leq 1 - \frac{C_2}{Q_{\text{tot}}}$ the initial flow on route 2 is higher than the capacity of route 2. This means that the second day more drivers will select route 1. This can result in $\beta^0 > \frac{Q_1^{\text{desired}}}{Q_{\text{tot}}}$, which situation is described above.



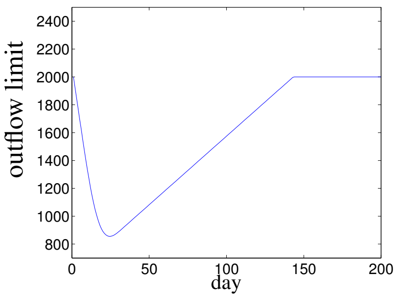


Fig. 3. Obtained turning fractions, desired turning fractions, and the outflow limit on route 1 as function of days, for the situation where the routes have the same free flow travel time and where the dynamics of the system lead to overshoot.

7. CONCLUSIONS

In this paper we have investigated how existing traffic control measures can influence the route choice of the drivers. The main goal of the paper was to get insight in the phenomena that take place when conventional control methods are used to influence route choice. To obtain this insight, we first have selected simple models to describe the travel times on two routes and the route choice behavior of the drivers. Then we have determined the equilibrium route choices that are reached when no control is applied. Next, we have presented possible types of control to influence the long-term route choice: outflow control and speed limit control. We have also illustrated the effects of an outflow controller by performing a case study. On a two route simulation network one basic linear controller was placed, with as goal to reach a desired flow at the first route. In all cases the controller could keep the flow on route 1 at the desired level or lower.

The insight obtained with the simple models and controller can be useful to interpret results of more advanced controller methods, such as Model Predictive Control or adaptive control. With more sophisticated controllers different control objectives can also be used. Possible goals are e.g. minimizing the flow on one route, minimizing the total travel time, minimizing the queue length, achieving a desired assignment etc.

In our future work we will investigate the use of more advanced, model-based predictive control methods. We will also look at traffic control using more realistic traffic models, and more advanced control measures, e.g. information panels, dedicated lanes, etc. Attention will also be paid to investigating stability regions, and to larger traffic networks.

REFERENCES

J. Barceló and J. L. Ferrer. AIMSUN2: Advanced interactive microscopic simulator for urban networks, user's manual. Technical report, Universitat Politècnica de Catalunya, 1997.

T. Bellemans, B. De Schutter, and B. De Moor. Anticipative model predictive control for ramp metering in freeway networks. In *Proceedings of the 2003 American Control Conference*, pages 4077–4082, Denver, Colorado, June 2003.

M. Ben-Akiva, A. De Palma, and I. Kaysi. Dynamic network models and driver information systems. *Transportation Research A*, 25(5):251–266, September 1991.

M.C.J. Bliemer. A quasi-variational inequality approach to multi-user class dynamic traffic assignment. In *Proceedings of the 79th Annual Meeting of the Transportation Research Board*, Washington, DC, USA, January 2000.

E. Bogers, F. Viti, and S.P. Hoogendoorn. Joint modeling of ATIS, habit and learning impacts on route choice by laboratory simulator experiments. In *Proceedings of the 84th Annual Meeting of the Transportation Research Board*, Washington DC, USA, January 2005.

M. Carey and Y.E. Ge. Comparing whole-link travel time models. *Transportation Research Part B*, 37:905–926, 2003.

C.F. Daganzo. The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research B*, 28B(4):269–287, 1994.

C.F. Daganzo and Y. Sheffi. On stochastic models of traffic assignment. *Transportation Science*, 11:253–274, 1977.

C. Fisk. Some developments in equilibrium traffic assignment. *Transportation Research B*, 14B:243–255, 1980.

H. Haj-Salem and M. Papageorgiou. Ramp metering impact on urban corridor traffic: Field results. *Transportation Research A*, 29(4):303–319, 1995.

A. Karimi, A. Hegyi, B. De Schutter, J. Hellendoorn, and F. Middelham. Integration of dynamic route guidance and freeway ramp metering using model predictive control. In *Proceedings of the 2004 American Control Conference (ACC 2004)*, pages 5533–5538, Boston, MA, USA, 2004.

H.S. Mahmassani, N.N. Huynha, K. Srinivasan, and M. Kraanc. Tripmaker choice behavior for shopping trips under real-time information: model formulation and results of stated-preference internet-based interactive experiments. *Journal of Retailing and Consumer Services*, 10(6):311–321, November 2003.

A. Messmer and M. Papageorgiou. METANET: A macroscopic simulation program for motorway networks. *Traffic Engineering and Control*, 31:466–470, 1990.

S. Peeta and H.S. Mahmassani. Multiple user classes real-time traffic assignment for online operations: a rolling horizon solution framework. *Transportation Research C*, 3(2):83–98, 1995.

Quadstone. Paramics system overview. Technical report, Quadstone limited, Edinburgh, Schotland, 2002. www.paramics-online.com.

H. Taale and H.J. van Zuylen. Traffic control and route choice: Occurrence of instabilities. In *TRAIL 5th Annual Congress 1999: Five Years Crossroads of Theory and Practice*, volume 2, pages 1–19. TRAIL, 1999.

Y. Wang and M. Papageorgiou. A predictive feedback routing control strategy for freeway network traffic. In *Proceedings of the American Control Conference*, pages 3606–3611, Anchorage, Alaska, USA, May 2002.

J.G. Wardrop. Some theoretical aspects of road traffic research. *Proceedings of the Institute of Civil Engineers, Part II*, 1(2): 325–378, 1952.