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Delft Center for Systems and Control  
Delft University of Technology  
Mekelweg 2, 2628 CD Delft  
The Netherlands  
phone: +31-15-278.24.73 (secretary)  
URL: <https://www.dcsc.tudelft.nl>

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# Model-based speed limit control with different traffic state measurements

M. Burger\* A. Hegyi\*\* B. De Schutter\*,\*\*\*

\* *Delft Center for Systems and Control, Delft University of Technology, The Netherlands, m.burger@tudelft.nl*

\*\* *Transportation and Planning, Delft University of Technology, The Netherlands, a.hegyi@tudelft.nl*

\*\*\* *Marine and Transport Technology Department, Delft University of Technology, The Netherlands, b@deschutter.info*

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**Abstract:** In this paper, traffic flow is controlled using dynamic speed limits, obtained by Model Predictive Control (MPC). MPC is a model-based approach, where the states of the system, influenced by control actions, are predicted over a certain time span. The states of the system are the mean speeds and densities on the motorway.

Traffic flow models typically use space mean speeds, while measurements on motorways are often time mean speeds. Several methods for obtaining estimates of the space mean speed based on the time mean speeds are discussed, and the possible performance loss of using another mean speed than the space mean speed for model-based traffic control is investigated.

The resulting controllers, using the different estimates, are evaluated for a scenario where speed limits are used to eliminate a shock wave from a motorway by comparing the achieved reduction in the total time that the vehicles spend on the motorway (TTS). The result show that the performance for the different estimation methods is comparable, and lead to an improvement of the TTS of around 14%.

*Keywords:* traffic and transportation; model predictive control; traffic control; dynamic speed limits; space mean speed; time mean speed

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## 1. INTRODUCTION

In many countries the demand on motorways exceeds the capacity regularly, which can lead to traffic jams causing economical losses, due to increasing travel times, and environmental problems since vehicles use more fuel, which results in an increasing emission of pollutant gasses. If increasing the number of lanes on a motorway is infeasible or not the desired solution for solving the capacity shortage problem, more advanced solutions are necessary, such as traffic control. A potential solution is model-based traffic control, which is capable of reducing or dissolving shock waves, e.g., by using dynamic speed limits (Hegyi et al., 2005; Zhang et al., 2005).

In the research discussed in this paper, Model Predictive Control (MPC) is used to determine dynamic speed limits on a part of the Dutch motorway A12. The objective of the traffic controller is to reduce the travel time. The current traffic state, consisting of densities and mean speeds, is needed as initial value to predict the future traffic states. Hence, measurements on the motorway are required for this control method.

Most often loop detectors are used to measure the traffic state on the motorway. Loop detectors typically count the number of passing vehicles, and determine the mean speed over a certain time.

In traffic flow models the speeds used are typically space mean speeds, whereas loop detectors typically return time

mean speeds. Using local measurements, it is impossible to calculate exact space mean speeds, but estimations of the space mean speed can be used in traffic flow models (Daganzo, 1997; Lint, 2004; Rakha and Zhang, 2005). In this paper, we present the results of an investigation of the prediction accuracy for six distinct mean speed calculation methods, and their suitability for use in model-based traffic control.

This paper is organised as follows. Section 2 discusses the modelling of traffic flows. The shock wave phenomenon in traffic flows is briefly discussed, and a model for predicting traffic flows is given. Several methods for obtaining mean speeds from individual vehicle speed measurements are discussed. In Section 3 we explain the MPC scheme which is used to reduce shock waves. The approach is given in Section 4, where the objective function for the controller, the performance criteria for traffic conditions, and the performance criteria for traffic model calibration are discussed. Section 5 gives details about the performed experiment. The results of the experiment are given in Section 6. The conclusions are discussed in Section 7.

## 2. TRAFFIC FLOW MODELLING

### 2.1 Shock waves

Shock waves are upstream propagating traffic jams, which often emerge from on-ramps and other types of bottlenecks. The outflow of a shock wave is usually about 70% of

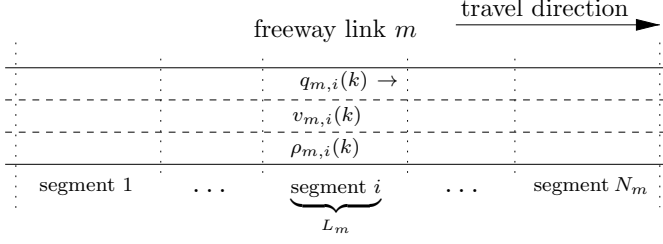


Fig. 1. In the METANET model, a motorway link is divided into segments.

the motorway capacity (Kerner and Rehborn, 1996), and resolving shock waves can greatly improve the motorway traffic flow. Since the vehicle speeds in the shock wave are low, travel times are higher compared to the free flow situation. Shock waves also cause potentially dangerous situations due to the speed differences, and air pollution is increased by slowly driving and accelerating vehicles.

Shock waves can be reduced or dissolved by applying dynamic speed limits on the motorway. Traffic upstream of the shock wave can be slowed down, thereby limiting the inflow to the shock wave. This will reduce the length of the shock wave, and can even dissolve it (depending on the traffic demand and length of the motorway stretch where dynamic speed limits are applied). The settings for the dynamic speed limits can, e.g., be determined using MPC (Hegyi et al., 2005), as is also done in this paper. The prediction model, which is used in the controller, should be able to represent the shock waves.

## 2.2 Prediction model

For the prediction of future traffic states, the macroscopic traffic flow model METANET is used (Kotsialos et al., 2002). Here we will give a brief description of the model, including the extensions proposed by Hegyi et al. (2005).

The METANET model divides a motorway network into multiple links  $m$ . A link is divided into multiple segments  $i$ , for which the state is given in terms of the density  $\rho_{m,i}(k)$ , mean speed  $v_{m,i}(k)$ , and outflow  $q_{m,i}(k)$  of the segment. Here  $k$  denotes the simulation step, with simulation time interval  $T$ . Figure 1 shows a link  $m$  which is divided into  $N_m$  segments. Each segment has a length  $L_m$ , and a number of lanes  $\lambda_m$ , which are constant for all segments in a link  $m$ .

(a) *Basic METANET model* The basic METANET model equations are given by (Kotsialos et al., 2002):

$$\begin{aligned} q_{m,i}(k) &= \rho_{m,i}(k)v_{m,i}(k)\lambda_m, \\ \rho_{m,i}(k+1) &= \rho_{m,i}(k) + \frac{T}{L_m\lambda_m} (q_{m,i-1}(k) - q_{m,i}(k)), \\ v_{m,i}(k+1) &= v_{m,i}(k) + \frac{T}{\tau} (V(\rho_{m,i}(k)) - v_{m,i}(k)) \\ &\quad + \frac{T}{L_m} v_{m,i}(k) (v_{m,i-1}(k) - v_{m,i}(k)) \\ &\quad - \frac{\eta T}{\tau L_m} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa}, \end{aligned}$$

and

$$V(\rho_{m,i}(k)) = v_{\text{free},m} \exp \left[ -\frac{1}{a_m} \left( \frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}} \right)^{a_m} \right], \quad (1)$$

where  $v_{\text{free},m}$  is the free flow speed in link  $m$ ,  $\rho_{\text{crit},m}$  is its critical density (threshold between free and congested traffic flow), and  $\tau$ ,  $\eta$ ,  $\kappa$  and  $a_m$  are model fitting parameters.

Origins are modelled using a simple queue model. The number of vehicles  $w_{o,m}(k+1)$  in the queue for link  $m$  at the next simulation step equals

$$w_{o,m}(k+1) = w_{o,m}(k) + T (d_{o,m}(k) - q_{o,m}(k)), \quad (2)$$

where  $w_{o,m}(k)$  is the previous queue length,  $d_{o,m}(k)$  is the demand, and  $q_{o,m}(k)$  is the outflow given by

$$q_{o,m}(k) = \min \left\{ d_{o,m}(k) + \frac{w_{o,m}(k)}{T}, q_{\text{on-ramp},m}(k) \right\}, \quad (3)$$

where

$$q_{\text{on-ramp},m}(k) = Q_{o,m} \frac{\rho_{\text{max}} - \rho_{m,1}(k)}{\rho_{\text{max}} - \rho_{\text{crit},m}}$$

is the maximum possible on-ramp flow,  $Q_{o,m}(k)$  is the on-ramp capacity under free flow conditions, and  $\rho_{\text{max}}$  is the maximum density occurring during a traffic jam.

(b) *Extended METANET model* Three extensions to the basic METANET model are proposed by Hegyi et al. (2005). A brief description of these extensions is given next.

The first extension adds the effect of using dynamic speed limits to the METANET model. The desired speed  $V(\rho_{m,i}(k))$  in (1) is replaced by

$$V_{\text{ext}}(\rho_{m,i}(k)) = \min \{ (1 + \alpha)v_{\text{ctr},m,i}(k), V(\rho_{m,i}(k)) \}, \quad (4)$$

where  $\alpha$  is the compliance factor to the speed limits, and  $v_{\text{ctr},m,i}(k)$  is the speed limit on link  $m$  at segment  $i$ , displayed at simulation step  $k$ .

The second extension is introduced to model the inflow of traffic at mainstream origins. The origin of link  $m$  is limited by either the demand, or the maximum inflow, given as

$$q_{o,m}(k) = \min \left\{ d_{o,m}(k) + \frac{w_{o,m}(k)}{T}, \gamma q_{\text{lim},m,1}(k) \right\}, \quad (5)$$

where  $\gamma$  is a model fitting parameter, and the maximum inflow  $q_{\text{lim},m,1}(k)$  is dependent on the limiting speed in the first segment of link  $m$ :

$$q_{\text{lim},m,1}(k) = \begin{cases} q_{\text{spd},m}(k) & \text{if } v_{\text{lim},m,1}(k) < V_{\text{ext}}(\rho_{\text{crit},m}) \\ q_{\text{cap},m} & \text{if } v_{\text{lim},m,1}(k) \geq V_{\text{ext}}(\rho_{\text{crit},m}) \end{cases},$$

where

$$\begin{aligned} q_{\text{cap},m} &= \lambda_m V_{\text{ext}}(\rho_{\text{crit},m}) \rho_{\text{crit},m} \\ q_{\text{spd},m}(k) &= \lambda_m v_{\text{lim},m,1}(k) \rho_{\text{crit},m} \sqrt[ a_m ]{ -a_m \ln \left( \frac{v_{\text{lim},m,1}(k)}{v_{\text{free},m}} \right) } \end{aligned}$$

and

$$v_{\text{lim},m,1}(k) = \min \{ v_{\text{ctr},m,1}(k), v_{m,1}(k) \}.$$

When the origin is an on-ramp, (3) is used.

The third extension improves the modelling of shock waves travelling upstream on the link. The anticipation behaviour of drivers may be different at the head and tail of a shock wave. The anticipation constant  $\eta$  in (1) is replaced by the density dependent parameter

$$\eta_{m,i}(k) = \begin{cases} \eta_{\text{high}} & \text{if } \rho_{m,i+1}(k) \geq \rho_{m,i}(k) \\ \eta_{\text{low}} & \text{if } \rho_{m,i+1}(k) < \rho_{m,i}(k) \end{cases} \quad (6)$$

in order to account for these anticipation differences.

### 2.3 Various mean speeds

In the field of motorway traffic flow modelling, two types of mean speeds are often used: *time mean* speed and *space mean* speed (Daganzo, 1997; May, 1990). Time mean speeds are based on measurements at a specific location that are averaged over a certain time span. These speeds can be measured by, e.g., loop detectors. Space mean speeds are based on measurements averaged over a motorway stretch, at a certain time instant. Video images can be used to obtain space mean speeds (Dailey et al., 2000).

Using loop detectors, individual vehicle speeds  $u_n$  can be obtained, where  $n$  is the vehicle index. The flow of the traffic can be obtained by dividing the number of observed vehicles  $N_{m,i}(k_s)$  on a motorway segment  $i$  of link  $m$ , in the period  $[k_s T_s, (k_s+1)T_s)$ , by the sampling time interval  $T_s$ . The relation between  $T$  and  $T_s$  is assumed to be given by  $T_s = M_s T$ , where  $M_s$  is a positive integer. Hence the flow can be determined using

$$q_{m,i}(k_s) = \frac{N_{m,i}(k_s)}{T_s}. \quad (7)$$

The density follows from the flow  $q_{m,i}(k_s)$ , mean speed  $v_{m,i}(k_s)$ , and number of lanes  $\lambda_m$ , as

$$\rho_{m,i}(k_s) = \frac{q_{m,i}(k_s)}{v_{m,i}(k_s)\lambda_m} \quad (8)$$

The mean speeds are calculated using six different methods, which are discussed next.

(a) *Time mean speed* The time mean speed is calculated using the **arithmetic mean** of the  $N_{m,i}(k_s)$  locally measured vehicle speeds  $u_n$ , measured over the sampling time interval  $T_s$  (Daganzo, 1997) as

$$v_{\text{tms}}(k_s) = \frac{1}{N_{m,i}(k_s)} \sum_{n=1}^{N_{m,i}(k_s)} u_n \quad (9)$$

(b) *Estimated space mean speed* For obtaining the space mean speed, the **harmonic mean** of the locally measured vehicle speeds is used by Daganzo (1997), given by

$$\hat{v}_{\text{sms}}(k_s) = \left( \frac{1}{N_{m,i}(k_s)} \sum_{n=1}^{N_{m,i}(k_s)} \frac{1}{u_n} \right)^{-1} \quad (10)$$

(c) *Geometric mean speed* Besides the arithmetic and harmonic mean, there is a third ‘classic’ Pythagorean mean, namely the **geometric mean** (Petz and Temesi, 2005). This mean can be calculated as

$$v_{\text{geo}}(k_s) = \sqrt[N_{m,i}(k_s)]{\prod_{n=1}^{N_{m,i}(k_s)} u_n} \quad (11)$$

(d) *Estimated space mean speed using instantaneous speed variance* A method for estimating the space mean speed, based on locally measured vehicle speeds  $u_n$ , is proposed by Lint (2004). It uses the relation between time mean speed  $v_{\text{tms}}$  (9), and the space mean speed  $v_{\text{sms}}$  (Wardrop, 1952), given by

$$v_{\text{tms}}(k_s) = \frac{\sigma_1^2(k_s)}{v_{\text{sms}}(k_s)} + v_{\text{sms}}(k_s) \quad (12)$$

where  $\sigma_1^2(k_s)$  is the variance of the instantaneously measured vehicle speeds. Since the instantaneous speed variance  $\sigma_1^2(k_s)$  cannot be determined exactly by local measurements, an estimation is needed, given by

$$\hat{\sigma}_1^2(k_s) = \frac{1}{2N} \sum_{n=1}^N \frac{\hat{v}_{\text{sms}}(k_s)}{u_n} (u_{n+1} - u_n)^2$$

where  $\hat{v}_{\text{sms}}(k_s)$  is given by (10). The estimated space mean speed becomes

$$\hat{v}_{\text{sms},\hat{\sigma}_1}(k_s) = \frac{1}{2} \left\{ v_{\text{tms}}(k_s) + \sqrt{v_{\text{tms}}^2(k_s) - 4\hat{\sigma}_1^2(k_s)} \right\} \quad (13)$$

(e) *Estimated space mean speed using local speed variance*

In (Rakha and Zhang, 2005), an estimate of the space mean speed is proposed, based on the time mean speed and variance of the locally measured speeds. The space mean speed estimate is determined by

$$\hat{v}_{\text{sms},\sigma_1}(k_s) = v_{\text{tms}}(k_s) - \frac{\sigma_1^2(k_s)}{v_{\text{tms}}(k_s)} \quad (14)$$

where  $v_{\text{tms}}(k_s)$  is calculated using (9), and

$$\sigma_1^2(k_s) = \frac{1}{N_{m,i}(k_s)} \sum_{n=1}^{N_{m,i}(k_s)} (u_n - v_{\text{tms}}(k_s))^2$$

Equation (14) is based on (12), where it is assumed that  $v_{\text{tms}} \approx v_{\text{sms}}$ , and  $\sigma_1^2 \approx \hat{\sigma}_1^2$ . This assumption only holds for stationary and homogeneous traffic flow, therefore this method is likely to give poor estimations of the space mean speed when shock waves are present in the traffic network.

(f) *Time average space mean speed* Using, e.g., video images it is possible to obtain the space mean speed  $v_{\text{sms}}(k_t)$  on a motorway segment for every second  $k_t$  (Dailey et al., 2000). Since the previous mean speed methods are all based on a time period  $[k_s T_s, (k_s+1)T_s)$ , we will take a time average of the obtained space mean speeds in the time period  $[k_t T_s, (k_t+M_t)T_s)$ , where  $k_t = M_t k_s$ :

$$\bar{v}_{\text{sms}}(k_s) = \frac{1}{M_t} \sum_{k_t=M_t k_s}^{M_t(k_s+1)} v_{\text{sms}}(k_t) \quad (15)$$

The six methods described above can be used to calculate the mean speed. Together with the density, which can be calculated using (7) and (8), these variables are used to describe the state of the traffic flow. This state is needed as initial starting point for the prediction in the model-based traffic controller.

## 3. MODEL PREDICTIVE CONTROL FOR TRAFFIC

We use an MPC scheme as shown in Figure 2 to solve the problem of optimal coordination of dynamic speed limits.

The traffic system is the physical traffic network, from which measurements are taken. The obtained mean speeds, flows, and densities are sent to the controller at each controller step  $k_c$ . The controller uses a macroscopic traffic flow model to predict the future traffic states over a prediction horizon of  $N_p$  controller steps. Using numerical

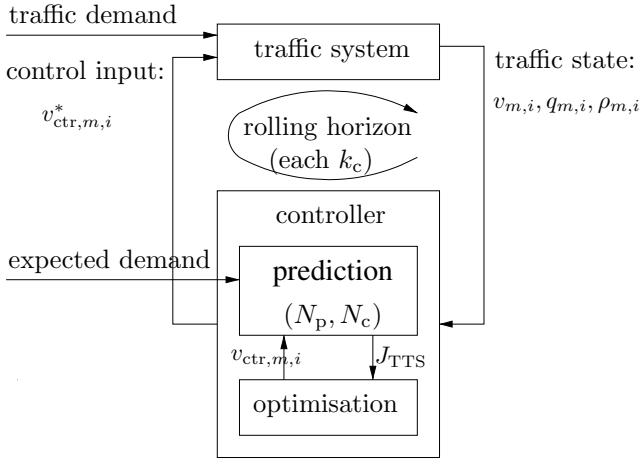


Fig. 2. Schematic view of the MPC approach.

optimisation, the optimal speed limits  $v_{ctr,m,i}^*$  are determined up to a control horizon of  $N_c$  controller steps. For further information on MPC, we refer the interested reader to Maciejowski (2002).

## 4. APPROACH

### 4.1 Objective function

The numerical optimisation is searching for a minimum value for the objective function

$$J_{TTS}(k_c) = T \sum_{\kappa=k_c+1}^{k_c+N_p} \sum_{(m,i) \in \mathcal{M}} \{ \rho_{m,i}(\kappa) \lambda_m L_m + w_{o,m}(\kappa) \} \quad (16)$$

where  $\kappa$  is a time index,  $\mathcal{M}$  is the set of segments, and  $w_{o,m}(j)$  is the number of vehicles unable to enter the simulated area of the motorway, due to congestion. Equation (16) calculates the total time that is spent by the vehicles (TTS) over the prediction horizon  $N_p$  on the measures area of the network.

### 4.2 Performance evaluation

For the finished simulations, the improvement in traffic conditions by using MPC is evaluated using the TTS. Comparing the TTS based on the number of vehicles in the measured area is not a good measure, since the controller will not only increase the outflow, but also the inflow when the shock wave is resolved successfully. The inflow is increased, because the incoming vehicles will not be blocked anymore, when the shock wave is dissolved. Therefore an equivalent formulation of the TTS is used, based on the demand  $q_{dem}(k)$  and the outflow  $q_{out}(k)$ :

$$J_{TTS} = TN_0K + T^2 \sum_{k=0}^{K-1} (K-k) (q_{dem}(k) - q_{out}(k)) \quad (17)$$

where  $N_0$  is the initial number of vehicles in the measured area of the motorway, and  $K$  is the final sample step in the scenario.

### 4.3 Performance criteria for model calibration

Since the model parameters in the traffic flow model are different for each traffic network, model calibration is

necessary. This is done by offline numerical optimisation using an objective function given by

$$J_{cal}(\theta) = \frac{1}{M_c N_p} \sum_{k_s=1}^{K_s - M_c N_p} \sum_{\kappa=k_s}^{k_s + M_c N_p} \check{J}_{cal}(\theta, k_s, \kappa), \quad (18)$$

where  $\theta$  is the set of model parameters,  $K_s$  is the number of sample steps for which measurement data is available,  $M_c$  is a positive integer value relating the controller steps and sample steps as  $k_c = M_c k_s$ , and  $\check{J}_{cal}(\theta, k_s, \kappa)$  is given by

$$\check{J}_{cal}(\theta, k_s, \kappa) = \sum_{(m,i) \in \mathcal{M}} \left\{ \left( \frac{v_{m,i}(\kappa) - \tilde{v}_{m,i}(\kappa)}{\bar{v}(k_s)} \right)^2 + \left( \frac{\rho_{m,i}(\kappa) - \tilde{\rho}_{m,i}(\kappa)}{\bar{\rho}(k_s)} \right)^2 \right\}^{\frac{1}{2}}, \quad (19)$$

where  $\bar{v}(k_s)$  and  $\bar{\rho}(k_s)$  are the average speed and density of the measured data from controller step  $k_c$  to  $k_c + N_p$ . A lower value of the objective function (18) means a better fit of the measured states  $\{v_{m,i}; \rho_{m,i}\}$ , with the predicted states  $\{\tilde{v}_{m,i}; \tilde{\rho}_{m,i}\}$  by the traffic flow model. Since the traffic controller will be optimising on the TTS, also the difference in measured and predicted TTS is determined, to judge the performance of the parameter values. The error is given by

$$E_{TTS}(\theta) = \frac{1}{K - N_p} \sum_{k_c=1}^{K - N_p} \frac{\tilde{J}_{TTS}(k_c) - J_{TTS}(k_c)}{J_{TTS}(k_c)} \quad (20)$$

which gives the average percentage of mismatch between the TTS of the measured data  $J_{TTS}(k_c)$  and the predicted data  $\tilde{J}_{TTS}(k_c)$  for the parameter set  $\theta$ .

## 5. EXPERIMENT

### 5.1 Traffic flow modelling and constants

Shock waves on motorways can be reduced and dissolved by using model predictive traffic control. The control scheme as shown in Figure 2 is used, where the traffic system is simulated using the micro-simulation program Paramics v5.1 from Quadstone (Quadstone, 2005). The traffic state is updated with a sampling time interval of  $T_s=60$  s.

The controller uses the METANET model (1), including the extensions (4-6), to predict the future traffic states, using a simulation time interval of  $T=10$  s. The controller time interval equals  $T_c=60$  s, the control horizon is  $N_c=10$  steps, and the prediction horizon equals  $N_p=20$  steps. The speed limits have values between 40 and 120 km/h, in steps of 10 km/h. To avoid reduce speed limit oscillations, a penalty on signal variations may be added in (16).

### 5.2 Numerical optimisation methods

Both the minimisation of (17) and (18) are nonlinear, nonconvex optimisation problems. Some methods to solve these problems are sequential quadratic programming, genetic algorithms, and pattern search (Pardalos and Resende, 2002). Multistart is needed, since the optimisation problems have many local minima. The offline calibration

of the METANET model, based on measured data from the micro-simulation, is done by minimising (18). For each of the six mean speed calculation methods, 100 optimisation runs are performed, using the MATLAB function `fmincon` (The MathWorks, 2007), using sequential quadratic programming. The on-line optimisation for the dynamic speed limits is also using a multistart approach. At each controller step  $k_c$ , 16 distinct initial value sets are used. The MATLAB function `fmincon` is used to perform the optimisation of (17).

### 5.3 Traffic network layout

For the traffic network, a part of the Dutch motorway A12 is implemented in Paramics. The stretch of interest is the part between Veenendaal and Maarsbergen, which is a motorway stretch without on-ramps and off-ramps. The loop detectors in the micro-simulator are placed at the locations of the existing loop detectors on the motorway. Since the distance between subsequent loop detectors is varying, the model consists of multiple links  $m$ , all containing one segment. The links are chosen such that the detectors are near the downstream boundary, in order to obtain accurate measurements of the outflow  $q_{m,i}(k_s)$  of the links. The link lengths vary between 545 m and 810 m, and the total length of the measured area is 17422 m, with a controlled area of 9775 m.

### 5.4 Traffic scenario

The traffic demand  $q_{\text{dem}}(k)$  on the motorway is set to 4400 veh/h, at which a shock wave will remain existent in the network when no control is applied. A shock wave is introduced by simulating an incident downstream of the measured area. One vehicle is stopped for a period of 5 minutes, during which one of the two lanes is blocked. This will create a traffic jam, which expands while the lane is blocked, and the shock wave moves upstream when both lanes are accessible again.

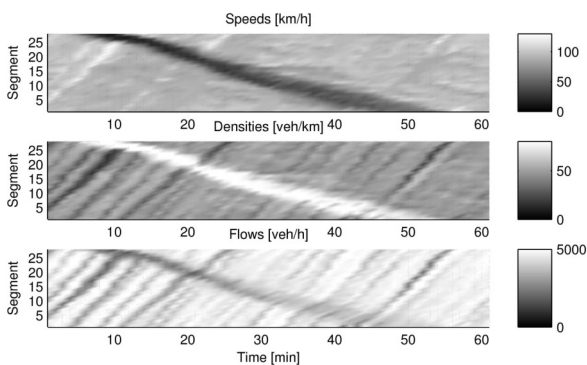


Fig. 3. Traffic condition without control.

Figure 3 shows measurements on the network. On the horizontal axis, the time is shown, and on the vertical axis, the segment numbers are given. Traffic flows from bottom to top. In the first subplot, the measured mean speeds are given, obtained using  $v_{\text{geo}}$  (11). Lighter colours represent higher mean speeds. The shock wave is clearly shown as the thick, dark stripe going upstream as time elapses. Also in the density plot (the second subplot), the shock wave

is clearly visible as the thick, light stripe representing high densities. The final subplot shows the flow, where it can be seen that due to the shock wave, the flow decreases. The thin stripes which are going downstream with elapsing time, are caused by differences in desired speed of individual vehicles.

### 5.5 Suitability of mean speed variants

The six variants for calculating the mean speed  $v_{m,i}(k)$ , discussed in Section 2.3, are used to calibrate the METANET model, resulting in different parameter sets. Using (18) we determine how well the model predicts future traffic states, using the different mean speeds. Using (20), the difference between the measured and predicted TTS is determined. For both  $J_{\text{cal}}(\theta)$  and  $E_{\text{TTS}}(\theta)$ , the values should be as small as possible. Based on the two values, the most appropriate mean speed variant for using MPC with the TTS as objective function, is determined.

## 6. RESULTS

### 6.1 Comparison of the various mean speeds

To increase the reliability of the results, five different random seeds are used to obtain different data sets, for both the calibrations and MPC simulations. The average values  $\bar{J}_{\text{cal}}(\theta)$  and  $\bar{E}_{\text{TTS}}(\theta)$  over the data sets are determined for the different mean speed types. The results are shown in Table 1. The time average space mean speed is used as the reference value (100%), since this method uses real space mean speeds, and therefore is expected to give the most accurate approximation of the space mean speed. The first column shows the average calibration errors, using (18). The second column shows the calibration errors as a percentage of the reference value, where a lower percentage means a better fit of the model to the measured data.

Table 1. Mean speed performance

	$\bar{J}_{\text{cal}}$	%	$\bar{E}_{\text{TTS}}$	%
$\bar{v}_{\text{sms}}$	<b>42.5</b>	<b>100</b>	<b>6.0</b>	<b>100</b>
$v_{\text{tms}}$	38.9	91.5	5.5	91.4
$\hat{v}_{\text{sms}}$	42.0	98.8	5.6	93.1
$v_{\text{geo}}$	39.4	92.7	5.1	85.4
$\hat{v}_{\text{sms},\hat{\sigma}_1}$	38.3	90.1	5.5	91.0
$\hat{v}_{\text{sms},\sigma_1}$	42.3	99.5	6.5	107.9

The third column contains the average error between the measured and predicted TTS, as obtained using (20). The fourth column shows the relative error based on the reference value, where a lower percentage means a better prediction of the TTS.

Based on these results, it is concluded that the geometric mean speed  $v_{\text{geo}}$  gives the best result, followed by the estimated space mean speed  $\hat{v}_{\text{sms},\hat{\sigma}_1}$ , and the time mean speed  $v_{\text{tms}}$ . These three mean speed variants are used in the model predictive traffic controller, to investigate which variant will give the best improvement in traffic conditions.

### 6.2 Improvement in traffic conditions

For the comparison of improvement in traffic conditions, the TTS is used. A lower value of  $J_{\text{TTS}}$  as given by

(17) represents better traffic conditions, since on average vehicles are spending less time in a certain area, indicating that the flow is higher. In the uncontrolled situation, as shown in Figure 3, the TTS is  $1068.0 \text{ veh}\cdot\text{h}$ .

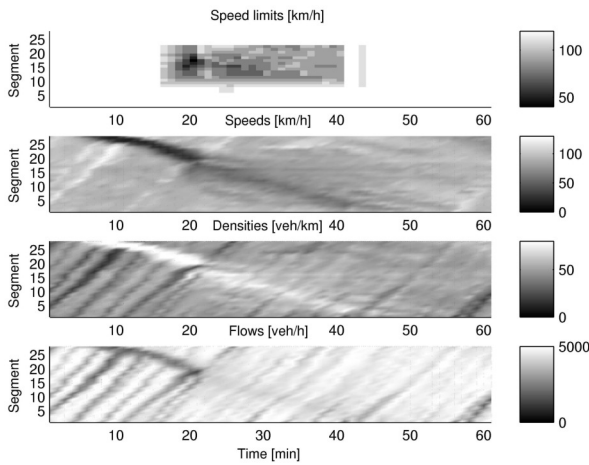


Fig. 4. Controlled traffic flow using time mean speeds as state variable.

Using an MPC-based traffic controller, the shock waves are dissolved, as shown in Figure 4. Using time mean speeds  $v_{\text{tms}}$  as state variables for the controller gives the largest improvement compared to the uncontrolled situation ( $J_{\text{TTS}}=901.1 \text{ veh}\cdot\text{h}$ , i.e., 15.6% improvement), followed by the estimated space mean speed  $\hat{v}_{\text{sms},\hat{\sigma}_1}$  ( $J_{\text{TTS}}=916.1 \text{ veh}\cdot\text{h}$ , i.e., 14.2% improvement), and the geometric mean speed  $v_{\text{geo}}$  ( $J_{\text{TTS}}=928.0 \text{ veh}\cdot\text{h}$ , i.e., 13.1% improvement). The average outflow of the network increases with 4.8%, from 4218 to 4422  $\text{veh}/\text{h}$ .

## 7. CONCLUSIONS

The effect of using different variants of calculating mean speeds is investigated. For dynamic speed limit control, using Model Predictive Control (MPC) with METANET as its prediction model, the time mean speed is the best mean speed to use for reducing the Total Time Spent (TTS). The most accurate prediction of the TTS is obtained by using the geometric mean speed.

Furthermore, the reduction of shock waves by applying dynamic speed limits on a motorway is discussed. For a specific case study, namely simulating a part of the A12 motorway in The Netherlands, we have shown that using an MPC approach, the TTS can be reduced significantly. Improvements up to 15.6% compared to the uncontrolled situation are reached. Reducing the shock waves also has a positive effect on the flow, which is increased by 4.8% using the time mean speed.

For statistically significant conclusions, more calibration and simulation runs are necessary. Based on this research it seems that the differences between using time mean speeds and space mean speeds is small, and in practise, the time mean speed can be used in model-based traffic control.

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