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A simplified macroscopic urban traffic network model for model-based predictive control

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Abstract: A model predictive control (MPC) approach offers several advantages for controlling and coordinating urban traffic networks. To apply MPC in large urban traffic networks, a fast model that has a low on-line computational complexity is needed. In this paper, a simplified macroscopic urban traffic network model is proposed and tested. Compared with a previous model, the model reduces the computing time by enlarging its updating time intervals, and preserves the computational accuracy as much as possible. Simulation results show that the simplified model reduces the computing time significantly, compared with the previous model that provided a good trade-off between accuracy and computational complexity. We also illustrate that the simplifications introduced in the simplified model have a limited impact on the accuracy of the simulation results. As a consequence, the simplified model can be used as prediction model for MPC for larger urban traffic network.

Keywords: Macroscopic traffic modeling; Urban traffic control; Model predictive control; Urban traffic network.

1. INTRODUCTION

In recent years, the number of vehicles has grown larger and larger, and the requirements for traveling by vehicles are getting more and more stringent. To reduce traffic jams and to promote efficiency in traveling, effective traffic control algorithms are necessary. Many control theories have been applied to control traffic (Kachroo and Özbay, 1999; Papageorgiou, 1983), like fuzzy control, PID control, model predictive control, and multi-agent control, in combination with different control structures like centralized, distributed, and hierarchical control.

We focus on model-based control methods, and on MPC in particular. Considering on-line computational complexity, macroscopic traffic models are usually used in MPC. However, for different model-based control approaches, there still exist different levels of requirements for the macroscopic model. Some models just need to express the relation between the input values and the performance indicators, but some are more detailed so as to describe the dynamics of the traffic evolution; some models are more precise in modeling the dynamics, while some are simpler so as to be fast for on-line computing. As a result, there exists a wide variety of macroscopic traffic models with different levels of detail. For different control methods, appropriate traffic models with the required modeling power need to be selected.

In the past few years, various macroscopic urban traffic models were developed and used for traffic control. The store-and-forward model, proposed by Gazis and Potts (1963) and later used by Diakaki et al. (2002), is a simple model with low computational complexity, but it can only be used for saturated traffic, i.e., if the vehicle queues resulting from the red phase cannot be dissolved completely at the end of the following green phase. The model proposed by Barisone et al. (2002) and extended by Dotoli et al. (2006) is computationally more intensive and it can describe different scenarios, but it is also more complicated. The model proposed by Kashani and Saridis (1983) has lower modeling power, but can not depict scenarios other than saturated traffic either. The model of van den Berg et al. (2003); Hegyi (2004); van den Berg et al. (2004) is capable of simulating the evolution of traffic dynamics in all traffic scenarios (unsaturated, saturated, and over-saturated traffic conditions) by updating the discrete-time model in small simulation steps. This model provides a good trade-off between accuracy and computational complexity compared with the microscopic model, which is tested and further extended in Lin and Xi (2008) and Lin et al. (2008).

In principle, a centralized MPC method guarantees globally optimal control actions for traffic networks. It can maximize the throughput of the whole network, and provide network-wide coordination of the traffic control measures. However, the problem is that the on-line computational complexity for centralized MPC grows significantly, when the network scale gets larger, the prediction time span gets longer, and the traffic model becomes more complex or gets a higher modeling power. There are two main approaches to address this problem: (1) simplifying the traffic model in order to reduce the computational complexity, and (2) cutting the traffic network into small subnetworks or even intersections, which are then controlled using distributed or multi-agent control. In this paper we consider

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the first approach, i.e., we develop simplified, yet sufficiently accurate, traffic models, in particular, for urban traffic networks.

We start with an urban traffic model based on previous work of Kashani and Saridis (1983); van den Berg et al. (2003); Lin and Xi (2008). To reduce the computational burden, the simplified model enlarges the simulation time interval to one cycle time. During each simulation time interval, traffic states are updated once in each link according to the average input and output traffic flow rates in the current cycle. To add flexibility, every intersection in the network can have a different cycle time, and the intersections share the same control time interval. This control time interval is the least common multiple of all the cycle times of the intersections in the network. It is necessary to define this common control time interval to keep the model running and communicating synchronously under both centralized control and distributed control. For a given link the average input traffic flow rates are provided by the upstream links, which transform their own output traffic flow rates into input flow rates for the given link taking the different simulation time intervals into account.

We will demonstrate with examples that this simplified traffic model reduces the simulation time significantly, compared with the model in van den Berg et al. (2003) and Lin and Xi (2008), with only a limited reduction in accuracy. This makes it possible to apply centralized MPC to larger urban traffic networks.

2. TWO MACROSCOPIC URBAN TRAFFIC NETWORK MODELS

In this section we present the original model of van den Berg et al. (2003) and Lin and Xi (2008) (indicated as the BLX model) as well as a new simplified model (called the S model). But first we introduce some common notation for both models.

Define J the set of nodes (intersections), and L the set of links (roads) in the urban traffic network. Link (u,d) is marked by its upstream node u ($u \in J$) and downstream node d ($d \in J$). The sets of input and output links for link (u,d) are $I_{u,d} \subset L$ and $O_{u,d} \subset L$ (e.g., for the situation of Fig. 1 we have $I_{u,d} = \{i_1, i_2, i_3\}$ and $O_{u,d} = \{o_1, o_2, o_3\}$).

In order to describe the evolution of the models, we first define some variables (see also Fig. 1):

$I_{u,d}$: set of input links of link (u,d),
$O_{u,d}$: set of output links of link (u,d),
k : simulation step counter for the urban traffic model,
$n_{u,d}(k)$: number of vehicles in link (u,d) at step k,
$q_{u,d}(k)$: queue length at step k in link (u,d), q_{u,d,o_m} is the queue length of the sub-stream turning to link o_m,
$m^{\mathrm{l}}_{u,d,o_m}(k)$: number of cars leaving link (u,d) and turning to o_m,
$m^{\mathrm{a}}_{u,d}(k)$: number of cars arriving *at the (end of the) queue* in link (u,d) at step k, $m^{\mathrm{a}}_{u,d,o_m}(k)$ is the number of arriving cars in the sub-stream towards o_m,
$S_{u,d}(k)$: available storage space of link (u,d) at step k expressed in number of vehicles,
$\alpha^{\mathrm{l}}_{u,d}(k)$: flow rate leaving link (u,d) at step k, $\alpha^{\mathrm{l}}_{u,d,o_m}(k)$ is the leaving flow rate of the sub-stream towards o_m,
$\alpha^{\mathrm{a}}_{u,d}(k)$: flow rate arriving at the end of the queue in link (u,d) at step k, $\alpha^{\mathrm{a}}_{u,d,o_m}(k)$ is the arriving flow rate of the sub-stream towards o_m,
$\alpha^{\mathrm{e}}_{u,d}(k)$: flow rate entering link (u,d) at step k,
$\beta_{u,d,o_m}(k)$: relative fraction of the traffic turning to o_m at step k,
$\mu_{u,d}$: saturated flow rate leaving link (u,d),
$g_{u,d,o_m}(k)$: green time length during step k for the traffic stream towards o_m in link (u,d),
$b_{u,d,o_m}(k)$: boolean value indicating whether the traffic signal at intersection d for the traffic stream in link (u,d) turning to o_m is green (1) or red (0) at step k,
$v^{\mathrm{free}}_{u,d}$: free-flow vehicle speed in link (u,d),
$C_{u,d}$: capacity of link (u,d) expressed in number of vehicles,
$N^{\mathrm{lane}}_{u,d}$: number of lanes in link (u,d),
$\Delta c_{u,d}$: offset between node u and node d,
l_{veh} : average vehicle length.

2.1 BLX model

In the BLX model a queue is modeled as follows. For the sake of simplicity, the assumption is made that at an intersection the cars going to the same destination move into the correct lane, so that they do not block the traffic flows going to other destinations. For each lane (or destination), a separate queue is constructed (with queue lengths denoted by q). Furthermore, the simulation time step T_{s} is typically set to 1 s and cars arriving at the end of a queue in simulation period $[kT_{\mathrm{s}}, (k+1)T_{\mathrm{s}})$ are allowed to cross the intersection in that same period (provided that they have green, that there is enough space in the destination link, and that there are no other restrictions).

Consider link (u,d) (see Fig. 1). For each $o_m \in O_{u,d}$ the number of cars leaving link (u,d) for destination o_m in the period $[kT_{\mathrm{s}}, (k+1)T_{\mathrm{s}})$ is given by

$$m^{\mathrm{l}}_{u,d,o_m}(k) = \begin{cases} 0 & \text{if } b_{u,d,o_m}(k) = 0 \\ \max\big(0, \min(q_{u,d,o_m}(k) + m^{\mathrm{a}}_{u,d,o_m}(k), \\ \quad S_{o_m}(k), \beta_{u,d,o_m}(k) \cdot \mu_{u,d} \cdot T_{\mathrm{s}})\big) & \text{if } b_{u,d,o_m}(k) = 1\,. \end{cases}$$

The traffic arriving at the end of the queue in link (u,d) is given by the traffic entering the link via the upstream intersection delayed by the time $\tau(k) \cdot T_{\mathrm{s}} + \gamma(k)$ needed to drive from the upstream intersection to the end of the queue in the link; to this extent $m^{\mathrm{a}}_{u,d}$ is updated as follows:

$$m^{\mathrm{a}}_{u,d}(k) = (1-\gamma(k)) \cdot \sum_{i_m \in I_{u,d}} m^{\mathrm{l}}_{i_m,u,d}(k - \tau(k)) + \gamma(k) \cdot \sum_{i_m \in I_{u,d}} m^{\mathrm{l}}_{i_m,u,d}(k - \tau(k) - 1)\,,$$

where

$$\tau(k) = \mathrm{floor}\left\{ \frac{\big(C_{u,d} - q_{u,d}(k)\big) \cdot l_{\mathrm{veh}}}{N^{\mathrm{lane}}_{u,d} \cdot v^{\mathrm{free}}_{u,d} \cdot T_{\mathrm{s}}} \right\},$$

$$\gamma(k) = \mathrm{rem}\left\{ \frac{\big(C_{u,d} - q_{u,d}(k)\big) \cdot l_{\mathrm{veh}}}{N^{\mathrm{lane}}_{u,d} \cdot v^{\mathrm{free}}_{u,d} \cdot T_{\mathrm{s}}} \right\},$$

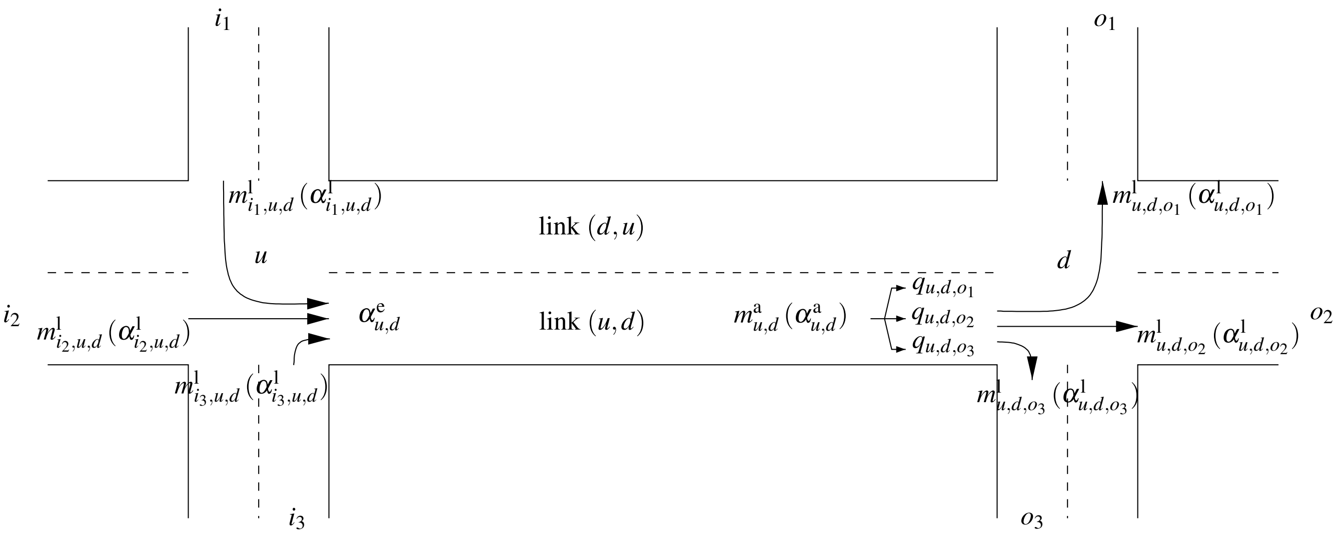

Fig. 1. A link connecting two traffic-signal-controlled intersections

with floor(x) referring to the largest integer smaller than or equal to x, and rem(x) is the remainder. The fraction of the arriving traffic in link (u,d) turning to $o_m \in O_{u,d}$ is

$$m^{\mathrm{a}}_{u,d,o_m}(k) = \beta_{u,d,o_m}(k)\cdot m^{\mathrm{a}}_{u,d}(k) \ .$$

The new queue lengths are given by the old queue lengths plus the arriving traffic minus the leaving traffic

$$q_{u,d,o_m}(k+1) = q_{u,d,o_m}(k) + m^{\mathrm{a}}_{u,d,o_m}(k) - m^{\mathrm{l}}_{u,d,o_m}(k)$$

for each $o_m \in O_{u,d}$, and

$$q_{u,d}(k) = \sum_{o_m\in O_{u,d}} q_{u,d,o_m}(k) \ .$$

The new available storage stage depends on the number of cars that enter and leave the link in the period $[kT_{\mathrm{s}},(k+1)T_{\mathrm{s}})$:

$$S_{u,d}(k+1) = S_{u,d}(k) - \sum_{i_m\in I_{u,d}} m^{\mathrm{l}}_{i_m,u,d}(k) + \sum_{o_m\in O_{u,d}} m^{\mathrm{l}}_{u,d,o_m}(k) \ .$$

2.2 Simplified Model (S Model)

In the simplified model, every intersection takes the cycle time as its simulation time interval. The cycle times for intersection u and d, which are denoted by c_u and c_d respectively, can be different from each other, as Fig. 2 illustrates. Moreover, the S model works with (average) flow rates rather than with number of cars for describing flows leaving or entering links.

Taking the cycle time c_d as the length of the simulation time interval for link (u,d) and k_d as the corresponding time step counter, the number of the vehicles in link (u,d) is updated according to the input and output average flow rate over c_d at every time step k_d by

$$n_{u,d}(k_d+1) = n_{u,d}(k_d) + \Big(\alpha^{\mathrm{e}}_{u,d}(k_d) - \alpha^{\mathrm{l}}_{u,d}(k_d)\Big)\cdot c_d \ . \qquad (1)$$

The leaving average flow rate is the sum of the leaving flow rates turning to each output link:

$$\alpha^{\mathrm{l}}_{u,d}(k_d) = \sum_{o_m\in O_{u,d}} \alpha^{\mathrm{l}}_{u,d,o_m}(k_d), \quad o_m \in O_{u,d} \ . \qquad (2)$$

The leaving average flow rate over c_d is determined by the capacity of the intersection, the number of cars waiting and/or arriving, and the available space in the downstream link:

$$\begin{aligned}\alpha^{\mathrm{l}}_{u,d,o_m}(k_d) = \min\Big(&\beta_{u,d,o_m}(k_d)\cdot\mu_{u,d}\cdot g_{u,d,o_m}(k_d)/c_d,\\ &q_{u,d,o_m}(k_d)/c_d + \alpha^{\mathrm{a}}_{u,d,o_m}(k_d),\\ &\beta_{u,d,o_m}(k_d)\left(C_{o_m} - n_{o_m}(k_d)\right)/c_d\Big) \ .\end{aligned} \qquad (3)$$

The number of vehicles waiting in the queue turning to link o_m is updated as

$$\begin{aligned} q_{u,d,o_m}(k_d+1) = q_{u,d,o_m}(k_d) + \\ \Big(\alpha^{\mathrm{a}}_{u,d,o_m}(k_d) - \alpha^{\mathrm{l}}_{u,d,o_m}(k_d)\Big)\cdot c_d \ .\end{aligned} \qquad (4)$$

Then, the number of waiting vehicles in link (u,d) is

$$q_{u,d}(k_d) = \sum_{o_m\in O_{u,d}} q_{u,d,o_m}(k_d) \ . \qquad (5)$$

The flow rate entered link (u,d) will arrive at the end of the queues after a time delay $\tau(k_d)\cdot c_d + \gamma(k_d)$, i.e.,

$$\begin{aligned}\alpha^{\mathrm{a}}_{u,d}(k_d) = (1-\gamma(k_d))\cdot\alpha^{\mathrm{e}}_{u,d}\,(k_d - \tau(k_d)) + \\ \gamma(k_d)\cdot\alpha^{\mathrm{e}}_{u,d}\,(k_d-\tau(k_d)-1)\,,\end{aligned} \qquad (6)$$

$$\begin{aligned}\tau(k_d) &= \mathrm{floor}\left\{\frac{\left(C_{u,d}-q_{u,d}(k_d)\right)\cdot l_{\mathrm{veh}}}{N^{\mathrm{lane}}_{u,d}\cdot v^{\mathrm{free}}_{u,d}\cdot c_d}\right\},\\ \gamma(k_d) &= \mathrm{rem}\left\{\frac{\left(C_{u,d}-q_{u,d}(k_d)\right)\cdot l_{\mathrm{veh}}}{N^{\mathrm{lane}}_{u,d}\cdot v^{\mathrm{free}}_{u,d}\cdot c_d}\right\}.\end{aligned} \qquad (7)$$

Before reaching the tail of the waiting queues in link (u,d), the flow rate of arriving vehicles need be divided by multiplying the turning rates:

$$\alpha^{\mathrm{a}}_{u,d,o_m}(k_d) = \beta_{u,d,o_m}(k_d)\cdot\alpha^{\mathrm{a}}_{u,d}(k_d). \qquad (8)$$

The flow rate entering link (u,d) is made up from the flow rates from all the input links:

$$\alpha^{\mathrm{e}}_{u,d}(k_d) = \sum_{i_m\in I_{u,d}} \alpha^{\mathrm{l}}_{i_m,u,d}(k_d). \qquad (9)$$

In this formula, we see that the flow rate entering link (u,d) is provided by the combination of the flow rates leaving the upstream links. Recall that we have different cycle times between

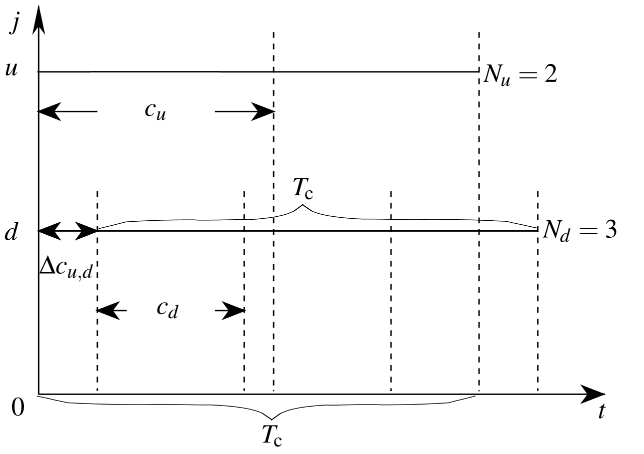

Fig. 2. Relationship between cycle times and control time interval

the upstream and downstream intersections, so the simulation time steps are not the same. Some operations need to be carried out to synchronize the leaving and entering flow rates.

In order to control the urban traffic network, a common control time interval need to be defined for the network model, so that intersections can communicate with each other and be synchronous.

$$T_c = N_j \cdot c_j, \quad \text{for } j \in J \tag{10}$$

with N_j an integer.

So T_c is the least common multiple of all the intersection cycle times in the traffic network. As Fig. 2 shows, we have

$$T_c = N_u \cdot c_u = N_d \cdot c_d. \tag{11}$$

For a given k_c the simulation time step counters for both intersections can range as follows:

$$\begin{array}{ll} k_u = N_u \cdot k_c + p_u, & p_u = 0, 1, \ldots, N_u - 1 \\ k_d = N_d \cdot k_c + p_d, & p_d = 0, 1, \ldots, N_d - 1. \end{array} \tag{12}$$

Now we show how the flow rates expressed in the timing of intersection u can be recast into the timing of intersection d. First, we smooth the leaving flow rates from the upstream links as

$$\alpha^{l}_{i_m,u,d}(t) = \alpha^{l}_{i_m,u,d}(k_u), \quad k_u \cdot c_u \le t < (k_u + 1) \cdot c_u, \tag{13}$$

and then sample them again to obtain the average flow rates in time step k_d so as to be able used by the downstream link, as Fig. 3 shows:

$$\alpha^{l}_{i_m,u,d}(k_d) = \frac{\displaystyle\int_{k_d \cdot c_d + \Delta c_{u,d}}^{(k_d+1)\cdot c_d + \Delta c_{u,d}} \alpha^{l}_{i_m,u,d}(t)}{c_d} \, \mathrm{d}t \; . \tag{14}$$

3. SIMULATION EXPERIMENTS

In centralized MPC, a fast running traffic network model is needed to satisfy the on-line optimization requirements. So, simulations are designed and carried out to verify whether the new simplified model (S model) can save time compared with the more detailed model (BLX model) while retaining a sufficiently high level of accuracy. The two models are compared for different network input flow rates, different prediction horizons, and different traffic network scales. During the experiment, the simulation time interval of the BLX model is set to 1 s, while the simulation time intervals of the S model are cycle times

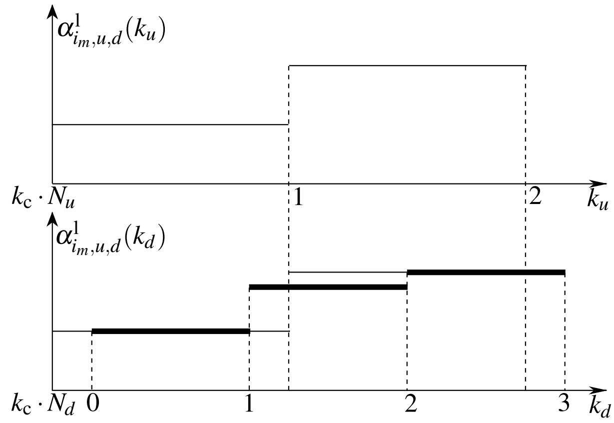

Fig. 3. Illustration for synchronizing flow rates

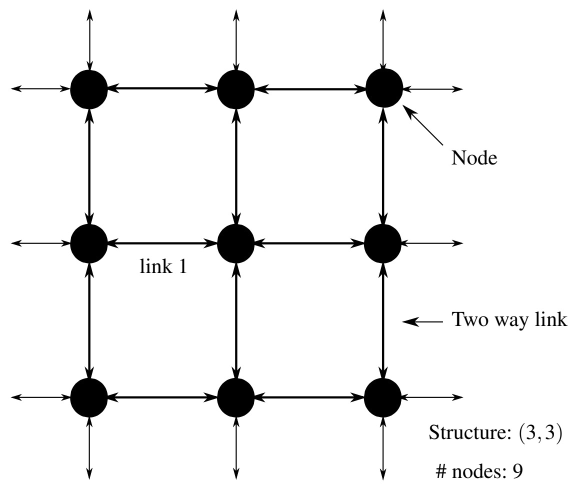

Fig. 4. The layout of a urban traffic network

which are 120 s, the same for all intersections in the network. The prediction horizons and traffic network scales are listed in Table 1.

Table 1. Traffic network characteristics and prediction horizon for each of the 5 simulation cases

Case number		1	2	3	4	5
Network	Structure	(1,2)	(3,3)	(8,8)	(13,13)	(18,18)
	# nodes	2	9	64	169	324
N_p		5	10	20	30	40

Each network considered is a grid-like network, where the "Structure" of the network is expressed as the number of nodes in each row and each column, and "# nodes" indicates the number of nodes. For example, Fig. 4 shows the layout of a $(3,3)$ network containing 9 nodes. "N_p" is the number of the control time intervals the model will run (i.e., simulation or prediction horizon expressed in steps of length T_c).

When using network 3, and $N_p = 10$, the computing times of the two models under different network input flow rates are shown in Fig. 5. The figure shows that the computing times are almost independent of the network input flow rates for both models. This means the traffic scenarios almost do not have any influence on the running time. Moreover, we can see from the figure that the S model required a much shorter computation time, around 0.5 s, while the BLX model took about 7 s, which is 14 times longer.

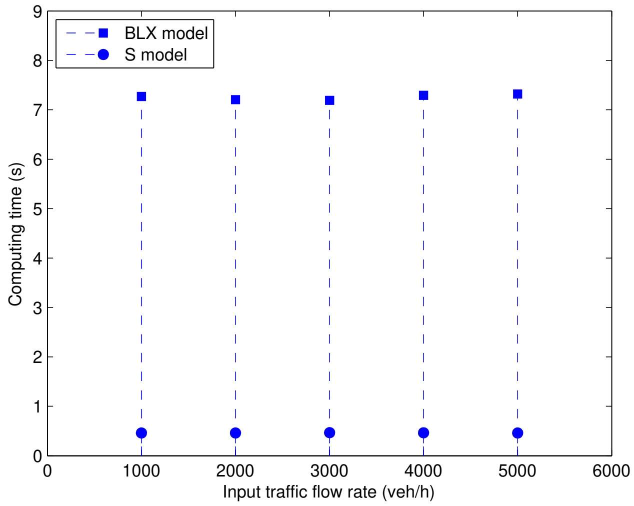

Fig. 5. The computing time consumed for different input flow rates of the traffic network

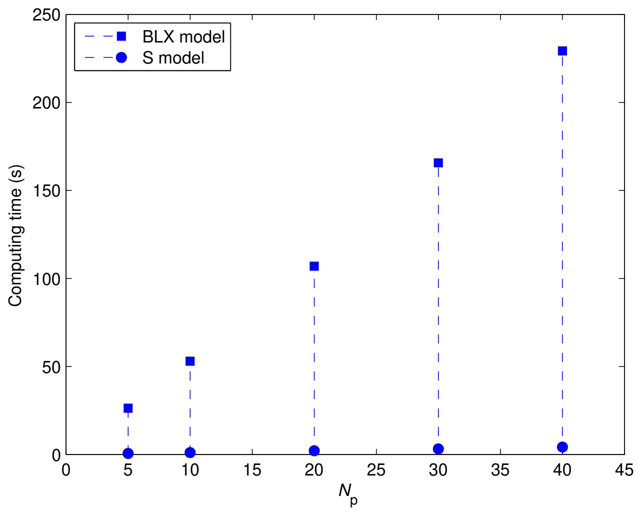

Fig. 6. The computing time consumed for different prediction horizons

In each step of MPC for traffic control, a numerical optimization problem needs to be solved to obtain the optimal input value for the next step (using, e.g., a multi-start Sequence Quadratic Programming (SQP) algorithm). During the optimization, the model may need run hundreds to thousands of times. Therefore, by decreasing the computing time of the model, the on-line optimization time in MPC can be dramatically reduced.

Fig. 6 shows the changing of the running time with N_p, when the traffic network is set to network 5. Fig. 7 shows the changing of the running time with network scale, when $N_p = 40$. From the two figures, we can see that the longer the model is predicting, the larger scale the network is set to, the more time that the S model will save. The same conclusions can also be drawn from Fig. 8.

The S model is much faster than the BLX model, especially for longer prediction horizons N_p and larger network scales, but this extra speed is obtained by ignoring some details when modeling. Therefore, we need to verify whether the S model can still satisfy the requirements of control. The number of leaving vehicles can reflect the control effect of traffic lights

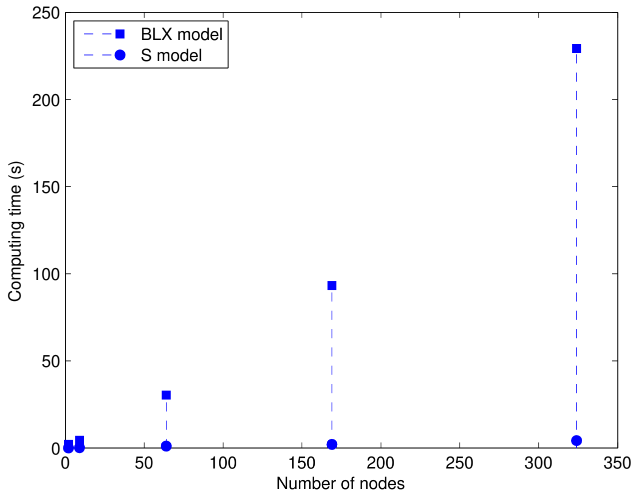

Fig. 7. The computing time consumed for different traffic network scales

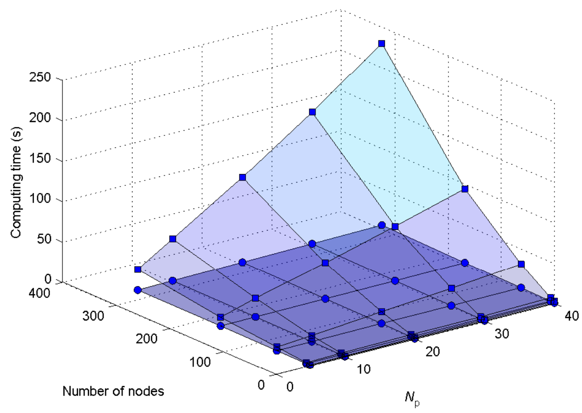

Fig. 8. The computing time consumed for both different prediction horizons and different traffic network scales

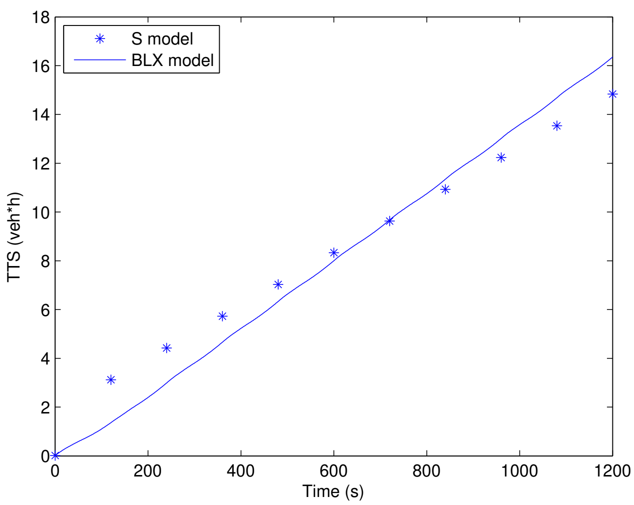

Fig. 9. The TTS for two models

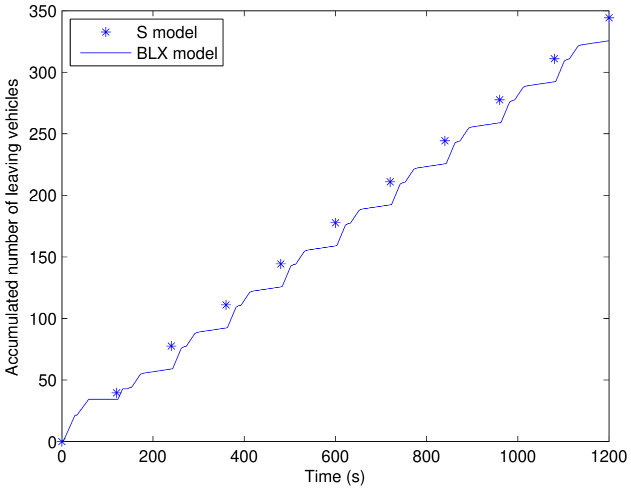

Fig. 10. The accumulated number of leaving vehicles for two models

on urban traffic, and Total Time Spent (TTS) is usually used as the control performance. If the S model shows behavior that is similar to that of the BLX model for these two indexes, then it can be used as urban traffic control model guaranteeing similar control effects but with less control efforts. Fig. 9 and 10 are drawn for link 1 of network 2 (see Fig. 4), and $N_\mathrm{p} = 10$. The figures show that the simplified model is accurate enough as a control model for urban traffic network.

4. CONCLUSIONS

A simplified macroscopic model has been established for controlling urban traffic network using model predictive control (MPC). This model takes the cycle times of the intersections as simulation time steps, where every intersection can have a different simulation time step. A control time interval, which is the least common multiply of all the cycle times, is defined to guarantee the communication and synchronization in the urban traffic network. The simplified model also describes how to ensure communication and synchronization between intersections with different simulation time steps.

The simplified model can take all typical traffic scenarios (saturated, unsaturated, and over-saturated traffic) into consideration, and is more flexible by having different cycle times. Moreover, it significantly reduces the computing time, which make it possible to be used for controlling larger urban traffic network.

However, the increasing of computing speed is obtained by enlarging the simulation time interval, which makes it lose some details and sacrifice some accuracy at the same time. But simulation results show that it guarantees enough accuracy to be used as the control model for urban traffic network.

Further research will focus on developing MPC algorithm to control urban traffic network based on this model, as well as an extensive assessment and comparison of the simplified model with a wide range of other traffic models for various network layouts and traffic demands when used for MPC-based traffic control.

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