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# Considerations for Model-Based Traffic Control

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## Abstract

The use of traffic control systems can potentially improve the traffic flows on traffic networks. However, for the implementation of such control systems —both in simulation and in practice— many steps should be taken, and many choices are to be made. In this paper a list of considerations is provided for developing model-based traffic control systems in general, with a more detailed discussion on the use of model-predictive control for traffic regulation. A case study of designing a traffic controller is provided for the Dutch A12 freeway.

*Keywords:* Traffic control, model-based control.

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## 1. Introduction

Due to increasing demands, current road networks often reach their limits in capacity, or the available capacity is not used efficiently. This can lead to congestion, which can lead to travel delays, noise nuisance, pollution, and dangerous traffic situations. Traffic control<sup>1</sup> measures have been developed to reduce the corresponding problems, such as variable speed limits (Smulders, 1990), on-ramp metering (Papageorgiou and Kotsialos, 2002), and route guidance (Messmer and Papageorgiou, 1994). These control measures influence the traffic in such a way that the existing road capacity is used more efficiently, thereby improving the throughput of the network. However, the performance of these control strategies largely depends on the choices made during the design process that precedes the implementation process.

The focus of this paper is on providing points of interest when designing and implementing a model-based traffic control system in general, with some more details on the use of model-predictive control for traffic regulation. As such it can be used as a guideline for more experienced traffic engineers and researchers, and it can be used as an introductory to traffic control for people who are new to the topic.

### 1.1. Considerations for model-based traffic control

In this paper some considerations for developing traffic controllers will be given by discussing several points of interest. A specific type of advanced controllers are model-based controllers (Brosilow and Joseph, 2002), where the known dynamics of the system and its response

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<sup>1</sup>The term “traffic controller” is used in a general sense, not only for model-based traffic control.

to actuation is taken into account in determining control actions. In the field of traffic control, some established systems using model-based control methods are e.g. UTOPIA (Peek Traffic, 2002), MITROP (Gartner et al., 1976), OPAC (Gartner et al., 2001), PRODYN (Henry et al., 1983), and RHODES (Mirchandani and Wang, 2005). We discuss model-based traffic controllers in general, which require consideration of many practical issues, partly due to the inherent differences between the model and the real world.

In this paper we provide an example using a specific form of model-based control, namely model predictive control (MPC) (Maciejowski, 2002; Rawlings and Mayne, 2009). Traffic controllers that are based on MPC use a model to predict the future evolution of the traffic flows. Based on this prediction, the controller determines dynamic settings for the traffic control measures. Advantages of this control approach are that different control measures can be integrated into one control system, and that the prediction allows for the investigation of the longer-term effects of the control actions and thus allows for the selection of dynamic control settings that are optimal over a longer period. Disadvantages of MPC are the often large computation times this method requires, possibly complex code that is needed to compute the optimal control values, and the problems it may cause for a network operator due to unfamiliarity with MPC.

The practical issues related to the deployment of traffic controllers in general—and model-based control using MPC in specific—are divided into two main classes: design issues and implementation issues. Design issues are often related to the policy of the road authority: which traffic flows should be controlled, which part of the traffic network is considered, which measurements can be obtained, what should be the objectives of the controller, what are the constraints within the system? Design issues related to technical details of model and controller choice are: what level of detail should the model provide, which modeling methods can be used to satisfy the desired specifications, which control methods are suitable for the problem at hand? For model-based traffic control, depending on the required level of detail versus requirements on computational efficiency, a choice should be made between using microscopic or macroscopic traffic flow models (Daganzo, 1997). Microscopic traffic flow models use details of individual drivers, and provide much detail but are computationally expensive. Macroscopic traffic flow models aggregate the details of individual drivers, thereby providing less detail but using lower computation times. Using all this information, a model of the traffic network can be made, and an appropriate control method can be chosen.

Once the design of the model-based controller is done, some implementation issues should be considered before the controller can be applied to a real traffic situation. The selected model should be calibrated and validated, a state estimation method should be selected, the expected demand must be estimated, and the controller must be tuned. The steps should first be taken in simulation (to avoid problems in the real network due to, e.g., errors in the model or badly tuned controllers) until the performance is satisfying, and they should be repeated for implementation on the real network, as differences between the model and the real system will most likely exist.

Many of the issues mentioned above strongly depend on the measurements that are available. A measurement structure should be designed, which includes selecting e.g. detectors, communication networks, data polishing<sup>2</sup> methods, and data handling methods. We will briefly discuss different types of detectors, after which we focus on speed measurements with loop detectors.

Model-based traffic controllers typically use mean speeds, hence when speed measurements of individual vehicles for a certain time period are available the measured speeds should be

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<sup>2</sup>Data polishing detrends the data, and removes outliers of the measured data.

averaged. Several methods exist for averaging measurements; for traffic two well-known types are the time-mean speed (obtained as the arithmetic mean of the measured speeds) and the space-mean speed (approximated by the harmonic mean of the measured speeds) (Daganzo, 1997).

As a case study we go through the process of developing a variable speed limit controller for the A12 freeway in the Netherlands. During the design of this controller the design issues and implementation issues are considered as far as they are useful for the discussed simulation study. The objective of the developed controller is to reduce traffic jams in the network, using variable speed limits as actuators.

### *1.2. Outline of the paper*

The remainder of this paper is organized as follows. Section 2 explains model-based control and describes the general process of controller development. Controller design issues are discussed in Section 3, followed by implementation issues in Section 4. In Section 5 a case study is provided for developing a traffic controller for a stretch of the Dutch freeway A12. Finally, conclusions are drawn in Section 6.

## **2. Model-based traffic control**

This section starts with a general description of model-based traffic control, where a general feedback control scheme is presented. Next a more detailed explanation of model predictive control (MPC) is given, which is a specific type of model-based control that is often used for traffic control. At the end of this section an overview of the procedure of developing a model-based traffic controller is given.

### *2.1. General scheme for model-based traffic control*

A general scheme for feedback control methods will be introduced first, as shown in Figure 1. In feedback control, measurements are taken from the process (the system under consideration) to determine the state of the system. Using the system state, a controller determines the control actions that need to be applied to the system in order to force the system towards some desired situation. The use of measurements for determining the control actions is referred to as closed-loop control. For traffic control purposes, the process consists of the traffic network and the traffic flows that should be controlled. The traffic flows propagate through the network depending on the traffic scenario and the control actions, which leads to a specific performance of the network. To obtain information that could be used to determine the performance, the current state of the network should be measured, or estimated based on measurements. These measurements can be performed using e.g. radar detectors, loop detectors, and cameras. The measured quantities can e.g. be flows, occupancies, and speeds. These measured values are fed into the controller, which determines the control signal based on these measurements and the desired performance of the network that is described by the objectives and constraints of the controller. The control actions consist of the dynamic settings for the traffic measures, such as ramp metering rates, speed limit values, or timings for traffic signals. These measures influence the process, and thus influence the performance. In this way, the controller is used to improve the network performance.

*Place Figure 1 about here*

How the controller determines the control signal depends on the type of controller; model-based controllers use an internal model of the traffic. For urban areas the control inputs for the traffic signals can be determined using queue length models, which is investigated in e.g. (Robertson and Bretherton, 1991; Wolson and Taylor, 1999; Peek Traffic, 2002). For freeways there are model-based controllers using variable speed limits, ramp metering installations, and peak lanes, as described in e.g. (Kotsialos et al., 2002; Hegyi et al., 2005a). Dynamic route guidance can be used for model-based route choice control, as described in e.g. (Jayakrishnan et al., 1994; Bellemans et al., 2003; Deflorio, 2003; Karimi et al., 2004).

Depending on the model of the process, different control methods can be used to obtain control actions. For linear models many classical control methods can be used, such as PID control or pole placement (Franklin et al., 2002). But when the dynamics of the traffic network cannot be captured by a linear model efficiently, the use of non-linear models and associated control strategies should be considered. Feedback linearization can be used to (partly) regain a linear model, but methods as optimal control, adaptive control, fuzzy control, or sliding mode control could be chosen for the control strategy as well (Khalil, 2002). When choosing a control method some issues that also should be taken into account are the robustness against model-mismatch, the required operator skills for the resulting controller, and the operator acceptance for working with the controller.

In Section 5 an example of model-based traffic control is given. There we will use model predictive control as an example of a model-based control method, for which we provide an introduction with a focus on traffic control next.

## 2.2. Model predictive control

This section considers a specific model-based control method called model predictive control (MPC) (Maciejowski, 2002; Rawlings and Mayne, 2009). MPC-based traffic controllers use a model to predict the evolution of the traffic flows, which are used to determine the optimal control signals based on the current state of the system. As such, MPC is a specific variant of optimal control. MPC has initially been developed for the process industry; the first applications for traffic control are described in (Gartner, 1984). Traffic controllers that explicitly use MPC are proposed in (Bellemans, 2003; Hegyi et al., 2005a), and other controllers that use similar schemes are presented in e.g. (Robertson and Bretherton, 1991; Peek Traffic, 2002; Diakaki et al., 2002; Wang et al., 2003). The latter controllers also use models and predictions to obtain the control settings, but they are not explicitly formulated corresponding to the MPC structure that will be described next.

When MPC is used, the controller in Figure 2 contains a state estimation algorithm, a prediction model, and an optimization algorithm. The measurements from the real network are used to obtain the estimated state of the network. These measurements are not fed to the controller continuously, but at certain time-instances referred to as *control steps*. Between consecutive steps there is a fixed amount of time referred to as *control periods*. Furthermore, *horizons* consist of a time period defined by the number of steps that are considered.

At a certain control step not only the desired control action for the upcoming period is determined, but several control actions over a *control horizon* of  $N_c$  control steps is determined based on the prediction of the system dynamics (under influence of the control actions) provided by the prediction model.

*Place Figure 2 about here*

Using the measurement information and the traffic model, the controller predicts the evolution of the traffic flows over a *prediction horizon* that has a length of  $N_p$  control steps, as illustrated in Figure 2. A larger prediction horizon will result in more detailed information for the controller, but this increase in possible solutions (due to the increase of control inputs  $u$  that need to be determined) can make the controller slow. Therefore, often a *control horizon* of  $N_c$  control steps is used to reduce the complexity. During the period from control step  $k$  to  $k + N_c$  the control signals can vary, while during the remainder of the prediction horizon the control signals are kept constant. The optimal values for the control signals during the current control step are applied to the real network, using the control signals of the current control step  $k$  only. At the next control step the procedure is started again, with the horizon shifted one step into the future. This is called the rolling horizon approach.

The use of a control horizon leads to a reduction of the number of optimization variables as compared to only using a prediction horizon (of the same size). This results in a decrease of the computational burden, a smoother controller signal (because of the emphasis on the average behavior rather than on aggressive noise reduction), and a stabilizing effect (since the output signal is forced to its steady-state value). For more information on MPC, we refer the interested reader to (Camacho and Bordons, 1995; Maciejowski, 2002; Rawlings and Mayne, 2009).

### 2.3. Controller development

The process of developing a model-based controller is a combination of design issues and implementation issues. Figure 3 presents an overview of the required steps. The process starts with the design issues, consisting of policy issues and technical issues. The policy issues consider the objectives and constraints of the controllers, the selection of the network, and the design of the measurement structure. If not all desired measures can be taken, these values can often be estimated. In this case a procedure should be developed to estimate the state of the traffic flows in the network, together with the demands.

When the design issues have been considered, the general design of the controller is available. Next the technical steps considering the selection of the control method and the model are discussed. Some guidelines for how the steps of the design process can be applied to real situations can be found in (Ministry of Transport, Public Works and Water Management, 2003).

*Place Figure 3 about here*

The model should be calibrated and validated, meaning that values for the parameters in the model should be determined based on measurements and tested for correctness using another set of measurements. When some of the desired state variables cannot be measured directly, the use of state estimation can provide the missing data. If the demands are not known beforehand, the use of demand estimators can improve the accuracy of the system by providing additional information. With these issues settled, the controller can be applied in a simulation environment to investigate the effects of policy/economic choices and the effects of choices regarding e.g. the number of measurements, control measures, and objectives. The simulation environment can also be used to tune the controller. If problems are encountered, parts of the design process should be reviewed again. In general, multiple iterations will be necessary before the main problems are solved. When the simulation gives good results, the controller is ready to be implemented in the real network. Once the controller is implemented, its performance can be evaluated by comparing measurements of the controlled situation with measurements of the uncontrolled situation, and with the results of the simulation experiments. Based on the performance, recalibration and

validation of the prediction model will be necessary to improve the performance of the controlled process until satisfying results are obtained.

### 3. Design issues

In this section the design issues that have been introduced in Section 2.3 (see also Figure 3) will be discussed in more detail.

#### 3.1. Formulation of the objectives and constraints

The objectives of a controller describe the goals that the controller should try to reach, and they have to be determined by the road authorities. Selecting the objectives is often a trade-off between the interests of different parties (e.g. drivers, road managers, pedestrians, environmentalists). Accordingly, controller objectives and constraints can focus on different topics:

**Efficiency:** The available road capacity should be used as efficient as possible. Possible network-wide objectives in this context are: reducing the total time spent, reducing the total travel time, increasing the throughput, and reducing delays.

**Safety:** Traffic controllers can improve safety by e.g. reducing speeds, creating homogeneous flows, increasing intersection clearance times, and reducing flows in residential areas.

**Environment:** Traffic generates noise and air pollution, and it consumes fuel. The environment benefits from e.g. reducing the number of stops, smoothening the flows, and decreasing the waiting time in the queues.

**Location:** When the traffic demand is so large that congestion cannot be prevented, the controller can try to put the congestion at a specific location where it causes the least problems, which can improve the situation in e.g. residential areas and nature reserves.

**Priority:** Some specific road users might be prioritized, such as trucks in industrial area's, public transport, or long distance traffic over short distance traffic. Also, some roads might be prioritized, e.g. to make sure the traffic on a ring road keeps flowing, one could assign a high priority to the traffic on the ring road.

In general, the overall objective of the controller will be formulated as a multi-objective criterion given as

$$J = \sum_{i=1}^{N_0} w_i J_i, \quad w_i \geq 0, \quad (1)$$

where  $J_i$  denotes one of the  $N_0$  objectives as mentioned above, and the scalars  $w_i$  are used to set the relative weight between the objectives, taking into account average values of the objectives (to avoid the dominance of objectives that return large values) and relative importance between the objectives. It is possible to add penalties related to the expected behavior of the controller, which can target e.g. variations in the control signal, and the traffic situation at the end of the prediction period. The use of multiple objectives in a controller results in a multi-objective optimization problem. Some methods to handle this kind of problems are: the weighted-sum method, the  $\epsilon$ -constraint method, and the goal attainment method (Miettinen, 1999).

Another way to implement the requirements resulting from traffic policies, such as service levels, protection/safety of traffic participants, safety around schools, etc., is to formulate them

as constraints for the optimization problem. This results in e.g. maximum or minimum values for travel times, flows, speeds, intersection clearance times, or queue lengths. It is also possible to formulate physical constraints for the controller that consider the limitations of the control measures and can result in e.g. minimum or maximum values of the control signal.

### *3.2. Selection of the network*

The decision to develop a traffic controller is often induced by a traffic network in which a problem occurs. However, the extent of the network that should be controlled in order to reduce the problems is not always evident. Some problems can be solved within a small network, while others require a larger area to be solved efficiently. The extent of the required network can depend on e.g. the ratio between local traffic and long distance traffic, the available measurements and their locations, the available traffic control measures, and the area on which the effects of the control measures appear. Note that a trade-off has to be made between choosing the size of the network (in traffic states and control possibilities rather than physical dimensions) and the possible control methods, in order to provide appropriate control signals within reasonable times. For some guidelines on selecting the network size, available literature in the area of hierarchical control can be used (see e.g. (Huang and Hsieh, 1995)), where large systems are divided into subsystems based on the influence that parts of the system have on each other.

### *3.3. Design of the measurement structure*

Once the network has been selected, the measurement structure should be designed. This includes selecting the measurement technology, the locations where the measurements are taken, the communication structure, the storage database, and the method for data polishing.

Existing measurement technologies for traffic networks include e.g. pneumatic sensors, radar detectors, infra-red sensors, video cameras, or inductive loops (Klein et al., 2006). The latter are the most commonly used sensors, and consist of inductive loops in the pavement that detect the presence of a vehicle, and thereby the occupancy<sup>3</sup>. They count the number of passing vehicles and average this over a time span (typically between 1 and 15 minutes). By using double loops the speed of each vehicle can be determined. Also pneumatic sensors located on the road can detect the presence of a vehicle. They are cheap but they are wearing fast, which is why they are mainly used for temporary measurements only. Radar detection determines the presence and speed of vehicles via radar waves. These detectors are mainly used to determine the speed of vehicles. Infra-red detectors determine the presence of a vehicle using infra-red light. There are passive sensors which measure the radiation of the vehicles, and active sensors that send out a pulse and determine whether there is a vehicle based on the reflection of this pulse. Video images can be used to measure the traffic flows as well. The advantages of video imaging are that many different measurements can be obtained, e.g. space mean speeds, density, vehicle positions, and vehicle types. The disadvantages are the sensitivity to rain, mist, or snow, and the relatively high costs of maintenance.

Furthermore, for new infrastructure the locations of the detectors should be determined<sup>4</sup>. On freeways, detectors are often placed at around 500 meters distance from each other, and near bottlenecks such as on-ramps, off-ramps, lane-drops, and weaving areas. In urban areas, queue

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<sup>3</sup>The occupancy is the percentage of time that the detector is occupied, which is representative for the density at the location of the detector.

<sup>4</sup>For existing infrastructure one can use the available detectors, and possibly extend it using new ones.

length detectors can be located at controlled intersections, and at the beginning and end of each link the number of entering and leaving vehicles can be measured respectively.

Then, the communication structure should be selected. The detectors and controllers can exchange data with their neighbors, or they can communicate with a central controller. The obtained data should be stored in a database. The structure of the database and the desired contents should be determined.

Finally, before the measured data can be used it should be polished (Ljung, 1999). Methods should be developed to remove outliers and sensor failures, and to address the uncertainty of the obtained measurements. For use in macroscopic models the data should be averaged to obtain mean values.

#### *3.4. Selection of the control method*

To determine the settings for the control measures, a control method should be selected. In the area of freeway traffic control there exist methods that use no models, of which ALINEA (Papageorgiou et al., 1991) —a ramp-metering strategy using classical feedback regulators— is the most well known. Furthermore, there are methods based on fuzzy learning or neural networks (Chen et al., 1990; Palacharla and Nelson, 1999; Yin et al., 2000), that do not need a model of the system beforehand, but they develop a model while being employed. Examples of model-based methods are presented in e.g. (Hu and Mahmassani, 1997; Busch and Kruse, 2001; Diakaki et al., 2002; Peek Traffic, 2002; Kotsialos et al., 2002; Hegyi et al., 2005a).

When the model-based approaches are used for optimal control (hence also for MPC), an optimization algorithm should be selected to determine the optimal values for the control measures. Which algorithm to selected depends on the type of optimization problem, which in turn depends on the selected model, the objective function, and the constraints. For convex problems many algorithms are available that will yield the global optimum (Pardalos and Resende, 2002; Bemporad and Morari, 1999; Pierre, 1986). However, a traffic control problem is nearly always non-convex (and hence non-linear), and therefore it can have many local optima. In that case a global optimization method can be used, such as genetic algorithms, simulated annealing, pattern search, or multi-start local optimization (Pardalos and Resende, 2002; Davis, 1991; Eglese, 1990; Glover and Laguna, 1997; Boggs and Tolle, 1995). These algorithms cannot guarantee that the global optimum is obtained, but they usually can obtain acceptable (sub-optimal) values. The use of these algorithms however increases the computation time, which is undesired for on-line computations. The selection of an optimization algorithm is thus based on the trade-off between the accuracy of the solution and the required computational effort. Some examples of optimization toolboxes are NAG (2013), OSL (1992), MINPACK (1980), MINOS (2008), and LANCELOT (1992).

#### *3.5. Selection of the traffic model*

Several models are available for use in model-based traffic controllers. An overview of traffic flow modeling in general is given by e.g. Daganzo (1997); Hoogendoorn and Bovy (2001b); Papageorgiou et al. (2007). Traffic models can be divided into categories based on the properties of the models. First the modeled application can be used as categorization criterion, e.g. traffic flow models (Messmer and Papageorgiou, 1990; Daganzo, 1994; Helbing et al., 2002; Ngoduy, 2006), travel time models (Carey and Ge, 2003; van Lint, 2004), and traffic assignment models (Peeta and Mahmassani, 1995; Bliemer, 2000; Florian et al., 2001). Second, the models can be stochastic (Cascetta, 1989; Maher, 1998), or deterministic (Messmer and Papageorgiou, 1990;

Bliemer, 2000). Third, the models can be grouped based on the level of detail. Three categories that can be distinguished are:

**Microscopic** models describe the behavior of individual vehicles in relation to the other vehicles and the infrastructure. Examples of commercially available models are Paramics (Quadstone, 2002), Vissim (PTV, 2003), and Aimsun (Barceló and Ferrer, 1997), while an overview of more theoretical models is given in (Hoogendoorn and Bovy, 2001b; Papageorgiou et al., 2007).

**Mesosopic** models describe the traffic in probabilistic terms, using probability distribution functions, see e.g. the gas-kinetic model of (Hoogendoorn and Bovy, 2001a). Some mesoscopic models use a mix between detailed descriptions of important properties and a more general overall formulation, see e.g. (Jayakrishnan et al., 1994; Celikoglu and DelÓrco, 2007).

**Macroscopic** models describe the traffic flows using aggregated values, e.g. average speeds, and average densities. Early macroscopic models are formulated in (Lighthill and Whitham, 1955; Richards, 1956). More recent models are METANET (Messmer and Papageorgiou, 1990), INDY (Bliemer, 2000), and the Cell Transmission Model (Daganzo, 1994). An overview of macroscopic models is presented in (Papageorgiou, 1998).

When a model should be selected for a model-based controller, attention should be paid to the features that are modeled. All features that are important for the controller should be modeled, including e.g. traffic flows, influence of control actions, and properties affecting the objective of the controller. Further, the required computational effort should be taken into account. For a controller, limited time is available for simulation of the model, since new control values are expected within a certain time. High accuracy and low computation times are conflicting goals, which makes a trade-off between the two criteria necessary. For on-line traffic controllers often macroscopic models are selected because they yield a reasonable accuracy within an acceptable computation time.

#### 4. Implementation issues

When the design of the controller has been completed, more practical issues should be investigated. The design of the measurement structure has already been discussed in Section 3.3. In this section implementation issues are discussed involving the use of measured data sets: calibration and validation, state estimation, demand estimation, and performance evaluation.

For the calibration and validation procedure a set of measurements is divided into two parts. One part is used for the calibration of the model by changing its parameters, while the other part is used for the validation of the parameters found, by determining the accuracy of the model in a different traffic situation. These data sets should be gathered under free-flow conditions as well as under congested conditions, in order to capture the full scope of possible traffic scenarios in the model (Ljung, 1999). For a detailed description on the process of calibration and validation we refer to the work of Hollander and Liu (2008); here a brief summary will be provided.

State estimation (Simon, 2006) requires real-time measurements of the system, while demand estimation (Zhou and Mahmassani, 2007) can be done based on real-time measurements or data sets with historical measurements. Since the aim of applying traffic control is to improve e.g. the

traffic flow, it should be possible to test whether or not the applied controller improves the situation. Therefore, data should be collected for both the original situation (without traffic control) and after implementation and tuning of the controller, such that a performance evaluation can be done on the improvement due to traffic control.

#### *4.1. Calibration and validation*

Calibration is the process of selecting values for the parameters of a model, such as e.g. the critical density, the desired speed, or the reaction time. The optimal parameter set minimizes the difference between the measured output and the output predicted by the model. The resulting optimization problem is in general non-linear and non-convex. The problem can be solved using the optimization methods mentioned in Section 3.4, as well as indicated methods for nonlinear least squares problems such as the Gauss-Newton method (see e.g. Gill and Murray (1978)).

Considering traffic flow models, macroscopic models are relatively easy to calibrate due to the limited number of variables. However, for large networks the required computation time could increase up to the point where the problem becomes intractable. Manual calibration of macroscopic traffic models is described in e.g. (Cremer and Papageorgiou, 1981). An example of an automated calibration procedure for macroscopic traffic flow models is described in (Ngoduy et al., 2003).

The calibration of microscopic models is more elaborate due to the large number of parameters that (in principle) can differ for each vehicle type. The calibration of microscopic models is considered in e.g. (Brockfeld et al., 2005; Ciuffo et al., 2008; Hollander and Liu, 2008), where different available calibration methods are summarized and compared.

In modern traffic surveillance systems calibration can be performed on-line (and is also called parameter estimation), see e.g. (Antoniou et al., 2005; Wang and Papageorgiou, 2005; Paz and Peeta, 2009). The advantage of this method is that the difference between the predictions and the real traffic situation will be as small as possible. For the on-line calibration the measurements of the real network during the last period (e.g. the last 15 minutes) are compared with the values that are predicted by the model. The difference between the two is minimized by optimizing the model parameters. These parameters are then used in the model, until a calibration for the next period is completed. An even more general tool that includes on-line calibration, traffic prediction, travel time estimation, queue length estimation, and incident detection is described in (Wang et al., 2006).

Once a model is calibrated, the next step is to validate the model in order to determine the quality of the obtained parameters. Examples of traffic model validation can be found in e.g. (Rakha et al., 1996; Brockfeld et al., 2004). During the validation, the parameters that are obtained from the model calibration are used in the traffic model. The model is then used to predict the traffic variables corresponding to a different data set than the one used for the calibration. The difference between the simulation data and the real data set for the considered situation gives an indication of the correctness of the obtained parameter values, and of the generalizability of the simulation results obtained with the selected parameter values.

#### *4.2. State estimation*

To be able to make predictions with a model, the current state of the network should be known. This state is determined based on the available measurements, and it is used as initial state for the model predictions. In Figure 4 a general state estimation procedure is shown.

*Place Figure 4 about here*

The measurements of the real network are compared with the measurements that are obtained by simulation. Based on this comparison, the state estimator determines the estimated state. State estimation is often done using Kalman filtering or one of its extensions (Jazwinski, 1970; Simon, 2006). For linear models, a Kalman filter adapts the estimated state in such a way that the mean of the error between predictions and measurements is minimized. For non-linear models, filters such as an extended Kalman filter, an unscented Kalman filter, or a particle filter could be used (Simon, 2006). For traffic flow models the use of extended Kalman filtering is described in e.g. (Wang and Papageorgiou, 2005; Gazis and Liu, 2003). The use of particle filtering in traffic control applications can be found in e.g. (Mihaylova et al., 2007; Hegyi et al., 2006). Particle filters use probabilistic models, and start with a distribution of possible states. For all of these possible states the likelihood that it corresponds to the current state is computed, based on a measurement function and on Bayes' rule. With each new set of measurements these likelihoods are updated. The most likely state is selected to be the estimated state.

#### 4.3. Demand estimation

To make a prediction of future traffic states, the future demand must be known. The demand can be obtained based on Origin-Destination (OD) matrices or upstream measurements:

**OD-matrices** OD-matrices contain the demand (in veh/h) from each origin to each destination. OD-matrices can be determined off-line or adapted periodically in an on-line setting. Off-line OD-matrices are obtained from e.g. surveys, historical measurements, and estimations based on the surroundings (residential areas, shopping centers, business areas) and the expected amount of drivers that want to visit these places (Kuwahara and Sullivan, 1987; Castillo et al., 2008). On-line updating of the OD-matrix is done based on measurements using a method that is similar to methods for on-line calibration. When the OD-matrix is determined, a traffic assignment algorithm can be used to divide the traffic over the network. This results in the expected flows on each link. This procedure is described by e.g. Zhou and Mahmassani (2007); Ashok and Ben-Akiva (2000).

**Upstream measurements** Traffic flows measured upstream of the controlled road section will arrive at the controlled section with a delay approximately equal to the expected travel time from the measurement location to the beginning of the controlled stretch. With measurements of the upstream flows, estimations of the flows at the controlled road section can be made, as presented by Papageorgiou (1988). The accuracy of the estimation is influenced by the distance between the upstream measurement and the controlled location, and by the number of intersections, on-ramps, and off-ramps on this stretch.

**Downstream measurements** Traffic jams propagate in an upstream direction, and influence the traffic state in this direction. Therefore, downstream measurements are also needed to obtain an accurate estimation of the demand.

#### 4.4. Controller tuning

A controller often has parameters that should be tuned. For MPC important parameters are the control period, the prediction and control horizon, and the possible weights in the cost function. Methods for tuning MPC controllers have been developed in the process industry (Lee and Yu, 1994; Al-Ghazzawi et al., 2001; van der Lee et al., 2008). There are methods for off-line as well as on-line tuning. However, most of these methods focus on linear MPC, whereas traffic controllers often require non-linear MPC due to the non-linearities in the traffic model.

Controller tuning starts with the selection of initial parameters, which are subsequently adjusted based on simulation results or real measurements. Related to traffic control using MPC, the initial values for the parameters can be selected as follows. The choice of the controller time step, the prediction horizon, and the control horizon are all dependent. The controller time step should be small enough to provide fast responses to changing situations, but large enough to capture the main effects of the control actions within the traffic network by the prediction horizon, without requiring a large control horizon, to be able to compute the control actions in real time. An initial value for the prediction horizon  $N_p$  is the time that a vehicle needs to drive through the selected network under congested conditions. This ensures that all the effects of the control actions on this vehicle are taken into account. To reduce computation times, the value of  $N_p$  can be reduced, while assuring (by simulation) that the main effects of the control actions are still captured by the selected prediction horizon. The length of the control horizon  $N_c$  mainly depends on the computational effort required to optimize the cost function. A longer control horizon leads to more parameters, which leads to longer computation times. However, when the control horizon is too short, the possible impact of the control actions will decrease. Some more detailed tuning rules for the horizons can be found in (Hegyi et al., 2005a). The weight  $w_i$  of each part  $J_i$  of the cost function  $J$  as defined in (1) should be based on the relative importance of the different parts, which should be determined by the road authorities. The weights have to be normalized, which can be done by dividing each part of the cost function by its nominal value.

#### 4.5. Performance evaluation

To evaluate the performance of the controller, the performance of the new, controlled situation should be compared with the initial, uncontrolled or —when some control measures have already been implemented— controlled situation. This means that first, before the deployment of the controller, the initial situation should be measured. Then, when the controller is installed a period should be selected during which the traffic can adapt to the controller. During this period the behavior of the drivers can change, and if necessary the controller should be adjusted. After this period the performance of the controller can be determined. This can be done by comparing measurements for the initial situation (i.e. before control was applied) with measurements in the new controlled situation. For this comparison a performance evaluation measure has to be defined. An obvious choice for such a measure is based on the cost function that is selected for the controller. For both situations (initial/new) the costs are computed, and the relative difference in the costs between the two situations represents the performance gain or efficiency of the controller. When multiple controllers are compared in a case study, for all controllers the resulting traffic situation should be measured. These measurements can then be used to compare the performance of the different controllers.

#### 4.6. Other issues

A brief list of other topics that are also relevant —but that will not be discussed in detail in this paper— for the implementation of traffic controllers is given next.

**Fault detection and fault tolerant control:** The availability and the failure probability of the equipment is important for the functioning of the controller. Missing measurements and wrong representations of the control signals can significantly influence the performance of the controller (Prakash et al., 2005; Blanke et al., 2006). By monitoring, the equipment failures can be noticed (or even predicted) and the controller can take the effects of the

failures into account. This allows the controller to reduce the influence of the failure and to prevent a large decrease in the performance of the network.

**Robustness:** A model is never an exact representation of reality. The sensitivity of the controller to errors in the model structure should be accounted for, as well as other uncertainties in the controller design, such as the error in the demand prediction, the state estimation, and the values of the model parameters. The effect of errors and uncertainties can be reduced by e.g. including demand prediction, using a smaller controller time step to decrease the deviation between the real state and the predicted state, by on-line calibration of the parameters, and by using robust control techniques (Landau, 1999; Wang and Rawlings, 2004).

**Stability:** The control actions explicitly influence the traffic flows. The control actions should lead to a stable traffic situation, without fast fluctuating control signals (Santos et al., 2008; Yin, 2008). Stable traffic situations are situations in which eventually the traffic flows stay around the same level, even if small disturbances of the flows occur. Fast fluctuating control signals can result in a fast changing traffic situation and thus in unstable traffic flows. Sometimes fast changing control signals will be beneficial, e.g. for preventing and dissolving traffic jams. But as human drivers might get confused and annoyed by fast changing signals, some limitation on the frequency and amount of changes is often desirable. Fast changing control signals can be prevented by including a penalty on changes in the control signal in the cost function, or by using larger thresholds with respect to the reactions on changes in the measurements.

**Averaging:** For macroscopic models the measured data should be averaged to obtain mean values. For speed averaging the mostly used means are the time-mean speed and the space-mean speed. The former can be determined from loop detector measurements by taking the arithmetic mean over the measured speeds at a certain detector over a certain period of time. The space-mean speed cannot be determined exactly from local measurements, but several approximations exist in the literature (Daganzo, 1997; van Lint, 2004; Rakha and Zhang, 2005).

## 5. Case study

In this section we will provide an example of the development cycle for model-based traffic control. Based on a specific part of the Dutch freeway A12 we will discuss the steps that are taken and the choices that are made, and apply the controller to the freeway stretch in simulation.

Since the simulations make use of stochastics for both the microscopic traffic simulator and the multi-start approach for the traffic controller, multiple runs of the same traffic condition should be performed to account for the randomness. Therefore, each result shown in this section is obtained by using the arithmetic mean of five simulation runs. Furthermore, in the controlled case several randomly chosen initial values are used for the multi-start approach of the model-predictive controller.

First the network and traffic scenario used in the case study are defined, followed by the development of a variable speed limit controller according to the steps described in Sections 3 and 4. For the simulations we first calibrate the prediction model with the obtained data, and perform a simulation of the case study.

### 5.1. Network and traffic scenario

For the traffic network, a part of the Dutch freeway A12 is selected, as shown schematically in Figure 5. The total length of the considered stretch is 17422 m. There are two on-ramps, near Veenendaal and near Maarsbergen. The major cause of delay on this stretch are shock waves. With shock waves we mean traffic jams that propagate in the opposite direction of the traffic flows (in the upstream direction), and often emerge from on-ramps and other types of bottlenecks. The outflow of a shock wave is usually about 70% of the freeway capacity (Kerner and Rehborn, 1996), and resolving shock waves can significantly improve the freeway traffic flow (Hegyi et al., 2005b).

*Place Figure 5 about here*

The network shown in Figure 5 is divided into 26 segments. Measurements will be taken on the whole stretch, over the length of 17422 m, from segment 1 up to segment 26. We will control the part between the on-ramps at Veenendaal and Maarsbergen from segment 7 up to segment 20, which results in a controlled stretch of 9775 m. Notice that the smallest segment has a length of 540 m (segment 2), and the largest segments have a length of 810 m (segments 9 and 25). For modeling simplicity the on-ramps are not used, but they are shown in the figure to indicate where they are on the freeway.

During the simulation of the freeway a shock wave is introduced by simulating an incident at 1500 meters downstream of segment 26. One vehicle is stopped downstream of the on-ramp near Maarsbergen at the beginning of the simulation for a period of 5 minutes, during which one of the two lanes is blocked. This will create a traffic jam that expands while the lane is blocked, and the shock wave starts to move upstream when both lanes are accessible again. The traffic demand  $q^{\text{dem}}(k)$  on the freeway is set to a constant value of 4400 veh/h, at which a shock wave will remain existent in the network when no control is applied. In the simulations there has been no inflow from the on-ramps.

#### 5.1.1. Representation of the real world

To represent the real world, a microscopic traffic simulator is used. Due to its availability, the selected freeway stretch is modeled with Paramics v5.1 from Quadstone (Quadstone, 2002), a microscopic traffic simulator. In this traffic simulator some parameters can be set to influence the simulations. We refer to (Pinna, 2007) for details of calibrating Paramics. Since Paramics uses a stochastic model, each simulation should be performed several times to obtain statistically significant results. For our model, we perform 5 runs for each simulation. The program uses stochastics to represent reality, e.g. to determine when new vehicles will enter the network, and to obtain the driver characteristics for each individual vehicle.

#### 5.1.2. Selection of the prediction model

For use in the model-based control approach, a macroscopic traffic model is used to obtain fast computations. The aim is to reduce traffic jams, which can be accomplished by reducing the TTS within the network. Therefore, we require knowledge of density, flow and speed in the network, which is why a traffic flow model is selected. Furthermore, we assume that the demand is known, and that a deterministic model can represent the real world well. Using these requirements, the METANET model presented in (Messmer and Papageorgiou, 1990) has been chosen to represent the real world traffic network in the model-based controller.

The METANET model introduces the division of a freeway network into multiple links and segments. A link is a stretch of freeway, e.g. between two crossings. Each freeway link  $m$  is divided into several segments  $i$ , as shown in Figure 6. For the case study, we consider three links, with in total 26 segments, as schematically represented in Figure 5. Segments 1 to 5 belong to link 1, segments 6 to 21 belong to link 2, and segments 22 to 26 belong to link 3. The segments are chosen such that the loop detectors are near the downstream boundary, in order to obtain accurate measurements of the outflow  $q_{m,i}(k_m)$  of the segments. This means that the segments have lengths  $L_{m,i}$  and thus that each segment should have its own values for the model parameters, which is not conform the original formulation of METANET. However, for simplicity, in this case study we assume that within the given link all segments use the same value for the model parameters. The on-ramp near Veenendaal is connected to segment 5, and the on-ramp near Maarsbergen to segment 22. Segments 7 to 20 will be controlled using variable speed limits.

*Place Figure 6 about here*

Within the METANET model the state of segment  $i$  of link  $m$  during the period  $[kT, (k+1)T]$  is given in terms of the density  $\rho_{m,i}(k)$ , the space mean speed  $v_{m,i}(k)$ , and the outflow  $q_{m,i}(k)$  of the segment. Here  $k$  denotes the simulation step, with simulation time step  $T$ . Each segment  $i$  of link  $m$  has a length  $L_{m,i}$ , while the number of lanes  $\lambda_m$  is equal for all segments in link  $m$ . The lengths  $L_{m,i}$  of the segments used in the simulations are shown in Figure 5. The METANET model equations (including an extension to model variable speed limits) are given by (Messmer and Papageorgiou, 1990; Hegyi et al., 2005a):

$$q_{m,i}(k) = \rho_{m,i}(k)v_{m,i}(k)\lambda_m, \quad (2a)$$

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_{m,i}\lambda_m} (q_{m,i-1}(k) - q_{m,i}(k)), \quad (2b)$$

$$v_{m,i}(k+1) = v_{m,i}(k) + \frac{T}{\tau_m} (V(\rho_{m,i}(k)) - v_{m,i}(k)) \quad (2c)$$

$$+ \frac{T}{L_{m,i}} v_{m,i}(k) (v_{m,i-1}(k) - v_{m,i}(k)) - \frac{\eta_m T}{\tau_m L_{m,i}} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa_m}, \quad (2d)$$

and

$$V(\rho_{m,i}(k)) = \min \left( v_m^{\text{free}} \exp \left[ -\frac{1}{a_m} \left( \frac{\rho_{m,i}(k)}{\rho_m^{\text{crit}}} \right)^{a_m} \right], (1 + \alpha)v_{m,i}^{\text{ctr}}(k_c) \right), \quad (2e)$$

where  $v_m^{\text{free}}$  is the free flow speed in link  $m$ ,  $\rho_m^{\text{crit}}$  is its critical density (i.e. the threshold between free and congested traffic flow), and  $\tau_m$ ,  $\eta_m$ ,  $\kappa_m$ , and  $a_m$  are model fitting parameters. The parameter  $\alpha$  expresses the disobedience of the drivers with respect to the applied speed limit. When the speed limits are enforced  $\alpha$  will be smaller since drivers will not exceed the speed limits as much as without enforcement. The index  $k_c$  counts the control time steps. The set of simulation time steps that correspond to the control time step  $k_c$  is given by  $[k_c M_c M_m, (k_c + 1) M_c M_m - 1]$ .

Origins are modeled using a simple queue model. The number of vehicles  $w_o$  in the queue at origin  $o$  evolves as follows:

$$w_o(k+1) = w_o(k) + T \left( q_o^{\text{dem}}(k) - q_o(k) \right), \quad (3)$$

where  $q_o^{\text{dem}}(k)$  is the demand at origin  $o$  at simulation step  $k$ , and  $q_o(k)$  is the outflow at origin  $o$  given by

$$q_o(k) = \min \left\{ q_o^{\text{dem}}(k) + \frac{w_o(k)}{T}, Q_o \frac{\rho_m^{\text{max}} - \rho_{m,1}(k)}{\rho_m^{\text{max}} - \rho_m^{\text{crit}}} \right\}, \quad (4)$$

where  $Q_o$  is the capacity of origin  $o$  under free flow conditions, and  $\rho_m^{\text{max}}$  is the maximum density at link  $m$ . For on-ramps (3) and (4) are also valid.

## 5.2. Design of a variable speed limit controller

To obtain the desired variable speed limit controller, the design steps described in Sections 3 and 4 will be followed. Note however that since the case study is performed in a simulation environment, not all steps are necessary.

### 5.2.1. Formulation of the cost function

The policy objective of the controller is chosen to be the reduction of the travel time on the freeway stretch (in order to reduce traffic jams). The long travel time in the uncontrolled situation is partly due to shock waves. Shock waves can be reduced or dissolved by applying variable speed limits on the freeway, see (Hegyi et al. (2005a,b)). Traffic upstream of the shock wave can be slowed down, thereby limiting the inflow to the shock wave. Since the outflow of the shock wave will remain (more or less) constant, this will reduce the length of the shock wave, and can even dissolve it. This effect can be reached by selecting the total time spent as cost function, which should be minimized.

When using the METANET model, the TTS can be computed as follows:

$$J^{\text{TTS}}(k_c) = T \sum_{k=k_c M_c M_s + 1}^{(k_c + N_p) M_c M_s} \left( \sum_{(m,i) \in \mathcal{M}} \rho_{m,i}(k) \lambda_m L_{m,i} + \sum_{o \in O} w_o(k) \right) \quad (5)$$

where  $\mathcal{M}$  is the set of pairs  $(m, i)$  of link indices and the corresponding segment indices. To avoid large speed limit differences between adjacent segments, we add the penalty terms. This cost function calculates the average of the sum of the squared differences between two speed limits, for the entire control horizon  $N_c$ :

$$J^{\text{SLD}}(k_c) = \frac{1}{N_m N_c} \sum_{m=1}^{N_m} \frac{1}{N_{\text{ctr},m} - 1} \sum_{k=k_c}^{k_c + N_c - 1} \sum_{i=1}^{N_{\text{ctr},m} - 1} (v_{i+1}^{\text{ctr}}(k_c) - v_i^{\text{ctr}}(k_c))^2, \quad (6)$$

where  $N_m$  denotes the number of links in the network, and  $N_{\text{ctr},m}$  is the number of controlled segments on link  $m$ . The two cost functions (5) and (6) can be combined to form the objective function

$$J(k_c) = J^{\text{TTS}}(k_c) + w J^{\text{SLD}}(k_c), \quad (7)$$

where  $w$  is a weighing factor to change the relative importance between the two costs. The *total* time spent  $J^{\text{TTS}}(k_c)$  for this example is in the order of 1000 veh·h, whereas  $J^{\text{SLD}}(k_c)$  represents the *average* squared difference of subsequent speed limits; a value of 100 would mean that on average there is a difference of 10 km/h between the speed limits at all adjacent segments at all times. Therefore, a value of  $w = 100$  is chosen to penalize  $J^{\text{SLD}}(k_c) = 10$  by the same amount as  $J^{\text{TTS}}(k_c) = 1000$ .

### 5.2.2. Formulation of the constraints

It is possible to define constraints for the controller. A possible policy constraint that can be selected is a maximum queue length at the origins:

$$w_o(k) \leq w_o^{\max} \quad \forall o \in O.$$

When speed limit control is applied, an individual speed limit  $v_i^{\text{ctr}}(k_c)$  can be applied to segment  $i$  at control step  $k_c$ . Constraints on the speed limits can be given by

$$v^{\min} \leq v_i^{\text{ctr}}(k_c) \leq v^{\max}$$

where  $v^{\min}$  and  $v^{\max}$  are the minimum and maximum allowed speed of the variable speed limit.

Before the speed limits are applied to the traffic network they are rounded off to steps of 10 km/h to mimic reality more closely. This increases the mismatch between the predicted and measured states, which decreases the performance of the controller. However, in reality the actual speed of drivers will vary stochastically around the presented speed limit, which decreases the negative effect of the rounding operation on the performance of the controller. Moreover, in (Hegyí et al., 2005b) it was found that when using the round operation the speeds that are obtained during a simulation are approximately equal to the speeds that are obtained during a simulation where no rounding is applied.

### 5.2.3. Obtaining measurements

To obtain the measurements loop detectors will be used to mimic the actual situation on the A12 freeway. The loop detectors in the micro-simulator are placed at the locations of the existing loop detectors on the freeway. The distances between subsequent loop detectors is varying between 540 m and 810 m, as shown in Figure 5. A measurement period of 10 s is chosen, and the arithmetic mean of the measured speeds within this measurement period are used as average speeds (representing the time-mean speed).

### 5.2.4. Selection of the control method

Due to prior experience, and the promising results that are obtained with the method, we select model predictive control (MPC) as model-based control method of choice for reducing shock waves in the traffic network example. This control method requires a model and an optimization algorithm. Using a standard MPC approach the controller might not be fast enough for real-time implementation, but with the use of explicit MPC (Mäder et al., 2007; Scibilia, 2010) this could be achieved.

The model selection will be described below. As optimization algorithm sequential quadratic programming (SQP) (Boggs and Tolle, 1995) will be used, by using the MATLAB function `fmincon` (The MathWorks, 2011) with a multi-start approach. This algorithm is selected since it can handle the non-convex, bounded optimization problem that should be solved by the MPC controller. At each control step  $k_c$ , 16 distinct initial value sets are used to reduce the effect of stochastic influences.

## 5.3. Simulation of real traffic networks

To obtain realistic data from the Paramics simulations, a simulation period of two hours is used. The first hour is used to create a network filled with traffic, and this data is discarded in the results. We use the measurement data of the second hour—in which the traffic state is well developed—for the parameter estimation of the METANET model.

Figure 7 shows the simulated measurements on the network when no control is applied on the freeway stretch presented in Figure 5. On the horizontal axis the time is shown, and on the vertical axis the locations are given. The first segment (near Veenendaal) is plotted at the bottom of the figure, and the last segment (near Maarsbergen) at the top.

The upper sub-plot shows the measured mean speeds. Lighter colors represent higher mean speeds. The shock wave is clearly visible as the thick, dark stripe going upstream as time elapses. Also in the density plot (the middle sub-plot), the shock wave is clearly visible as the thick, light stripe representing high densities. The bottom sub-plot shows the flow, where it can be seen from the dark color that due to the shock wave the flow decreases. Note that the thin dark stripes that are going downstream as time elapses, are caused by differences in speed between individual vehicles.

*Place Figure 7 about here*

### 5.3.1. Calibration and validation

Calibration is done by off-line numerical optimization using an objective function given by

$$J^{\text{cal}}(\theta) = \frac{1}{k_c - N_p} \sum_{k_c=1}^{k_c - N_p} \sum_{k_m=M_c k_c}^{M_c(k_c + N_p)} \sum_{k=M_m k_m+1}^{M_m(k_m+1)} \check{J}^{\text{cal}}(\theta, k_m, k), \quad (8)$$

where  $\theta$  is the set of model parameters consisting of  $v_m^{\text{free}}$ ,  $\rho_m^{\text{crit}}$ ,  $\rho_m^{\text{max}}$ ,  $\tau_m$ ,  $\eta_m$ ,  $\kappa_m$ ,  $\rho_m$ , and  $a_m$  for each of the three links,  $k_m$  is the number of sample steps for which measurement data is available, and where  $\check{J}^{\text{cal}}(\theta, k_m, k)$  is given by:

$$\check{J}^{\text{cal}}(\theta, k_m, k) = \sum_{(m,i) \in \mathcal{M}} \left\{ \left( \frac{v_{m,i}^{\text{sample}}(k_m) - \bar{v}_{m,i}(k)}{\bar{v}(k_m)} \right)^2 + \left( \frac{\rho_{m,i}^{\text{sample}}(k_m) - \bar{\rho}_{m,i}(k)}{\bar{\rho}(k_m)} \right)^2 \right\},$$

where  $\bar{v}(k_m)$  and  $\bar{\rho}(k_m)$  are the average speed and density of the measured data from control step  $k_c$  to  $k_c + N_p$ . The error in the predictions that are made by the controller at every control step is computed, and then the average of the errors of all prediction periods is taken. A lower value of the objective function (8) means a better fit of the measured states  $\{v_{m,i}^{\text{sample}}; \rho_{m,i}^{\text{sample}}\}$  by the predicted states  $\{\bar{v}_{m,i}; \bar{\rho}_{m,i}\}$  reproduced by the traffic flow model.

Another option is to compare the two data sets with respect to the value of the cost function. When the cost function is selected to be the TTS (as in (5)), the difference between measured and predicted TTS can be determined to judge the performance of the parameter values. The error is given by

$$E^{\text{TTS}}(\theta) = \frac{1}{k_c - N_p} \sum_{k_c=1}^{k_c - N_p} \left| \frac{\check{J}^{\text{TTS}}(k_c) - J^{\text{TTS}}(k_c)}{J^{\text{TTS}}(k_c)} \right| \quad (9)$$

which gives the average percentage of mismatch between the TTS for the measured data  $J^{\text{TTS}}$ , and the TTS for the predicted data  $\check{J}^{\text{TTS}}$  for the parameter set  $\theta$ , and where  $k_c$  is the final control step in the scenario.

This off-line calibration of the METANET model is performed with the MATLAB function `fmincon` (The MathWorks, 2011) which implements SQP, which is the same algorithm that will be used in the on-line controller. We use the algorithm in a multi-start configuration with 100 different initial values, which increases the probability of finding the global (or best local)

optimum. To deal with the stochasticity of Paramics (see Section 5.1) we use 10 different random seeds, 5 for the calibrations and 5 for the MPC simulations.

The cost functions  $J^{\text{cal}}(\theta)$  and  $E^{\text{TTS}}(\theta)$  are determined at each control step, for the prediction that is made at this step. For the calibration of the total model the average value  $\bar{J}^{\text{cal}}(\theta)$  and  $\bar{E}^{\text{TTS}}(\theta)$  over the results of all control steps are determined.

### 5.3.2. State estimation

The state of the network consists of the average speed, average density, and average flow. The speeds are measured by loop detectors, and averaged using the arithmetic mean. The flow at a loop detector  $d$  can be calculated from these measurements using

$$q_d^{\text{sample}}(k_m) = \frac{N_d(k_m)}{T_m}, \quad (10)$$

where  $N_d(k_m)$  is the number of vehicles that passed the detector during sampling period  $k_m$  and  $T_m$  is the sampling time step. The density at the segment associated with the loop detector  $d$  follows from the flow  $q_d^{\text{sample}}(k_m)$ , the mean speed  $v_d^{\text{sample}}(k_m)$ , and the number of lanes  $\lambda$ , as

$$\rho_d^{\text{sample}}(k_m) = \frac{q_d^{\text{sample}}(k_m)}{v_d^{\text{sample}}(k_m)\lambda}. \quad (11)$$

At every controller time step a new estimation of the current state is obtained, based on the last available measurements.

### 5.3.3. Demand estimation

The demand of the Paramics model is set to 4400 veh/h, which means that it randomly introduces vehicles with a mean of 4400 vehicles per hour. To make predictions with the METANET model, an estimation of this stochastic demand should be made. However, for simplicity we use the known average value of 4400 veh/h as estimation of the demand during the case study.

### 5.3.4. Controller tuning

The METANET model (2) uses a simulation time step of  $T=10$  s. This period is small enough to ensure that the vehicles cannot drive through a whole segment in one simulation step, and large enough to prevent unnecessary long computation times. The controller time step equals  $T_c=60$  s, which prevents fast switching between control values and which is a reasonable time to perform the on-line optimization. The prediction horizon equals  $N_p=20$  steps and the control horizon is  $N_c=10$  steps, which forms a trade-off between the required computation time and the detectability of the effects of the control actions.

### 5.3.5. Performance evaluation

Now we will illustrate the selection of the evaluation function, using the speed limit control example. Since the objective is to minimize the TTS, we determine the improvement in traffic conditions by comparing the obtained values for the TTS. However, only comparing the TTS based on the number of vehicles in the measured area is not a good measure, since the controller will not only increase the outflow, but also the inflow when the shock wave is resolved successfully. This is due to the fact that the incoming vehicles will not be blocked any more when the

shock wave is dissolved. To take into account this change in the demand a different formulation of the TTS is used, based on the demand  $q^{\text{dem}}(k)$  and the outflow  $q^{\text{out}}(k)$  (Hegyí et al., 2005a):

$$J^{\text{TTS}} = TN_0M_cM_m k_c + T^2 \sum_{k=0}^{M_cM_m k_c - 1} (M_cM_m k_c - k) \left( q^{\text{dem}}(k) - q^{\text{out}}(k) \right) \quad (12)$$

where  $N_0$  is the initial number of vehicles in the freeway stretch under consideration.

#### 5.4. Results

The results of the case study are presented below. First, we briefly discuss the calibration results, followed by the effect of using the model-based traffic controller in simulation.

##### 5.4.1. Calibration results

First a calibration of the prediction model is performed, as described in Section 5.3.1. The average calibration error  $\bar{J}_{\text{cal}}$  between the measurements in the traffic network and the predictions from the METANET model are computed using (8), resulting in an average of 38.9. The average error between the measured and predicted TTS is  $\bar{E}_{\text{TTS}} = 5.5\%$ , which is obtained using (9).

##### 5.4.2. Performance evaluation

For the identification of improvement in traffic conditions, the TTS is used. A lower value of  $J^{\text{TTS}}$  as given by (12) represents better traffic conditions, since on average vehicles are spending less time in a certain area, indicating that the outflow is higher. In the uncontrolled situation—shown in Figure 7—the TTS is 1068.0 veh·h.

*Place Figure 8 about here*

Using the MPC-based traffic controller, the shock waves can be dissolved, as shown in Figure 8. The speed limits lower the inflow to the shock wave by delaying the upstream traffic. In this way the inflow of the shock wave is lower than the outflow, which reduces the shock wave. Using time-mean speeds  $v^{\text{tms}}(k_m)$  as state variables for the controller gives an improvement of 15.6 % compared to the uncontrolled situation; the TTS is reduced to 901.1 veh·h. The variable speed limit controller increases the outflow of the network with 4.8% on average, from 4218 to 4422 veh/h.

## 6. Conclusions

The use of model-based traffic control systems can significantly improve the performance of a traffic network. However, implementing such controllers is not straightforward. Therefore, we have investigated issues that are important when developing traffic control systems. A general overview of the process of developing and implementing such a controller has been presented.

Within the general overview, first several issues were discussed related to the design of a model-based controller. In particular, the selection of the network, measurements, control objective, system model, control method, and the formulation of constraints have been considered. Next, issues related to the implementation of model-based controllers have been presented. More specifically, calibration and validation, state estimation, demand estimation, controller tuning, and performance evaluation have been addressed.

For a specific simulation case study involving a part of the A12 freeway in The Netherlands, we have illustrated design procedure and the performance of a variable speed limit controller that applies model predictive control (MPC) with METANET as its prediction model. Using the controller, the TTS could be reduced significantly. In this case study improvements of 15.6% in the TTS compared to the uncontrolled situation are reached. Reducing the shock waves also has a positive effect on the flow, which has increased by 4.8%.

## Appendix

### Metanet model

$T$	simulation period (h)
$k$	simulation step counter, for period $[kT, (k+1)T)$
$\mathcal{M}$	set of pairs $(m, i)$ of freeway links $m$ and corresponding segments $i$
$\lambda_m$	number of lanes of freeway link $m$ (lanes)
$L_{m,i}$	length of segment $i$ of freeway link $m$ (km)
$\rho_{m,i}(k)$	density in segment $i$ of freeway link $m$ at simulation step $k$ (veh/km/lane)
$q_{m,i}(k)$	outflow of segment $i$ of freeway link $m$ at simulation step $k$ (veh/h)
$v_{m,i}(k)$	mean speed on freeway segment $i$ of link $m$ at simulation step $k$ (km/h)
$w_o(k)$	queue length at origin $o$ at simulation step $k$ (veh)
$q_o^{\text{dem}}(k)$	traffic demand at origin $o$ during the $k$ -th simulation period (veh/h)

### Measurements

$T_m$	sampling period (h)
$k_m$	sampling step counter, for period $[k_m T_m, (k_m+1)T_m)$
$M_m$	ratio between the sample (i.e. measurement) period $T_m$ and the simulation period $T$ (-)
$q_d^{\text{sample}}(k_m)$	flow on the freeway near detector $d$ during the $k_m$ -th sampling period (veh/h)
$v_d^{\text{sample}}(k_m)$	mean speed on the freeway near detector $d$ during the $k_m$ -th sampling period (km/h)
$\rho_d^{\text{sample}}(k_m)$	density at the freeway stretch near detector $d$ during the $k_m$ -th sampling period (veh/km/lane)
$u_{d,n}(k_m)$	speed of vehicle $n$ measured by detector $d$ during sampling period $k_m$ (km/h)
$l_d$	distance between the consecutive inductive loops of detector $d$ for determining the vehicle speed $u_{d,n}$ (km)
$t_{d,n}$	time-difference between detection of vehicle $n$ at consecutive inductive loops of detector $d$ (h)
$N_d(k_m)$	number of observed vehicles at detector $d$ during the $k_m$ -th sampling period (veh)
$v_d^{\text{tms}}(k_m)$	time mean speed at detector $d$ during the $k_m$ -th sampling period (veh/h)

### Calibration and control

$T_c$	control period (h)
$k_c$	control step counter, for period $[k_c T_c, (k_c+1)T_c)$
$M_c$	ratio between the control period $T_c$ and the sample period $T_m$ (-)
$N_p$	prediction horizon (in number of control steps)
$N_c$	control horizon (in number of control steps)
$\theta$	set of model parameters that should be calibrated

$J^{\text{TTS}}(k_c)$	total time spent in the network and at origin queues for control period $k_c$ (veh-h)
$J^{\text{cal}}(\theta)$	calibration cost for parameter set $\theta$ (-)
$E^{\text{TTS}}(\theta)$	difference between measured and predicted TTS for $\theta$ (%)

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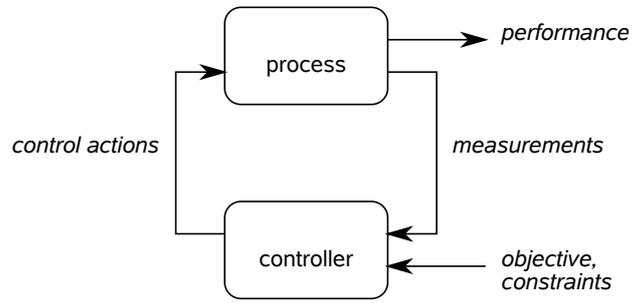


Figure 1: A general scheme for feedback control methods.

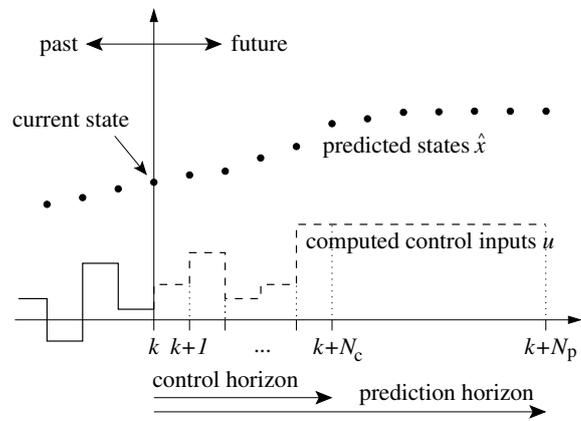


Figure 2: Graphical overview of terms and variables related to Model Predictive Control.

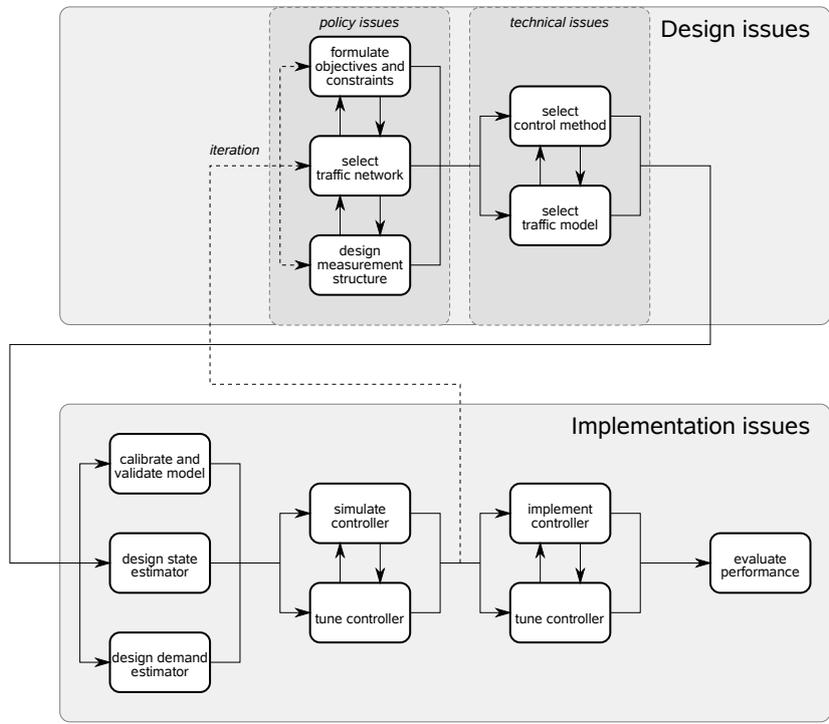


Figure 3: Overview of the controller development process.

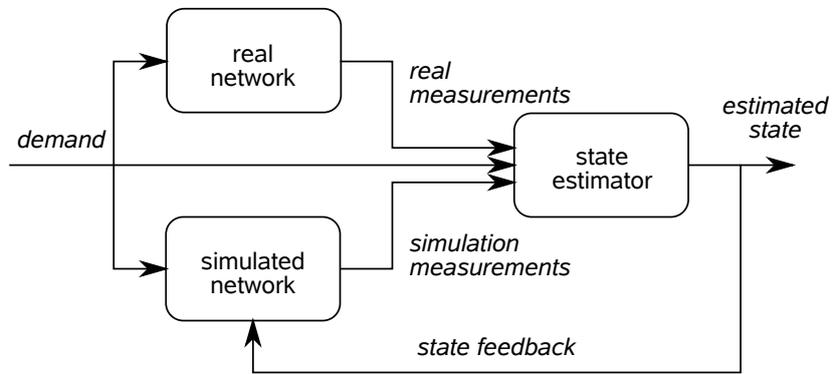


Figure 4: The state estimation procedure.

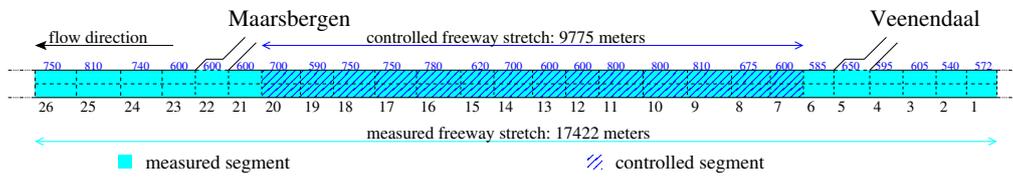


Figure 5: Schematic representation of the network stretch used in the case study. Segment numbers and lengths are shown below and above the freeway stretch respectively.

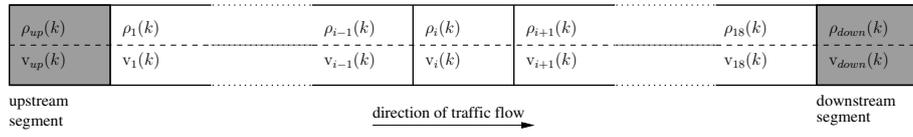


Figure 6: In the METANET model, a freeway link is divided into segments.

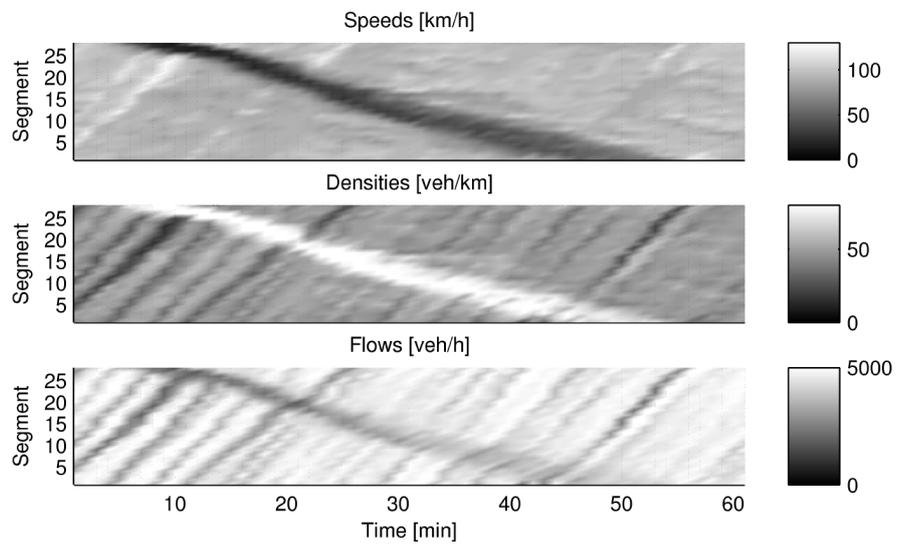


Figure 7: Traffic condition without control. A traffic jam moves from the top-left part to the bottom-right part of the subplots, and results in low speeds (top), high densities (middle), and low flows (bottom).

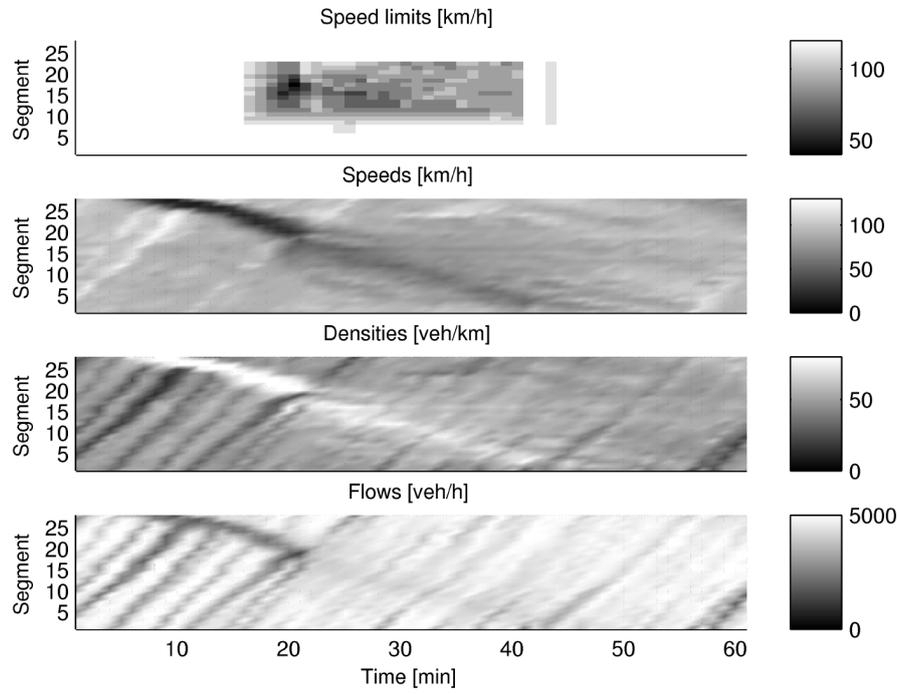


Figure 8: Controlled traffic flow using time mean speeds. The speed limits are only present in segments 7 to 20, and active during a period of 25 minutes. The traffic jam has been reduced compared to the uncontrolled case shown in Figure 6.

Figure captions:

- Figure 1: A general scheme for feedback control methods.
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- Figure 3: Overview of the controller development process.
- Figure 4: The state estimation procedure.
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