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# Offtake Feedforward Compensator Design for an Irrigation Channel with Distributed Control

Yuping Li and Bart De Schutter

**Abstract**—In the control of irrigation channels, there exists a tradeoff between the local performance of regulating the water-level in each pool to a given setpoint and rejecting offtake disturbances, on the one hand, and decoupling between pools, on the other. Such a tradeoff is well managed by a distributed control that inherits the interconnection structure of the plant. Furthermore, to decrease the water-level deviation from its setpoint in response to large offtakes, i.e. to improve the transient response of the local closed-loop system, the knowledge of the offtakes can be fed forward. This paper explores the designing of a feedforward compensator to improve local control performance while maintaining the good management of closed-loop coupling between pools by distributed control.

## I. INTRODUCTION

Water is a scarce resource all over the world, and management of the water resources has become an important issue. The water losses in irrigation channels are large, but it is recognised that these losses can be substantially reduced by employing improved control systems. Fig. 1 shows the topview of an irrigation network. Water flows out from the

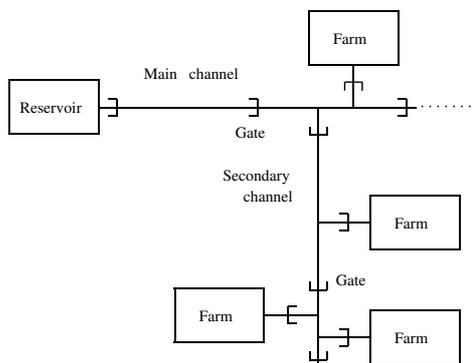


Fig. 1. Topview of an irrigation network

reservoir and is distributed through the main and secondary channels to farms. Mechanical gates are installed along the channels to control the flows. The stretch of water between two neighbouring gates is called a pool. The network is largely gravity fed (i.e. there is no pumping). To satisfy water demands from farms and to decrease water losses, it is

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important to regulate the water-level of each pool at a certain setpoint. Typically, most farms are situated at the downstream end of each pool. For an efficient water distribution, distant-downstream control, i.e. use the upstream gate to control the downstream water-level of each pool, is implemented (see [1]). Further, an irrigation channel is a system presenting strong interactions between pools, i.e. the flow out of a pool is equivalent to the flow into its downstream pool. With the distant-downstream control structure, when a water offtake occurs in a downstream pool, such interactions cause amplification of control actions (i.e. the flow over upstream gates) and water-level error propagation towards upstream pools. Indeed, there exists a tradeoff between local performance, i.e. regulating the water-level to a setpoint and rejecting offtake disturbances in each pool, and decoupling of the closed-loop system. Such a tradeoff can be well-managed by a special structured distributed control (see [2], [3]).

On the other hand, the internal time-delays, i.e. the time for transportation of water from the upstream end to the downstream of each pool, limit the performance of the distant-downstream control. The transient deviation of the water-level in a pool from the setpoint caused by large disturbances sometimes constrains the water service to offtake points. Such an issue can be coped with, on the higher level of an hierarchical control system, by scheduling of the offtake demands [4]. On the lower level of the control system, since the offtakes are usually measured, it is possible to decrease the transient water-level deviation by feeding forward the known disturbances. In this paper, designing of a disturbance feedforward compensator for an irrigation channel with distributed control is discussed. The main objective is to improve the local control performance in terms of a better transient response to offtake disturbances, while maintaining the good management of closed-loop coupling between pools by distributed control. Two approaches to designing the feedforward compensator are explored. First, an ad-hoc method is introduced based on exploring an alternative realisation of the distributed controller. Then, a systematic approach is studied by changing the configuration of the distributed controller. Simulation results show that both approaches achieve a better performance in rejecting local offtakes while maintain the good management of coupling between pools obtained by the nominal distributed control. The paper is organised as follows. Section II discusses the feedforward compensation problem. An ad-hoc and a systematic approach to feedforward compensator design are given in Section III and Section IV respectively. Section V concludes the paper.

## II. PROBLEM FORMULATION

An irrigation channel under decentralised feedback control with decentralised feedforward compensation is discussed in [5]. The feedforward compensator is generally used to decouple the interactions between pools, see Fig. 2. Note

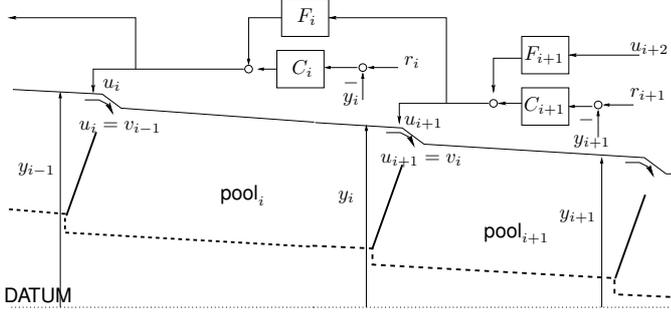


Fig. 2. Decentralised feedback control with feed-forward

a perfect decoupling of the interaction cannot be realised even if the internal delays are exactly known [6]. A systematic approach to designing the decoupling compensator is discussed in [2], [3], which proposes the distributed control structure shown in Fig. 3. Such a distributed control presents a performance advantage over the decentralised feedback with feedforward control. It involves interconnections be-

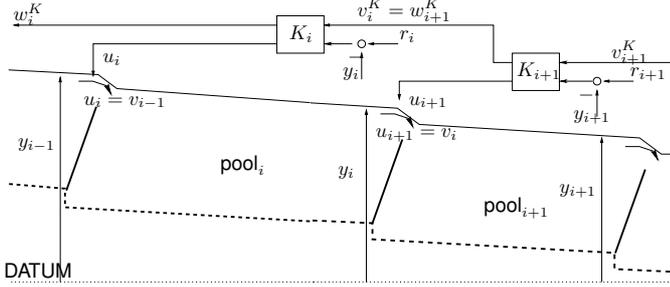


Fig. 3. Distributed control of an open water channel

tween sub-controllers as a function of the water-level errors. The controller is synthesised by solving a structured  $\mathcal{H}_\infty$  optimisation problem to cope with the tradeoff between local performance and the closed-loop coupling between subsystems, see Section III. Note the decentralised feedback control with feedforward compensation in Fig. 2 is a special realisation of the distributed controller, i.e.  $K_i = \begin{bmatrix} F_i & C_i \\ F_i & C_i \end{bmatrix}$ .

An irrigation channel is in fact a string of pools. A simple model of the water level in pool<sub>*i*</sub> can be obtained by conservation of mass [2], [7]:

$$\alpha_i \dot{y}_i(t) = u_i(t - \tau_i) - v_i(t) - d_i(t), \quad (1)$$

where  $y_i$  is the downstream water-level,  $u_i$  is the flow over the upstream gate,  $v_i$  the flow over the downstream gate,  $d_i$  models the offtake load-disturbances from pool<sub>*i*</sub>;  $\tau_i$  is the transport delay of water from upstream gate to downstream gate of the pool,  $\alpha_i$  a measure of the pool surface area. Note the interconnection  $v_i = u_{i+1}$ , i.e. the flow out from pool<sub>*i*</sub>

equals the flow into pool<sub>*i+1*</sub>. Taking the Laplace transform of (1), yields

$$P_i : y_i(s) = \frac{1}{s\alpha_i} (e^{-s\tau_i} u_i - v_i - d_i)(s). \quad (2)$$

Note the offtakes  $d_i$  have the same impact as  $v_i$  on the water-level  $y_i$ . In practice, water offtakes are usually measured and hence known disturbances. With the structure of decentralised feedback with feedforward compensation, this information can be straightly fed through the feedforward term and a good rejection of the disturbance is expected if the feedforward compensator is properly designed. In this case,

$$\begin{pmatrix} w_i^K \\ u_i \end{pmatrix} = \begin{bmatrix} F_i & F_i & C_i \\ F_i & F_i & C_i \end{bmatrix} \begin{pmatrix} v_i \\ d_i \\ e_i \end{pmatrix} \quad (3)$$

Such a straight feedforward implementation, on the contrary, cannot be involved in the general distributed controller shown in Fig. 3. In fact, the physical interpretation of the interconnection between sub-controllers,  $v_i^K$ , is not clear. To obtain a proper feedforward controller for the disturbance, one may explore the relationship between  $v_i^K$  and  $v_i$ .

## III. AN AD-HOC APPROACH TO DESIGNING THE FEEDFORWARD COMPENSATOR

Fig. 4 is the localised portion of a channel under distributed distant-downstream control given in [3]. In the

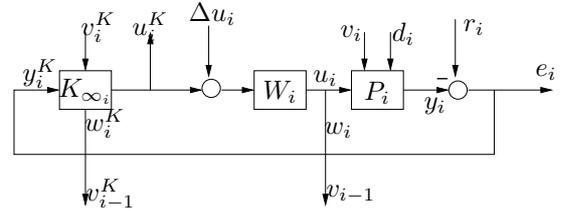


Fig. 4. Localised portion of distributed controller design

figure,  $P_i$  is the nominal model (2) for pool<sub>*i*</sub>, and  $K_i$  in Fig. 3 is split into a loop-shaping weight  $W_i$  and a compensator  $K_{\infty i}$  (with  $u_i^K$  and  $y_i^K$ , the input to and the output from the shaped plant, respectively). Note the constraint on the interconnection between controllers  $v_i^K = w_{i+1}^K$ . Designing of the distributed controller consists of the following three steps, which is consistent with the well-known  $\mathcal{H}_\infty$  loop-shaping approach [12].

- 1) Design  $W_i$  to shape  $P_i$  based on local performance. Typical offtakes  $d_i$  are step disturbance; based on the internal model principle [8], a simple selection could be  $W_i = \frac{\kappa_i}{s}$  for a zero steady-state water-level error. For robust stability,  $\kappa_i$  is selected such that the local crossover frequency  $\omega_{c_i}$  is less than  $1/\tau_i$  (see [9]). Denote  $n_i := (r_i, d_i, \Delta u_i)^T$  and  $z_i := (e_i, u_i^K)^T$ , with  $r_i$  the water-level setpoint,  $e_i := r_i - y_i$  the water-level error in pool<sub>*i*</sub> and  $\Delta u_i$  modelling additional uncertainty in the control. For a channel of  $N$  pools,

let  $G_s := (G_{s_1}, \dots, G_{s_N})$  denote the interconnection of the shaped plant

$$G_{s_i} := \begin{pmatrix} v_i \\ n_i^K \\ u_i^K \end{pmatrix} \mapsto \begin{pmatrix} w_i \\ z_i^K \\ y_i^K \end{pmatrix} = \begin{bmatrix} 0 & (0 \ 0 \ W_i) & W_i \\ \left(\frac{1}{s\alpha_i}\right) & \left(1 \ \frac{1}{s\alpha_i} \ \frac{e^{-s\tau_i}}{-s\alpha_i} W_i\right) & \left(\frac{e^{-s\tau_i}}{-s\alpha_i} W_i\right) \\ 0 & 0 & 1 \end{bmatrix}$$

with  $v_i = w_{i+1}$  and boundary condition  $v_N = 0$ . Note that such a boundary condition is possible with distant-downstream control.

- 2) Synthesise  $K_{\infty_i}$  to cope with the tradeoff between local performance and closed-loop coupling.<sup>1</sup> Let  $K_{\infty} = (K_{\infty_1}, \dots, K_{\infty_N})$  denote the interconnection of

$$K_{\infty_i} := \begin{pmatrix} v_i^K \\ y_i^K \end{pmatrix} \mapsto \begin{pmatrix} w_i^K \\ u_i^K \end{pmatrix}$$

with  $v_i^K = w_{i+1}^K$  and boundary condition  $v_N^K = 0$ ; and let  $H(G_s, K_{\infty})$  denote the closed-loop transfer function from  $(n_1, \dots, n_N)^T$  to  $(z_1, \dots, z_N)^T$ . The synthesis problem is formulated as

$$\begin{aligned} & \min_{K_{\infty} \in \mathcal{K}_{\text{syn}}} \gamma \\ & \text{subject to} \\ & \|H(G_s, K_{\infty})\|_{\infty} < \gamma \end{aligned} \quad (4)$$

where  $\mathcal{K}_{\text{syn}}$  represents the set of stabilising  $K_{\infty}$ 's. Note that we use  $\|\cdot\|_{\infty}$  to denote the  $\mathcal{H}_{\infty}$  norm of a transfer function. Such a structured optimisation problem can be solved by employing the technique in [11], see [3].

- 3) The final distributed controller is given by

$$\begin{aligned} K_i & := \begin{pmatrix} v_i^K \\ e_i \end{pmatrix} \mapsto \begin{pmatrix} w_i^K \\ u_i^K \end{pmatrix} = \begin{bmatrix} 1 & W_i \end{bmatrix} K_{\infty_i} \\ & = \begin{bmatrix} K_i^{11} & K_i^{12} \\ K_i^{21} & K_i^{22} \end{bmatrix} = \begin{bmatrix} K_{\infty_i}^{11} & K_{\infty_i}^{12} \\ \frac{\kappa_i}{s} K_{\infty_i}^{21} & \frac{\kappa_i}{s} K_{\infty_i}^{22} \end{bmatrix}. \end{aligned} \quad (5)$$

Comparing to (3), the distributed controller with additional offtake feedforward compensation can be constructed as

$$\begin{pmatrix} w_i^K \\ u_i^K \end{pmatrix} = \begin{bmatrix} K_{\infty_i}^{11} & E_i & K_{\infty_i}^{12} \\ \frac{\kappa_i}{s} K_{\infty_i}^{21} & F_i & \frac{\kappa_i}{s} K_{\infty_i}^{22} \end{bmatrix} \begin{pmatrix} v_i^K \\ d_i \\ e_i \end{pmatrix} \quad (6)$$

One may simply select  $E_i = 0$ . A properly designed  $F_i$  can help to improve local performance, i.e. to attain a smaller  $e_i$ . On the other hand, this could degrade the global performance achieved by the nominal implemented distributed control. In fact, the knowledge of  $d_i$  is not used for decoupling of the multi-loop system. Then, is there a simple realisation of  $E_i$  such that the good management of the tradeoff between local and global performance of the distributed control is maintained?

An ad-hoc idea is: If the relationship between  $w_i^K$  and  $u_i^K$  can be approximated by  $w_i^K \approx f_i(u_i^K)$  with  $f_i$  linear, set  $E_i = f_i F_i$ .

<sup>1</sup>For local performance, one considers  $e_i$  be small; while closed-loop coupling is caused by control action  $u_i$  to compensate  $e_i$ . As shown in [10], in purely decentralised feedback control for a string of identical pools, given the local controllers are selected the same for each pool, there exists the following tradeoff between local performance and the coupling between the pools:  $T_{d_i \mapsto e_i} + T_{e_i \mapsto e_{i-1}} e^{-s\tau_i} = 1$ .

A typical bode-plot of the synthesised  $K_{\infty_i}$  is shown in Fig 5. We see that the four entries of  $K_{\infty_i}$  are all lead-lag

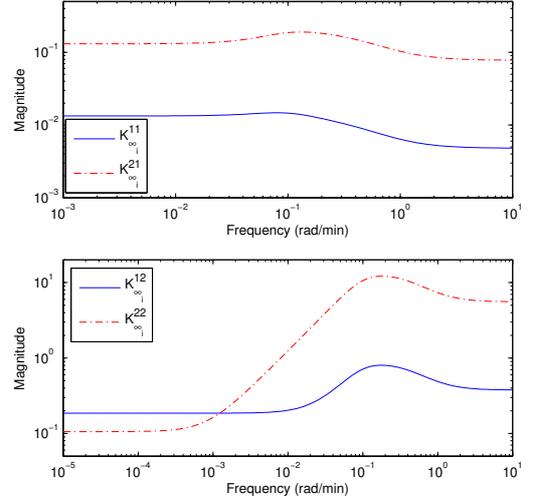


Fig. 5. The synthesised compensator  $K_{\infty_i}$

compensators, then from (5)  $f_i$  should involve a derivative  $s$ . Following the above ad-hoc idea, this requires that  $E_i$  involves an  $s$ ,<sup>2</sup> which contributes little to decoupling in the face of step disturbance  $d_i$ . Instead, an alternative realisation of  $K_i$  is next studied.

#### A. An alternative realisation of $K_i$

Pull the integrator in the 21-block of  $K_i$  to the first output of  $K_{i+1}$ . Let  $\hat{v}_i^K = \frac{1}{s} v_i^K$ . The distributed controller is then composed of the following sub-controllers:

$$\begin{aligned} \hat{K}_1 & := \begin{pmatrix} \hat{v}_1^K \\ e_1 \end{pmatrix} \mapsto u_1 = \begin{bmatrix} \kappa_1 K_{\infty_1}^{21} & \frac{\kappa_1}{s} K_{\infty_1}^{22} \end{bmatrix} \\ & \text{with } \hat{v}_1^K = \hat{w}_2^K, \\ \hat{K}_i & := \begin{pmatrix} \hat{v}_i^K \\ e_i \end{pmatrix} \mapsto \begin{pmatrix} \hat{w}_i^K \\ u_i^K \end{pmatrix} = \begin{bmatrix} K_{\infty_i}^{11} & \frac{1}{s} K_{\infty_i}^{12} \\ \kappa_i K_{\infty_i}^{21} & \frac{\kappa_i}{s} K_{\infty_i}^{22} \end{bmatrix} \\ & \text{with } \hat{v}_i^K = \hat{w}_{i+1}^K, \text{ for } i = 2, \dots, N-1 \\ \hat{K}_N & := e_N \mapsto \begin{pmatrix} \hat{w}_N^K \\ u_N^K \end{pmatrix} = \begin{bmatrix} \frac{1}{s} K_{\infty_N}^{12} \\ \frac{\kappa_N}{s} K_{\infty_N}^{22} \end{bmatrix} \end{aligned}$$

*Remark 1:* One should prove that changing the 12 and 21-block to have an alternative realisation of  $K_i$  does not change the closed-loop stability (Section 4.7 of [9]).

Indeed, from (2), for a channel of  $N$  pools

$$\begin{pmatrix} y_1 \\ \vdots \\ y_{N-1} \\ y_N \end{pmatrix} = \begin{bmatrix} G_1 & \tilde{G}_1 & & \\ & \ddots & \ddots & \\ & & G_{N-1} & \tilde{G}_{N-1} \\ & & & G_N \end{bmatrix} \begin{pmatrix} u_1 \\ \vdots \\ u_{N-1} \\ u_N \end{pmatrix} + \begin{bmatrix} \tilde{G}_1 & & \\ & \ddots & \\ & & \tilde{G}_N \end{bmatrix} \begin{pmatrix} d_1 \\ \vdots \\ d_N \end{pmatrix}$$

<sup>2</sup>For closed-loop stability,  $F_i$  cannot involve an integrator.

with  $G_i = \frac{1}{s\alpha_i} e^{-s\tau_i}$  and  $\tilde{G}_i = -\frac{1}{s\alpha_i}$ . Further,

$$\begin{pmatrix} u_1 \\ \vdots \\ u_N \end{pmatrix} = \begin{bmatrix} \hat{K}_{11} & \cdots & \hat{K}_{1N} \\ & \ddots & \vdots \\ & & \hat{K}_{NN} \end{bmatrix} \begin{pmatrix} e_1 \\ \vdots \\ e_N \end{pmatrix},$$

where  $\hat{K}_{ii} = \hat{K}_i^{22}$  taking care of the local performance, and additional decoupling terms

$$\begin{aligned} \hat{K}_{i,i+1} &= \hat{K}_i^{21} \hat{K}_{i+1}^{12}, \\ \hat{K}_{ij} &= \hat{K}_i^{21} \left( \prod_{k=i+1}^{j-1} \hat{K}_k^{11} \right) \hat{K}_j^{12} \text{ for } j > i + 1. \end{aligned}$$

Then the closed-loop relationship between the water-level errors and the offtake disturbances is:

$$\begin{pmatrix} e_1 \\ \vdots \\ e_N \end{pmatrix} = \begin{bmatrix} M_{11} & \cdots & M_{1N} \\ & \ddots & \vdots \\ & & M_{NN} \end{bmatrix} \begin{pmatrix} d_1 \\ \vdots \\ d_N \end{pmatrix}$$

where for  $i = 1, \dots, N$ ,  $M_{ii} = -\tilde{G}_i (1 + G_i \hat{K}_{ii})^{-1}$  and for  $j \geq i + 1$

$$M_{ij} = M_{ii} \sum_{k=i+1}^j \left( \hat{K}_{i+1,k} - \hat{K}_{ik} e^{-s\tau_i} \right) M_{kj}.$$

We see that the closed-loop transfer matrix is upper-triangular, hence the multivariable system inherits the local stabilities. That is, the multivariable system is stable *if and only if* all monovariate systems are stable. Note  $M_{ii}$  just involves the 22-block of  $\hat{K}_i$ 's, which is the same as the 22-block of the nominal realisation of  $K_i$ 's, see (5). From Fig. 5, it can be expected that  $\hat{K}_i$  is dominated by the second column, which comprises the synthesised  $\begin{bmatrix} K_{\infty_i}^{12} \\ K_{\infty_i}^{22} \end{bmatrix}$  and an integrator, in the frequency range of interest. One then makes the following approximation<sup>3</sup>

$$\hat{w}_i^K \approx \frac{1}{\kappa_i} K_{\infty_i}^{12} (K_{\infty_i}^{22})^{-1} u_i. \quad (7)$$

The following case study compares the LHS and the RHS of (7), see the red dash-dotted line and the blue solid line in Fig. 6 respectively. In this simulation, initially, no offtake

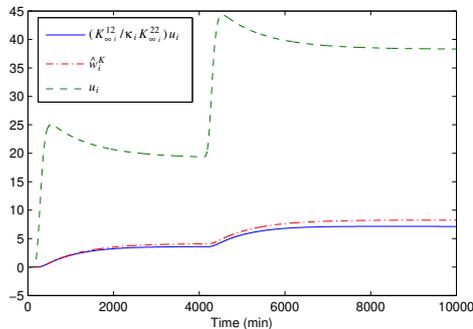


Fig. 6. The relationship between  $\hat{w}_i^K$  and  $u_i$

<sup>3</sup>In the approximation, the impact of the first column of  $K_{\infty_i}$  is thus omitted.

$i$	$\tau_i$	$\alpha_i$	$\psi_i$
1	6 mins	10344 m <sup>2</sup>	0.349 rad/min
2	25 mins	39352 m <sup>2</sup>	0.084 rad/min
3	15 mins	26317 m <sup>2</sup>	0.140 rad/min

TABLE I  
POOL MODEL PARAMETERS: DELAY ( $\tau_i$ ), SURFACE AREA ( $\alpha_i$ ) AND WAVE FREQUENCY ( $\psi_i$ )

occurs. At 150 min, an offtake of 19 MI/day starts. Then at 4100 min, the offtake increases to 38 MI/day. We see that for both disturbances the trend of the blue solid line (representing  $\frac{1}{\kappa_i} K_{\infty_i}^{12} (K_{\infty_i}^{22})^{-1} u_i$ ) closely follows the trend of the red dash-dotted line (representing  $\hat{w}_i^K$ ).

In the next subsection, we construct an offtake feedforward compensation, based on the above linear relationship between  $\hat{w}_i^K$  and  $u_i$  (i.e. the approximation (7)).

### B. The ad-hoc feedforward compensation

Following the ad-hoc idea for feedforward design given previously, it is straightforward to have the following representation of the distributed controller (with additional feedforward compensation):

$$\begin{pmatrix} \hat{w}_i^K \\ u_i \end{pmatrix} = \begin{bmatrix} K_{\infty_i}^{11} & \frac{1}{\kappa_i} K_{\infty_i}^{12} (K_{\infty_i}^{22})^{-1} F_i & \frac{1}{s} K_{\infty_i}^{12} \\ \kappa_i K_{\infty_i}^{21} & F_i & \frac{\kappa_i}{s} K_{\infty_i}^{22} \end{bmatrix} \begin{pmatrix} \hat{d}_i^K \\ d_i \\ e_i \end{pmatrix} \quad (8)$$

To see the performance of the additional feedforward compensation, we study the time responses of a string of three pools with distributed control. The three pools are taken from Eastern Goulburn No 12, Victoria, Australia. Table I gives the identified model parameters [13]. To shape the plant, we choose  $W_1 = \frac{12.605}{s}$ ,  $W_2 = \frac{2.761}{s}$ ,  $W_3 = \frac{5.130}{s}$ . A  $\gamma = 6.3$  is achieved by solving the structured optimisation problem (4). In the simulation, we simply select  $F_i = 0.75$  for  $i = 1, 2, 3$ .<sup>4</sup> Fig. 7 shows the transfer functions from  $d_3$  to  $\hat{w}_3^K$  and  $u_3$  respectively. Note that  $T_{d_3 \rightarrow \hat{w}_3^K}$  is non-minimum phase.

Fig. 8 shows the simulation results. In the simulation, the water-level setpoints in the three pools are set as  $r_{1,2,3} = 10$  m. At 100 min an offtake of 100 MI/day starts at pool<sub>3</sub>. With the additional feedforward compensation, the local performance is improved. Indeed, the maximal water-level error in pool<sub>3</sub> is 0.07 m, see the solid blue line; while without feedforward,  $\max_t |e_3(t)| = 0.11$  m. The better local performance from the additional feedforward did not impact the global performance. On the contrary, simulation results show a slight performance improvement in the upstream two pools, i.e. smaller water-level errors and slighter control actions are achieved with the feedforward compensation.

<sup>4</sup>In practice, hydraulic engineers use a constant gain (e.g.  $F_i = 0.5 \sim 1$ ) as feedforward compensator. To decrease the impact of delay uncertainty, a low pass filter should be added to the feedforward, see [6]

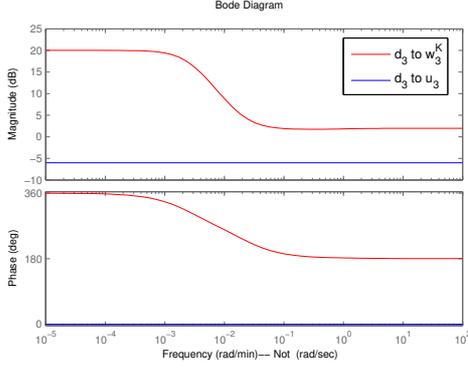


Fig. 7. The impact of  $d_3$  on  $\hat{w}_3^K$  and  $u_3$ , with feedforward compensation by the ad-hoc approach

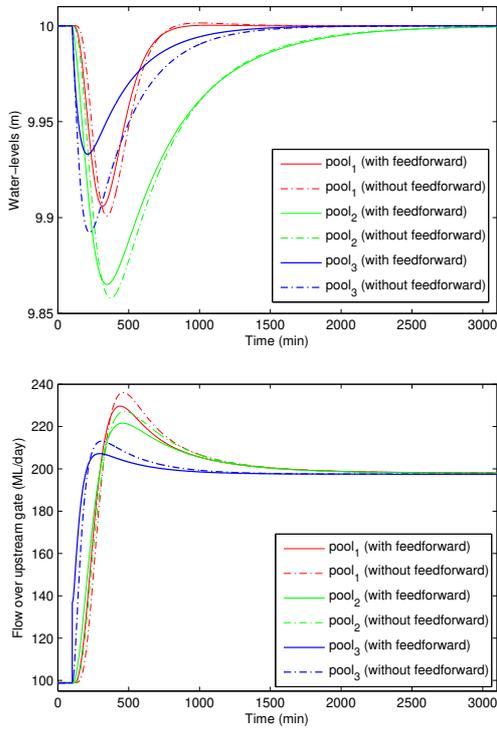


Fig. 8. The improvement of control performance with additional feedforward compensation (by the ad-hoc approach)

#### IV. A SYSTEMATIC APPROACH TO DESIGNING THE OFFTAKE FEEDFORWARD COMPENSATOR

In this section, a systematic approach to designing the feedforward is discussed. We follow the three-step procedure for designing distributed controller given in Section III. Fig. 9 shows the localised portion of a channel under distributed control with additional offtake feedforward.

- 1) Select  $W_i = \frac{\kappa_i}{s}$  for zero steady-state error. The local

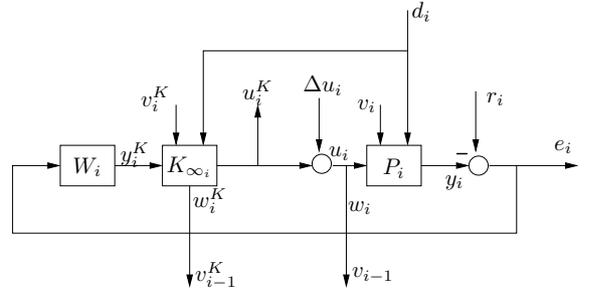


Fig. 9. Distributed controller design with  $d_i$  fed forward

shaped plant is given by

$$G_{s_i} := \begin{pmatrix} v_i \\ n_i \\ u_i^K \end{pmatrix} \mapsto \begin{pmatrix} w_i \\ z_i \\ d_i \\ y_i^K \end{pmatrix} = \begin{bmatrix} 0 & (0 \ 0 \ 1) & 1 \\ \left(\frac{1}{s\alpha_i}\right) & \begin{pmatrix} 1 & \frac{1}{s\alpha_i} & \frac{e^{-s\tau_i}}{-s\alpha_i} \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} e^{-s\tau_i} \\ -s\alpha_i \end{pmatrix} \\ \left(\frac{W_i}{s\alpha_i}\right) & \begin{pmatrix} 0 & 1 & 0 \\ W_i & \frac{W_i}{s\alpha_i} & \frac{e^{-s\tau_i}W_i}{-s\alpha_i} \end{pmatrix} & \begin{pmatrix} 1 \\ 0 \\ e^{-s\tau_i}W_i \\ -s\alpha_i \end{pmatrix} \end{bmatrix}$$

with  $v_i = w_{i+1}$ .

- 2) Synthesise  $K_{\infty_i}$  by solving the optimisation problem (4). Note that compared to the distributed controller  $K_{\infty_i}$  in Fig. 3, the synthesised controller has one more input  $d_i$ :

$$K_{\infty_i} : \begin{pmatrix} v_i^K \\ d_i \\ u_i^K \end{pmatrix} \mapsto \begin{pmatrix} w_i^K \\ u_i^K \end{pmatrix} = \begin{bmatrix} K_{\infty_i}^{11} & K_{\infty_i}^{12} & K_{\infty_i}^{13} \\ K_{\infty_i}^{21} & K_{\infty_i}^{22} & K_{\infty_i}^{23} \end{bmatrix}.$$

- 3) The final distributed controller can then be constructed:

$$\tilde{K}_i := \begin{pmatrix} v_i^K \\ d_i \\ e_i \end{pmatrix} \mapsto \begin{pmatrix} w_i^K \\ u_i^K \end{pmatrix} = K_{\infty_i} \begin{bmatrix} 1 & 1 \\ & W_i \end{bmatrix} \quad (9)$$

$$= \begin{bmatrix} K_{\infty_i}^{11} & K_{\infty_i}^{12} & \frac{\kappa_i}{s} K_{\infty_i}^{13} \\ K_{\infty_i}^{21} & K_{\infty_i}^{22} & \frac{\kappa_i}{s} K_{\infty_i}^{23} \end{bmatrix}.$$

Note that in Fig. 9 the position of  $W_i$  and that of  $K_{\infty_i}$  are swapped, compared to the nominal configuration of distributed controller shown in Fig. 3. This is important since otherwise  $\tilde{K}_i = \begin{bmatrix} K_{\infty_i}^{11} & K_{\infty_i}^{12} & K_{\infty_i}^{13} \\ \frac{\kappa_i}{s} K_{\infty_i}^{21} & \frac{\kappa_i}{s} K_{\infty_i}^{22} & \frac{\kappa_i}{s} K_{\infty_i}^{23} \end{bmatrix}$ ; the local feedforward from  $d_i$  to  $u_i$  (i.e. the 22-block) then involves an integrator and hence zero steady-state water-level error cannot be achieved when a step offtake disturbance occurs.

The performance of such designed distributed controller is studied next by simulation on the string of three pools introduced in Section III-B. The shaping functions are selected as  $W_1 = \frac{87.206}{s}$ ,  $W_2 = \frac{20.887}{s}$ ,  $W_3 = \frac{32.626}{s}$ . A  $\gamma = 3$  is achieved by solving the structured optimisation problem (4). Fig. 10 shows the synthesised transfer functions from  $d_3$  to  $w_3^K$  and  $u_3$  respectively. Both exhibit the properties of a low-pass filter. In the low and mid-frequency range where  $d_3$  is significant,  $T_{d_3 \rightarrow u_3}$  is a constant ( $= 0.37$ ). Fig. 11 shows the simulation results. At 100 min an offtake of 100 ML/day starts at pool<sub>3</sub>. With the additional feedforward compensation, the local performance is improved. The biggest water-level error in pool<sub>3</sub> is 0.08 m, see the solid blue line; while without feedforward,  $\max_t |e_3(t)| = 0.11$  m. We also see

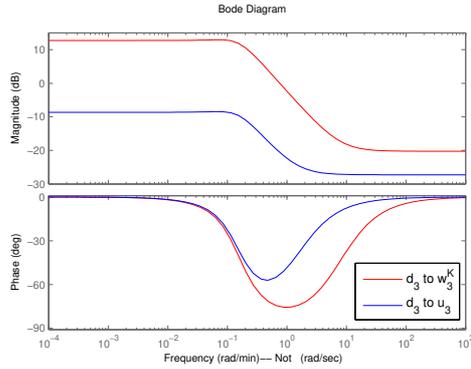


Fig. 10. The impact of  $d_3$  on  $w_3^K$  and  $u_3$ , with feedforward compensation by the systematic approach

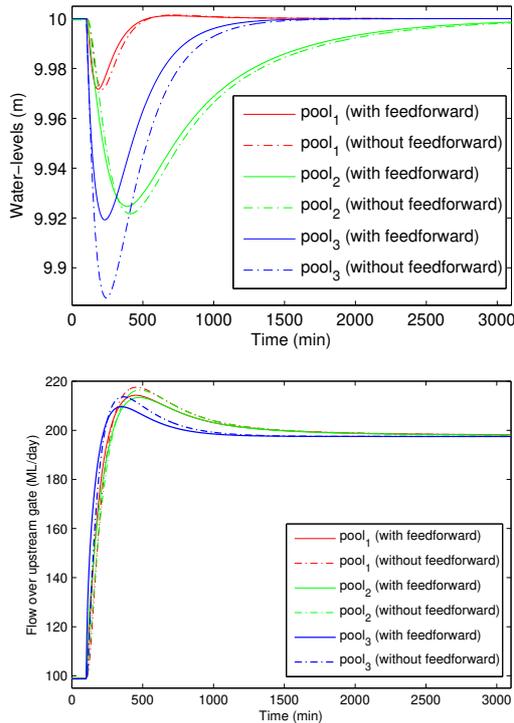


Fig. 11. The improvement of control performance with additional feedforward compensation (by the systematic approach)

a slight performance improvement in the upstream pool<sub>1,2</sub>, i.e. smaller water-level errors and slighter control actions are achieved with the feedforward compensation.

## V. SUMMARY

This paper explores the designing of a feedforward compensator to improve the local control performance for an irrigation system while maintaining the good management of closed-loop coupling between pools achieved by distributed control. The idea is that the water offtakes are usually measured and hence the knowledge of the offtakes can be fed forward to improve the transient response of local closed-loops, e.g. to attain a smaller deviation of water-level from the setpoint and a smaller flow change over the

upstream gate. Two approaches to designing the feedforward compensator are discussed. One approach tries to build a linear relationship between the control action ( $u_i$ ) and the interconnection between sub-controllers ( $w_i^K$ ) by changing the realisation of the distributed controller. Such a relationship is used to design the feed-forward compensator in an ad-hoc way. The other systematic approach, employing a different configuration of the distributed controller, synthesises the feed-forward compensation by solving a structured  $\mathcal{H}_\infty$  optimisation problem with local offtake disturbance  $d_i$  as an additional input to the controller  $K_{\infty_i}$ . Simulation results show that for both approaches, local performance is improved significantly with the additional feedforward compensation. Slight improvement of global performance, i.e. decoupling between subsystems, is also achieved.

It is worth to point out that the ad-hoc approach for designing the feedforward compensator gives a solution to extending the already designed distributed control system to a larger network. For example, when combining a distributed control system for a secondary channel with an already existed distributed control system for the main channel, with the ad-hoc approach, one just has to take the flow out to the secondary channel as a disturbance to the main channel. Hence there is no requirement to modify the already implemented controller.

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