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Control of a String of Identical Pools Using Non-Identical Feedback Controllers

Yuping Li and Bart De Schutter

***Abstract*— In the distant-downstream control of irrigation channels, the interactions between pools and the internal time-delay for water to travel from upstream to downstream, impose limitations on global performance, i.e. there exist propagation of water level errors and amplification of flows over gates in the upstream direction. This paper analyses these coupling properties for a string of identical pools, both with identical feedback controllers and with non-identical feedback controllers. The definition of string stability in terms of bounded water level errors and bounded flows is given. It is shown that for a string of infinite number of pools, string stability cannot be achieved by decentralised distant-downstream feedback control. However, for a string of finite number of pools, a better global performance can be achieved by non-identical feedback controllers such that the closed-loop bandwidths of the subsystems increase from downstream to upstream.**

I. Introduction

When designing decentralised feedback controllers for irrigation networks, one usually only takes local performance into account, i.e. regulating the water-level in a pool at its setpoint while rejecting offtake disturbances. Such a design might present very bad global performance, e.g. in response to offtake disturbances in the downstream pools, the gates in the upstream pools may go beyond saturation or the water-levels in the upstream pools may drop too low to satisfy the water demands. Therefore, in this paper, design of decentralised feedback controllers is discussed based on global performance considerations.

In large-scale irrigation networks, water is often distributed via open water channels under the power of gravity (i.e. there is no pumping). The flow of water through the network is then regulated by automated gates positioned along the channels [2], [6], [12]. The stretch of a channel between two gates is commonly called a pool. Water offtake points to farms and secondary channels are distributed along the pools.[1] As such, an important control objective is setpoint regulation of the water-levels immediately upstream of each gate, which enables flow demand at the (often gravity-powered) offtake points to be met without over-supplying. When the number of pools to be controlled is large and the gates are widely dispersed, it is natural to employ a decentralised control structure. In practice, a distant-downstream control structure (i.e. using the upstream gate to control the downstream water-level of a pool) is implemented for management of water service and water distribution efficiency [8]. Fig. 1 shows a side view of a channel under decentralised distant-downstream control. For such a control structure, when offtake disturbances occur in the downstream pools, the interactions between pools, due to the fact that the flow into one pool equals to the flow supplied by its upstream pool, and the internal time-delay for the transportation of water from upstream to downstream put requirements on managing the water-level error propagation and attenuating the amplification of flows over gates in the upstream direction, see [2], [3] for analysis of coupling between pools with distant-downstream control.

This paper studies the global control performance problem by analysing decentralised feedback control of a string of identical pools, for which we suggest a control strategy of using non-identical feedback controllers. A definition of string stability in terms of bounded water level errors and bounded flows is given. It is shown that string stability cannot be achieved for infinite number of pools with decentralised distant-downstream feedback control. However, for finite number of pools (which is true in practice), by designing the non-identical feedback controllers such that the closed-loop bandwidths of the subsystems increase from downstream to upstream, a much better global performance than that with identical feedback controllers can be achieved. Furthermore, we extend the analysis result to a string of heterogeneous pools and give guidelines for designing feedback controllers based on global performance.

The paper is organised as follows. Section II gives the definition of string stability in terms of bounded water level errors and bounded flows. Both the cases of a string of identical pools with identical feedback controllers and with non-identical feedback controllers are discussed. The global performance analysis is extended to a string of heterogeneous pools in Section III. Section IV shows simulation results. A brief summary is finally given in Section V.

II. Bounded water level errors and bounded flows

Consider $n+1$ pools. Denote the first downstream pool G_0, the second downstream pool G_1, and so on, till the most upstream pool, G_n. The sideview of the interconnected closed-loop system is shown in Fig. 1, where y_i is the water level in pool$_i$ and h_i is the head over gate$_i$.

Based on mass balance, a simple model of the water-level in pool$_i$ that captures the dynamics at low frequencies is

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[1]Typically at the downstream end of pools.

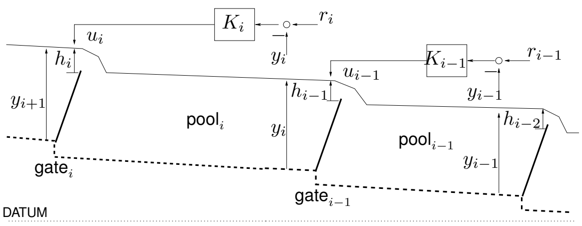

Fig. 1. Decentralised control of an open water channel

obtained (see [11]):

$$G_i:\ y_i(s) = \frac{c_i e^{-\tau_i s}}{s} u_i(s) - \frac{c_{i-1}}{s}\left(u_{i-1} + d_i\right)(s), \qquad (1)$$

where c_i and c_{i-1} are discharge coefficients, function of the pool surface area and the width of upstream and downstream gates respectively, and τ_i is the internal time-delay that the water takes to travel from the upstream end to the downstream end of the pool, $u_i := h_i^{3/2}$ is proportional to the flow over gate$_i$,[2] and d_i is the water offtake disturbance. Denote the water level error as $e_i := r_i - y_i$, where r_i is the water level setpoint. Essentially, the decentralised controller K_i is a PI compensator:

$$K_i : u_i(s) = \left(\kappa_i + \frac{\phi_i}{s}\right) e_i(s), \qquad (2)$$

with $\kappa_i > 0$ and $\phi_i > 0$; the integrator is included for zero steady-state water-level error in rejection to step load disturbance d_i, the phase-lead term helps for closed-loop stability.

As previously mentioned, the interaction between pools (i.e. the flow out of pool$_i$ equals to the flow into pool$_{i-1}$) influences the global performance of the closed-loop system. This is represented by the propagation of water level errors and the amplification of control actions in the upstream direction. To analyse the above coupling properties between pools, we study a string of identical pool with decentralised feedback control. In such case,

$$G_i:\ y_i(s) = \frac{c e^{-\tau s}}{s} u_i(s) - \frac{c}{s}\left(u_{i-1} + d_i\right)(s), \qquad (3)$$

with $c := c_i$ and $\tau := \tau_i$ for $i = 0, \dots, n$.

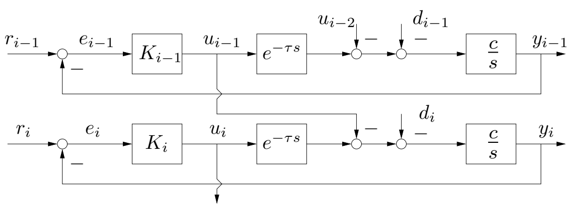

Fig. 2. Identical pools with decentralised feedback control

Fig. 2 shows the configuration of a string of identical pools with decentralised feedback control. The coupling transfer function from one closed-loop subsystem to the next one can be obtained as follows from the above pool model and the feedback controller:

$$T_{ee,i}(s) := \frac{e_i(s)}{e_{i-1}(s)} = \frac{\frac{c\left(\kappa_{i-1}s+\phi_{i-1}\right)}{s^2}}{1+\frac{c(\kappa_i s+\phi_i)}{s^2}e^{-s\tau}} \qquad (4)$$

$$T_{uu,i}(s) := \frac{u_i(s)}{u_{i-1}(s)} = \frac{\frac{c(\kappa_i s+\phi_i)}{s^2}}{1+\frac{c(\kappa_i s+\phi_i)}{s^2}e^{-s\tau}} \qquad (5)$$

Denote the coupling transfer functions from the first downstream pool to the n^{th} pool as $E_n(s) := \frac{e_n(s)}{e_0(s)}$ and $F_n(s) := \frac{u_n(s)}{u_0(s)}$.

Definition 2.1: Given a string of $n+1$ pools under centralised or decentralised control, if $\lim_{n\to\infty} |E_n(j\omega)| < \infty$ and $\lim_{n\to\infty} |F_n(j\omega)| < \infty$ for all $\omega \ge 0$, the system is said to be string stable in terms of bounded water level errors and bounded flows. ∎

For a string of pools with decentralised control one has

$$E_n(s) = \prod_{i=1}^{n} T_{ee,i}(s) = \prod_{i=1}^{n} \frac{c\left(\kappa_{i-1}s+\phi_{i-1}\right)}{s^2 + c\left(\kappa_i s+\phi_i\right)e^{-s\tau}} \qquad (6)$$

$$F_n(s) = \prod_{i=1}^{n} T_{uu,i}(s) = \prod_{i=1}^{n} \frac{c\left(\kappa_i s+\phi_i\right)}{s^2 + c\left(\kappa_i s+\phi_i\right)e^{-s\tau}}. \qquad (7)$$

A. Coupling of pools with identical feedback controllers

If one designs the decentralised controller based on local performance and if one takes the interaction between pools as an unknown disturbance, then for identical pools, it is natural to select K_i in (2) the same for $i = 0, \dots, n$, i.e.

$$u_i(s) = \left(\kappa_0 + \frac{\phi_0}{s}\right) e_i(s), \qquad (8)$$

where κ_0 and ϕ_0 are selected by just considering the local closed-loop system: regulating the water level in a pool to its setpoint while rejecting the offtake disturbances in the pool. Then the couplings between neighbouring pools are:

$$T_{ee}(s) = T_{uu}(s) = \frac{\frac{c(\kappa_0 s+\phi_0)}{s^2}}{1+\frac{c(\kappa_0 s+\phi_0)}{s^2}e^{-s\tau}} \qquad (9)$$

Similar as Lemma 1 in [2], we have the following result.

Lemma 2.2: For a string of identical pools with identical feedback controllers, there exists an $\omega > 0$ such that $|T_{ee}(j\omega)| > 1$ and $|T_{uu}(j\omega)| > 1$. ∎

Proof. The proof follows the lines of the proof for Lemma 9.3 of [4].

We first prove $\int_0^\infty \ln|T_{ee}(j\omega)|\frac{d\omega}{\omega^2} \ge 0$. Denote $L(s) := \frac{c(\kappa_0 s+\phi)}{s^2}$, then $T_{ee} = \frac{L(s)e^{-s\tau}}{1+L(s)e^{-s\tau}}e^{s\tau}$. Correspondingly,

$$|T_{ee}(j\omega)| = \left|\frac{L(j\omega)\exp(-j\tau\omega)}{1+L(j\omega)\exp(-j\tau\omega)}\right| \qquad (10)$$

for all $\omega \in \mathbb{R}$. Applying Cauchy's Theorem to the integral of the function $F(s) := \frac{1}{s^2}\ln\left(\frac{L(s)\exp(-\tau s)}{1+L(s)\exp(-\tau s)}\right)$ along the standard Nyquist contour C with infinitesimal indentation C_ϵ around the origin, we have

$$\oint_C F(s)ds = 0 = \int_{C_{i-}} F(s)ds + \int_{C_\epsilon} F(s)ds + \int_{C_\infty} F(s)ds,$$

where C_{i-} is the imaginary axis minus the indentation C_ϵ. Since $L(s)$ has two poles at the origin, the integral along C_ϵ

[2]We consider here open water channels with overshot gates, for which the flow over gate$_i$ can be approximated by $c_i h_i^{3/2}(t)$ [1]. Note that the discussions in this paper also work for channels with undershot gates.

is 0. By straightforward calculation, the integral along C_∞ is equal to $j\pi\tau$. Using the conjugate symmetry of the integrand and rearranging terms, yields

$$\int_0^\infty \ln\left|\frac{L(j\omega)\exp(-j\tau\omega)}{1+L(j\omega)\exp(-j\tau\omega)}\right|\frac{d\omega}{\omega^2} = \frac{\pi\tau}{2} > 0. \tag{11}$$

Indeed, $L(s)$ is strictly proper, hence $\ln|T_{ee}(j\omega)| < 0$ at high frequencies. It follows from (11) that there exists an $\omega_0 \in (0,\infty)$, such that $|T_{ee}(j\omega_0)| > 1$. From (9), $|T_{uu}(j\omega_0)| > 1$. ∎

Note that for the string of pools with identical feedback controllers, $E_n(s) = (T_{ee}(s))^n$. Hence there exists an $\omega > 0$ such that $\lim_{n\to\infty}|E_n(j\omega)|$ is unbounded. Similarly, there exists an $\omega > 0$ such that $\lim_{n\to\infty}|F_n(j\omega)|$ is unbounded. Following Definition 2.1,

Theorem 2.3: Consider a string of infinite number of pools (3) controlled by identical decentralised feedback controllers (8), the closed-loop system is string unstable. ∎

Let us consider a numerical example for a string of 101 identical pools. The model of the pools is given in (3) with the coefficient $c = 0.68$ and the transportation time delay $\tau = 20$ min. For local performance, select $\kappa_0 = 0.31$ and $\phi_0 = 8.2\times10^{-4}$ for the feedback controller (8). The magnitudes of the coupling transfer functions $T_{ee,i}(s)$ and $T_{uu,i}(s)$, for $i = 1,\ldots,100$, are shown in Fig. 3. It is observed that

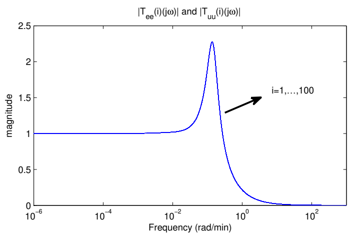

Fig. 3. Closed-loop coupling with identical feedback controllers

$\max_\omega|T_{ee,i}(j\omega)| \approx 2.28$. The maximum occurs at the same frequency, around 0.14 rad/min for all $i = 1,\ldots,100$. Hence $\max_\omega \frac{|e_{101}|}{|e_0|} = 2.28^{100}$ (and $\max_\omega \frac{|u_{101}|}{|u_0|} = 2.28^{100}$), which is intolerable in practice.

B. Coupling of pools with non-identical feedback controllers

In fact, a string of $n+1$ identical pools with identical feedback controllers involves the strongest coupling between pools, e.g. $\max_\omega|T_{ee,i}(j\omega)|(>1)$ occurs at the same ω for all i, which makes bounded water level errors impossible. To decouple the interaction and hence for global closed-loop performance, we consider non-identical feedback controllers as follows:

$$K_0 : u_0(s) = \left(\kappa_0 + \frac{\phi_0}{s}\right)e_0(s), \tag{12}$$

$$K_i : u_i(s) = \left((\kappa_0+\alpha i) + \frac{\phi_0}{s}\right)e_i(s) \text{ for } i = 1,\ldots,n \tag{13}$$

with $\alpha > 0$. Substituting (12-13) into (6-7) results in[3]

$$|E_n(j\omega)|^2 = \prod_{i=2}^{n}|T_{ee,i}(j\omega)|^2 = \prod_{i=2}^{n}\frac{i^2+A_e i+B_e}{i^2+Ci+D} \tag{14}$$

$$|F_n(j\omega)|^2 = \prod_{i=1}^{n}|T_{uu,i}(j\omega)|^2 = \prod_{i=1}^{n}\frac{i^2+A_f i+B_f}{i^2+Ci+D} \tag{15}$$

for $\omega > 0$, where

$$A_e = \frac{2\alpha c^2(\kappa_0-\alpha)\omega^4 + \frac{8\alpha c^2(\kappa_0-\alpha)}{\tau^2}\omega^2}{\alpha^2c^2\omega^4 + \frac{4\alpha^2c^2}{\tau^2}\omega^2}$$

$$B_e = \frac{(\kappa_0-\alpha)^2c^2\omega^4 + \left(\phi_0^2 + \frac{4(\kappa_0-\alpha)^2}{\tau^2}\right)c^2\omega^2 + \frac{4\phi_0^2c^2}{\tau^2}}{\alpha^2c^2\omega^4 + \frac{4\alpha^2c^2}{\tau^2}\omega^2}$$

$$A_f = \frac{2\kappa_0\alpha c^2\omega^4 + \frac{8\kappa_0\alpha c^2}{\tau^2}\omega^2}{\alpha^2c^2\omega^4 + \frac{4\alpha^2c^2}{\tau^2}\omega^2}$$

$$B_f = \frac{\kappa_0^2c^2\omega^4 + \left(\phi_0^2 + \frac{4\kappa_0^2}{\tau^2}\right)c^2\omega^2 + \frac{4\phi_0^2c^2}{\tau^2}}{\alpha^2c^2\omega^4 + \frac{4\alpha^2c^2}{\tau^2}\omega^2}$$

$$C = \frac{\left(2\kappa_0\alpha c^2 - \frac{8\alpha c}{\tau}\right)\omega^4 + \frac{8\kappa_0\alpha c^2}{\tau^2}\omega^2}{\alpha^2c^2\omega^4 + \frac{4\alpha^2c^2}{\tau^2}\omega^2}$$

$$D = \frac{\omega^6 + \left(\kappa_0^2c^2 - \frac{8\kappa_0 c}{\tau} + \frac{4}{\tau^2} + 2\phi_0 c\right)\omega^4}{\alpha^2c^2\omega^4 + \frac{4\alpha^2c^2}{\tau^2}\omega^2} + \frac{\left(\frac{4\kappa_0^2c^2-8\phi_0 c}{\tau^2} + \phi_0^2c^2\right)\omega^2 + \frac{4\phi_0^2c2}{\tau^2}}{\alpha^2c^2\omega^4 + \frac{4\alpha^2c^2}{\tau^2}\omega^2}$$

Note that for $\omega = 0$, $\lim_{n\to\infty}|E_n(j\omega)| = \lim_{n\to\infty}|F_n(j\omega)| = 1$.

The following sufficient conditions for bounded water level errors (in Lemma 2.4) and bounded flows (in Lemma 2.5) use properties (a) and (b) of the Gamma function defined in (16).

(a) If the real part of the complex number x is positive (i.e. Re$[x] > 0$), then the integral

$$\Gamma(x) := \int_0^\infty e^{-t}t^{x-1}dt \tag{16}$$

converges absolutely.

(b) Using integration by parts, $\Gamma(x+1) = x\Gamma(x)$.

Lemma 2.4: For a fixed $\omega > 0$, $\lim_{n\to\infty}|E_n(j\omega)|$ exists if $A_e(\omega) > 0$, $C(\omega) > 0$, $D(\omega) > 0$ and $A_e(\omega) < C(\omega)$. ∎

Proof. For the case of $A_e(\omega) = C(\omega)$, one has

$$|E_n(j\omega)|^2 = \prod_{i=1}^{n}\left[1 + \frac{B_e - D}{(i-y_1)(i-y_2)}\right]. \tag{17}$$

When $n\to\infty$, expression (17) corresponds to equation (89.5.7) of [5], which gives

$$\lim_{n\to\infty}|E_n(j\omega)|^2 = \frac{\Gamma(1-y_1)\Gamma(1-y_2)}{\Gamma(1-x_1)\Gamma(1-x_2)}. \tag{18}$$

Note the convergence of the RHS of (18) is not ensured. Indeed, even for the case that $A_e(\omega) > 0$ (and equivalently

[3]Here we use a first-order Padé approximation to represent the transportation time-delay τ. Such an approximation does not change the analysis result in practice given that the offtake disturbance that induces e_0 is significant at low frequency range, while the high-frequency resonances caused by time-delay are dampened by the feedback controller with a low-pass filter, see Section IV.

$C(\omega) > 0$) and $D(\omega) > 0$,[4] and hence $\mathrm{Re}[x_1] < 0$, $\mathrm{Re}[x_2] < 0$, $\mathrm{Re}[y_1] < 0$, $\mathrm{Re}[y_2] < 0$; one only has that $\Gamma(1-x_1)\Gamma(1-x_2)$ and $\Gamma(1-y_1)\Gamma(1-y_2)$ converge respectively, based on the previous property (a) of the Gamma function. However, $\frac{\Gamma(1-y_1)\Gamma(1-y_2)}{\Gamma(1-x_1)\Gamma(1-x_2)}$ might still diverge.

For $A_e(\omega) \neq C(\omega)$, one has

$$|E_n(j\omega)|^2 = \prod_{i=1}^{n} \left(1 + \frac{y_1 - x_1}{i - y_1}\right)\left(1 + \frac{y_2 - x_2}{i - y_2}\right) \tag{19}$$

where x_1, x_2 are the roots of $x^2 + A_e x + B_e = 0$, and y_1, y_2 are the roots of $y^2 + Cy + D = 0$. Note that expression (19) corresponds to equation (89.9.1) of [5], i.e.

$$|E_n(j\omega)|^2 = \frac{\Gamma(1-y_1)\Gamma(1-y_2)}{\Gamma(1-x_1)\Gamma(1-x_2)} \cdot \frac{\Gamma(n+1-x_1)\Gamma(n+1-x_2)}{\Gamma(n+1-y_1)\Gamma(n+1-y_2)}. \tag{20}$$

Applying the previous properties (a) and (b) of the Gamma function to the RHS of (20), it is direct that when $0 < A_e(\omega) < C(\omega)$ and $D(\omega) > 0$, $\lim_{n\to\infty} |E_n(j\omega)|^2 = 0$; while the limitation does *not* exist for the case of $A_e(\omega) > C(\omega)$. The lemma is thus proved. ∎

Remark 1: a) For all $\omega > 0$, the condition $A_e(\omega) > 0$ holds *if and only if* $\kappa_0 > \alpha$.

b) For all $\omega > 0$, the condition $A_e(\omega) < C(\omega)$ holds *if and only if* $-2\alpha^2 c^2 \omega^4 - \frac{8\alpha^2 c^2}{\tau^2}\omega^2 < -\frac{8\alpha c}{\tau}\omega^4$, which is equivalent to

$$\alpha c - \frac{4}{\tau} > -\frac{4\alpha c}{\tau^2}\omega^{-2}. \tag{21}$$

Since $\alpha > 0$, $c > 0$ and $\omega > 0$, (21) holds if $\alpha c - \frac{4}{\tau} \geq 0$.

c) Note the denominator of $D(\omega) > 0$ for $\omega > 0$. The numerator of $D(\omega)$ can be written as

$$\left(\omega^3 - \frac{2\kappa_0 c}{\tau}\omega\right)^2 + \left(\frac{2}{\tau}\omega^2 - \frac{2\phi_0 c}{\tau}\right)^2 + \left(2\phi_0 c\omega^2 + \phi_0^2 c^2\right)\omega^2 + \left(\kappa_0^2 c^2 - \frac{4\kappa_0 c}{\tau}\right)\omega^4.$$

For all $\omega > 0$, the condition $D(\omega) > 0$ holds if $\kappa_0 \geq \frac{4}{c\tau}$.

From the above points a), b) and c), if $\kappa_0 > \alpha \geq \frac{4}{c\tau}$, then the conditions $0 < A_e(\omega) < C(\omega)$ and $D(\omega) > 0$ hold for all $\omega > 0$. ◦

Similarly, one has the following result for bounded flows.

Lemma 2.5: For a fixed $\omega > 0$, $\lim_{n\to\infty} |F_n(j\omega)|$ exists if $0 < A_f(\omega) < C(\omega)$ and $D(\omega) > 0$. ∎

Proof. The proof follows the same lines as the proof of Lemma 2.4. ∎

Remark 2: For $\omega > 0$, the condition $A_f(\omega) < C(\omega)$ holds *if and only if* $0 < -\frac{8\alpha c}{\tau}\omega^4$, which is impossible given the assumption that $\alpha > 0$. ◦

In fact, under distant-downstream control, to compensate the influence of the internal time-delay, the amplification of control action in the upstream direction is unavoidable. This is shown in Fig. 4. Initially, the system is at steady-state. At time t_s, the flow out of pool$_i$ increases, see the change of u_{i-1} (the dashed line in Fig. 4(a)). To compensate for the influence of u_{i-1} on y_i, the flow into the pool, u_i, also increases (the solid line in Fig. 4(a)). However, the influence of u_i on the downstream water-level y_i will be τ_i (min) later than that of u_{i-1} on y_i (see Fig. 4(b)). For zero steady-state error of y_i from r_i (see Fig. 4(c)), u_i should be greater than u_{i-1} for some time such that the area of A_{u_i} is equivalent to the area of $A_{u_{i-1}}$. Hence, there exists $\omega > 0$ such that $\lim_{n\to\infty} |F_n(j\omega)|$ is unbounded. Then to have bounded water level errors for infinite number of identical pools with decentralised control, the energy of the control action goes to infinity, which is impossible in practice. Indeed, for robust stability of the closed-loop, one has the condition on the closed-loop bandwidth such that $\omega_b < \frac{1}{\tau}$ (see [10]). However, with the condition that $\alpha \geq \frac{4}{c\tau}$, the bandwidths of the string of pools increase from downstream to upstream. Hence, for a string of infinite number of pools, there exists an $N < \infty$, such that the temporal stability condition for the subsystems $i > N$ is not satisfied.

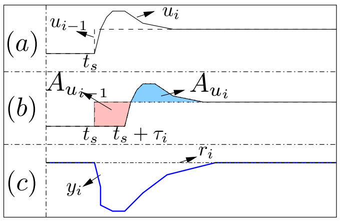

Fig. 4. Control actions for zero steady-state water-level error

From the above discussions and Definition 2.1, the following conclusion is obtained.

Theorem 2.6: For a string of infinite number of pools (3) controlled by the decentralised feedback controller (12-13), the closed-loop system is string unstable. ∎

Consider the numerical example given in Section II-A for a string of 101 identical pools. Select $\kappa_0 = 0.31$, $\phi_0 = 8.2 \times 10^{-4}$ and $\alpha = 0.29$ for the feedback controller in (12-13). The magnitudes of the coupling transfer functions $T_{ee,i}(s)$ and $T_{uu,i}(s)$, for $i = 1, \ldots, 100$, are shown in Fig. 5. The decoupling function of applying non-identical feedback controller is observed. Indeed, for all $i = 1, \ldots, 100$, $|T_{ee,i}(j\omega)| \leq 1$ for all $\omega \geq 0$. Hence, we can expect a decreasing propagation of the water-level errors in the upstream direction, which is confirmed by the top graph of $|E_n(j\omega)|$ in Fig. 6. Furthermore, an attenuation of the amplification of the control action (i.e. flows over gates) is also achieved, see in the bottom graph in Fig. 6 that $\max_\omega \frac{|u_{101}|}{|u_0|} = 17.3$, while as analysed in Section II-A, $\max_\omega \frac{|u_{101}|}{|u_0|} = 2.28^{100}$ for the case with identical feedback controllers.

[4] Note $Be(\omega) > 0$ for all $\omega > 0$.

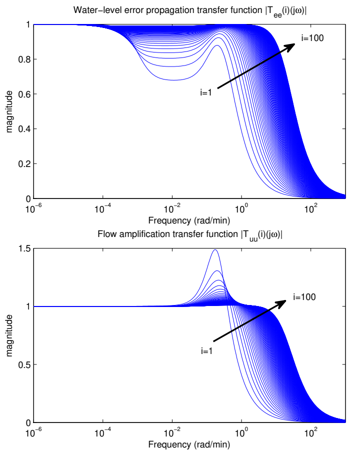

Fig. 5. Closed-loop coupling with non-identical feedback controllers

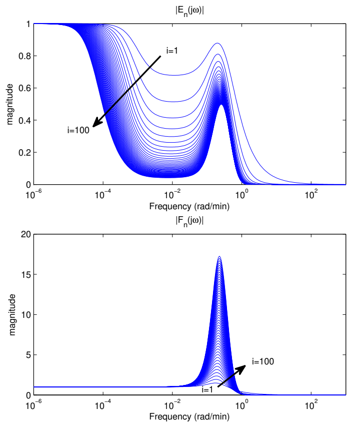

Fig. 6. $|E_n(j\omega)|$ and $|F_n(j\omega)|$ for $n = 1, \ldots, 100$ with non-identical feedback controllers

III. Further Discussion

In reality, the number of the pools in a channel is finite. When designing decentralised feedback control for a string of $n+1$ $(< \infty)$ similar pools (in terms of e.g. pool lengths, gate properties, etc.), one can first select κ_0, ϕ_0 such that the local performance in terms of setpoint regulation is guaranteed. Then select an $\alpha > 0$ such that the tradeoff between local and global performance is managed, i.e. with the selected α, the bandwidth of the upstream closed-loop $\omega_{b,n} < \frac{1}{\tau_n}$. Indeed, by including an $\alpha > 0$ in the non-identical feedback controllers, the bandwidths of the closed-loop subsystems increase in the upstream direction; hence, one can expect a faster response of the interconnected system to the offtake disturbance in the downstream pools than the case with identical feedback controllers. Note that in the distributed control strategy discussed in [2], [7], such a speeding up of the closed-loop response is achieved by involving the known interaction between neighbouring pools in the input signals to be rejected and by solving an optimisation problem to manage the tradeoff between local and global performance.

One can extend the analysis results to channels with heterogeneous pools: For a channel with distant-downstream control, given that the temporal stability is ensured for each subsystem, one can guarantee good global performance, i.e. management of the water level error propagation and attenuation of the amplification of flows over gates in the upstream direction, by ensuring that the closed-loop bandwidths increase from downstream to upstream.

Remark 3: For a channel in which the pool lengths increase from upstream to downstream, the above condition that the closed-loop bandwidths increase from downstream to upstream can be satisfied even by simply designing the decentralised feedback controllers just based on local performance. In reality, based on the consideration of storing water to satisfy demands from farms, civil engineers design irrigation networks such that the pool lengths, in general, tend to decrease from upstream to downstream. However, the previous guidelines for decentralised feedback control design should still be kept in mind for a good tradeoff between local and global performance. ◦

IV. Simulation Results

In this section, simulation results are shown for the case of a string of 5 identical pools with identical feedback controllers and for the case with non-identical feedback controllers. In the simulations, a third-order model that captures the dominant wave-frequency dynamics in the pools is used. The parameters of the pool is given in Table I.[5] Saturations are set for gate positions and flows over gates.

TABLE I
Parameters of the Pool and Saturation Values Set

Pool length	Wave frequency	τ	c
3129 m	0.20 rad/min	16 min	0.0092
Saturations of gate positions		Saturations of flows	
max (m)	1.487	max (Ml/day)	300
min (m)	0	min (Ml/day)	0

Correspondingly, the feedback controllers involve an extra low-pass filter $\frac{1}{s+0.125}$ to guarantee no excitement of waves, i.e. a low gain at high frequencies. Hence, a) the identical feedback controllers are set as

$$K_i(s) = \left(0.1050 + \frac{0.0008}{s}\right)\frac{1}{s+0.125} \text{ for } i = 0, 1, \ldots, 4;$$

b) while the non-identical feedback controllers are set as

$$K_0(s) = \left(0.1050 + \frac{0.0008}{s}\right)\frac{1}{s+0.125}, \text{ and for } i = 1, \ldots, 4$$

$$K_i(s) = \left((0.1050 + 0.1i) + \frac{0.0008}{s}\right)\frac{1}{s+0.125}.$$

Fig. 7 and 8 give the closed-loop responses to an offtake disturbance in the downstream pool. Clearly, a much better decoupling performance is obtained by the strategy with the non-identical feedback controllers.

Fig. 7 shows the water level errors in the five pools when an offtake of 75 Ml/day at the downstream pool begins at time 200 min. The water level setpoints for the pools are set

[5] The parameters of the pool is the same as that identified for pool_{10} of the Haughton Main Channel, see [9].

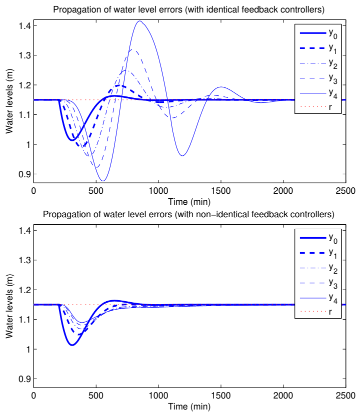

Fig. 7. Water-level error propagation, with identical feedback controllers (top graph) and with non-identical feedback controllers (bottom graph)

the same: $r = 1.15$ m. Note that the local water level error in the downstream pool (i.e. $r - y_0$) is the same for identical and non-identical feedback controllers. With identical feedback controllers (the top graph), the water level errors in the pools increase in the upstream direction. In the upstream pool, the maximum water level error caused by the offtake is 0.28 m. In contrast, with the non-identical feedback controllers (the bottom graph), the water level errors in the pools decrease in the upstream direction. In the upstream pool, the maximum water level error caused by the offtake is 0.06 m.

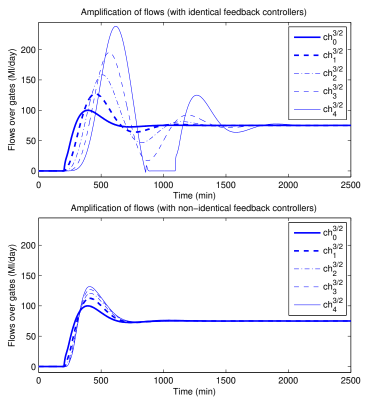

Fig. 8. Flow amplification, with identical feedback controllers (top graph) and with non-identical feedback controllers (bottom graph)

Fig. 8 shows the amplification of flows to compensate the influence of the offtake of 75 Ml/day at the downstream pool begins at time 200 min. With identical feedback controllers (top graph), the amplification of flows is significant, e.g. the maximum flow over the most upstream gate is 240 Ml/day around 600 min; more seriously, the flow over the most upstream gate goes beyond saturation from 870 min to 1170 min. While with non-identical feedback controllers (bottom graph), the amplification of flows over gates is well attenuated, e.g. the maximum flow over the most upstream gate is 130 Ml/day around 450 min. Note that, as expected, the control actions in the upstream pools, i.e. $ch_i^{3/2}(t)$ for $i = 1, \ldots, 4$, in response to the offtake disturbance are faster than the case with identical controllers.

V. Conclusions

This paper discusses the designing of decentralised feedback controllers for a string of identical pools based on the global performance of managing water-level error propagation and attenuating the amplification of flows over gates in the upstream direction. A definition of string stability in terms of bounded water level errors and bounded flows is given. It is shown that for infinite number of pools with decentralised distant-downstream feedback control, the closed-loop bandwidth limitation of each subsystem, imposed by the internal time-delay, makes it impossible to achieve string stability. However, for finite number of pools, by selecting non-identical feedback controllers such that the closed-loop bandwidths of the subsystems increase from downstream to upstream, a better global performance than that with identical feedback controllers is achieved. Furthermore, the analysis result is extended to a string of heterogeneous pools: In general, for distant downstream control, the management of water-level error propagation and the attenuation of the amplification of flows over gates in the upstream direction require the closed-loop bandwidths to increase from downstream to upstream.

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