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Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: https://www.dcsc.tudelft.nl

Nonlinear MPC for the improvement of dispersion of freeway traffic emissions

S. K. Zegeye * **B. De Schutter** * **J. Hellendoorn** * **E. A. Breunesse** **

* *Delft University of Technology, 2628 CD Delft, The Netherlands (e-mail: s.k.zegeye@gmail.com, b.deschutter@tudelft.nl, j.hellendoorn@tudelft.nl).*
** *Shell Nederland B.V., Carel van Bylandtlaan 30, 2596 HR The Hague, The Netherlands, (e-mail: ewald.breunesse@shell.com)*

Abstract: In this paper a model-based traffic control is used to design variable speed limits and on-ramp metering rates in order to reduce road traffic generated area-wide emissions near freeways. First an area-wide emission model is proposed and next a nonlinear model predictive control (MPC) approach is applied. The objectives of the MPC controller considered are the emissions, dispersions of emissions in a public area near a freeway, travel times, or the combination of these performance indicators. We compare different controlled scenarios with respect to the uncontrolled case and with respect to each other. The simulation-based case studies indicate that balanced solutions can be obtained using the proposed nonlinear MPC control strategy.

Keywords: Model-based control, nonlinear MPC, traffic control, emissions, dispersion, travel time

1. INTRODUCTION

Due to several reasons, it is believed that intelligent transportation systems are one of the most sound approaches to address most traffic related problems Kotsialos et al. (2002). In intelligent transportation systems, different traffic flow control measures (such as traffic signal, ramp metering, speed limits, route guidance, etc.) can be used to minimize the impact of traffic jams (such as area-wide emissions, fuel consumption, and longer travel times) and to optimize the performance of the traffic network.

In traditional traffic-assignment problems, only a single objective — travel time or travel cost — is considered Benedek and Rilett (1998); Tzeng and Chen (1993). However, a reduction of the travel time or an improvement in the throughput of traffic does not always guarantee reduction of the traffic-related nuisances such as air pollution, noise, and fuel consumption Ahn and Rakha (2008); Benedek and Rilett (1998). In this context De Schutter et al. (2010) have illustrated that it is possible to obtain a balanced trade-off between traffic generated emissions and travel time. In that work, the dispersion of the emissions to the neighborhoods of the freeways was not investigated. In Zegeye et al. (2010) it has been demonstrated that a model predictive controller can be used to also reduce the dispersion levels using only variable speed limits under the assumption that the wind speed and wind direction is constant.

Furthermore, although reducing the amount of emitted gases of each link in the traffic network will improve the overall total emissions of the whole traffic network, the dispersion of the emissions can be distributed unevenly and consequently higher emission levels can be experienced in some areas. For example, public areas could face higher emission levels (pollution) despite the reduced total emission levels, because other factors such as wind and temperature can affect the concentration of the locally emitted gases.

Moreover, most often public areas (such as schools, hospitals, parks) are small in size as compared to the nearby freeways. In such cases, by simply reducing the total emissions of the freeways one can reduce the dispersion levels in the public areas, but the traffic flow can be affected unnecessarily, because the reduction of emissions will focus on the reduction of the emissions over the whole nearby collection of freeways. However, the part of the freeway that affects the public area may vary with other factors, predominantly with the variation of the wind speed and the wind direction. Therefore, it is unwise to affect the traffic flow (or compromise the travel time) of the whole collection of freeways at all times. It is better to only focus on the part of the freeway that affects the public area dynamically. This could be done by predicting the wind speed and wind direction and identifying the freeway segments that affect the public area. In such a way, only the part of the freeway that has a negative impact on the public areas can be controlled dynamically to attain reduced traffic pollution and improved travel times or a good balance between these two objectives.

To this end, we propose to use a model-based traffic control approach to dynamically control variable speed limits and on-ramp metering rates of a freeway traffic system. In particular nonlinear model predictive control is proposed to provide a balanced trade-off between emissions, dispersion of emissions, and travel time. In order to predict and estimate the dispersion of the emissions, a simple area-wide emission model is presented. Unlike the model used in De Schutter et al. (2010), the area-wide emission model proposed in this paper accounts for the variation of the speed and direction of the wind. We further illustrate the proposed control approach and the dispersion model with a simulation-based case study by using variable speed limits and on-ramp metering.

2. MPC FOR TRAFFIC SYSTEMS

Model predictive control (MPC) Camacho and Bordons (1995) is a dynamic control approach that optimizes the control inputs

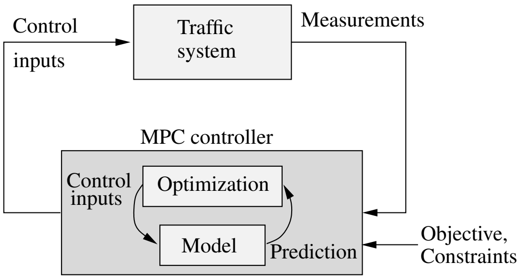

Fig. 1. Conceptual representation of model predictive control.

based on predicted and current states of the system. The basic concept can be explained with the help of Fig. 1 as follows. The MPC controller incorporates models of the traffic flow, emission, and dispersion of emissions. At control time step k_c (corresponding to the time instant $t = k_\mathrm{c}T_\mathrm{c}$ where T_c is the control sampling time), the controller collects measurements (such as density, flow, emission, etc.) of the traffic system through sensors. Based on the obtained or estimated current state and using the models, the controller predicts the future evolution of the traffic states up to time step $k_\mathrm{c} + N_\mathrm{p}$, where N_p denotes the prediction horizon. Using on-line optimization techniques the controller generates a sequence of traffic control inputs that minimize a given objective function over the prediction period. But, only the first control input is applied to the traffic system. At the next control time step $k_\mathrm{c}+1$, the controller again collects the newly changed traffic state, and it repeats the above process all over again. In this way, the controller continuously updates the control inputs based on the continuously changing demand and traffic dynamics of the system.

MPC has already been applied for traffic systems in the literature Bellemans et al. (2006); Gartner (1984); Hegyi et al. (2005); Kotsialos et al. (2002). It has proved, based on simulations, to be a potential control solution for traffic systems. Its main advantages are that it can handle constraints (such as maximum emission levels), it can be applied to nonlinear models (e.g. nonlinear traffic flow, emission, and dispersion models), and it can also be used to address multi-criteria optimization (such as minimization of emissions, dispersion, and travel time).

The performance measure for an MPC controller can be defined in different ways. One can consider a weighted sum of the travel time, emissions, fuel consumptions, and dispersion level. This could mathematically be described as

$$\begin{aligned} J(k_\mathrm{c}) = &\zeta_1 \frac{\mathrm{TTS}(k_\mathrm{c})}{\mathrm{TTS}_\mathrm{n}} + \zeta_2 \frac{\mathrm{TE}(k_\mathrm{c})}{\mathrm{TE}_\mathrm{n}} + \zeta_3 \frac{\mathrm{TFC}(k_\mathrm{c})}{\mathrm{TFC}_\mathrm{n}} \\ &+ \zeta_4 \frac{\mathrm{DL}(k_\mathrm{c})}{\mathrm{DL}_\mathrm{n}} + \zeta_5 \frac{\Delta(k_\mathrm{c})}{\Delta_\mathrm{n}} \end{aligned} \tag{1}$$

where $\zeta_n \geq 0$ for $n = 1,2,3,4,5$ are weight coefficients, $\mathrm{TTS}(k_\mathrm{c})$, $\mathrm{TE}(k_\mathrm{c})$, $\mathrm{TFC}(k_\mathrm{c})$, and $\mathrm{DL}(k_\mathrm{c})$ are respectively the total time spent, the total emissions, the total fuel consumption, and the dispersion level over the period $[T_\mathrm{c}k_\mathrm{c}, T_\mathrm{c}(k_\mathrm{c}+N_\mathrm{p})]$, $\Delta(k_\mathrm{c})$ denotes the change of the control input over time and space, and the subscript 'n' is used to denote the nominal values of the respective variables (i.e. the values of the variables obtained under nominal operation of the system, where no controller is implemented).

The dispersion level $\mathrm{DL}(k_\mathrm{c})$ can be described in different ways depending on the intentions of the performance criterion. In some cases it may be important to focus only on the reduction of the maximum dispersion levels; in some other cases it can be equally or more important to reduce the cumulative exposure of an area to dispersion of emissions; or a combination of the two cases could be required. So, depending on the regulations to be adopted the formulation of $\mathrm{DL}(k_\mathrm{c})$ in (1) can be different.

In the case study of Section 5 we will use a traffic flow model and an emission and fuel consumption model described in Section 3, and an area-wide emission model presented in Section 4. Note however that the MPC approach is generic and can also accommodate other, more complex traffic flow, emission, and dispersion models.

3. MODELS

3.1 METANET traffic flow model

The METANET model Messmer and Papageorgiou (1990) is a second-order macroscopic traffic flow model that describes the average behavior of vehicles in a traffic network. In this model, a freeway is divided into links, where a link is a part of the freeway with homogeneous geometric characteristics. A node is placed at the points where links join together. The links are further divided into a number of segments where the traffic behavior in each segment is described by a system of dynamic equations. These dynamic equations describe the density, flow, and space-mean speed of the traffic flow in each segment at every simulation time step k. The simulation time step k and the control time step k_c are assumed to be related as $k_\mathrm{c} = \lfloor \frac{k}{M} \rfloor$, where M is a positive integer and $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

In METANET, the dynamic equations that govern the traffic variables for every segment i of link m are given by

$$q_{m,i}(k) = \lambda_m \rho_{m,i}(k) v_{m,i}(k) \tag{2}$$

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_m \lambda_m}\left[q_{m,i-1}(k) - q_{m,i}(k)\right] \tag{3}$$

$$\begin{aligned} v_{m,i}(k+1) = v_{m,i}(k) &+ \frac{T}{\tau}[V[\rho_{m,i}(k)] - v_{m,i}(k)] \\ &+ \frac{T v_{m,i}(k)\left[v_{m,i-1}(k) - v_{m,i}(k)\right]}{L_m} \\ &- \frac{T\eta\left[\rho_{m,i+1}(k) - \rho_{m,i}(k)\right]}{\tau L_m\left(\rho_{m,i}(k) + \kappa\right)} \end{aligned} \tag{4}$$

$$V[\rho_{m,i}(k)] = v_{\mathrm{free},m} \exp\left[-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{\mathrm{cr},m}}\right)^{a_m}\right] \tag{5}$$

where $q_{m,i}(k)$, $\rho_{m,i}(k)$, and $v_{m,i}(k)$ denote the flow, density, and space-mean speed of segment i of link m at simulation time step k, $V[\rho_{m,i}(k)]$ denotes the desired speed of the drivers in segment i of link m, T denotes the simulation time, L_m denotes the length of the segments of link m, and λ_m denotes the number of lanes of the link. Furthermore, $v_{\mathrm{free},m}$ is the free-flow speed, $\rho_{\mathrm{cr},m}$ the critical density, τ a time constant, η the anticipation constant, a_m the parameter of the fundamental diagram, and κ is a model parameter.

If a segment i of link m is controlled by a variable speed limit $u_{m,i}(k)$, the desired speed in (5) is modified as Hegyi et al. (2005)

$$\begin{aligned} V[\rho_{m,i}(k)] = \min\Bigg\{ &v_{\mathrm{free},m} \exp\left[-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{\mathrm{cr},m}}\right)^{a_m}\right], \\ &(1+\alpha_m) u_{m,i}(k) \Bigg\} \end{aligned} \tag{6}$$

where α_m is the drivers' non-compliance factor.

Since the demand at the origin of a link (or freeway) or at the on-ramp can exceed the capacity or the number of vehicles that can enter the freeway, a queue may develop. The dynamics of the queue w_o is modeled as

$$w_o(k+1) = w_o(k) + T(d_o(k) - q_o(k)) \tag{7}$$

where $d_o(k)$ is the demand at the origin and $q_o(k)$ is the outflow of the origin. The outflow $q_o(k)$ is expressed as

$$q_o(k) = \min\left[d_o(k) + \frac{w_o(k)}{T}, r_o(k)C_o, C_o\left(\frac{\rho_{\mathrm{jam},m} - \rho_{m,1}(k)}{\rho_{\mathrm{jam},m} - \rho_{\mathrm{cr},m}}\right)\right] \tag{8}$$

with C_o is the capacity flow of the origin, $\rho_{\mathrm{jam},m}$ is the maximum density of link m, and $r_o(k)$ the on-ramp metering rate. The on-ramp metering rate is set to be $r_o(k) = 1, \forall k$ for uncontrolled on-ramps and mainstream origins, and for controlled on-ramps we have $r_o(k) \in [0,\ 1]$.

3.2 *Emission and fuel consumption model*

Traffic emission and fuel consumption models calculate the emissions produced and the fuel consumed by vehicles based on the operating conditions of the vehicles. The main inputs to the models are the operating conditions of the vehicle (such as speed, acceleration, engine load) Heywood (1988).

Since the inputs of emission and fuel consumption models are the outputs of traffic flow models, the choice of traffic flow models dictates the type of emission and fuel consumption models that has to be used. Since the METANET traffic flow model is a macroscopic model, we choose VT-macro Zegeye et al. (2009) as emission and fuel consumption model. The VT-macro model is a macroscopic emission and fuel consumption model that we have in particular developed for the METANET traffic flow model. The model takes the dynamics of the average space-mean speed of the traffic flow model into account. The inputs of the VT-macro model are the average space-mean speed, average acceleration, and the number of vehicles subject to the speed and acceleration pairs. These variables are computed from the space-mean speed, density, and flow variables of the METANET model.

Mathematically, the VT-macro model can be compactly described as

$$J_{y,m,i}(k) = f(v_{m,i}(k), v_{m,i}(k+1), v_{m,i+1}(k+1), \rho_{m,i}(k)) \tag{9}$$

where $J_{y,m,i}(k)$ [kg/s or l/s] is the estimate or prediction of the variable $y \in \mathscr{Y} = \{\mathrm{CO}, \mathrm{NO}_x, \mathrm{HC}, \mathrm{CO}_2, \text{Fuel consumption}\}$ for vehicles in segment i of link m during the time period $[kT, (k+1)T]$ and f is a nonlinear mapping (for a detailed discussion we refer to Zegeye et al. (2009)).

4. EMISSION DISPERSION MODELING

The dispersion of vehicular emissions is affected by several factors. The main factors are the speed of the vehicles, the weather conditions (such as rain, wind, and temperature), and the geometry of the freeway area. In a region far from the road, the dispersion of the emissions is primarily dependent on the speed and direction of the wind and the temperature of the atmosphere Baker (1996).

In this section we model the dispersion of emissions (i.e. area-wide emissions) to a specific location at some distance from a freeway. Since the distance is considered to be large, the effect of the speed of the vehicles on the dispersion of the emissions is assumed to be negligible Baker (1996). Moreover, the wavefronts of the emissions at these locations are approximated by lines that are orthogonal to the wind direction. The wavefronts describe the front of the air (or emission) particles that propagate through space in the direction of the wind. The emissions are also assumed to emanate from the center point of the segments of the links introduced in Section 3.1.

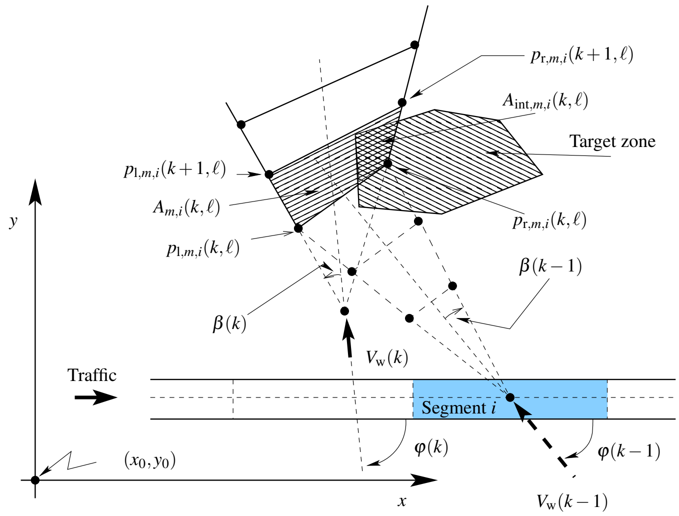

Fig. 2. Schematic representation of horizontal dispersion of vehicle emissions with varying wind speed and wind direction.

We define $V_{\mathrm{w}}(k)$ as the wind speed in the time interval $[kT, (k+1)T]$ and $\varphi(k)$ as the direction of the wind in the same time interval. Fig. 2 shows the propagation of emissions of vehicles from segment i of link m at time steps $k-1$, k, and $k+1$. The emissions propagate with a wavefront line in the direction of the wind. Since the emissions from vehicles are relatively denser and have higher temperature than the air particles, the emitted gases also expand sideways. The expansion of the emissions is inversely related to the wind speed (Baker, 1996). We model this phenomenon with a divergence angle β that depends on k. At time step k it represents the divergence angle that corresponds to half of the angle of the dispersion cone (see Fig. 2). Then, it is given by the expression

$$\beta(k) = \frac{\beta_{\max}}{1 + \beta_0 V_{\mathrm{w}}(k)} \tag{10}$$

where $\beta_{\max} \in [0,\ \pi]$ is the maximum angle at which the emissions are dispersed β_0 is model parameter.

Here we approximate wavefronts emanating from segment i of link m by lines with $p_{\mathrm{l},m,i}(k,\ell)$ as left-most point of the emission front at time step k that was released from segment i of link m at time step ℓ, and $p_{\mathrm{r},m,i}(k,\ell)$ as right-most point of the emission front at time step k that was released from segment i of link m at time step ℓ. A quantity of vehicular emissions emitted at time step ℓ from segment i of link m of the freeway arrives at time step k at the wavefront formed by the line segment joining the points $p_{\mathrm{l},m,i}(k,\ell)$ and $p_{\mathrm{r},m,i}(k,\ell)$. During the next time period $[kT,\ (k+1)T]$ each point of the line between $p_{\mathrm{l},m,i}(k,\ell)$ and $p_{\mathrm{r},m,i}(k,\ell)$ results in a small dispersion cone due to the combined wind and dispersion effect. But, it suffices to consider the left-most point $p_{\mathrm{l},m,i}(k+1,\ell)$ and the right-most point $p_{\mathrm{r},m,i}(k+1,\ell)$ to approximately describe the evolution of the wavefront of the emissions. Then, the evolution of the end points of the wavefronts $p_{\mathrm{l},m,i}(k+1,\ell) = (x_{\mathrm{l},m,i}(k+1,\ell), y_{\mathrm{l},m,i}(k+1,\ell))$ and $p_{\mathrm{r},m,i}(k+1,\ell) = (x_{\mathrm{r},m,i}(k+1,\ell), y_{\mathrm{r},m,i}(k+1,\ell))$ is respectively modeled as

$$x_{\mathrm{l},m,i}(k+1,\ell) = x_{\mathrm{l},m,i}(k,\ell) - TV_{\mathrm{w}}(k)\frac{\cos(\varphi(k)-\beta(k))}{\cos(\beta(k))},$$

$$y_{\mathrm{l},m,i}(k+1,\ell) = y_{\mathrm{l},m,i}(k,\ell) + TV_{\mathrm{w}}(k)\frac{\sin(\varphi(k)-\beta(k))}{\cos(\beta(k))},$$

and

$$x_{\mathrm{r},m,i}(k+1,\ell) = x_{\mathrm{r},m,i}(k,\ell) - TV_{\mathrm{w}}(k)\frac{\cos(\varphi(k)+\beta(k))}{\cos(\beta(k))},$$

$$y_{\mathrm{r},m,i}(k+1,\ell) = y_{\mathrm{r},m,i}(k,\ell) + TV_{\mathrm{w}}(k)\frac{\sin(\varphi(k)+\beta(k))}{\cos(\beta(k))}.$$

for $\cos(\beta(k)) \neq 0$.

Consider the wavefront formed by $p_{\mathrm{l},m,i}(k,\ell)$ and $p_{\mathrm{r},m,i}(k,\ell)$ and let $E_{y,m,i}(p_{\mathrm{l},m,i}(k,\ell), p_{\mathrm{r},m,i}(k,\ell))$ be the corresponding emission level for $y \in \mathscr{Y}$. Then the emission level for the next wavefront is

$$E_{y,m,i}(p_{\mathrm{l},m,i}(k+1,\ell), p_{\mathrm{r},m,i}(k+1,\ell)) = \gamma E_{y,m,i}(p_{\mathrm{l},m,i}(k,\ell), p_{\mathrm{r},m,i}(k,\ell)) \tag{11}$$

where $0 < \gamma \leq 1$ is a factor that characterizes the vertical dispersion.

Then the area that is subject to the emissions $E_{y,m,i}(p_{\mathrm{l},m,i}(k+1,\ell), p_{\mathrm{r},m,i}(k+1,\ell))$ during the time period $[kT,\ (k+1)T]$ is the tetragon formed by the points $p_{\mathrm{l},m,i}(k,\ell)$, $p_{\mathrm{l},m,i}(k+1,\ell)$, $p_{\mathrm{r},m,i}(k+1,\ell)$, and $p_{\mathrm{r},m,i}(k,\ell)$. The area of this tetragon is denoted by $A_{m,i}(k,\ell)$. The areal-density of the emissions in the time period is then given by

$$E_{\mathrm{ad},y,m,i}(k+1,\ell) = \frac{E_{y,m,i}(p_{\mathrm{l},m,i}(k+1,\ell), p_{\mathrm{r},m,i}(k+1,\ell))}{A_{m,i}(k,\ell)}. \tag{12}$$

The area at time step k of the intersection of the target zone and the tetragon formed by the emission wavefront generated at time step ℓ from segment i of link m is denoted by $A_{\mathrm{int},m,i}(k,\ell)$. We can then compute the amount of emissions dispersed to the target area as

$$E_{\mathrm{disp},y,m,i}(k,\ell) = A_{\mathrm{int},m,i}(k,\ell)E_{\mathrm{ad},y,m,i}(k,\ell). \tag{13}$$

Since the wavefronts are emanating from segment i of link m at each time step, the total dispersion level on the target area is computed by lumping all the emissions contributed by the segments of the different links due to the emissions generated over the past time steps. This is given by

$$J_{\mathrm{D,t},y}(k) = \sum_{(m,i)\in\mathscr{I}_{\mathrm{all}}} \sum_{\ell\in\mathscr{L}_{m,i,\mathrm{all}}(k)} E_{\mathrm{disp},y,m,i}(k,\ell) \tag{14}$$

where $\mathscr{L}_{m,i,\mathrm{all}}(k)$ denotes the set of all past time steps during which emissions generated from segment i of link m have crossed the target area at time step k and $\mathscr{I}_{\mathrm{all}}$ is the set of all segment-link pairs.

5. CASE STUDY

5.1 Freeway set up

In order to illustrate the proposed control approach and the area-wide emission modeling, we consider a case study with a 12 km three-lane freeway stretch. The freeway is divided into 12 segments with an on-ramp at the sixth segment from the left (see Fig. 3) and each segment is provided with a variable speed limit. The variable speed limits are grouped in groups of two so that each segment in a group will have the same speed limit signs. This implies the MPC controller optimizes only 6 speed limits $u_{\mathrm{SL},1}(k)$, $u_{\mathrm{SL},2}(k)$, …, $u_{\mathrm{SL},6}(k)$ and one on-ramp metering rate $r(k)$. So, in total there are 7 control variables at each control step.

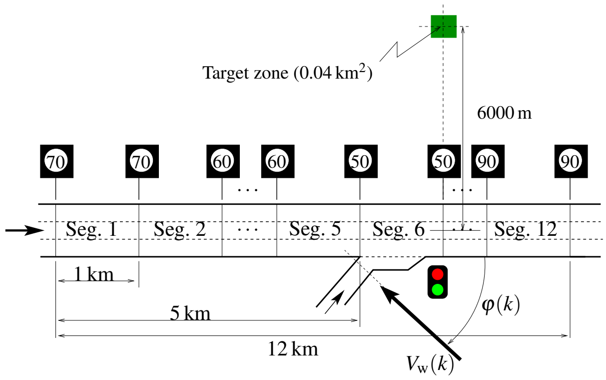

Fig. 3. A 12 km freeway with 12 variable speed limits and one on-ramp.

The freeway is subject to wind with speed and direction (see Fig. 3) given by

$$V_{\mathrm{w}}(k) = 7 + 2\sin(0.005\pi k + \pi/6)\sin(0.01\pi k) \tag{15}$$

$$\varphi(k) = \frac{2\pi}{5} + \frac{\pi}{4}\cos(0.004\pi k) \tag{16}$$

where the wind speed $V_{\mathrm{w}}(k)$ is expressed in m/s and the wind direction (angle) $\varphi(k)$ in radians. The case study is simulated for an hour with a time-varying demand profile.

5.2 Performance measures

We consider a multi-objective performance criterion that accommodates the emissions, dispersion of emissions, and travel time. The multi-objective function is defined as a weighted sum of the three performance measures similar to (1). In particular, we consider the objective function

$$J(k_{\mathrm{c}}) = \zeta_1\frac{\mathrm{TTS}(k_{\mathrm{c}})}{\mathrm{TTS_n}} + \zeta_2\frac{\mathrm{TE}(k_{\mathrm{c}})}{\mathrm{TE_n}} + \zeta_3\frac{\mathrm{DL}(k_{\mathrm{c}})}{\mathrm{DL_n}} + \zeta_4\frac{\Delta(k_{\mathrm{c}})}{\Delta_{\mathrm{n}}} \tag{17}$$

where

$$\mathrm{TTS}(k_{\mathrm{c}}) = T\sum_{k=Mk_{\mathrm{c}}}^{M(k_{\mathrm{c}}+N_{\mathrm{p}})-1}\ \sum_{(m,i)\in\mathscr{I}_{\mathrm{all}}} \lambda_m L_m \rho_{m,i}(k) + T\sum_{k=Mk_{\mathrm{c}}}^{M(k_{\mathrm{c}}+N_{\mathrm{p}})-1}\ \sum_{o\in\mathscr{O}_{\mathrm{all}}} w_o(k),$$

$$\mathrm{TE}(k_{\mathrm{c}}) = \sum_{y\in\mathscr{Y}'}\frac{\mathrm{TE}_y(k_{\mathrm{c}})}{\mathrm{TE}_{y,\mathrm{n}}}, \qquad \mathrm{DL}(k_{\mathrm{c}}) = \sum_{y\in\mathscr{Y}'}\frac{\mathrm{DL}_y(k_{\mathrm{c}})}{\mathrm{DL}_{y,\mathrm{n}}},$$

$$\Delta(k_{\mathrm{c}}) = \sum_{\ell=k_{\mathrm{c}}}^{k_{\mathrm{c}}+N_{\mathrm{p}}-1}\Bigg\{\sum_{s=1}^{6}\alpha_{u_{\mathrm{SL}}}(u_{\mathrm{SL},s}(\ell)-u_{\mathrm{SL},s}(\ell-1))^2 + \sum_{s=2}^{6}\alpha_{u_{\mathrm{SL}}}(u_{\mathrm{SL},s}(\ell)-u_{\mathrm{SL},s-1}(\ell))^2 + (r(\ell)-r(\ell-1))^2\Bigg\},$$

with

$$\mathrm{TE}_y(k_\mathrm{c}) = \sum_{k=Mk_\mathrm{c}}^{M(k_\mathrm{c}+N_\mathrm{p})-1} \sum_{(m,i)\in\mathscr{I}_\mathrm{all}} J_{y,m,i}(k),$$

$$\mathrm{DL}_y(k_\mathrm{c}) = \|[J_{\mathrm{D,t,y}}(Mk_\mathrm{c}) \ldots J_{\mathrm{D,t,y}}(MN_\mathrm{p}-1)]^\top\|_\infty,$$

where $\mathscr{Y}'$ is the set $\mathscr{Y}$ without fuel consumption, $\mathscr{O}_\mathrm{all}$ is the set of all origins in the traffic network, $\mathscr{S}_\mathrm{all}$ is the set of all speed limits, and $\alpha_{u_\mathrm{SL}} = \frac{1}{6N_\mathrm{p}v_\mathrm{step}^2}$ is the normalization of the speed limits with v_step denoting a nominal maximum change of speed limit between different segments and time steps. Moreover, the nominal values of the TTS, TE, TE_y, DL, DL_y, and Δ are computed by simulating the uncontrolled traffic system with all speed limits set to 120 km/h.

5.3 *Results and discussions*

We simulate the system for the uncontrolled and the controlled situation. In the controlled cases we consider four different scenarios by varying the weightings of the objective function given in (17). For all the controlled cases a nonlinear MPC controller is used. For the MPC controller we use the prediction horizon $N_\mathrm{p} = 15$ min, the control horizon $N_\mathrm{c} = 10$ min, and the control time step $T_\mathrm{c} = 2$ min. The simulation time step for all the models is set $T = 10$ s. The simulation results for these scenarios are tabulated in Table 1.

Table 1 compares the performance of the MPC controller with respect to the uncontrolled scenario. The performance measures considered are the total time spent (TTS), the total emissions (TE), and the total dispersion level (DL). As can be seen, when the objective of the controller is the TTS, both the TE and the total DL over the target area are worsened compared to the uncontrolled case. Similarly, if the objective of the controller is to reduce the TE or the DL, the TTS gets worse than the uncontrolled case. But, there is important difference between the two scenarios. The TTS gets much more worse when the objective of the controller is TE than when it is DL. This is because reducing the dispersion of the emissions over the target area does not necessarily mean reducing the total emission over the stretch of the whole freeway.

More can be observed by looking into the evolution of the dispersion levels. We give the evolution of the total dispersion level in the target area in Fig. 4. The figure depicts the total dispersion for the different controlled scenarios and for the uncontrolled case. To have more insight on what is happening we also depicted the space-mean speed of the complete freeway in Fig. 5.

From Fig. 5 one can see that the two shock waves of the traffic flow observed in the uncontrolled case are dissolved when the objective of the controller is to reduce only the total time spent. Under this situation the dispersion level becomes even more worse than for the uncontrolled case (see Fig. 4), i.e. the dispersion level is increased by 39% while the TTS is reduced by 49%. However, if the objective of the controller is set to be the dispersion level, we see that the traffic flow gets worse by 15% (see Fig. 5 and Table 1), but the dispersion level is reduced by 28%.

When we consider the objective of the controller to be the weighted sum of the TTS, TE and DL, the TTS is improved relative to both the uncontrolled case and to the case where only TE or DL are the objective of the controller. We see that the

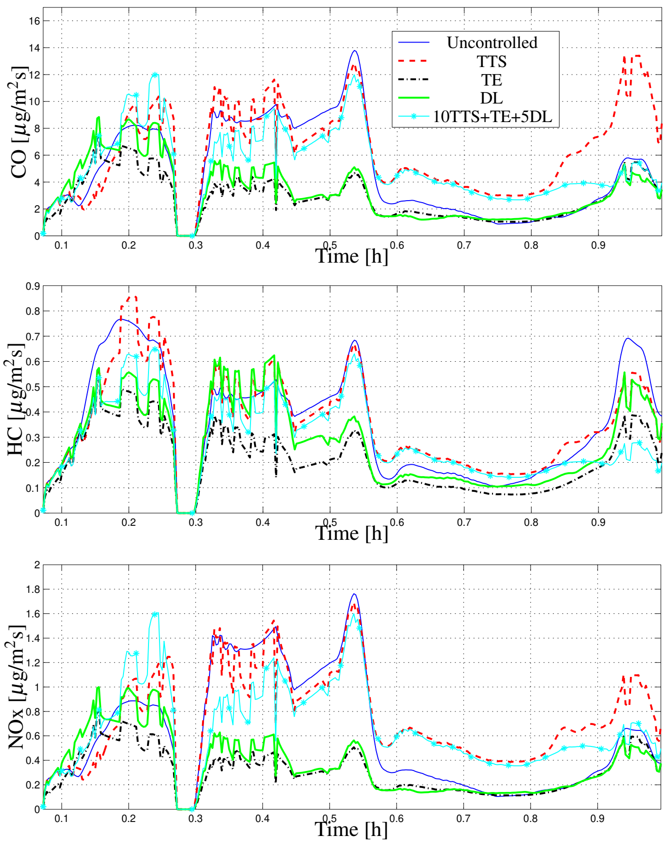

Fig. 4. Dispersion level for different control objectives.

DL has increased compared to the uncontrolled case. However, it is better than the case where the focus of the controller is only TTS. In general, we observe that as the weighting of the performance indicators varies, the controller can shift its focus towards the improvement of the indicators with higher weighting. In this way the model predictive controller can be used to specifically improve the traffic performance.

6. CONCLUSIONS

We have proposed a model-based traffic control approach, in particular nonlinear model predictive control, to optimize traffic control measures to provide a balanced trade-off between reducing dispersion of traffic emissions and minimizing travel time. The approach is based on a new emission dispersion model that includes the variation of the dispersion level in a target area due to the speed and direction of the wind.

Based on the simulations, we have shown that it is possible that the travel time is increased when a control strategy that simply focuses on the total emissions of the network is implemented. The same is true if the focus of the controller is to reduce the dispersion of the traffic emissions. In general, we have illustrated the potential of nonlinear MPC control approach in providing balanced trade-off between several performance objectives using a case study.

In our future work, we will study the stability of the closed-loop system due to the mismatch between the model and real system, we will use more extensive models, and we will consider more complex case studies.

Table 1. Simulation results for different scenarios.

Scenarios	Performance measure					
	TTS		TE		Total DL	
	[veh.h]	(g%)	[kg]	(g%)	[mg/m^{2}s]	(g%)
Uncontrolled	1362	(–)	127.5	(–)	1.8	(–)
TTS	692	(-49)	148.2	(+16)	2.5	(+39)
TE	1621	(+20)	65.6	(-49)	1.1	(-39)
DL	1606	(+15)	71.8	(-44)	1.3	(-28)
10TTS+TE+5DL	749	(-45)	109.4	(-15)	2.1	(+17)

The (g%) value denotes the percentage change of the variables with respect to the uncontrolled scenario ('-' means decrement and '+' means increment).

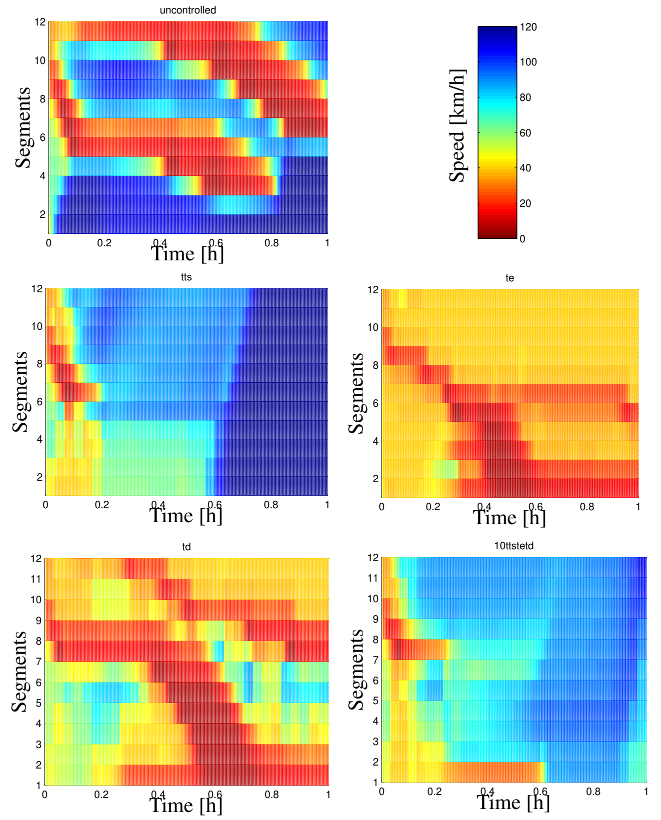

Fig. 5. Space-mean speed over segments and time for different control objectives.

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