

Technical report 11-046

Optimization of asset management of transformers based on a predictive health model of paper degradation*

G. Bajracharya, T. Koltunowicz, R.R. Negenborn, D. Djairam,
B. De Schutter, and J.J. Smit

If you want to cite this report, please use the following reference instead:

G. Bajracharya, T. Koltunowicz, R.R. Negenborn, D. Djairam, B. De Schutter, and J.J. Smit, "Optimization of asset management of transformers based on a predictive health model of paper degradation," *Proceedings of the 17th International Symposium on High Voltage Engineering*, Hannover, Germany, 6 pp., Aug. 2011. Paper G-002.

Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: <https://www.dcsc.tudelft.nl>

*This report can also be downloaded via https://pub.deschutter.info/abs/11_046.html

Optimization of asset management of transformers based on a predictive health model of paper degradation

Gautam Bajracharya^{1,*}, Tomasz Koltunowicz¹, Rudy R. Negenborn¹,
Dhiradj Djairam¹, Bart De Schutter¹ and Johan J. Smit¹

¹ Delft University of Technology, the Netherlands

* Email: g.bajracharya@tudelft.nl

Abstract: In this paper, a model-based predictive framework is proposed to optimize the operation and maintenance actions for power system equipment which operates in a changing environment of the future grid. In this framework, a predictive health model is proposed that predicts the health state of this equipment based on its operation and maintenance actions. In particular, this framework is used to predict the health state of transformers based on their usage and operating environment. The hot-spot temperature of the transformer is predicted from the expected loading of the transformer. Based on the hot-spot temperature predictions, the allowed loading limits of the transformers are determined. In the case of absence of the anticipated loading of the transformer, a maximum allowable loading limit of the transformer is estimated.

1 Introduction

Electrical power systems have been changing drastically in recent years, especially due to the introduction of deregulation in the power industry and the increase of distributed generation. Moreover, a significant portion of the electrical infrastructures are reaching the end of their operational age within the coming decade [1]. On the one hand, the impending replacement wave of these infrastructures will require extensive investments in the near future. On the other hand, the aging infrastructures are degrading the reliability of the system. There is a greater need for reducing the risk of the aging related failures and at the same time deferring the new investments by extending the life of the aging infrastructures. In addition, power equipment of the future grid will need to work with distributed generation, deregulation, and accelerated aging. So there is a need for maximum utilization of equipment without degrading the reliability of the system beyond the acceptable level [2].

The utilization of equipment should be based on its actual operating conditions as the operating conditions change significantly due to the introduction of distributed generation and renewable energy sources. The evolution of the health state of the component due to changing operation conditions should be tracked. For optimal utilization of equipment, the operational, maintenance, and planning decisions should be based on its health state. The health state of the equipment can be estimated by monitoring the equipment's operational condition and its condition parameters. With the knowledge of its health state, operational, maintenance, and planning actions can be taken when they are required.

A proper and efficient framework is required to incorporate the health information in these actions. A

framework for modeling the health state of power system equipment was proposed in [2]. This framework is used for predicting the hot-spot temperature of the transformer. Based on the hot-spot temperature prediction, the dynamic loading limit of the transformer is determined. The dynamic loading limit is determined for two cases. In the first case the predicted loading of the transformer is available where as in the second case the predicted loading of the transformer is not available.

2 Framework for model-based optimization

A framework for model-based optimization consists of a predictive health model and an optimizer [2]. The framework also defines the cost function for the optimization. Below, the components of this framework are outlined briefly.

2.1 Predictive health model

The predictive health model in the framework includes a dynamic stress model, a failure model, and a model for the estimation of cumulative stresses. As equipment ages, various stresses, such as electrical, thermal, mechanical, and environmental stresses, weaken the strength of the equipment. The cumulative stresses of the equipment depend on the usage pattern (e.g., the loading) and the maintenance actions (e.g., the replacement of parts) performed on the equipment. The health state of the equipment is represented by the cumulative stresses. Their dynamics can be described using a dynamic stress model such as the following discrete-time

state-space model:

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k)), \quad (1)$$

where $\mathbf{u}(k) = [\mathbf{u}_a^T(k) \quad \mathbf{u}_d^T(k)]^T$. At discrete time step k , the future cumulative stresses $\mathbf{x}(k+1)$ are predicted based on the usage of the equipment $\mathbf{u}_d(k)$, the maintenance actions $\mathbf{u}_a(k)$, and the current cumulative stresses $\mathbf{x}(k)$.

As the cumulative stresses increase over time, the probability of failure of the equipment also increases. The relationship between the cumulative stresses and the failure rate of the equipment is described in a failure model. The failure model uses the predicted cumulative stresses to predict the failure rate of the equipment. The failure model directly maps the cumulative stresses \mathbf{x} to the failure rate y as follows:

$$y(k) = g(\mathbf{x}(k)). \quad (2)$$

The cumulative stresses \mathbf{x} can be estimated by condition parameters of the equipment, such as the partial discharge, temperature measurements, etc. Different online and offline monitoring systems can detect these condition parameters. In practice, only a few condition parameters (such as the electrical and thermal stresses) are measured by monitoring systems. Estimates of the monitored cumulative stresses \mathbf{x}_e can be made based on measurements \mathbf{c} of the monitoring systems as follows:

$$\mathbf{x}_e(k) = \mathbf{h}_x(\mathbf{c}(k)). \quad (3)$$

The estimated cumulative stresses \mathbf{x}_e can be used to update the corresponding cumulative stresses \mathbf{x} . The remaining unmonitored cumulative stresses are predicted by the dynamic stress model (1).

2.2 Optimization of maintenance and usage

Typically, maintenance improves the health state of the equipment, which, in turn, reduces its failure rate. An optimal maintenance action balances the economical cost of the maintenance, the improvement of the health state, and the reduction in the failure rate of the equipment.

The total cost of the usage and the maintenance actions consist of three sub-cost functions. The sub-cost function of the planned usage and the maintenance actions J_a incorporates the economical cost of the planned usage and the maintenance. The sub-cost function of the failure rate J_f takes into account the cost associated with the failure of the equipment. The sub-cost function of the cumulative stresses J_{cs} incorporates the cost of the deterioration of the equipment. The summation of these three sub-cost functions gives the total cost of a particular maintenance action in a particular state.

The optimization of the usage and the maintenance actions is considered over a given predicted time frame

Table 1: Suggested maximum loading types based on the hot-spot temperature [3].

Loading types	Maximum hot-spot temperature (°C)
Normal life expectancy loading	120
Planned loading beyond nameplate	130
Long-time emergency loading	140
Short-time emergency loading	180

of N steps in the future, such that future usage and future maintenance actions can be optimized. The total cost over the predicted time frame is considered in the optimization. Hence, the model-based optimization problem is formulated as follows:

$$\min_{\mathbf{u}(k), \dots, \mathbf{u}(k+N-1)} \sum_{l=0}^{N-1} [J_a(\mathbf{u}(k+l)) + J_f(y(k+l+1)) + J_{cs}(\mathbf{x}(k+l+1))] \quad (4)$$

subject to

$$\begin{aligned} \mathbf{x}(k+l+1) &= \mathbf{f}(\mathbf{x}(k+l), \mathbf{u}(k+l)) \\ y(k+l+1) &= g(\mathbf{x}(k+l+1)) \\ &\text{for } l = 0, \dots, N-1 \end{aligned}$$

The predictive health model is thus used to predict the cumulative stresses and the failure rates for the planned usage and the future maintenance actions. The total cost is evaluated for different future usage and maintenance actions over the predicted time frame. The optimal usage and maintenance actions minimizing the total cost over the time horizon is searched for.

3 Thermal loading of transformer

The framework of model-based optimization is applied in the dynamic loading of the transformer based on its thermal performance.

The maximum allowable loading of a transformer mainly depends on the thermal performance of the transformer. IEEE C57.91 [3] defines four types of loading regimes, for which the suggested maximum hot-spot temperature is given in Table 1.

Under normal life expectancy loading, the maximum hot-spot temperature allowed is 120°C. The planned loading beyond the nominal rating is suggested for a planned, repetitive load, provided that the transformer is not loaded continuously at the rated load. The long-time emergency loading is suggested only for rare emergency conditions. The short-time emergency loading is only suggested for a short time in a few abnormal emergency conditions. Normal life expectancy loading is considered risk free [3]. This loading regime is considered in this paper.

4 Thermal model in the framework of model-based optimization

The top-oil temperature of a transformer is calculated based on the ambient temperature and on the dynamics of the heat transfer from the oil to the environment through the radiators. Similarly, the hot-spot temperature is calculated based on the top-oil temperature and on the dynamics of the heat transfer between the windings and the oil. The time constants of the dynamics of the top-oil and hot-spot temperatures are the top-oil time constant and hot-spot time constant, respectively.

IEEE C57.91 [3] suggests a top-oil time constant based on the mass of different parts and on the cooling type of the transformer. The winding time constant is estimated based on the cooling experiments. Swift et al. [4] propose a thermal model based on heat transfer theory, which includes thermal capacitances and non-linear thermal resistances. Their approach is extended by Susa [5] by considering the oil viscosity changes and the loss variation with the temperature.

The differential equations describing the dynamics of the top-oil and the hot-spot temperatures are discretized by using the forward Euler approximation [6]. The resulting thermal models can be converted to the dynamic stress model (1) of the model-based optimization framework as follows [6]. The top-oil temperature $x_{\theta, \text{oil}}$ and the hot-spot temperature $x_{\theta, \text{hs}}$ are taken as cumulative stresses. The load factor u_I is taken as the usage. The ambient temperature $u_{\theta, \text{amb}}$ is taken as the exogenous input. The discretized top-oil model is given by:

$$\begin{aligned} & \frac{1 + R \cdot u_I(k)^2}{1 + R} \cdot \mu_{\text{pu}}(k)^n \cdot \Delta\theta_{\text{oil, rated}} \\ &= \mu_{\text{pu}}(k)^n \cdot \tau_{\text{oil, rated}} \cdot \frac{x_{\theta, \text{oil}}(k+1) - x_{\theta, \text{oil}}(k)}{h} \\ & \quad + \frac{(x_{\theta, \text{oil}}(k) - u_{\theta, \text{amb}}(k))^{n+1}}{\Delta\theta_{\text{oil, rated}}^n}, \end{aligned} \quad (5)$$

where R is the ratio of the load losses at the rated current and the no-load losses, $\Delta\theta_{\text{oil, rated}}$ is the rated top-oil temperature rise over the ambient temperature, $\tau_{\text{oil, rated}}$ is the rated top-oil time constant, n is a constant that depends on the type of cooling, h is the time step, and μ_{pu} is the variable oil viscosity in pu given by:

$$\mu_{\text{pu}}(k) = \frac{\exp(2797.3/(x_{\theta, \text{oil}}(k) + 273))}{\exp(2797.3/(\theta_{\text{oil, rated}} + 273))}. \quad (6)$$

The discretized hot-spot model is then given by:

$$\begin{aligned} & u_I(k)^2 \cdot P_{\text{cu, pu}}(k) \cdot \mu_{\text{pu}}(k)^n \cdot \Delta\theta_{\text{hs, rated}} \\ &= \mu_{\text{pu}}(k)^n \cdot \tau_{\text{wdg, rated}} \cdot \frac{x_{\theta, \text{hs}}(k+1) - x_{\theta, \text{hs}}(k)}{h} \\ & \quad + \frac{(x_{\theta, \text{hs}}(k) - x_{\theta, \text{oil}}(k))^{n+1}}{\Delta\theta_{\text{hs, rated}}^n}, \end{aligned} \quad (7)$$

where $\Delta\theta_{\text{hs, rated}}$ is the rated hot-spot temperature rise over the top-oil temperature, $\tau_{\text{wdg, rated}}$ is the rated hot-spot time constant, and $P_{\text{cu, pu}}$ is the variable load losses in pu given by:

$$\begin{aligned} P_{\text{cu, pu}}(k) &= P_{\text{cu, dc, pu}} \frac{235 + x_{\theta, \text{hs}}(k)}{235 + \theta_{\text{hs, rated}}} \\ & \quad + P_{\text{cu, eddy, pu}} \frac{235 + \theta_{\text{hs, rated}}}{235 + x_{\theta, \text{hs}}(k)}. \end{aligned} \quad (8)$$

This thermal model is used for dynamic loading of transformer. Two cases are considered in this paper. In the first case, the predicted loading of the transformer is known. In the second case, the absence of the predicted loading is considered. These cases are presented in the following sections.

5 Dynamic loading based on predicted loading

The predicted loading of the transformer can be obtained from the power flow calculations based on anticipated generations and loads. Based on this predicted loading, a time-varying maximum loading limit is calculated. This loading limit is assumed to be respected by controlling the power flow of the transformer in the network.

For the transformer, there are two different kinds of scenarios which might occur in determining the loading limit, which are:

1. Case 1: The hot-spot temperature for the given predicted loading is below the maximum limit. Then there is no need for reducing the power flow of the transformer.
2. Case 2: The hot-spot temperature for the given predicted loading is above the maximum limit. In this case, the loading of the transformer has to be reduced by rerouting the power through other parts of the network.

In Case 1, there is no issue regarding violation of the maximum hot-spot temperature limit. However, there is a possibility of an increase (or decrease) in the loading of the transformer due to unforeseen events, for example, another transformer in the network could be overloaded and the resulting excess power could be rerouted through the former transformer. In such a case, the maximum loadability of the transformer should be known. This can be achieved by giving a maximum loading limit for which the maximum hot-spot limit is not violated within the predicted time horizon. This maximum loading limit is chosen such that it is proportional to the predicted loading.

In Case 2, the hot-spot temperature will exceed the maximum limit if the predicted loading is allowed through the transformer. In this case, the only option

is to reduce the loading of this particular transformer by rerouting the power through other parts of the network. A ‘safe’ loading limit should be provided so that the hot-spot temperature does not exceed the maximum limit. At the same time, there is a need for the maximum utilization of the loading capability of the transformer, so that the net burden of rerouting of the power through other parts of the network is minimized. Thus, for this case, a maximum loading limit is provided which follows the predicted loading as much as possible. In other words, the difference between the maximum loading and the predicted loading is kept at the minimum level.

The desired operation can be translated into the cost function of the optimization. These two different cases clearly indicate that the cost function of the optimization problem should have ‘a kind of’ double term to deal with them. The optimization problem is summarized as follows:

$$\min_{\alpha, u_{1,\max}(k), \dots, u_{1,\max}(k+N-1)} \sum_{l=0}^{N-1} \left[c_1 \left(u_{1,\max}(k+l) - \alpha u_{1,\text{pred}}(k+l) \right)^2 \right] - c_2 \alpha, \quad (9)$$

subject to

$$\begin{aligned} x_{\theta,\text{oil}}(k+l+1) &= f_{\text{oil}}(x_{\theta,\text{oil}}(k+l), u_{\theta,\text{amb}}(k+l), u_{1,\max}(k+l)) \\ x_{\theta,\text{hs}}(k+l+1) &= f_{\text{hs}}(x_{\theta,\text{hs}}(k+l), x_{\theta,\text{oil}}(k+l), u_{1,\max}(k+l)) \\ x_{\theta,\text{hs}}(k+l+1) &\leq x_{\theta,\text{hs},\max} \\ \alpha &\geq 1 \\ &\text{for } l = 0, \dots, N-1, \end{aligned}$$

where $u_{1,\text{pred}}$ is the predicted loading and $u_{1,\max}$ is the maximum loading. c_1 and c_2 are coefficients of the two terms of the cost function. $x_{\theta,\text{hs},\max}$ is the maximum hot-spot temperature. The functions f_{oil} and f_{hs} represent the top-oil temperature model (5) and the hot-spot temperature model (7), respectively. α is a slack variable.

The slack variable α is used to deal with the dual scenarios mentioned in Case 1 and Case 2. The optimization tries to minimize the quadratic term $(u_{1,\max}(k+l) - \alpha u_{1,\text{pred}}(k+l))^2$ and maximizes the slack variable α . In Case 1, the maximum loading limit can be more than the predicted loading. Thus, the optimization would maximize the slack variable α while keeping the quadratic term to the minimum, i.e. 0. Effectively, this means that the maximum load will be larger than the predicted load by a factor of the slack variable α which is greater than 1. As a result, a maximum loading limit $u_{1,\max}$ which is proportional to the predicted loading profile $u_{1,\text{pred}}$ is obtained.

In Case 2, maximization of the slack variable α is not possible as the maximum loading should be less than

the predicted load. Thus the slack variable α is set to its minimum value which is 1. Then the optimization attempts to minimize the difference between the maximum loading and the predicted loading while keeping the hot-spot temperature below its maximum limit.

The optimization problem (9) consists of non-linear constraints. The optimization therefore is solved by a non-linear solver, SNOPT [7]. This solver is used through the Tomlab v6.1 [8] interface in Matlab v7.5. Analytically computed gradients of the constraints and the cost function are supplied to the solver in order to reduce the execution time of the optimization.

5.1 Simulation

The optimization presented above is solved for the transformer with the parameters given in [6]. A daily load profile of the transformer is assumed based on the energy demand data for an average Dutch household as given in [9]. The loading regime of normal life expectancy loading (Table 1) is considered for the maximum hot-spot temperature, i.e. $x_{\theta,\text{hs},\max} = 120^\circ\text{C}$.

The proposed algorithm is simulated for 24 hours. The simulation results for the period from 960 minutes to 1380 minutes are shown in Figure 1. A prediction horizon N of 60 (minutes) is considered for the simulation.

Between the time interval of 1080 minutes and 1259 minutes, α cannot be increased beyond its minimum value of 1 due to the maximum hot-spot temperature constraint. Thus, the maximum loading is less than the predicted loading and the difference between them is minimized as far as possible. In the remainder of the time interval, the maximum hot-spot constraint is not violated by the predicted loading, thus the slack variable α takes a value greater than 1. Thus, the maximum loading is larger than the predicted loading given by a factor of α .

6 Dynamic loading in absence of predicted loading

In the case of lack of predictions of the loading of the transformer, a number of assumptions should be made in order to safely control the loading. Predicting the loading of a transformer could be difficult in the case of a network with renewable energy sources whose generations are stochastic in nature. In the power distribution system, the load forecasting system is not present so the loading of a transformer cannot be predicted accurately. Moreover, the control equipment used in such a system would have a limited computational capability due to the cost of such equipment.

In such a system, the working principle of the dynamic loading should be simple enough so that the control equipment can take the dynamic loading into consideration along with its other functions, such as the me-

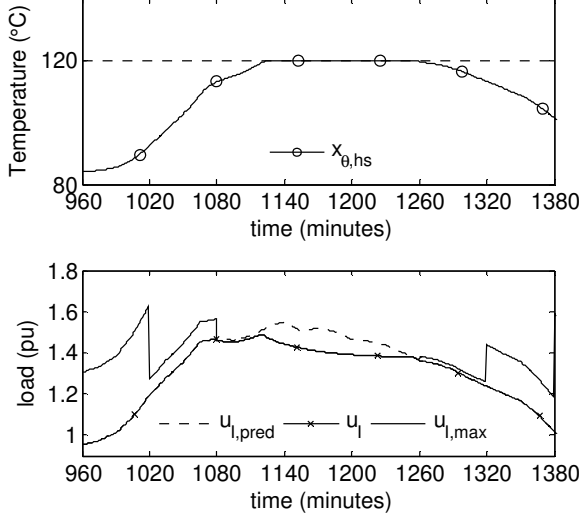


Figure 1: Simulation of the dynamic loading of the transformer for peak load period. The prediction horizon is 60 minutes. Between the time interval of 1080 minutes and 1259 minutes, the actual loading u_l of transformer is less than the predicted loading $u_{l,pred}$.

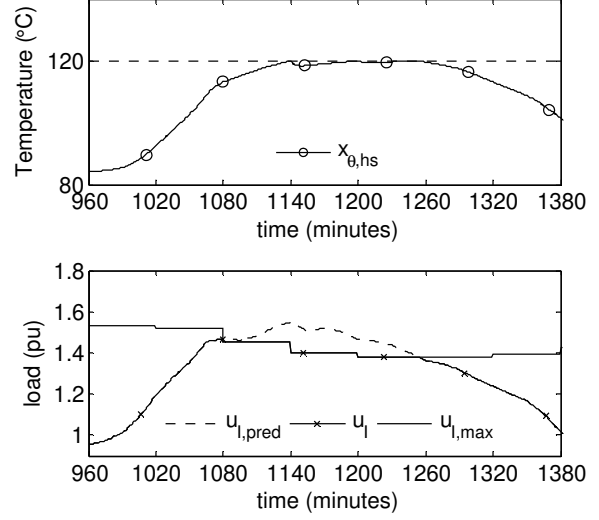


Figure 2: Simulation of the dynamic loading of the transformer without the information of the load prediction. A constant maximum loading limit $u_{l,max}$ is provided for the period of the prediction horizon of 60 minutes.

tering, the control and protection of the network, and the information exchange support. In the absence of the predicted loading, a constant loading limit based on the current hot-spot temperature is provided for a certain time period. This constant loading limit is the maximum loading that the transformer could supply for the time period considered, without exceeding the maximum hot-spot temperature limit. The optimization problem for this kind of dynamic loading can be formulated as follows:

$$\min_{u_{l,max}(k), \dots, u_{l,max}(k+N-1)} \sum_{l=0}^{N-1} -(u_{l,max}(k+l))^2, \quad (10)$$

subject to

$$\begin{aligned} x_{\theta,oil}(k+l+1) &= f_{oil}(x_{\theta,oil}(k+l), u_{\theta,amb}(k+l), u_{l,max}(k+l)) \\ x_{\theta,hs}(k+l+1) &= f_{hs}(x_{\theta,hs}(k+l), x_{\theta,oil}(k+l), u_{l,max}(k+l)) \\ x_{\theta,hs}(k+l+1) &\leq x_{\theta,hs,max} \\ &\text{for } l = 0, \dots, N-1. \end{aligned}$$

The solution to the optimization problem (11) gives the maximum loading limit. This algorithm requires less coding and less computation power, compared to the algorithm of solving the optimization problem (9). By simplifying the optimization problem, the computational requirement of the system is reduced. Thus, the system can be incorporated in the existing control system without taking a major portion of the computation power of the system.

6.1 Simulation

The optimization problem (11) is solved for the transformer given in the previous section with the same operating conditions. The simulation results for a prediction horizon N of 60 (minutes) is presented in Figure 2. As observed in the figure, the maximum loading limit $u_{l,max}$ provided by this optimization is a constant level for the prediction horizon of 60 (minutes). The optimization does not require the predicted loading $u_{l,pred}$, however, it is plotted in the figure to provide an impression of the required rerouting of the transformer load. As seen in the figure, the hot-spot temperature $x_{\theta,hs}$ is maintained below the maximum limit of 120°C.

7 Comparison of simulation results

The dynamic loading of a transformer is considered for two cases. In Section 5, the predicted loading of the transformer is taken into account while determining the maximum loading limit of the transformer. In Section 6, the maximum loading limit is generated based on the current condition of the transformer without the knowledge of the predicted loading.

The amount of energy rerouted for the simulations presented in these two sections is summarized in Table 2. As seen in the table, the simulation without the knowledge of the predicted loading (Section 6), with a prediction horizon of 1 minute gives the least energy rerouting requirement. This is because the optimization calculates the new loading limit after each time step. This means the communication between the transformer

Table 2: Total energy required to be rerouted during the simulation of 24 hours.

Prediction Horizon N	Energy Rerouting Required $\sum_{24 \text{ hours}} (u_{I,\text{pred}}(k) - u_I(k)) h$
Based on predicted loading	
60 minutes	43.7 MWh
15 minutes	42.8 MWh
Without predicted loading	
60 minutes	48.3 MWh
15 minutes	43.7 MWh
1 minute	42.7 MWh

controller and the power flow controller has to be done in a time interval of 1 minute. As the prediction time is increased from 1 minute to 15 minutes and 60 minutes, the rerouted energy increases.

When the predicted loading is taken into account (Section 5), the energy rerouted is less than the case without predictions (Section 6) for the same prediction horizon. In addition, for a larger prediction horizon, the required rerouted energy does not increase as much as in Section 6.

8 Conclusion

A model-based predictive optimization framework has been applied for the optimization of the loading of a transformer. The proposed method optimizes the utilization of the transformer by recommending load changes when required and by keeping the temperature within the safe limits. Scenarios of availability and absence of predicted loading of the transformer were considered. For the both scenarios, the hot-spot temperature was maintained below the allowed limit.

For the same prediction horizon, the required load control in the first case is less. The second case has the advantage that it is simpler for implementation.

9 Acknowledgments

This research is supported by the SenterNovem Sinergie project EOSLT04034 and the VENI project “Intelligent multi-agent control for flexible coordination of transport hubs” (project 11210) of the Dutch Technology Foundation STW, a subdivision of the Netherlands Organisation for Scientific Research (NWO).

References

[1] Commission of the European Communities, “Green paper: A European strategy for sustainable, competitive and secure energy,” Tech.

Rep. COM (2006) 105 final, 2006, available at http://ec.europa.eu/energy/green-paper-energy/doc/2006_03_08_gp_document_en.pdf.

[2] G. Bajracharya, T. Koltunowicz, R. R. Negenborn, Z. Papp, D. Djairam, B. De Schutter, and J. J. Smit, “Optimization of maintenance for power system equipment using a predictive health model,” in Proceedings of the 2009 IEEE Bucharest PowerTech Conference, Bucharest, Romania, July 2009, pp. 1–6.

[3] “IEEE guide for loading mineral-oil-immersed transformers,” IEEE Std C57.91-1995, April 1996.

[4] G. Swift, T. S. Molinski, and W. Lehn, “A fundamental approach to transformer thermal modeling. i. theory and equivalent circuit,” IEEE Transactions on Power Delivery, vol. 16, no. 2, pp. 171–175, April 2001.

[5] D. Susa, “Dynamic thermal modelling of power transformers,” Ph.D. dissertation, Helsinki University of Technology, Helsinki, Finland, August 2005.

[6] G. Bajracharya, T. Koltunowicz, R. R. Negenborn, Z. Papp, D. Djairam, B. De Schutter, and J. J. Smit, “Optimization of condition-based asset management using a predictive health model,” in Proceedings of the 16th International Symposium on High Voltage Engineering 2009, Cape Town, South Africa, August 2009.

[7] R. Fletcher and S. Leyffer, “Nonlinear programming without a penalty function,” Mathematical Programming, vol. 91, no. 2, pp. 239–269, January 2002.

[8] K. Holmström, A. O. Göran, and M. M. Edvall, “User’s guide for Tomlab/SNLP,” Tech. Rep., February 2008.

[9] R. R. Negenborn, “Multi-agent model predictive control with applications to power networks,” Ph.D. dissertation, Delft University of Technology, Delft, The Netherlands, December 2007.