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A Predictive Traffic Controller for Sustainable Mobility Using Parameterized Control Policies

S. K. Zegeye, B. De Schutter *Senior IEEE member*, J. Hellendoorn, E. A. Breunesse, and A. Hegyi

***Abstract*—We present a freeway-traffic control strategy that continuously adapts the traffic control measures to the prevailing traffic conditions and that features faster computation speed than conventional model-based predictive control (MPC). The control approach is based on the principles of state feedback control and MPC. Instead of computing the control input sequence, the proposed controller optimizes the parameters of control laws that parameterize the control input sequences. In this way, the computational burden of the controller is reduced substantially. We demonstrate the proposed control approach on a calibrated model of a part of the Dutch A12 freeway using variable speed limits and ramp metering rate.**

***Index Terms*—Traffic control, MPC, parameterized controller, sustainable mobility, variable speed limits, ramp metering**

I. Introduction

CONTROLLING the traffic network to get a system or user optimum flow is one of the challenges that traffic practitioners face every day. The demand to increase the traffic safety and the increasingly stringent environmental policies exacerbate the task of traffic control engineers. The multi-faceted control objectives of traffic systems also vary both spatially and temporally. Moreover, depending on the traffic conditions the multiple objectives of a traffic controller may be conflicting or compatible [1], [31].

There are several research results on the design of traffic controllers that can improve the traffic flow under certain traffic conditions [6], [9], [11], [15], [26], [27] and on the design of vehicle control strategies that can improve the fuel efficiency [34]. The freeway traffic controller in [15] is designed to eliminate shock waves on freeways. The freeway controllers designed in [6], [9], [26], [27] are state feedback control policies where the parameters of the control policies are determined using off-line optimization approaches based on simulation or historical data. This means that the control policies perform well for the specific scenarios they are optimized to. However, in general the traffic conditions may change so frequently that the performance of the controllers is most often reduced. Moreover, it is not possible to either change the desired control objective at any time or to adapt the control policies on-line to the prevailing traffic conditions.

An excellent traffic control solution for freeway traffic problems is a controller that takes the current and future traffic situation into account and that predicts the consequences of its control actions. One such traffic control strategy, based on model predictive control (MPC), has been independently proposed and re-proposed by several authors in the past decades [4], [8], [10], [22]–[24]. MPC can handle model uncertainties, include constraints, support multi-objective performance criteria, and can be used with nonlinear models [19]. Moreover, in several case studies MPC has proven to yield significant gains in the performance of the traffic network [5], [13], [33]. However, this comes with one main limitation: the computation time is very large, which makes the approach intractable in practice [11].

There are many advancements in the literature (e.g. [12], [16], [18], [28]) to address the computational complexity problems of general MPC. Almost all available literature deals either with linear time-invariant systems or specific classes of nonlinear systems (such as linear time-varying, linear parameter-varying, and piecewise affine systems). However, traffic systems are too complex and nonlinear to fall within the specific classes of nonlinear systems for which the methodologies to reduce the computation time are developed.

This paper contributes a new general theoretical approach for the design of a centralized traffic controller that yields fast computation speeds. The paper proposes a receding-horizon parameterized (RHP) traffic control approach that combines the advantages of conventional MPC (i.e., prediction, adaptation, and handling constraints, multi-objective criteria, and nonlinear models) and the advantages of state feedback controllers (i.e., faster computation speed and easier implementation). Moreover, the paper applies the theoretical approach to design control laws for variable speed limits and ramp metering. Furthermore, the paper illustrates the proposed control approach with a simulation-based case study on a part of the Dutch A12 freeway. In addition to the substantial reduction of computation times, the case study also demonstrates that for the given scenario the proposed RHP traffic controller performs almost the same as conventional MPC. This paper further considers different objective functions to demonstrate how the controller can be used to obtain a balanced trade-off between emissions, fuel efficiency, and travel times.

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II. MODELS

Since the proposed control strategy is model-based, this section gives a brief account of the traffic flow, emission, and fuel consumption models used in this paper is given. In particular, we discuss the METANET [21] traffic flow model and the VT-macro [31] emission and fuel consumption model. Note, however, that the control approach presented is generic; hence, it can also be used with other, more complex models.

A. METANET

METANET [17], [21] is a macroscopic second-order traffic flow model. The model describes the evolution of the average density ρ [veh/km/lane], average flow q [veh/h], and average space-mean speed v [km/h] as nonlinear difference equations. The METANET model is both temporally and spatially discretized. The model uses directed graphs to represent the traffic network. In the graph, a node is placed wherever there is a change in the geometry of a freeway (such as a lane drop, on-ramp, off-ramp, or a bifurcation). A homogeneous freeway stretch that connects such nodes is represented by a link. Links are further divided into segments of equal length (typically 300-500 m). The equations that describe the traffic dynamics in segment i of link m are given by

$$q_{m,i}(k) = \lambda_m \rho_{m,i}(k) v_{m,i}(k) \tag{1}$$

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_m \lambda_m}\big(q_{m,i-1}(k) - q_{m,i}(k)\big) \tag{2}$$

$$\begin{aligned} v_{m,i}(k+1) =& v_{m,i}(k) + \frac{T}{\tau}\big(V[\rho_{m,i}(k)] - v_{m,i}(k)\big) \\ &+ \frac{T}{L_m} v_{m,i}(k)\big(v_{m,i-1}(k) - v_{m,i}(k)\big) \\ &- \frac{T\eta}{\tau L_m}\frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k)+\kappa} \end{aligned} \tag{3}$$

$$V[\rho_{m,i}(k)] = v_{\mathrm{free},m} \exp\left[-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{\mathrm{cr},m}}\right)^{a_m}\right] \tag{4}$$

where $q_{m,i}(k)$ denotes the outflow of segment i of link m during the time period $[kT, (k+1)T)$, $\rho_{m,i}(k)$ denotes the density of segment i of link m at simulation time step k, $v_{m,i}(k)$ denotes space-mean speed of segment i of link m at simulation time step k, L_m denotes the length of the segments of link m, λ_m denotes the number of lanes of link m, and T denotes the simulation time step (a typical value for T is 10 s). Furthermore, $\rho_{\mathrm{cr},m}$ is the critical density, $v_{\mathrm{free},m}$ is the free-flow speed of link m, τ a time constant, η the anticipation constant, a_m the parameter of the fundamental diagram, and κ is a model parameter.

The desired speed $V[\rho_{m,i}(k)]$ in (4) is given for the uncontrolled case, where the speed limit is kept constant and equal to $v_{\mathrm{free},m}$. For the controlled case, where the speed limit is dynamic, the desired speed is determined according to [13]

$$\begin{aligned} V[\rho_{m,i}(k)] = \min\Bigg\{ &(\alpha_m+1)u_{\mathrm{sl},m,i}(k), \\ &v_{\mathrm{free},m}\exp\left[-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{\mathrm{cr},m}}\right)^{a_m}\right]\Bigg\} \end{aligned} \tag{5}$$

where $u_{\mathrm{sl},m,i}(k)$ is the speed limit of segment i of link m and α_m is the drivers' non-compliance factor.

For origins (such as on-ramps and mainstream entry points) a queue model is used. The dynamics of the queue length w_o at the origin o are modeled as

$$w_o(k+1) = w_o(k) + T(d_o(k) - q_o(k)) \tag{6}$$

where $d_o(k)$ and $q_o(k)$ denote respectively the demand and outflow of origin o. The outflow $q_o(k)$ is given by

$$\begin{aligned} q_o(k) = \min\Bigg[&d_o(k) + \frac{w_o(k)}{T},\ r_o(k)C_o, \\ &C_o\left(\frac{\rho_{\mathrm{jam},m} - \rho_{m,1}(k)}{\rho_{\mathrm{jam},m} - \rho_{\mathrm{cr},m}}\right)\Bigg], \end{aligned} \tag{7}$$

with $r_o(k)$ the ramp metering rate (where $r_o(k) \in [0\ 1]$ for a metered on-ramp and $r_o(k) = 1$ for an unmetered on-ramp or mainstream origin), $\rho_{\mathrm{jam},m}$ the maximum density of link m, and C_o the capacity of origin o. For refinements and extensions we refer an interested reader to [14], [17], [21].

B. VT-macro

The VT-macro model [31] is a macroscopic emission model that in particular developed for the METANET traffic flow model. The model takes the dynamics of the space-mean speed of the traffic flow model into account. The inputs of the model are the average space-mean speed, average acceleration, and the number of vehicles subject to the speed and acceleration pairs. The average acceleration and number of vehicles are computed from the space-mean speed v, density ρ, and flow q variables of the METANET model. A detailed explanation of the way to compute the average acceleration and number of vehicles subject to it is provided in [31]. In the sequel we briefly describe the formulas for these quantities.

Since the METANET model is discrete both in time and in space, the acceleration and the number of vehicles that are subject to the acceleration are differentiated as segmental and cross-segmental variables. The segmental acceleration $a_{\mathrm{seg},m,i}$ is the rate of change in space-mean speed of the vehicles in one simulation time step within segment i of link m. The segmental acceleration of the vehicles in the segment i of link m at time step k is given by

$$a_{\mathrm{seg},m,i}(k) = \frac{v_{m,i}(k+1) - v_{m,i}(k)}{T} \tag{8}$$

and the corresponding number of vehicles subject to the segmental acceleration is given by

$$n_{\mathrm{seg},m,i}(k) = L_m \lambda_m \rho_{m,i}(k) - T q_{m,i}(k). \tag{9}$$

The cross-segmental acceleration is the change in speed experienced by vehicles moving from one segment of a link to another segment of the same or a different link. The expression for the cross-segmental acceleration of the vehicles depends on the geometry of the traffic network: the formula is different for vehicles in a link, on-ramp, off-ramp, merging links, and splitting links. However, all expressions have a similar structure of the form

$$a_{\mathrm{cross},\alpha,\beta}(k) = \frac{v_\beta(k+1) - v_\alpha(k)}{T} \tag{10}$$

for vehicle going from one segment α, which can represent a segment-link pair or an on-ramp to the next segment β, a representing segment-link pair or an off-ramp.

The number of vehicles $n_{\text{cross},\alpha,\beta}$ that are subject to the cross-segmental acceleration in (10) is obtained as

$$n_{\text{cross},\alpha,\beta}(k) = Tq_{\alpha}(k). \quad (11)$$

With the segmental and cross-segmental variables as input, the VT-macro model is given as

$$J^{\text{seg}}_{y,m,i}(k) = n_{\text{seg},m,i}(k) \exp\left(\tilde{v}_{m,i}(k) P_y \tilde{a}^T_{\text{seg},m,i}(k)\right) \text{ or } \quad (12)$$
$$J^{\text{cross}}_{y,\alpha,\beta}(k) = n_{\text{cross},\alpha,\beta}(k) \exp\left(\tilde{v}_{\alpha}(k) P_y \tilde{a}^T_{\text{cross},\alpha,\beta}(k)\right) \quad (13)$$

where $J^{\cdot}_{y,\cdot,\cdot}(k)$ is the estimate or prediction of the variable $y \in \mathcal{Y} = \{\text{CO}, \text{NO}_x, \text{HC}\}$ for the given segment, on-ramp, or off-ramp from time step k to time step $k+1$, the operator $\tilde{x}$ defines a vector of the scalar variable x as $\tilde{x} = [1\ x\ x^2\ x^3]^\top$, and P_y denotes the model parameter matrix for the variable y. The values of the entries of P_y are given in [32].

By summing $J^{\text{seg}}_{y,m,i}(k)$ and $J^{\text{cross}}_{y,\alpha,\beta}(k)$ for all segments, on-ramps, and off-ramps we get the total emissions $J^{\text{tot}}_{y}(k)$ for the entire network during the time period $[kT, (k+1)T)$.

Note that the VT-macro model may not reflect the real world conditions, as this is also the case with any other model. There is always a trade-off between accuracy and computational efficiency. In [32] the VT-macro traffic emissions and fuel consumption model has been validated and has been compared with the VT-micro and COPERT [25] models based on a case study. Moreover, the error introduced due to the integration of macroscopic traffic flow variables to microscopic emissions and fuel consumption model has been analyzed in [32].

III. Model predictive control

A. Conventional MPC

Conventional Model Predictive Control (MPC) is a control approach in which the current control action to be applied is obtained by solving an on-line open-loop optimal control problem using the current state of the plant as the initial state. The optimization process yields an optimal control sequence and the first control input in the sequence is applied to the plant [19]. In other words, unlike other control methods, conventional MPC does not use a precomputed (or pre-designed) control law to yield control signals [20]. MPC is capable of controlling multivariable and nonlinear systems, and it can handle state and input constraints. Moreover, conventional MPC can to some extent address the prediction errors resulting from model mismatches (or uncertainity) and disturbances due to the feedback and receding horizon approaches [19]. By introducing an on-line identification of parameters for freeway traffic, the issues due to model mismatches can also be handled [3].

In the context of MPC, dynamic systems are often described or approximated by a system of ordinary difference equations

$$x(k+1) = f(x(k), u(k)), \quad y(k) = h(x(k)), \quad (14)$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$, and $y(k) \in \mathbb{R}^{n_y}$ are respectively the state, input, and output vectors, $f(\cdot) \in \mathbb{R}^{n_x}$ is the state vector field, and $h(\cdot) \in \mathbb{R}^{n_y}$ is the output vector field, with n_x, n_u, and n_y being positive integer numbers.

At each control step the MPC controller determines a sequence of control inputs that optimize a given performance criterion over a given prediction period N_{p} control steps ahead, subject to model equation (14) and operational constraints.

Next, only the value of the first control input of the optimal sequence is applied to the system until the control time step, after which the MPC controller repeats the above process all over again using a rolling horizon approach.

MPC, and in particular, MPC for nonlinear systems has certain disadvantages. The main disadvantage emanates from the nonlinear and nonconvex optimization problem involved. Such optimization problems do not only pose difficulty in computing optimal solutions, but also the computation time involved to get the (sub-)optimal solutions is very high. Usually, the computation time exponentially increases as the number of control inputs or the prediction horizon increases. To alleviate the computational problems several approaches can be used. E.g., in order to limit the number of variables to be optimized, a positive integer control horizon N_{c} is defined after which the control input is kept constant, i.e. $\hat{u}(k_{\text{c}} + \ell | k_{\text{c}}) = \hat{u}(k_{\text{c}} + \ell - 1 | k_{\text{c}})$ for $\ell = N_{\text{c}}, \ldots, N_{\text{p}} - 1$, where $N_{\text{c}} \leq N_{\text{p}}$ and k_{c} is the control time step counter which is related to the simulation time step k as $k = Mk_{\text{c}}$ for a positive integer M [19].

Another interesting approach that can reduce the computation time considerably is the parameterization of the control inputs (by a small number of parameters) [12], [16], [18], [28], so that the controller optimizes a set of parameters instead of optimizing a sequence of control inputs as in the case of conventional MPC. This approach is discussed next.

B. Parameterized MPC

In parameterized MPC, the control signals are parameterized according to some control laws. The parameters of the control policies are optimized over certain time horizon to reduce defined objective (cost) function [12], [16], [18], [28]. The parameters of the control policies can be optimized in such a way that they are constant over the prediction horizon [16], [18], [28] or they can be considered to be time-varying over the prediction horizon [12]. However, most of the work on MPC with feedback control policies focus on linear time-invariant [12], [18], [28], linear time-varying [16], and linear parameter-varying [7], [30] systems. The general class of nonlinear systems (such as traffic systems) has not yet been addressed.

IV. Rolling-horizon parameterized (RHP) traffic controller

A. Philosophy of RHP traffic controller

The concept of RHP can be illustrated with the schematic diagram depicted in Fig. 1. The system block designates the real traffic system where the measurements of the traffic states (such as speed, flow, emissions, and fuel consumption) are fed to the RHP controller. The RHP controller contains the model of the traffic system, the optimization tool, and the control law blocks. Using the current measurements of the traffic states as the initial states of the model, the RHP controller predicts the evolution of the system states. The optimization block optimizes a set of parameters that describe the control policy in such a way that the defined objective function is reduced while the constraints are met. The optimal set of parameters is fed to the control policy block that uses them along with the predict traffic states from the model to generate the traffic control measures (such as speed limits, ramp metering rate, and route guidance signals). The parameters are optimized and generated as to be used for the whole prediction horizon. However, the RHP controller applies only the first parameter set to the system. At the next control time step, the prediction horizon shifts one control time step, and the controller repeats the optimization process all over again.

By defining the control policy in different ways, the concept presented above can be realized in three different ways. This is a consequence of the fact that only the first parameter set—out of the parameter sets optimized for the entire prediction horizon—is used for controlling the system. These three different ways are:

1. Control policies with constant parameters. This option is used in most of the literature. In this approach a constant state feedback controller (control law) is designed. The parameters of the control law do not change over the prediction horizon. Thus, the number of parameters is smaller, which leads to reduced computation time. In general, since this approach limits the domain of the parameters, it is conservative, and could have lower performance than the two approaches presented next.
2. Control policies with variable parameters. In this approach the parameters of the control policies vary over the whole prediction horizon. Hence, the space of the control inputs is larger. This control approach requires higher computation times but in general yields better performance than the first option. However, in general this approach is still faster than the conventional MPC approach provided that the number of parameters is less than the number of control measures (see Remark 1).
3. The third approach combines the characteristics of both control policies with constant and variable parameters. In this approach different options can be used to vary the control law parameters while keeping the variation limited. One way is to use blocking, i.e., to force the parameters to remain constant during some pre-defined non-uniform intervals over the prediction horizon [19]. Another approach is to allow the parameters to vary for certain control time horizon N_c that is smaller than the prediction horizon N_p and to keep them constant for the rest of the prediction period.

B. General formulation of RHP for traffic systems

In general, the traffic system can be described by the systems of nonlinear difference equations given in (14). Depending on the model type, the state vector $x(\cdot)$ represents the dynamic states of the traffic system. For example for macroscopic traffic models, it contains the average speeds, flows, densities, and queue lengths. The variable $u(\cdot)$ represents the control signals (such as the dynamic speed limits and the ramp metering rates) and the variable $y(\cdot)$ contains the outputs of the traffic system. This could be the travel time, throughput, and emissions.

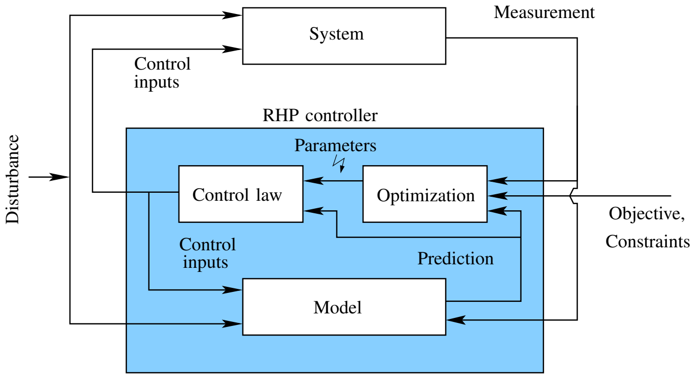

Fig. 1. Model-based Receding-Horizon Parameterized (RHP) control.

We denote[1] $x_\mathrm{me}(k_\mathrm{c})$ as the measured or estimated value of the traffic system state $x(\cdot)$ at time instant $t = k_\mathrm{c}T_\mathrm{c}$. Similarly, we denote $y_\mathrm{me}(k_\mathrm{c})$ as the measured or estimated value of the traffic system output $y(\cdot)$ at time $t = k_\mathrm{c}T_\mathrm{c}$. In the RHP control formulation, the discrete-time control input $u_\mathrm{c}(k_\mathrm{c})$ for the time period $[T_\mathrm{c}k_\mathrm{c}, T_\mathrm{c}(k_\mathrm{c}+1))$ can be defined as a parameterized function of the measured or estimated traffic state vector $x_\mathrm{me}(k_\mathrm{c}-1)$, output vector $y_\mathrm{me}(k_\mathrm{c}-1)$, and parameter vector $\theta(k_\mathrm{c}-1)$. So, the RHP control law is in general given by

$$u_\mathrm{c}(k_\mathrm{c}+1+j'|k_\mathrm{c}) = F\big(\hat{x}_\mathrm{me}(k_\mathrm{c}+j'|k_\mathrm{c}), \hat{y}_\mathrm{me}(k_\mathrm{c}+j'|k_\mathrm{c}), \theta(k_\mathrm{c}+j'|k_\mathrm{c})\big) \tag{15}$$

for $j' = 0, 1, \ldots, N_\mathrm{p}-1$, where F is a user-defined mapping and the notation $\hat{x}_\mathrm{me}(k_\mathrm{c}+j'|k_\mathrm{c})$ and $\hat{y}_\mathrm{me}(k_\mathrm{c}+j'|k_\mathrm{c})$ denote the predicted values of x and y at time step $k_\mathrm{c}+j'$ using the information available at time step k_c.

At every control time step k_c, the RHP controller collects all the parameters of the control law (15) and it optimizes the control objective over the parameters in the same way as the conventional MPC.

Next, only the first value of the parameter is implemented for traffic system, until the next control time step $k_\mathrm{c}+1$, at which the RHP controller repeats the above process all over again using a receding horizon approach as described in the previous sections.

Since the RHP optimization problem is nonconvex and nonlinear, global or multi-start local optimization methods are required, such as multi-start sequential quadratic programming (SQP), pattern search, generic algorithms, or simulated annealing.

[1]We use the subscript 'me' in order make a distinction between the real (or simulated) traffic state x at time step k and its measured (or estimated) value at every control time step k_c. Since we make measurements and apply new control measures to the systems only once every $M \in \mathbb{N}$ simulation time steps, we use a different counter k_c to denote the control time step as opposed to the simulation time step counter k.

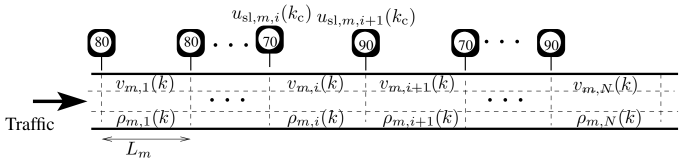

Fig. 2. A freeway link with N segments and variable speed limits.

V. RHP FOR VARIABLE SPEED LIMIT CONTROL

A. General variable speed limit control law

Although one can design an RHP controller for any dynamic traffic model here we design the RHP controller based on a macroscopic model. Actually, since speed limits are used to limit the speed of all vehicles within the same segment of a link, it is logical to use the macroscopic variables such as the average speed, density, or flow of the vehicles to design the RHP controller for speed limit control. So, in this particular case we consider a link with a number of segments as depicted in Fig. 2. There are two ways to control the variable speed limits on a link of the freeway. One option is to control the variable speed limits of each segment independently; and the second option is to group a number of neighboring variable speed limits together and assign them the same value. The general strategy to be presented below holds for both options. So, we will first present the first option and the second option easily follows.

From traffic theory, we know that the density of every segment is affected by the density of the downstream segment (cf. (2)). Hence, it is important to consider the density of the downstream segment in determining the speed limit of the segment. However, density alone cannot describe the flow relation of the two consecutive segments. Therefore, we also need to use the speed to determine the control signal that can produce the desired flow. In fact, it is logical to take the downstream speed, because drivers can only adapt their speeds to the speeds of the leading vehicles. This means that in addition to the downstream density, the downstream speed is also important in the parameterization of the control signals.

In the sequel we give a general approach for designing the dynamic speed limit controller for a link of a freeway depicted in Fig. 2. The dynamic speed limit $u_{\mathrm{sl},m,i}$ of segment i at control time step $k_{\mathrm{c}}+1$ can thus be expressed as

$$u_{\mathrm{sl},m,i}(k_{\mathrm{c}}+1) = f_m\big(v_{\mathrm{me},m,i}(k_{\mathrm{c}}), v_{\mathrm{me},m,i+1}(k_{\mathrm{c}}), \rho_{\mathrm{me},m,i}(k_{\mathrm{c}}), \rho_{\mathrm{me},m,i+1}(k_{\mathrm{c}}), \theta_m(k_{\mathrm{c}})\big) \tag{16}$$

where $f_m(\cdot)$ is a general mapping that determines the control law of the speed limit of segment i of link m, $\theta_m(k_{\mathrm{c}})$ is the parameter vector, $v_{\mathrm{me},m,i}(k_{\mathrm{c}})$ is the measured or estimated average speed of segment i of link m at time $t = k_{\mathrm{c}}T_{\mathrm{c}}$, and $\rho_{\mathrm{me},m,i}(k_{\mathrm{c}})$ is the measured or estimated average density of segment i of link m at time $t = k_{\mathrm{c}}T_{\mathrm{c}}$.

The number of the parameters needed to describe f_m should be less than the number of the variable speed limits, because in general, the RHP controller results in lower computation time than the cMPC controller provided that the control polices are defined in such a way that the number of parameters is smaller than the number of control signals.

In general one can use different relations. Here we propose a variable speed limit control policy where the parameterization describes the relation of the speeds and densities in a linear form. In this form we can easily describe the functions pertaining speed or density separately as

$$\begin{aligned} u_{\mathrm{sl},m,i}(k_{\mathrm{c}}+1) &= \theta_{0,m}(k_{\mathrm{c}})u_{\mathrm{sl,ref},m}(k_{\mathrm{c}}) \\ &\quad + \theta_{1,m}(k_{\mathrm{c}})f_{1,m}(v_{\mathrm{me},m,i}(k_{\mathrm{c}}), v_{\mathrm{me},m,i+1}(k_{\mathrm{c}})) \\ &\quad + \theta_{2,m}(k_{\mathrm{c}})f_{2,m}(\rho_{\mathrm{me},m,i}(k_{\mathrm{c}}), \rho_{\mathrm{me},m,i+1}(k_{\mathrm{c}})) \end{aligned} \tag{17}$$

where $u_{\mathrm{sl,ref},m}(k_{\mathrm{c}})$ is the reference speed that can be either the maximum speed limit of the link or the speed limit that was one control time step back and $f_{1,m}(\cdot)$, $f_{2,m}(\cdot)$ are respectively any state feedback functions that relate the speeds and densities to the variable speed limit control, and $\theta_{0,m}(k_{\mathrm{c}})$, $\theta_{1,m}(k_{\mathrm{c}})$ and $\theta_{2,m}(k_{\mathrm{c}})$ are time-dependent parameters that parameterize the speed limit control signals. In the sequel we provide a specific example of the speed limit controller given in (17).

B. Speed limit controller

The intention of traffic control solutions using speed limits is to attain homogenized traffic flows (such as homogenizing speed and density of vehicles on the freeway [2], [29]) so that the desired performance measure can be met. The speed limit controller presented in (17) is general. The functions $f_{1,m}(\cdot)$ and $f_{2,m}(\cdot)$ could be defined in different ways. Here, we define the two functions by considering the relative difference of the corresponding variables of the functions. The function $f_{1,m}(\cdot)$ is defined as the relative-speed difference of a segment with respect to the speed of the downstream segment, and the function $f_{2,m}(\cdot)$ is defined as the relative-density difference of a segment with respect to the density of the downstream segment. Then the controller can make its decision based on these quantities in order to minimize the differences in speed and density between the segments. Mathematically, these functions are given by

$$f_{1,m}(v_{\mathrm{me},m,i}(k_{\mathrm{c}}), v_{\mathrm{me},m,i+1}(k_{\mathrm{c}})) = \frac{v_{\mathrm{me},m,i+1}(k_{\mathrm{c}}) - v_{\mathrm{me},m,i}(k_{\mathrm{c}})}{v_{\mathrm{me},m,i+1}(k_{\mathrm{c}}) + \kappa_v}, \tag{18}$$

$$f_{2,m}(\rho_{\mathrm{me},m,i}(k_{\mathrm{c}}), \rho_{\mathrm{me},m,i+1}(k_{\mathrm{c}})) = \frac{\rho_{\mathrm{me},m,i+1}(k_{\mathrm{c}}) - \rho_{\mathrm{me},m,i}(k_{\mathrm{c}})}{\rho_{\mathrm{me},m,i+1}(k_{\mathrm{c}}) + \kappa_\rho}, \tag{19}$$

where κ_v and κ_ρ are fixed parameters introduced to prevent division by 0.

We chose the reference speed $u_{\mathrm{sl,ref},m}(k_{\mathrm{c}})$ in (17) to be constant independent of time and it is taken to be equal to the maximum speed limit $v_{\mathrm{ref},m}$ of the traffic network. Hence, the RHP speed limit controller becomes

$$\begin{aligned} u_{\mathrm{sl},m,i}(k_{\mathrm{c}}+1) &= \theta_{0,m}(k_{\mathrm{c}})v_{\mathrm{ref},m} \\ &\quad + \theta_{1,m}(k_{\mathrm{c}})\frac{v_{\mathrm{me},m,i+1}(k_{\mathrm{c}}) - v_{\mathrm{me},m,i}(k_{\mathrm{c}})}{v_{\mathrm{me},m,i+1}(k_{\mathrm{c}}) + \kappa_v} \\ &\quad + \theta_{2,m}(k_{\mathrm{c}})\frac{\rho_{\mathrm{me},m,i+1}(k_{\mathrm{c}}) - \rho_{\mathrm{me},m,i}(k_{\mathrm{c}})}{\rho_{\mathrm{me},m,i+1}(k_{\mathrm{c}}) + \kappa_\rho}. \end{aligned} \tag{20}$$

The proposed controller has only $3 \times N_{\mathrm{p}}$ parameters[2] to be optimized in the RHP control strategy. This means that this speed limit controller can reduce the computation time if it is used with a freeway link with at least four independent variable speed limits ($4 \times N_{\mathrm{p}}$ speed limit variables over the prediction horizon).

Usually, the speed limits are constrained. The speed limit constraint $L_{\mathrm{l},m} \leq u_{\mathrm{sl},m,i}(k_{\mathrm{c}}+1) \leq L_{\mathrm{u},m}$ where $L_{\mathrm{l},m}$ and $L_{\mathrm{u},m}$ are respectively the lower and upper speed limits, can also be recast as constraints for the parameterization vector $\theta_{\cdot,m}(k_{\mathrm{c}})$.

VI. RHP FOR RAMP METERING CONTROL

A. General ramp metering control law

The design of an RHP controller for ramp metering is similar to the way we design the RHP variable speed limit controller. In the case of ramp metering control, the main goal of the controller is to increase the throughput of the on-ramp without affecting the traffic flow in the freeway. The on-ramp flow is basically dependent on the current density and critical density of the freeway. Hence, the ramp metering control signal has to consider these effects. We then propose a ramp metering control which is affine with respect to the time-dependent parameter $\theta_{3,m}(k_{\mathrm{c}})$ as

$$\begin{aligned} u_{\mathrm{r},m,i}(k_{\mathrm{c}}+1) =& u_{\mathrm{r,ref},m}(k_{\mathrm{c}}) \\ &+ \theta_{3,m}(k_{\mathrm{c}}) f_{3,m}(\rho_{\mathrm{me},m,i}(k_{\mathrm{c}}), \rho_{\mathrm{cr},m}) \end{aligned} \quad (21)$$

where $u_{\mathrm{r,ref},m}(k_{\mathrm{c}})$ is the reference ramp metering rate and $f_{3,m}(\cdot)$ is a mapping.

The reference ramp metering rate $u_{\mathrm{r,ref},m,i}(k_{\mathrm{c}})$ can be either the maximum rate that results in desired flow or the rate one control time step back. One possible specific on-ramp controller is presented in the sequel.

B. On-ramp controller

With similar reasoning as in (18) and (19), for the on-ramp control, we define the RHP ramp metering controller to be

$$\begin{aligned} u_{\mathrm{r},m,i}(k_{\mathrm{c}}+1) =& u_{\mathrm{r},m,i}(k_{\mathrm{c}}) \\ &+ \theta_{3,m}(k_{\mathrm{c}}) \frac{\rho_{\mathrm{cr},m} - \rho_{\mathrm{me},m,i}(k_{\mathrm{c}})}{\rho_{\mathrm{cr},m}}. \end{aligned} \quad (22)$$

In this controller, the reference $u_{\mathrm{r,ref},m,i}(k_{\mathrm{c}})$ is taken to be the ramp metering rate one control time step back. The idea behind the structure of the controller is the same as that of ALINEA [26]. In the RHP approach, $\theta_{3,m}(k_{\mathrm{c}})$ is updated every control time step and it is optimized on-line, while in ALINEA the parameter is optimized off-line and is constant irrespective of the prevailing traffic conditions.

Similar to the speed limit control, the ramp metering rate is also constrained as $0 \leq u_{\mathrm{r},m,i}(k_{\mathrm{c}}+1) \leq 1$, which can be recast as a constraint on the parameter $\theta_{3,m}(k_{\mathrm{c}})$.

[2]This is so for the second option in Section IV-A. But, if option 1 or 3 in Section IV-A are used, the number of parameter can be lower than $3 \times N_{\mathrm{p}}$.

VII. CASE STUDY

A. Freeway stretch

The freeway stretch that we consider to illustrate the proposed control approach is a part of the Dutch A12 freeway going from the connection with the N11 at Bodegraven up to Harmelen, and is shown in Fig. 3. The freeway has three lanes in each direction. The part that we consider is approximately 14650 m long and it has two on-ramps and three off-ramps. The stretch is equipped with double-loop detectors at a typical distance of 500 to 600 m, measuring the average speed and flow every minute. It has 24 segments, each equipped with a dynamic speed limit. In [15] real-life data of the freeway has been used to calibrate the METANET model. In our case study we will use the parameters that have been obtained in [15].

B. Control objective

We define our objective function as a weighted sum of the following traffic performance measures: travel time, emissions, and the variance in the control signals (because fast and big changes in the control signals can increase the nervousness of the drivers.)Mathematically, the objective function as computed based on the prediction model is given by

$$J(k_{\mathrm{c}}) = \alpha_1 \frac{\mathrm{TTS}(k_{\mathrm{c}})}{\mathrm{TTS_n}} + \alpha_2 \mathrm{TE}(k_{\mathrm{c}}) + \alpha_3 \frac{\Delta(k_{\mathrm{c}})}{\Delta_{\mathrm{n}}} \quad (23)$$

where α_1, α_2, and α_3 are weighting factors, and

$$\begin{aligned} \mathrm{TTS}(k_{\mathrm{c}}) &= T \sum_{k \in \mathcal{K}(k_{\mathrm{c}},N_{\mathrm{p}})} \left[\sum_{(m,i)\in\mathcal{I}_{\mathrm{all}}} \lambda_m L_m \rho_{m,i}(k) + \sum_{o\in\mathcal{O}_{\mathrm{all}}} w_o(k) \right], \\ \mathrm{TE}(k_{\mathrm{c}}) &= \sum_{y\in\mathcal{Y}} \mu_y \frac{J_y(k_{\mathrm{c}})}{J_{y,\mathrm{n}}}, \qquad J_y(k_{\mathrm{c}}) = \sum_{k\in\mathcal{K}(k_{\mathrm{c}},N_{\mathrm{p}})} J_y^{\mathrm{tot}}(k), \\ \Delta(k_{\mathrm{c}}) &= \sum_{\ell=k_{\mathrm{c}}}^{k_{\mathrm{c}}+N_{\mathrm{p}}-1} \left[\sum_{s\in\mathcal{S}_{\mathrm{all}}} \alpha_{\mathrm{s}} \big(u_s(\ell) - u_s(\ell-1)\big)^2 \right. \\ &\quad + \sum_{(s_1,s_2)\in\mathcal{P}_{\mathrm{all}}} \alpha_{\mathrm{cs}} \big(u_{s_1}(\ell) - u_{s_2}(\ell)\big)^2 \\ &\quad \left. + \sum_{r\in\mathcal{R}_{\mathrm{all}}} \alpha_{\mathrm{r}} \big(u_r(\ell) - u_r(\ell-1)\big)^2 \right] \end{aligned}$$

with $J_y^{\mathrm{tot}}(k)$ denoting the sum of the emissions given by (12) and (13), $\mathcal{K}(k_{\mathrm{c}}, N_{\mathrm{p}})$ denoting the set defined as $\mathcal{K}(k_{\mathrm{c}}, N_{\mathrm{p}}) = \{Mk_{\mathrm{c}}, \ldots, M(k_{\mathrm{c}}+N_{\mathrm{p}})-1\}$, μ_y denoting the weights of the emissions and fuel consumption (in particular we consider $\mu_y = 1$ in our case study), $\mathcal{I}_{\mathrm{all}}$ denoting the set of all segments of links in the traffic network, $\mathcal{O}_{\mathrm{all}}$ denoting the set of all origins in the traffic network, $\mathcal{S}_{\mathrm{all}}$ denoting the set of all speed limits, $\mathcal{P}_{\mathrm{all}}$ denoting pairs of consecutive speed limits, $\mathcal{R}_{\mathrm{all}}$ denoting the set of all controlled on ramps, $\alpha_{\mathrm{s}} = (\#(\mathcal{S}_{\mathrm{all}}) v_{\mathrm{step}}^2)^{-1}$, $\alpha_{\mathrm{cs}} = (\#(\mathcal{P}_{\mathrm{all}}) v_{\mathrm{step}}^2)^{-1}$, and $\alpha_{\mathrm{r}} = (\#(\mathcal{R}_{\mathrm{all}}))^{-1}$ are respectively the normalization factors of the variation of the speed limits over time, the variation speed limits in space, and the variation of the ramp metering rate over time, $\#(\cdot)$ denoting the set cardinality, and the subscript 'n' denoting the nominal value of the quantities TTS, J_y, and

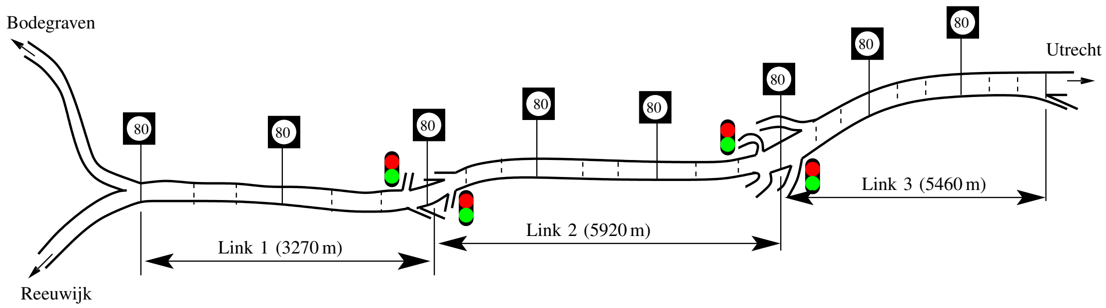

Fig. 3. A part of the Dutch A12 highway considered in the case study.

Δ. For our case study, the nominal values of TTS and J_y are computed by simulating the uncontrolled traffic system (which is equivalent to a control system with all speed limits set to $v_{\mathrm{free},m} = 120\,\mathrm{km/h}$ and all ramp metering rates equal to 1). The nominal value for Δ is equal to N_{p}.

C. Optimization

In this particular case study we use the first approach of the RHP control strategy of Section IV-A, where the parameters of the control policy are maintained constant over the prediction horizon N_{p}. This means that the number of optimization variables is equal to the number of the controller parameters $\theta_m(k_{\mathrm{c}})$, i.e. 4.

We use the upper speed limit $L_{\mathrm{u},m} = v_{\mathrm{ref},m} = 120\,\mathrm{km/h}$ and the lower speed limit $L_{\mathrm{l},m} = 40\,\mathrm{km/h}$. The ramp metering rate is also constrained $0 \leq u_r(k) \leq 1$. To solve the optimization problem we use multi-start SQP. We use 5 initial starting points of which 1 is random, while the rest consists of the lower bounds of the optimization variables, the upper bounds, the average of the lower and upper bounds, and the one time- step forward-shifted version of the solution of the previous optimization step.

D. Comparison of conventional MPC and RHP

In order to compare the conventional MPC (cMPC) and RHP traffic controllers, we simulate the case study for three different sets of weights for the performance measures of the objective function given in (23). Since our main objective is to optimize the control measures such that the TTS or TE can be reduced, we assign smaller value for α_3 than α_1 and α_2. In this way the fluctuation of the control measures over time and space is not overemphasized. For all the cases the weight of the control input α_3 is assigned the same value, and it is equal to $\alpha_3 = 0.01$. The three cases considered to compare the cMPC and RHP are:

1) $\alpha_1 = 1$ and $\alpha_2 = 0$, i.e., the focus of the controllers is to reduce only the total time spent (TTS).
2) $\alpha_1 = 0$ and $\alpha_2 = 1$. In this case the focus of the controllers is to reduce the total emissions (TE).
3) $\alpha_1 = 9$ and $\alpha_2 = 1$, i.e., the controllers will focus on the reduction of the TTS and the TE with a different degree of emphasis. This combination should result in a balanced trade-off between travel time and emissions.

The 24 speed limits of the freeway stretch are coupled in groups of three, where each speed limit control in the group is assigned the same value (see Fig. 3). Thus there are 8 variable speed limit values. Moreover, the two ramp meters are controlled independently. Thus, in total there are 10 (8 speed limit values and 2 ramp metering rates) control variables. This means that the number of the optimization variables of the cMPC controller is $10 \times N_{\mathrm{c}}$ per control time step. For the RHP controller we use the control laws given in (20) and (22). Thus, due to the parameterization of control inputs, the RHP controller will have $4 \times N_{\mathrm{c}}$ optimization variables.

We consider the traffic flow in one direction (from left to right in Fig. 3). We use $T_{\mathrm{c}} = 60\,\mathrm{s}$, $N_{\mathrm{p}} = 15$ control time steps (15 min), $N_{\mathrm{c}} = 10$ control time steps (10 min), and $T = 10\,\mathrm{s}$. We simulate the evolution of the case study over 1 h of the real system. Moreover, the conventional MPC (cMPC) is also simulated under constrained optimization time, where the cMPC is allowed to optimize the traffic control variable as long as the computation time is less than the control time step. This option is called constrained optimization time cMPC (COT-cMPC).

The results of the simulations of the three cases are presented in Table I. The results give the values of each performance indicator for each controlled case. For comparison reasons, Table I presents the simulation results of the cMPC, COT-cMPC, and the RHP controllers. It can be seen that the performance measures (TTS and TE) for the cMPC and RHP control approaches is almost the same for the three different combinations of weights. However, the difference in the average computation time (CPU Time) per control step of the cMPC and RHP controllers is significant. The RHP controller improves the computation times of all cases by more than 96%. Note, however, that it is possible that the performance (in terms of TTS or TE) of the RHP controller can be worse than the performance of the cMPC controller as the RHP controller is an approximation of the cMPC controller.

In the case where the computation time of the cMPC controller is not allowed to exceed the control time step T_{c}, the COT-cMPC controller performs badly. In all the three cases presented in Table I, the TTS is worst than the uncontrolled case. The simulation results presented in Fig. 7 also conform to these results.

Moreover, the on-ramp queue length produced under the RHP and cMPC TTS-controlled scenarios and for the uncontrolled scenario are plotted in Fig. 4. The plots show that

TABLE I
SIMULATION RESULTS FOR THE CONVENTIONAL MPC (CMPC) TRAFFIC CONTROLLER AND THE RHP TRAFFIC CONTROLLER.

Objective	Controller	TTS (veh·h)	TE (kg)	CPU Time (seconds)
Uncontrolled	—	1045.8	219.5	0.0
TTS	RHP	841.0	216.9	10.9
	cMPC	811.5	230.5	375.2
	COT-cMPC	1295.2	164.5	60.0
TE	RHP	2377.5	82.7	25.7
	cMPC	2283.1	92.0	1862.7
	COT-cMPC	1317.5	153.5	60.0
9TTS+TE	RHP	842.7	213.9	40.4
	cMPC	807.4	214.0	1809.3
	COT-cMPC	1271.6	148.4	60.0

Performance measures: TTS, TE, CPU Time.

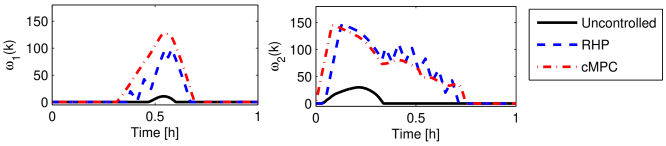

Fig. 4. On-ramp queue length of TTS controlled scenarios.

the queue-length formed using either the RHP or the cMPC controllers are almost the same for this particular case study. The control inputs of the two controllers are also plotted in Fig. 6 for the TTS-controlled case. Fig. 6 shows no relation between the control inputs generated by the RHP controller and the control inputs of the cMPC controller. The variation of the parameters of the RHP controller that resulted in the control inputs in Fig. 6 are also depicted in Fig. 5. One can clearly see that the variation of the parameters and the control inputs are not explainable. At the same time, for the same parameter the control inputs of each segment are different (compare Fig. 6 and Fig. 5). Due to limited space, the queue length, control inputs, and parameters of the different control scenarios are not displayed.

E. RHP and cMPC under varying weightings

Furthermore, we use the RHP, cMPC, and COT-cMPC controllers to simulate several possible weight combinations of the TTS and the TE. We consider the relation $\alpha_2 = 10-\alpha_1$, where $\alpha_1 = 0, 1, 2, ..., 10$, and $\alpha_3 = 0.01$. The results of the simulation for the different weight combinations are depicted

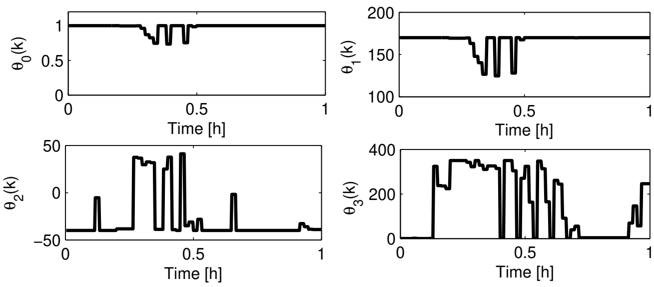

Fig. 5. The parameters of the RHP controller for TTS controlled case.

in Fig. 7. The figure provides the percentage reduction or increment of the TTS and TE as the weights vary.

The figure shows that the TTS can be reduced by a factor of more than 19% and the TE by less than 2% when the focus of the RHP controller is on TTS ($\alpha_1 = 10, \alpha_2 = 0$) only. Moreover, it shows that the TE can be reduced by more than 58% if the focus of the RHP controller is on TE ($\alpha_1 = 0$, $\alpha_2 = 10$) only, but then the TTS increases by more than 120%. The figure also indicates that a reduction of more than 30% in emissions can be attained without affecting the travel time if the relative weight of the TTS is about $\alpha_1 = 4$ while the TE has a weight of $\alpha_2 = 6$ when the controller is RHP and if the relative weight of the TTS is about $\alpha_1 = 2.5$ while the TE has a weight of $\alpha_2 = 7.5$ when the controller is cMPC.

Although the performance of the cMPC controller under the unlimited computation time is almost similar even in many of the cases better than the RHP controller, the COT-cMPC performs worst than the RHP controller. In all the cases, the performance of the COT-cMPC is worst than even the uncontrolled scenario. In this particular case study, the results show that limiting the optimization time of the cMPC controller can negatively impact the traffic flow. However, with the use of RHP and with appropriate definition of the control laws, the computation time can be reduced significantly while the performance of the controller is negligibly degraded.

VIII. CONCLUSIONS

We have presented a new traffic control approach (called rolling-horizon parameterized (RHP) traffic control) that considers the prevailing traffic conditions and the consequences of its control actions to optimize the performance of the traffic network. We have discussed the general philosophy behind the RHP approach and its possible derivatives. Since the presented control strategy is general, we have selected a specific model class, in particular macroscopic models, in order to illustrate how to design and use the controller for traffic systems. In this respect, we have given a general description of variable speed limit control and ramp metering control design strategies derived from a general RHP problem formulation.

We have demonstrated the control approach for a simulation based case study on a part of the Dutch A12 freeway. With this case study we have compared the performance of the proposed RHP controller with the conventional MPC controller. The comparison shows that the RHP controller results in a performance that is almost the same as that of conventional MPC, while it only requires very low computation times, which is important for the realization of the controller in practice.

We will also extend the proposed control strategy to include the dispersion of emissions as additional performance criterion, we will use other traffic control measures, and we will consider other more complex case studies.

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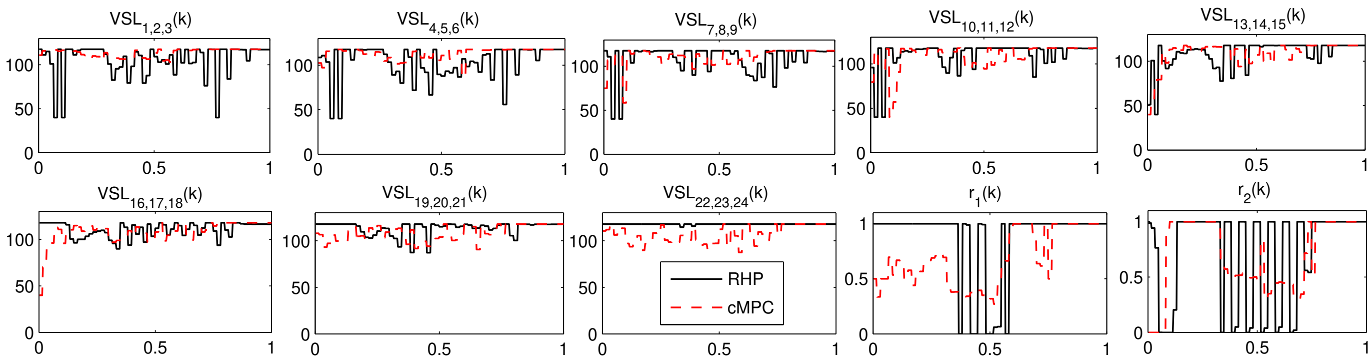

Fig. 6. Evolution of the variable speed limits and ramp metering rates for the TTS-controlled cases when RHP and cMPC controllers are used. The $\mathrm{VSL}_{(\cdot)}(k)$ are in km/h, $r_{(\cdot)}(k)$ are dimensionless, and the $x-$axis is the time in hours.

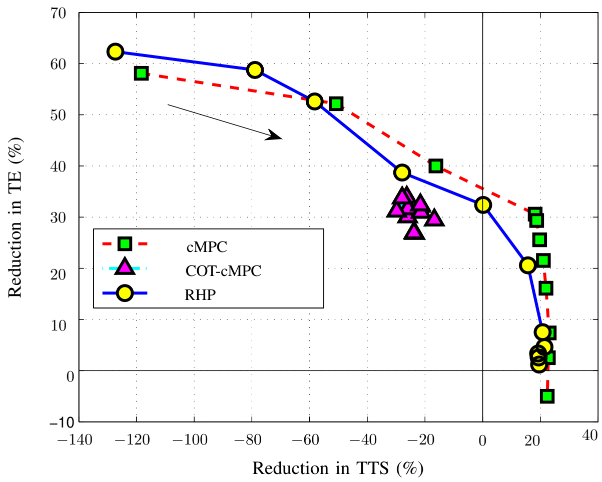

Fig. 7. Trade-off curve of the objective function $J = \alpha_1 \mathrm{TTS} + (10 - \alpha_1)\mathrm{TE} + 0.01\Delta$ for α_1 increasing in the direction of the arrow. Each point on the graph indicates the relative reduction of the variables with respect to the uncontrolled case. Negative values indicate an increment of the variables.

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