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# Benchmarking the Operation of a Hydro Power Network Through the Application of Agent-Based Model Predictive Controllers

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## Abstract

This paper presents a comparison between a decentralized and a distributed model-based predictive controller on a hydro power valley (HPV) benchmark recently proposed. The HPV is composed by three lakes and a river that is divided in six reaches that terminate with dams equipped with turbines for power production. The lakes and the river reaches are connected in three different ways: by a duct, ducts equipped with a turbine, and ducts equipped with a pump and a turbine. The river is fed by upstream inflows and tributary flows. In order to test the controllers, the following test scenario is considered: the power output of the system should follow a given reference while keeping the water levels in the lakes and at the dams as constant as possible. Finally, a 24 hour simulation for the two controllers has been carried out in order to compare both methods using several performance indices.

## 1 Introduction

A hydro power network consists of various sub-systems that can transform water flow into energy with the main objective of generating as much energy or financial gain as possible. At the same time, these flows influence water levels in the environmental water systems they belong to. These levels need to be controlled in order to satisfy other objectives such as flood protection or water availability for irrigation and navigation, all with a dynamic, yet predictable behavior. The control infrastructure consists of pumps, turbines, and gates that are all characterized by limited capacities. As such, the control of a hydro power network can be considered a constrained optimization problem that needs to be solved in real-time. In literature and more and more in practice, model predictive control (MPC) is proposed as the methodology that is well fit for this task [7, 8, 13] due to its capability to structurally handle anticipation and problems with limited capacities and conflicting objectives. Moreover, MPC is able to handle multi-variable interactions and constraints on the process variables in a systematic manner. To this end, MPC uses a mathematical model of the system to calculate the optimal control sequence signal that will steer the system along a given prediction horizon and according to a given cost function.

Unfortunately, optimization comes with a computational burden that hinders real-time application on larger networks [6]. At the same time, within these networks, the subsystems may need to be controlled by a control system that preferably is installed locally. This is especially important in systems that are geographically distributed; a centralized control solution for such systems requires that it is possible to gather and process all the information at a single point, and to calculate and transmit the optimal control actions within the time sample. These assumptions may not hold and for this reason it may be necessary to implement decentralized or distributed control strategies. Hence, the way to tackle these challenges is by configuring the controller of each sub-system as an agent that may or may not coordinate its operations with the other agents in the network. In case that the agents do not exchange any kind of information among them, we speak of decentralized MPC. Under this approach each agent sees the actions carried out by other agents as mere disturbances. Alternatively, the agents can share information in order to coordinate their actuation. In this case we speak of distributed MPC (DMPC). The way the centralized problem is distributed and the type and amount of information that the agents exchange before attaining a solution to the control problem depend on the particular DMPC algorithm used. For example, in [5] dual decomposition is used in order to distribute the centralized control problem. Another way to distribute the problem is the Jacobi algorithm, whose description can be found in [2] (see pages 219-223). This algorithm is the core idea of one of Venkat's feasible cooperation-based MPC [9, 12]. There are many other approaches to distribute the centralized control problem. See [11] for an extensive survey of DMPC techniques. In this paper we will compare the performance of one of these techniques with a decentralized approach in a benchmark [10] that belongs to the European project HD-MPC<sup>1</sup>. One of the goals of this project has been the design and implementation of a variety of distributed control algorithms in benchmarks. Previous results of this project can be seen in [1], where a four-tank plant was used as benchmark. In this case, different schemes will be tested in a hydro power plant composed by 3 lakes and a river that is divided in 6 reaches that terminate with dams equipped with turbines for power production. The lakes and the river reaches are connected by ducts equipped with turbines and pumps. Notice that the switching between generating energy using the turbines and consuming energy using the pumps in combination with the continuous water flow and level variables makes this a hybrid optimization problem, which considerably increments the complexity of the overall control problem. In particular, we will compare the following techniques:

- Decentralized MPC: the agents do not share information with each other. For this reason, the performance of this approach can be used as a lower bound.
- DMPC based on agent negotiation: this scheme was proposed in [4] and it is the extension of the scheme of [3]. It is tailored for distributed control problems in which the agents have little information on the overall system and must reach a cooperative solution with a low number of communications.

These two schemes will be tested using a power tracking scenario in which the power output of the overall system should follow a given reference while keeping the water levels in the lakes and at the dams as constant as possible. Four quantitative indices will be used to analyze the performance of the approaches: mean absolute tracking error, mean quadratic tracking error, and two economic power reference tracking indices.

The rest of the paper is organized as follows. Section 2 introduces the HPV model. Section 3 presents the application of a decentralized MPC scheme to the HPV. Section 4 deals with the applica-

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<sup>1</sup>More information about this project can be found at [www.ict-hd-mpc.eu](http://www.ict-hd-mpc.eu).

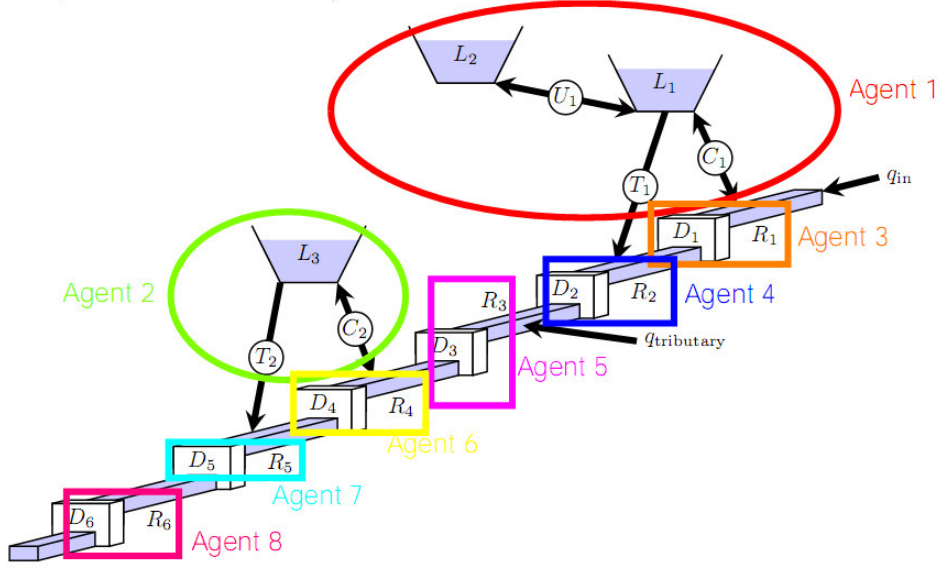


Figure 1: Hydro power valley.

tion of the DMPC based on agent negotiation presented in [4] to the HPV. Finally, Section 5 analyzes the results obtained by these schemes.

## 2 Hydro power valley

The system we consider is a hydro power plant composed by several subsystems connected together. Figure 1 gives an overview of the HPV, which is composed by 3 lakes ( $L_1$ ,  $L_2$ , and  $L_3$ ) and a river that is divided in 6 reaches ( $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$ ,  $R_5$ , and  $R_6$ ) that terminate with dams equipped with turbines for power production ( $D_1$ ,  $D_2$ ,  $D_3$ ,  $D_4$ ,  $D_5$ , and  $D_6$ ). The lakes and the river reaches are connected by a duct ( $U_1$ ), ducts equipped with a turbine ( $T_1$  and  $T_2$ ), and ducts equipped with a pump and a turbine ( $C_1$  and  $C_2$ ). The river is fed by the flows  $q_{in}$  and  $q_{tributary}$ . The system is partitioned into 8 agents as shown in Figure 1. In this paper, we will work with the complete mathematical model given in [10], which is based on first principle equations. For example, the reaches are modeled using the Saint Venant's equations [10]. These equations are discretized and linearized in order to obtain a simpler model more suitable for our control purposes.

## 3 Control test scenario: Power reference tracking

The power reference to be followed is known in advance. In particular, there are patterns on the power consumption behavior that allow us to assume that the power reference is known 24 hours in advance (86400 s). We can use the knowledge of the reference and the model that is presented in [10] to calculate the optimal control inputs that have to be applied to the HPV during the next 24 hours. In particular, we recalculate every 30 minutes (1800 s) the optimal inputs that will be applied to the system during the next 24 hours. Naturally, only the control actions that correspond to the first 30 minutes are actually implemented and the rest are discarded. All the calculations are repeated every 30 minutes in a rolling horizon fashion, so that the controller can take into account the new information

that is available with time. This is the key idea behind MPC and in our case it is translated into the solution of the following optimal control problem:

$$\begin{aligned} \min_{x_i, u_i} \int_0^{86400} \gamma \left| p_r(t) - \sum_{i=1}^8 p_i(x_i(t), u_i(t)) \right| dt + \sum_{i=1}^8 \int_0^{86400} (x_i(t) - x_{ss,i})^T Q_i (x_i(t) - x_{ss,i}) dt \\ \text{s.t.} \\ \dot{x}(t) = f(x(t), u(t)) \end{aligned} \quad (1)$$

where  $p_r(t)$  is the given power reference (piecewise constant),  $p_i(x_i(t), u_i(t))$  is the power generated by agent  $i$  and depends respectively on its state  $x_i(t)$  and input  $u_i(t)$ ,  $x_{ss,i}(t)$  is the steady state of agent  $i$ , and  $f$  is a function that represents the dynamics of the whole system and that depends respectively on the state  $x(t)$  and the input  $u(t)$  of the overall system. Notice that this function is weighted by a constant  $\gamma$  and the matrices  $Q_i$ , which have the proper size to weight the components of state vectors of the agents  $x_i(t)$ . The following indices will be calculated in order to compare the decentralized and the distributed controller:

- Mean absolute tracking error (MAE) in MW: the absolute value of the tracking error integrated during the whole simulation.
- Mean quadratic tracking error (MQE) in MW<sup>2</sup>: the square error of the tracking error integrated during the whole simulation.
- Power reference tracking index 1 ( $J_1$ ) in Euros:

$$\int_0^{86400} c(t) \left| p_r(t) - \sum_{i=1}^8 p_i(x_i(t), u_i(t)) \right| dt \quad (2)$$

where  $c(t)$  is the cost of the electricity at time  $t$ .

- Power reference tracking index 2 ( $J_2$ ) in Euros: another option that will be used to test the economic performance of the scheme is given by the following expression, which penalizes less the overproduction of power:

$$\begin{aligned} \int_0^{86400} c(t) \max \left( p_r(t) - \sum_{i=1}^8 p_i(x_i(t), u_i(t)), 0 \right) dt + \\ 0.5 \int_0^{86400} c(t) \max \left( \sum_{i=1}^8 p_i(x_i(t), u_i(t)) - p_r(t), 0 \right) dt \end{aligned} \quad (3)$$

## 4 Simulations

In this section, we show the results obtained for all the control schemes that have been considered. The numerical values of the parameters that are used in the simulations can be found in [10].

### 4.1 Decentralized MPC

The decentralized MPC is defined as follows:

- There are 8 local controllers, each of them is responsible for the control of one subsystem. Each one can only measure its own output and can only control its own manipulator(s).

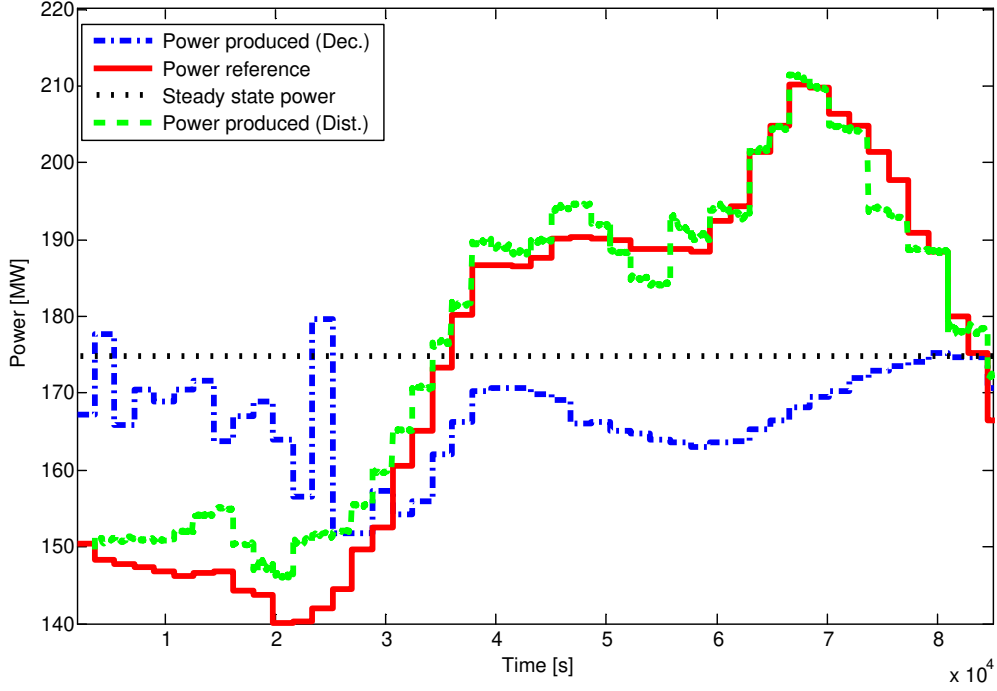


Figure 2: Total power reference tracking with decentralized and distributed MPC

- The controllers use linearized local models with double-flow technique (see p. 5 of [10]) in order to apply discrete-time linear MPC control to the local problem.
- No information exchange is allowed. The steady-state inputs and states are the only common information of the local controllers. Any subsystem interaction will be modeled by using the steady-state variables of the other models.
- The power reference tracking is separated into local tracking, with the local power references proportional to the steady-state power of the corresponding subsystem.

With the above conditions, we treat the control problem proposed in the previous section. In particular, each local MPC controller tries to solve the following problem:

$$\min_{x_i, u_i} \int_0^{86400} \gamma |p_{r,i}(t) - p_i(x_i(t), u_i(t))| dt + \int_0^{86400} (x_i(t) - x_{ss,i})^T Q_i (x_i(t) - x_{ss,i}) dt, \quad (4)$$

where  $p_{r,i}(t)$  is the local power reference. The values of  $p_{r,i}(t)$  are computed such that the following condition is maintained:

$$\frac{p_{r,i}(t)}{p_r(t)} = \frac{p_{ss,i}}{p_{ss}}, \quad \forall i, \forall t \quad (5)$$

in which  $p_{ss,i}$  and  $p_{ss}$  respectively represent the steady-state power generated by the subsystem  $i$  and the whole plant. For the decentralized simulation, we use the sampling time  $T = 1800$  s and  $\gamma = 500$ . The number of time samples that the controller calculates each time is 10, i.e., the control horizon is  $N_c = 10$ . The results are plotted in Figure 2.

## 4.2 DMPC based on agent negotiation

This scheme is proposed in [4] and its goal is to minimize a global performance index defined as the sum of each of the local cost functions. The local cost function of agent  $i$  based on the predicted trajectories of its state and inputs is defined as

$$J_i(x_i, \{U_j\}_{j \in n_i}) = \sum_{k=0}^{N-1} L_i(x_{i,k}, \{u_{j,k}\}_{j \in n_i}) \quad (6)$$

where  $U_j = \{u_{j,k}\}$  is the future trajectory of input  $j$ ,  $n_i$  is the set of inputs that affect the dynamics of agent  $i$ ,  $N$  is the prediction horizon, and  $L_i(\cdot)$  is the stage cost function defined as

$$L_i(x_i, \{u_j\}_{j \in n_i}) = (x_i - x_{ss,i})^T Q_i (x_i - x_{ss,i}) + \sum_{j \in n_i} u_j^T S_{ij} u_j \quad (7)$$

with  $Q_i > 0$ ,  $S_{ij} > 0$ . The term  $x_{ss,i}$  stands for the agent  $i$  reference. We use the notation  $x_{i,k}$  to denote the state  $i$ ,  $k$ -steps in the future obtained from the initial state  $x_i$  applying the input trajectories defined by  $\{U_j\}_{j \in n_i}$ .

We define next the proposed distributed MPC scheme:

- Step 1: Each agent  $p$  measures its current state  $x_p$ . The initial value for the decision control vector  $U^d(t)$  is set to the value of the shifted input trajectories (the components corresponding to the first time sample are discarded and the last value of these trajectories is repeated), that is,  $U^d(t) = F_{\text{shift}}(U^d(t-1))$ .
- Step 2: Randomly, each agent asks the neighbors affected if they are free to evaluate a proposal (each agent can only evaluate a proposal at the time). If all the neighbors acknowledge the petition, the algorithm continues. If not, the agent waits a random time before trying again. We will use the superscript  $p$  to refer to the agent which is granted permission to make a proposal.
- Step 3: In order to make its proposal, agent  $p$  solves:

$$\{U_j^p(t)\}_{j \in n_p} = \arg \min_{\{U_j\}_{j \in n_p}} J_p(x_p, \{U_j\}_{j \in n_p}) \quad (8)$$

subject to the system model, the operational constraints and the information regarding the previous agreed trajectories. From the centralized point of view, the proposal at time step  $t$  of agent  $p$  is defined as

$$U^p(t) = \{U_j^p(t)\}_{j \in n_p} \cup U^d(t) \quad (9)$$

where  $\cup$  stands for the update of the control actions affected by the proposal in  $U^d(t)$ .

- Step 4: Each agent  $i$  affected by the proposal evaluates the difference between the cost of the new proposal  $U^p(t)$  and the cost of the current accepted proposal  $U^d(t)$  as

$$\Delta J_i^p(t) = J_i(x_i(t), \{U_j^p(t)\}_{j \in n_i}) - J_i(x_i(t), \{U_j^d(t)\}_{j \in n_i}) \quad (10)$$

This difference  $\Delta J_i^p(t)$  is sent back to the agent  $p$ . If the proposal does not satisfy the constraints of the corresponding local optimization problem, an infinite cost increment is assigned. This implies that infeasible proposals will never be chosen.

Table 1: Table of the quantitative benchmark indices of each tested controller

<b>Control performance</b>	$J_1$	$J_2$	MAE	MQE
DMPC based on agent negotiation	5.75e3	4.19e3	3.72	19.88
Decentralized MPC	1.65e5	1.39e5	104.36	13.52e3

- Step 5: Once agent  $p$  receives the local cost increments from each neighbor, it can evaluate the impact of its proposal  $\Delta J^p(t)$ , which is given by the following expression

$$\Delta J^p(t) = \sum_{i \in m(n_p)} \Delta J_i^p(t) \quad (11)$$

where  $m(n_p)$  is the set of agents affected by the inputs in the set  $n_p$ . This global cost increment is used to make a cooperative decision on the future input trajectories. If  $\Delta J^p(t)$  is negative, the agent will broadcast the update on the control actions involved in the proposal and the joint decision vector  $U^d(t)$  will be updated to the value of  $U^p(t)$ , that is  $U^d(t) = U^p(t)$ . Else, it is discarded.

- Step 6: The algorithm goes back to Step 1 until the maximum number of proposals have been made or the time available for placing proposals is over. We denote the optimal cost corresponding to the decided inputs as

$$J(t) = \sum_{i=1}^8 J_i(x_i(t), \{U_j^d(t)\}_{j \in n_i}) \quad (12)$$

- Step 7: The first input of each optimal sequence in  $U^d(t)$  is applied and the procedure is repeated at the next sampling time instant.

In Figure 2, the results of the HPV controlled in closed loop with the distributed controller based on agent negotiation are shown.

## 5 Conclusions

In Table 1 the performance indices for both the decentralized and the distributed controllers are shown. Notice that there are very important differences among them. For example, the values of the two economical indices of the distributed scheme are two orders of magnitude better than the corresponding values of the decentralized scheme. Hence, the communication between the controllers translates into a significant improvement of the economical indices.

Our results show clearly the advantages derived from the cooperation among the different controllers. In Figure 2 the power produced by the agents hardly follows the reference. On the other hand, it can be seen in the same figure how a joint actuation of the agents allows them to follow the reference. Even when the distributed MPC scheme presented in [4] is tailored for agents that have access only to their local model, state and objective, the negotiation procedure among the agents allows them to calculate an actuation that improves the overall performance of the system as a whole.



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