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Technical report 12-016

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M. Rinaldi, L. Capisani, A. Ferrara, A. Núñez, M. Hajiahmadi, and B. De Schutter, "Distributed identification of the cell transmission traffic model: A case study," *Proceedings of the 2012 American Control Conference*, Montréal, Canada, pp. 6545–6550, June 2012.

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* This report can also be downloaded via https://pub.bartdeschutter.org/abs/12_016.html

Distributed Identification of the Cell Transmission Traffic Model: A Case Study

Marco Rinaldi, Luca Capisani, Antonella Ferrara, Alfredo Núñez, Mohammad Hajiahmadi, Bart De Schutter

Abstract— **The problem of the distributed identification of a macroscopic first-order traffic model, viz. the Cell Transmission Model (CTM), is considered in the paper. The parameters to be identified characterize the dynamics of the density in different sections of the freeway (cells). We explore different distributed identification schemes. The purposes of the approach are mainly to obtain good prediction models through the minimization of the one-step ahead prediction error of the densities of the cells, and to reduce the computational time and the effort required to perform the identification. The methodology is validated relying on real-life data measured on a portion of the A12 freeway in The Netherlands. An evaluation of the performance of the identified model used as a set of virtual sensors in different scenarios is presented.**

I. INTRODUCTION

The design of advanced control and prediction systems that are able to regulate or predict the flow of vehicles on a freeway requires the knowledge of a suitable model of the specific portion of the freeway to be considered, see e.g. [3], [8]. The choice of the model is dictated by the particular requirements to be satisfied. While microscopic models take into account the behavior and the dynamics of each single vehicle present on the freeway, macroscopic models are particularly useful when little computation power is available. Those models are based on the vehicle conservation principle and are able to model the dynamics of the average flow, density, and velocity of different segments of the freeway. In particular, macroscopic first-order models [6], [13] describe the dynamics of the densities in different segments of the freeway called "cells", and are based on the fundamental assumption that the relation between the density and the flows in each cell is given by a pre-specified fundamental diagram, see [2], [5], [9].

In this paper we deal with the distributed identification of a macroscopic first-order traffic density model, viz. the Cell Transmission Model (CTM), developed in [5], [6]. This model is a linear switching system [11], and the parameters to be identified characterize the dynamics of different cells. Those parameters are: the free velocity, the maximum density, and the backward congestion propagation speed.

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All the authors of this paper acknowledge the financial support from FP7 European Research Network, HYCON2

In cases when, for example, an accident occurs, or there are changes in the weather, a fast and good identification method is essential to update, for example, the model used in a model predictive control (MPC) scheme so to decide how to modify a reconfigurable controller, or to trigger a fault detection scheme. To perform the identification for the whole highway section in those cases is computationally prohibitive as the optimization problem we solve is highly non-linear, [1], [4], [10]. Therefore, in this paper the purpose is to extend the results obtained in [14], [15], estimating a set of parameters that minimize a proposed objective function in a fast and distributed way. This function has a twofold objective. On one hand, to obtain parameters suitable for prediction, monitoring, and control, it takes into account the minimization of the one-step ahead prediction error of the densities of the cells. On the other hand, to decide how to properly split the identification problem (a nonlinear optimization problem), into smaller but more tractable ones, it changes the interactions between different hierarchical and distributed optimization problems. Some consequences of the topology we select in this step are also related to the robustness of the identification procedure, the total computational time required to identify the whole highway, the level of communication between the distributed problems, the number of parameters to be obtained, etc. It is important to highlight that while most of the literature focuses on the problem of distributed control and state estimation design, in this paper we focus on the distributed identification problem. Even if we validate the procedure reference to a specific class of hybrid non-linear systems, the methodology we propose in the paper is general and can be used for the identification of other classes of distributed parameter systems [12].

The outline of the paper is as follows. In Section 2, the CTM is presented. In Section 3, the identification procedure is proposed. In Section 4, we use real data to identify the parameters of the model, and we evaluate the resulting models under different scenarios. Finally, conclusions and future works are highlighted.

II. CELL TRANSMISSION MODEL (CTM)

The CTM [6] represents a network with a set of elementary components called cells, interconnected by junctions, containing on-ramps and off-ramps. Within each cell the density of vehicles is homogeneous. The network is represented by links divided into N segments of length L with λ lanes. Each segment i represents one cell. The cells are characterized by the traffic density $\rho_i(k)$ $(veh/lane/km)$ and the flow in the junction $q_i(k)$ (veh/h). In each cell at most one type of ramp

can exist (or not), where the flow entering via an on-ramp or the flow leaving the cell via an off-ramp is denoted by $u_i(k)$ (veh/h). An extra variable $\gamma_i \in \{-1,0,1\}$ is introduced in order to indicate whether the flow variable $u_i(k)$ represent the flow leaving the cell via an off-ramp ($\gamma_i = -1$), the entering flow via an on-ramp ($\gamma_i = 1$), or the cell does not have any ramps at all ($\gamma_i = 0$). For each cell i, the following equations hold:

$$\begin{aligned} \rho_i(k+1) &= \rho_i(k) + \frac{T}{L\lambda}(q_i(k) - q_{i+1}(k) + \gamma_i u_i(k)), \\ q_i(k) &= \min\{v_{i-1}\rho_{i-1}(k), q_{i,\mathrm{M}}, w_i(\rho_{i,\mathrm{M}} - \rho_i(k))\}, \end{aligned} \tag{1}$$

where k indicates the time instant $t = kT$, and T is the time step used for the simulation of the traffic flow (in our case $T = 10s$), $v_i(km/h)$ is the free flow speed, $q_{i,\mathrm{M}}(veh/h)$ is the maximum flow between the upstream cell and the downstream cell, $w_i(km/h)$ is the backward congestion propagation speed, and $\rho_{i,\mathrm{M}}(veh/km)$ the maximal density of cell i. In Fig. 1 the elements of the CTM are depicted.

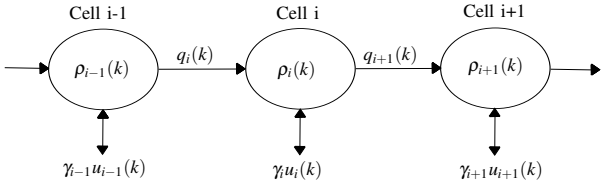


Fig. 1. Cell Transmission Model.

In this paper, the identification procedure will be explained with a single link case study, with no on-ramps or off-ramps. Next we write a synthetic representation of the CTM, which is an important hybrid system in the field of traffic. The PWA structure of the CTM could be exploited in further considerations of controller designs, closed loop analysis, etc. Let us introduce a state vector $\mathbf{x}^T(k) = [\rho_1(k), \rho_2(k), ..., \rho_N(k)] \in \mathbb{R}^N$, the input vector $\mathbf{u}^T(k) = [q_1(k), q_{N+1}(k)] \in \mathbb{R}^2$ for the whole freeway section, and the vector of unknown parameters $\theta^T = [\mathbf{v}^T, \mathbf{w}^T, \boldsymbol{\rho_M}^T] \in \mathbb{R}^{3N}$, with $\mathbf{v}^T = [v_1, v_2, ..., v_{N-1}] \in \mathbb{R}^{N-1}$, $\mathbf{w}^T = [w_2, w_3, ..., w_N] \in \mathbb{R}^{N-1}$, $\boldsymbol{\rho_M}^T = [\rho_{2,\mathrm{M}}, \rho_{3,\mathrm{M}}, ..., \rho_{N-1,\mathrm{M}}] \in \mathbb{R}^{N-1}$. From (1), a nonlinear dynamic state equation can be derived having the general form:

$$\begin{cases} \mathbf{x}(k+1) = f_{\mathrm{CTM}}(\mathbf{x}(k), \mathbf{u}(k), \theta), \\ \mathbf{y}(k) = C^T \mathbf{x}(k), \end{cases} \tag{2}$$

where $f_{\mathrm{CTM}}(\cdot)$ is a non-linear function defined by a Piece-Wise Affine (PWA) model. For the identification purposes, we assume to have one sensor for each state ($C^T = I_N$, with I_N the identity matrix). A PWA model has the following structure [7]:

$$\begin{cases} \mathbf{x}(k+1) = A_j \mathbf{x}(k) + B_j \mathbf{u}(k) + f_j, \\ \text{if} \quad G_j^x \mathbf{x}(k) \le G_j^c \end{cases} \tag{3}$$

where the subindex j takes values from 1 to 2^N, so the number of PWA dynamics is 2^N, each one defined over a set χ_j of a polyhedral partition. Every set χ_j of the partition defines the state-input space over which the different dynamics are active. The dynamics are defined by the matrices A_j, B_j, and vector f_j. The sets of the partition are defined by hyper-planes given by matrices G_j^x, and G_j^c. The model (3) is supposed to be well-posed, and then the regions χ_j form a complete partition of the output regressor set χ, i.e., $\bigcup_{j=1}^{2^N} \chi_j = \chi$ and $\chi_j \cap \chi_k = \emptyset$, $\forall j \neq k$. Then, the set of inequalities $G_j^x \mathbf{x}(k) \le G_j^c$ in the system (3), should be split in strict inequalities ($<$) and non-strict inequalities ($\le$). For simplicity in the notation this issue will be neglected. The vectors and matrices that define (3) for $j = 1, ..., 2^N$ are in the Appendix.

The region χ_j represents one of the 2^N discrete states of the traffic in the highway. Next we will define the meaning of the state j. Let us first obtain the number j in terms of binary variables $j = [1, 2, 2^2, ..., 2^{N-1}]\boldsymbol{\delta}^{\mathbf{j}}$, where $\boldsymbol{\delta}^{\mathbf{j}} = [\delta_1^j, \delta_2^j, ..., \delta_{N-1}^j, \delta_N^j]^T \in \{0,1\}^N$. If $\delta_i^j = 1$ we will assume that in the region j the cell i is in free flow. If $\delta_i^j = 0$, then in the region j the cell i is congested. For example, if $N = 8$, there are 2^8 regions. For the region $j = 100$, $\boldsymbol{\delta}^{\mathbf{100}} = [1,1,0,0,1,0,0,0]^T$, so the cells 1, 2 and 5 are in free flow state, while the rest are under congestion. In the notation used in [11], $\boldsymbol{\delta}^{\mathbf{100}} = [F,F,C,C,F,C,C,C]^T$ where F stands for free flow and C for congested flow.

III. IDENTIFICATION PROCEDURE

Throughout this paper we assume that N_{d} input/output data have been collected:

$$\Phi = \begin{bmatrix} (\mathbf{x}^1)^T & (\mathbf{u}^1)^T \\ (\mathbf{x}^2)^T & (\mathbf{u}^2)^T \\ \vdots & \vdots \\ (\mathbf{x}^{N_{\mathrm{d}}})^T & (\mathbf{u}^{N_{\mathrm{d}}})^T \end{bmatrix}_{N_{\mathrm{d}}+1, N+2}, \tag{4}$$

where N_{d} denotes the number of data samples, $(\mathbf{x}^k)^T = \mathbf{y}(k) = [\rho_1(k), \rho_2(k), ..., \rho_N(k)] \in \mathbb{R}^N$ are the outputs, and $(\mathbf{u}^k)^T = [q_1(k), q_{N+1}(k)] \in \mathbb{R}^2$ are the inputs. All those variables were measured at time steps $k = 0, ..., N_{\mathrm{d}}$.

Next, an identification procedure is carried out by minimizing an objective function with respect to the unknown parameters θ of the CTM. In a PWA black-box identification, we would need data from each of the $j = 2^N$ regions, in order to obtain the parameters. However, since each of the regions is actually defined by just $3(N-1)$ parameters (for each cell the parameters are $[v_{i-1}, w_i, \rho_{i,\mathrm{M}}]^T$), it is not necessary to have data in each of the 2^N regions. The collected data should contain enough information for each cell, to make the modeling and prediction of the congested and the free flows cells possible.

The fastest and most simple approach to calibrate the CTM is based on comparing the fundamental diagram with a real one [14], [15], minimizing the following function:

$$\begin{aligned} V_{\mathrm{static}}(\theta) = \frac{1}{N_{\mathrm{d}} N} \sum_{k=1}^{N_{\mathrm{d}}} \sum_{i=1}^{N} \big(q_i(k) - \\ \max\{0, \min\{v_{i-1}\rho_i(k), w_i(\rho_{i,\mathrm{M}} - \rho_i(k))\}\}\big)^2 \end{aligned} \tag{5}$$

This approach is static, because it does not consider the dynamics of the system, but only the interpretability of the fundamental diagram assumed for the CTM (triangular shape). In order to obtain a good prediction capability of the model (e.g. for control purposes), we should consider explicitly the prediction capabilities of the CTM, minimizing an objective function such as:

$$V(\theta) = \frac{1}{N_{\mathrm{d}} N} \sum_{k=1}^{N_{\mathrm{d}}} \sum_{i=1}^{N} (\rho_i(k) - \widehat{\rho}_i(k|k-1))^2 \qquad (6)$$

where $\widehat{\rho}_i(k|k-1)$ is the one-step ahead prediction given by the CTM. In large highway networks, performing the identification for the whole highway section may take several minutes, unacceptable in a control setting. To deal with this problem, in this paper we propose to use distributed optimization schemes. The identification procedure is divided in two steps. Firstly, we split the optimization problem into smaller but more tractable ones. This step is actually an integer optimization problem that could be further generalized, as it involves many variables like communication, computational time, computational effort, the topology of the network, etc. In the second step, given the distributed scheme, we will obtain the parameters of the CTM that minimize the one-step ahead prediction of the densities.

A. Distributed schemes: Information exchange

For cell i the parameters needed to completely define the CTM model are $\theta_i = [(\theta_i^1)^T, (\theta_i^2)^T]^T$, where $\theta_i^1 = [v_{i-1}, w_i, \rho_{i,\mathrm{M}}]^T$ are the shared parameters with the model of the cell $i-1$ and $\theta_i^2 = [v_i, w_{i+1}, \rho_{i+1,\mathrm{M}}]^T$ the shared parameters with model of the cell $i+1$. In the model for cell 1 we only need to identify $\theta_1^2 = [v_1, w_2, \rho_{2,\mathrm{M}}]^T$, and in the cell N the parameters are $\theta_N^1 = [v_{N-1}, w_N, \rho_{N,\mathrm{M}}]^T$. Note that to obtain the fundamental diagram of cell i we use θ_i^2.

The distributed identification problem of the highway can be represented using an interconnected network of subsystems (the identification procedure of the parameters θ_i needed for cell i), that can be described by a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, where the nodes in $\mathscr{V}$ are the subsystems and the edge (j,i) in the set $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ models how the j-th subsystem communicates directly its common parameters with the i-th subsystem.

Assuming we use one-step ahead prediction, we will have four cases for the communication between the identification procedures of the parameters θ_i needed for the model of cell i and the parameters θ_j needed for the model of cell j (let us assume θ_i^2 and θ_j^1 to be the common parameters between the optimization procedures for cell i and j). If $(j,i) \notin \mathscr{E}$ and $(i,j) \notin \mathscr{E}$ then the parameters of cell i and j will be obtained independently. If $(j,i) \in \mathscr{E}$ and $(i,j) \in \mathscr{E}$ then the identification procedures for cells i and j will be performed in a group optimization algorithm that includes the constraint $\theta_i^2 = \theta_j^1$, meaning that the shared parameters between i and j will be obtained at the same time, with a consensus algorithm, or in joint optimization. If $(j,i) \in \mathscr{E}$ and $(i,j) \notin \mathscr{E}$ then the optimization procedure of cell j will hierarchically communicate the shared parameters, so the procedure of cell i will wait to receive from j the information (θ_j^{1*}) to assume $\theta_i^2 = \theta_j^{1*}$ and then it will obtain the remaining parameters. The fourth case is $(i,j) \in \mathscr{E}$ and $(j,i) \notin \mathscr{E}$, analogous to the previous one.

In total, for the CTM identification we may have 4^{N-1} information exchange combinations. To select an optimal structure, some criteria could be the following: computational time, decay rate of transmission, CPU times, RMS of the output, etc. To limit the number of possible solutions, we can, for example, avoid groups of more than 3 cells. In this paper, we solve the identification problem for few but representative cases (see Figure 2): decentralized, hierarchical, and mixed hierarchical-distributed scheme.

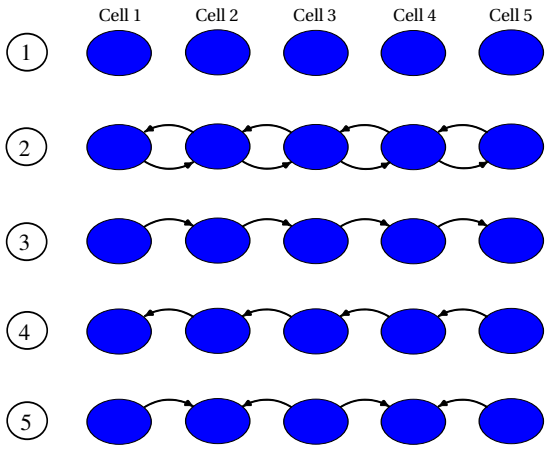


Fig. 2. Distributed schemes, 1) Decentralized, 2) Centralized, 3) Hierarchical forward, 4) Hierarchical backward, 5) Mixed hierarchical-decentralized

B. Distributed schemes: Parameters identification

We will say that the optimization problems of the consecutive cells i_{ini} until the cell i_{fin} will form a group if: 1) they are completely connected, so the edges $(i,i+1)$ and $(i+1,i)$ are in $\mathscr{E}$, $i = i_{\mathrm{ini}}, ..., i_{\mathrm{fin}} - 1$, and 2) at least one of the edges in the cell i_{ini} (edges $(i_{\mathrm{ini}} - 1, i_{\mathrm{ini}})$ or $(i_{\mathrm{ini}}, i_{\mathrm{ini}} - 1)$) and at least one of the edges in the cell i_{fin} (the edges $(i_{\mathrm{fin}} + 1, i_{\mathrm{fin}})$ or $(i_{\mathrm{fin}}, i_{\mathrm{fin}} + 1)$) do not belong to $\mathscr{E}$. In this case, the identification of the cells will be solved including as constraints the fact that there are shared parameters between consecutive optimization problems.

Once the communication structure $\mathscr{E}$ is determined, the following algorithm determines the order according to which each optimization problem has to be solved:

Step 1: Define $\mathscr{V}_{\mathrm{c}}$ the set nodes (identification problems) whose parameters θ_i are known. Set $l = 0$ the iteration of this algorithm.

Step 2: If $\mathscr{V}_{\mathrm{c}} \setminus \mathscr{V} = \{\}$ then stop. Otherwise, $l = l + 1$, and choose the nodes i belonging to $\mathscr{V}_{\mathrm{c}} \setminus \mathscr{V}$ that are in a group of cells (or alone) not waiting to receive information. Call this set of optimization problems $\mathscr{O}_l$.

Step 3: For each of the groups in the set $\mathscr{O}_l$, let say the group $\{i_{\mathrm{ini}}, ..., i_{\mathrm{fin}}\}$, solve the following optimization problem to identify the parameters:

$$
\begin{aligned}
&\min_{\{\theta_{i_{\text{ini}}},\ldots,\theta_{i_{\text{fin}}}\}} \sum_{i=i_{\text{ini}}}^{i_{\text{fin}}} V_i(\theta_i) \\
&V_i(\theta_i) = \frac{1}{N_{\text{d}}} \sum_{k=1}^{N_{\text{d}}} (\overline{\rho}_i(k+1) - \widehat{\rho}_i(k+1|k))^2 \\
&\widehat{\rho}_i(k+1|k) = \overline{\rho}_i(k) + \frac{T}{L_i}(q_i(k) - q_{i+1}(k)) \\
&q_i(k) = \begin{cases} \min\{v_{i-1}\overline{\rho}_{i-1}(k), w_i(\rho_{i,M} - \overline{\rho}_i(k))\}, & i > 1 \\ \overline{q}_1(k), i = 1 \\ \overline{q}_{N+1}(k), i = N+1 \end{cases} \\
&\theta_j^2 = \theta_{j+1}^1, \quad j = i_{\text{ini}}, \ldots, i_{\text{fin}} - 1 \\
&if \quad (i_{\text{ini}} - 1, i_{\text{ini}}) \in \mathscr{E} \Rightarrow \theta_{i_{\text{ini}}}^1 = \theta_{i_{\text{ini}}-1}^{*2} \\
&if \quad (i_{\text{fin}}, i_{\text{fin}} + 1) \in \mathscr{E} \Rightarrow \theta_{i_{\text{fin}}}^2 = \theta_{i_{\text{fin}}+1}^{*1}
\end{aligned} \tag{7}
$$

where the data collected from $k = 1,\ldots,N_{\text{d}}$ needed is $\phi_i(k) = [\overline{\rho}_{\text{i}_{\text{ini}}-1}(k),\ldots,\overline{\rho}_{\text{i}_{\text{fin}}+1}(k)]^T$ if optimization problems for cells 1 and N are not in the group, $\phi_i(k) = [\overline{q}_1(k),\overline{\rho}_1(k),\ldots,\overline{\rho}_{\text{i}_{\text{fin}}+1}(k)]^T$ if the optimization problem for 1 belongs to the group, and similarly, $\phi_i(k) = [\overline{\rho}_{\text{i}_{\text{ini}}-1}(k),\ldots,\overline{\rho}_N(k),\overline{q}_{N+1}(k)]^T$ if the optimization problem for cell N belongs to the group. Note that if $(\text{i}_{\text{ini}} - 1, \text{i}_{\text{ini}}) \in \mathscr{E}$ or $(\text{i}_{\text{fin}}, \text{i}_{\text{fin}} + 1) \in \mathscr{E}$, then the optimization problem cannot be solved until the optimal parameters coming from the neighbors cells are known ($\theta_{i_{\text{ini}}-1}^{*2}$ and $\theta_{i_{\text{fin}}}^{*1}$ respectively). Note that this situation (to wait for the parameters from other groups) cannot happen within the loop of this algorithm.

Step 4: Include all the already solved nodes i in the set $\mathscr{V}_{\text{c}}$ and go to Step 2.

To clarify this algorithm, let us consider the different cases in the Figure 2:

Case 1 (decentralized): In this case the algorithm first will suggest to solve all the problems that are not waiting to receive information. As this is the decentralized scheme, with no connections between optimization problems of neighbors, then $\mathscr{E}_1 = \{\{1\},\{2\},\{3\},\{4\},\{5\}\}$, and all those problems are solved in parallel. The computational time of this case is the maximum of all the times spent in each problem (if they are solved in parallel), and the number of variables obtained in each problem is 6. Half of the parameters obtained in each problem are not used for validation.

Case 2 (centralized): In this case the algorithm first will suggest to solve all the problems that are not waiting to receive information. All the problems are connected, being all the nodes a group. Then $\mathscr{E}_1 = \{\{1,2,3,4,5\}\}$, and all those problems are solved in the same group optimization, including the constraints of shared parameters. The computational time of this case is the time spent in solving the group optimization. All the parameters obtained from this problem are used in the final model.

Case 3 (hierarchical forward): In this case the algorithm first will suggest to solve the optimization problem 1, then $\mathscr{E}_1 = \{\{1\}\}$. Once this problem is solved, sequentially the consecutive problems are solved: $\mathscr{E}_2 = \{\{2\}\}$, $\mathscr{E}_3 = \{\{3\}\}$, $\mathscr{E}_4 = \{\{4\}\}$, and finally $\mathscr{E}_5 = \{\{5\}\}$. The computational time of this case is the sum of the times spent in solving all problems (they have to be solved sequentially). All the parameters obtained from this problem are used in the final model.

Case 4 (hierarchical backward): Analogous to Case 3, but $\mathscr{E}_1 = \{\{5\}\}$, $\mathscr{E}_2 = \{\{4\}\}$, $\mathscr{E}_3 = \{\{3\}\}$, $\mathscr{E}_4 = \{\{2\}\}$, and finally $\mathscr{E}_5 = \{\{1\}\}$.

Case 5 (mixed hierarchical-distributed): In this case the algorithm first will suggest to solve the optimization problems $\mathscr{E}_1 = \{\{1\},\{3\},\{5\}\}$ in a distributed way. Once those problems are solved, the next ones are: $\mathscr{E}_2 = \{\{2\},\{4\}\}$. The computational time of this case is the maximum of the time spent in solving the problems $\{1\}$, $\{3\}$ or $\{5\}$ (they can be solved in parallel). The optimization problems $\{2\}$, $\{4\}$ are easily solved as the shared parameters are obtained from the neighbors (the parameters of 2 come from the optimization of 1 and 3, and the parameters of 4 from the optimization of 3 and 5). All the parameters obtained from this problem are used in the final model.

IV. EXPERIMENTAL RESULTS

In this section we summarize the simulation tests conducted to show the application of the identification procedure to a real scenario. A 2.55 *km* long stretch of the A12 freeway, in The Netherlands has been used as test field to validate the identification method. This road connects the city of The Hague, with the German border, near Zevenaar. The stretch we use is in the segment that crosses the Dutch province of South Holland. In Fig. 3 the scheme of the stretch is depicted.

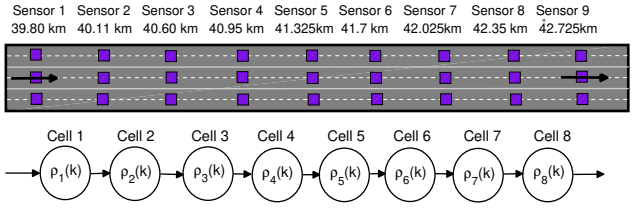


Fig. 3. Schematic sensor positions and cells of the A12 freeway in The Netherlands.

A period of eight hours (4:00-12:00) representative of typical working Monday will be modeled. Data of two days were used for the identification. For validation, data from Monday 12 of October was used (see Figure 4). We use the command *lsqnonlin* from the optimization toolbox of Matlab®.

1) Model identification: We will consider the distributed cases 1, 3, 4 and 5 of Figure 2. In Figure 5, the one-step ahead prediction for cell 5 of the distributed schemes using the validation data is shown.

The sum of the Root Mean Square (RMS) of the prediction error of densities in the cells for the cases 1 (Decentralized), 3 (Hierarchical Forward), 4 (Hierarchical backward) and 5 (Mixed hierarchical-decentralized) is 15.9396, 16.7916, 16.0512, and 16.5784 respectively. The standard deviation for the respectively cases is: 1.4689, 1.3886, 1.4722, 1.4622.

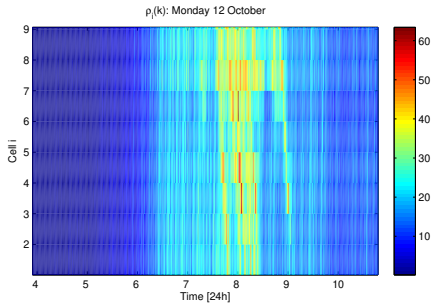


Fig. 4. Validation data measured the Oct. 12, 2009.

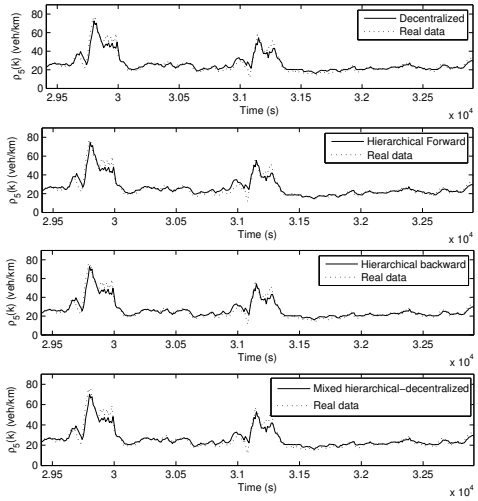


Fig. 5. One-step ahead prediction cell 5, cases 1) Decentralized, 2) Hierarchical Forward, 3) Hierarchical backward, 4) Mixed hierarchical-decentralized.

The computational time required to solve the optimization problems, in an AMD Athlon $x26000+$, @$3.1GHz$, 4GB RAM, are:

Decentralized: for the identification of the central cells (from cell 2 to cell 7), $\approx 107\ (s)$. For the cells in the borders (they require half the parameters to be identified), $\approx 18\ (s)$. So the total time required to solve the problem in parallel was $\approx 107\ (s)$.

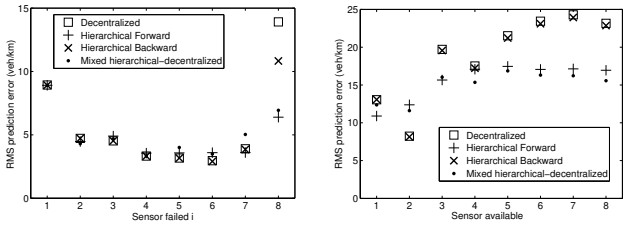
Hierarchical forward: for the cell 1 the time was $\approx 18\ (s)$, and for each of the following six cells $\approx 18\ (s)$. So the total time was $\approx 126\ (s)$. Note that the last cell does not need any optimization procedure. The times and procedure are similar for the hierarchical backwards (total $\approx 126\ s$).

Mixed hierarchical-decentralized: $\approx 107\ (s)$ for the identification of cells 3, 5 and 7; $\approx 18\ (s)$ for the identification of cell 1. Only those 4 optimizations problem are needed to be solved.

Centralized: Just to highlight the important reduction of the computational time, the centralized solution's computational time was $\approx 1500\ (s)$ for the 8 cells. We also tried 10 and 11 cells and the computational time was $\approx 1996\ (s)$ and $\approx 3225\ (s)$ respectively, increasing exponentially with the number of cells.

2) Virtual Sensor: failure in the sensor of cell i: In this sub-section, we analyze the effect of a failure in the sensor i for the cases 1 (Decentralized), 3 (Hierarchical Forward), 4 (Hierarchical backward) and 5 (Mixed hierarchical-decentralized). We assume the failure is detected immediately, and we replace the sensor i with the CTM identified in the previous section, one sensor at the time. In Fig. 6(a) we show the RMS of the virtual sensor error, when cell i has failed. The sum of the Root Mean Square (RMS) for the cases is 45.4500, 39.0148, 42.2545, 40.6964 and the standard deviation is 4.0570, 2.4123, 3.2730, 2.4650 respectively. From the figure, we can see when the sensors in the borders fail, the most important errors happen. When the sensor of the middle fails, it generates a smaller error.

3) Virtual Sensor: Only the sensor of cell i is available: We consider now the case when only the density sensor of cell i is available, so the output equation is $y(k) = \rho_i(k)$. In Fig. 6(b) we show the RMS of the virtual sensor errors, when only the sensor of cell i is working. The sum of the Root Mean Square (RMS) for the different cases in each case is 150.8778, 124.6164, 149.1010, 120.3082 and the standard deviation is 8.2253, 5.7030, 8.1012, 5.3341 respectively. From the figure, the methods hierarchical-forward and mixed hierarchical-decentralized are the ones that generate the best estimation of the whole section in terms of the mean value and standard deviation.



(a) (b)

Fig. 6. Identification data measured: (a) RMS for failure in cell i. (b) Sum of the RMS from virtual sensors, when only the sensor i is available.

V. CONCLUSIONS AND FUTURE WORK

In this paper, the distributed identification of the parameters of the Cell Transmission Model (CTM) for a chosen freeway has been analyzed. Those parameters are: the free velocity, the maximum density and the backward congestion propagation speed. The purpose of the proposed identification approach is to minimize the one-step ahead prediction error of the densities of the cells; while, on the other hand, reducing the computational time and effort to obtain them. By relying on experimental data measured on a portion of the A12 freeway of The Netherlands, we have shown that it is possible to find a good solution. The parameters obtained in some of the sections (especially the backpropagation waves' speeds) resulted to be inaccurate, mainly due to the overabundance of free flow vs. congestion measurements in the data set. The purpose of this paper is the empirical exploration of a possible solution to reduc-

ing model calibration computational time, the mathematical soundness of this approach will be object of future studies.

Finally, to show the good properties of the identified model, an evaluation of the performance of the identified model used as a set of virtual sensors in several scenarios was made. As the CTM belongs to the class of linear Piecewise Affine (PWA) systems, a topic for future work is to compare the obtained models to a more general PWA model. The method we described can be used to generate an initial solution to be transferred to other iterative optimization procedures that could converge to the centralized solution. Convergence and properties of the methods are topics for further research. More sophisticated schemes of failures and detection can also be analyzed in the future, as well as on-line identification.

VI. ACKNOWLEDGMENTS

The traffic data for the case study in Section IV was provided by dr. Andreas Hegyi from Delft University of Technology, Faculty of Civil Engineering. Research supported by the European 7th Framework Network of Excellence Highly complex and networked control systems (HYCON2), the European 7th framework STREP project "Hierarchical and distributed model predictive control of large-scale systems (HD-MPC)", and the European COST Action TU1102.

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APPENDIX

From (1), the following vectors and matrices define (3) for $j = 1,...,2^N$:

$$(A_j)_{i,k} = \begin{cases} 1 - \dfrac{T}{L_1}(\delta_2^j v_1) & \text{if} \quad i = k = 1 \\ \dfrac{T}{L_1}(1-\delta_2^j) w_2 & \text{if} \quad i = 1, k = 2 \\ \dfrac{T}{L_i}\delta_i^j v_{i-1} & \text{if} \quad 1 < i < N, k = i-1 \\ 1 - \dfrac{T}{L_i}((1-\delta_i^j) w_i + \delta_{i+1}^j v_i) & \\ \qquad\qquad \text{if} \quad 1 < i < N, i = k & \\ \dfrac{T}{L_i}(1-\delta_{i+1}^j) w_{i+1} & \\ \qquad\qquad \text{if} \quad 1 < i < N, k = i+1 & \\ \dfrac{T}{L_N}\delta_N^j v_{N-1} & \text{if} \quad i = N, k = N-1 \\ 1 - \dfrac{T}{L_N}((1-\delta_N^j) w_N) & \\ \qquad\qquad \text{if} \quad i = N, k = N & \\ 0 \quad \text{otherwise.} & \end{cases} \tag{8}$$

$$(B_j)_{i,k} = \begin{cases} \dfrac{T}{L_1} & \text{if} \quad i = 1, k = 1 \\ \dfrac{-T}{L_N} & \text{if} \quad i = N, k = 2 \\ 0 \quad \text{otherwise.} & \end{cases} \tag{9}$$

$$(f_j)_{i,1} = \begin{cases} \dfrac{T}{L_1}((1-\delta_2^j) w_2 \rho_{2,\mathrm{M}}), & \text{if} \quad i = 1, \\ \dfrac{T}{L_i}((1-\delta_i^j) w_i \rho_{i,\mathrm{M}} + (1-\delta_{i+1}^j) w_{i+1} \rho_{i+1,\mathrm{M}}), & \\ \qquad \text{if} \quad i = 2,...,N-1, & \\ \dfrac{T}{L_N}((1-\delta_N^j) w_N \rho_{N,\mathrm{M}}), & \text{if} \quad i = N, \end{cases} \tag{10}$$

$$(G_j^x)_{i,k} = \begin{cases} (2\delta_i^j - 1) v_i & \text{if} \quad i = 1,...,N-1, k = i \\ (2\delta_i^j - 1) w_{i+1} & \text{if} \quad i = 1,...,N-1, k = i+1 \\ 0 \quad \text{otherwise.} & \end{cases} \tag{11}$$

$$(G_j^c)_{i,1} = \quad (2\delta_i^j - 1) w_i \rho_{i,\mathrm{M}} \tag{12}$$