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# Achieving System-Optimal Splitting Rates in a Freeway Network Using a Reverse Stackelberg Approach

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**Abstract:** A game-theoretical method is proposed to achieve a system-optimal distribution of traffic over a freeway network. In particular, the road authority is represented by a leader player and each follower player embodies a group of drivers with the same value of time that plan to travel between a given origin and destination. In the proposed reverse Stackelberg approach, the leader presents a function to each follower that maps a vector of splitting rates over possibles routes to a monetary incentive. The follower then decides upon a splitting rate and the associated monetary incentive that yield the minimum weighted measure of travel time and monetary fees. In this manner, the road authority can compose an optimal leader function under which the followers will behave as desired, i.e., to achieve the system-optimal splitting rates.

*Keywords:* traffic control; hierarchical decision making, reverse Stackelberg game

## 1. INTRODUCTION

It is well-known that the uncontrolled behavior of individual vehicles in a freeway network in general does not result in an optimal distribution of traffic over the available routes (Wardrop, 1952). Several methods aim at bringing this so-called user equilibrium closer to the system-optimal traffic distribution. One of such methods consists in providing travel time information on dynamic route information panels (DRIPs) (Deflorio, 2003), which can be integrated with variable speed limits (van den Berg et al., 2004) and ramp metering (Karimi et al., 2004). Other methods assign tolls to different freeway stretches, either as fixed, time-dependent tolls (Joksimovic et al., 2005) or as flow-dependent tolls (Staňková et al., 2009).

In the DRIP-based approaches however, the indicated travel times on the alternative routes should be similar for drivers to deviate from the shortest, popular routes. This causes inefficiency because congestion should apply in order for some routes to sufficiently increase in average travel time. The same holds for the time-dependent tolls on individual highway stretches, which will in general not yield a system-optimal distribution either.

Hence, in order to optimize the use of alternative routes in a traffic network by minimizing the total time that traffic spends in the system, we propose a reverse Stackelberg game approach with monetary incentives. Here, we treat the traffic control problem as a leader-follower game where the road authority as a leader provides the drivers (followers) with a leader function that assigns a monetary incentive, positive or negative, to each possible combination of splitting rate values that the followers can choose. The leader does so based on her computation of the desired, system-optimal equilibrium state. By composing an optimal leader function, the rational follower response will be to adopt system-optimal splitting rates.

The motivation for this reverse Stackelberg approach to deal with splitting rates is as follows. In previous work, we have proposed a reverse Stackelberg approach in which a follower's decision variable is the travel time in which he aims to reach his destination (Groot et al., 2012). While this yields a system-optimal distribution in case the players make fully rational decisions, the difficulty is in realizing the drivers' desired individual travel times in case they differ from the optimal, rational response.

Hence, in order to more easily deal with such deviations, in this paper a method is proposed in which a follower group of motorists decides upon a certain splitting of traffic over the different routes, where any (suboptimal) splitting rate can be realized in practice. In other words, instead of the leader determining the optimal splitting rate and the according leader functions to achieve this situation, now, we adopt the splitting rate as a follower decision variable. This means that a homogeneous group of drivers needs to make a collective decision on how to distribute this group over the available routes. This division of individual drivers of the follower group over the alternative routes according the splitting rates is a separate, lower-level problem that we leave for further research. Here, we can assume that vehicles are equipped with an on-board unit that allows for communication between the vehicles.

The remainder of the paper is built up as follows. First, the reverse Stackelberg game is described in Section 2. After a statement of the traffic control problem, we describe how the game can be translated to fit the traffic situation in Section 3. Here, first the basic elements of the game are linked to the context, and then the dynamic framework is explained. In Section 4 a simple case-study is presented to clarify the performance of the proposed reverse Stackelberg approach as compared to the use of DRIPs. The paper is concluded in Section 5.

## 2. THE REVERSE STACKELBERG GAME

The basic reverse Stackelberg game is a hierarchical game that can be described as follows. A leader player proposes a leader function $\gamma_{\mathrm{L}} : \Omega_{\mathrm{F}} \to \Omega_{\mathrm{L}}$, with the leader decision variable $u_{\mathrm{L}} \in \Omega_{\mathrm{L}} \subseteq \mathbb{R}^{n_{\mathrm{L}}}$ and the follower decision variable $u_{\mathrm{F}} \in \Omega_{\mathrm{F}} \subseteq \mathbb{R}^{n_{\mathrm{F}}}$. Based on this leader function $\gamma_{\mathrm{L}}(u_{\mathrm{F}})$, the follower determines his optimal response $u_{\mathrm{F}}^* \in \Omega_{\mathrm{F}}$, which yields the associated leader decision variable $u_{\mathrm{L}} = \gamma_{\mathrm{L}}(u_{\mathrm{F}}^*)$.

Here, the leader aims to achieve the desired reverse Stackelberg equilibrium or her globally optimal solution

$$(u_{\mathrm{L}}^{\mathrm{d}}, u_{\mathrm{F}}^{\mathrm{d}}) := \arg \min_{u_{\mathrm{L}} \in \Omega_{\mathrm{L}}, u_{\mathrm{F}} \in \Omega_{\mathrm{F}}} \mathcal{J}_{\mathrm{L}}(u_{\mathrm{L}}, u_{\mathrm{F}}),$$

where $\mathcal{J}_{\mathrm{L}} : \Omega_{\mathrm{L}} \times \Omega_{\mathrm{F}} \to \mathbb{R}$ denotes the follower's cost function. Similarly, given the leader function $\gamma_{\mathrm{L}}(u_{\mathrm{F}})$, the follower optimizes his objective function $\mathcal{J}_{\mathrm{F}}(\gamma_{\mathrm{L}}(u_{\mathrm{F}}), u_{\mathrm{F}})$.

A well-known special case of this leader-follower game is the original Stackelberg game (von Stackelberg, 1952). In this game, the follower player determines his optimal decision variable $u_{\mathrm{F}} \in \Omega_{\mathrm{F}}$ as a direct response to the leader's – constant – decision variable $u_{\mathrm{L}} \in \Omega_{\mathrm{L}}$, thus not to the more general leader function $\gamma_{\mathrm{L}}(u_{\mathrm{F}})$. The reverse Stackelberg game has an important advantage to the regular Stackelberg game, as there, the leader cannot sufficiently influence the follower's response in case it is not unique. The following simple example illustrates the reverse Stackelberg concept:

*Example 1.* (Adopted from Olsder (2009)). Consider the following simple static, single-leader single-follower situation as also depicted in Fig. 1. Let the objective functions of leader and follower be respectively:

$$\mathcal{J}_{\mathrm{L}}(u_{\mathrm{L}}, u_{\mathrm{F}}) = (u_{\mathrm{F}} - 5)^2 + u_{\mathrm{L}}^2,$$
$$\mathcal{J}_{\mathrm{F}}(u_{\mathrm{L}}, u_{\mathrm{F}}) = u_{\mathrm{L}}^2 + u_{\mathrm{F}}^2 - u_{\mathrm{L}} u_{\mathrm{F}},$$

with decision variables $u_{\mathrm{L}} \in \mathbb{R}, u_{\mathrm{F}} \in \mathbb{R}$. The leader's global optimum is $(u_{\mathrm{L}}^{\mathrm{d}}, u_{\mathrm{F}}^{\mathrm{d}}) = (0, 5)$. In the original Stackelberg game formulation, the follower's response to the desired variable $u_{\mathrm{L}}^{\mathrm{d}} = 0$ would be the suboptimal $u_{\mathrm{F}}^* = 1/2 u_{\mathrm{L}} = 0$.

However, under the leader function

$$u_{\mathrm{L}} = \gamma_{\mathrm{L}}(u_{\mathrm{F}}) = 2u_{\mathrm{F}} - 10,$$

the follower's response will be:

$$\arg \min_{u_{\mathrm{F}}} \mathcal{J}_{\mathrm{F}}(u_{\mathrm{F}}) = \arg \min_{u_{\mathrm{F}}} (2u_{\mathrm{F}} - 10)^2 + u_{\mathrm{F}}^2 + (2u_{\mathrm{F}} - 10)u_{\mathrm{F}} = 5.$$

In Fig. 1 several level curves of $\mathcal{J}_{\mathrm{F}}$ are plotted; the leader's optimum $(u_{\mathrm{L}}^{\mathrm{d}}, u_{\mathrm{F}}^{\mathrm{d}})$ is in the center of the dotted level curves for $\mathcal{J}_{\mathrm{L}}$. The contours centered around the four corners of the plotted decision space represent the level curves of $\mathcal{J}_{\mathrm{F}}$. The follower's optimal response to $u_{\mathrm{L}}^{\mathrm{d}}$ and $\gamma_{\mathrm{L}}(u_{\mathrm{F}})$ are respectively $u_{\mathrm{F}}^*$ and $u_{\mathrm{F}}^{\mathrm{d}}$ for the original versus the reverse Stackelberg game.

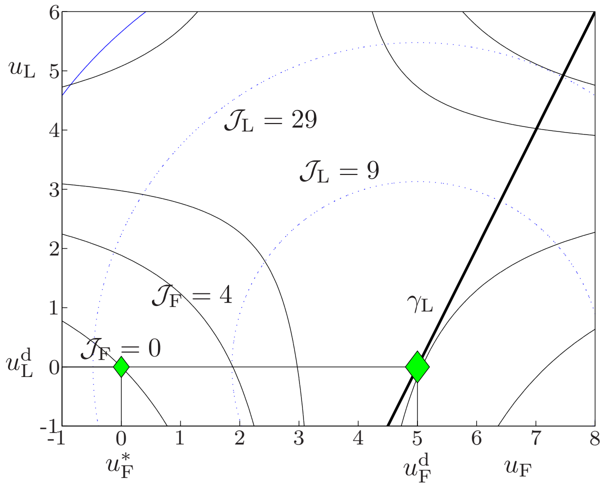



Fig. 1. Graphical representation of Example 1

## 3. THE REVERSE STACKELBERG ROUTING APPROACH

### 3.1 Problem Statement

The aim of the control problem of our interest is to achieve a system-optimal distribution of traffic over a network, i.e., to make optimal use of the available routes in the sense that the total time spent (TTS) of traffic in the particular network is minimized. As has been indicated in the introduction, some current methods cannot bring the user equilibrium traffic assignment to coincide with the system-optimal assignment. In the remainder of this section, we propose a method based on game theory that can accomplish this.

First, we model the traffic network as a directed graph with a set of origin and destination nodes $\mathcal{O}$ and $\mathcal{D}$ respectively. Homogeneous freeway stretches are represented by links $\ell \in \mathcal{L}$, that connect origins, destinations, and internal nodes of the set $\mathcal{N}$. We define the node sets such that $\mathcal{O} \cap \mathcal{N} = \emptyset$ and $\mathcal{D} \cap \mathcal{N} = \emptyset$. Since $\mathcal{O} \cap \mathcal{N} = \emptyset$, w.l.o.g. we assume that each origin $o \in \mathcal{O}$ has a single outgoing link $\ell_{\mathrm{out}}(o)$. In case of multiple outgoing links out of a given origin, a single virtual link can be created with zero length and zero travel time that connects the given origin to a virtual internal node from which the multiple outgoing links then depart. When adopting a traffic prediction model, the links can be further divided into road segments $r \in \mathcal{R}$ of equal length, in order to achieve an accurate prediction of the traffic behavior. For clarity, we however use link indices throughout the paper.

A receding horizon approach is adopted, where $k_{\mathrm{c}}$ indicates the time instant $t = k_{\mathrm{c}} T_{\mathrm{c}}$, with $T_{\mathrm{c}}$ the sample or control time step of the dynamic routing approach. Similarly, $k$ indicates the time instant $t = kT$, with $T$ the time step for the simulation of the traffic flow behavior based on a prediction model; a time horizon $[kT, (k + N_{\mathrm{p}})T]$ is thus considered with $N_{\mathrm{p}}$ the prediction horizon. Finally, $T = MT_{\mathrm{c}}, M \in \mathbb{N}$.

Further, $q_{\ell,d}(k)$ denotes the traffic flow on link $\ell \in \mathcal{L}$ traveling towards destination $d$. The in-flow of origins can be written

$$q_{\mathrm{in}}^{o,d}(k) = q_{\ell_{\mathrm{out}(o)},d}(k), \tag{1}$$

with $q_{\mathrm{in}}^{o,d}$ the demand for traveling from $o$ to $d$. Then, the total inflow of node $n \in \mathcal{N}$ at time step $k$ that has a destination $d$ is denoted by

$$Q_{n,d}(k) = \sum_{\ell \in I(n)} q_{\ell,d}(k), \tag{2}$$

with $I(n)$ the set of incoming links of node $n$. Similarly, $O(n)$ denotes the set of outgoing links for node $n$, with a traffic flow

$$q_{\ell,d}(k) = \beta_{n,\ell,d}(k) Q_{n,d}(k), \tag{3}$$

where $\beta_{n,\ell,d}(k) \in [0,1]$ represents the splitting rate for link $\ell$ at node $n$ with the destination $d$. We can now write the total flow $q_\ell(k)$ on link $\ell$ by

$$q_\ell(k) = \sum_{d \in \mathcal{D}} q_{\ell,d}(k). \tag{4}$$

The system-optimal traffic distribution is such that the TTS over a given prediction horizon $N_\mathrm{p}$ is minimized, i.e., the cost function can be described by

$$\mathcal{J}(k) = T \sum_{j=1}^{N_\mathrm{p}} \sum_{\ell \in \mathcal{L}} q_\ell(k+j) \tau_\ell(k+j), \tag{5}$$

with $\tau_\ell(k)$ the mean travel time associated with link $\ell \in \mathcal{L}$ that is determined using a traffic prediction model. Here, the prediction horizon is incorporated to take into account not only the present but also the future traffic conditions.

### 3.2 Reverse Stackelberg Approach

In order to introduce the reverse Stackelberg approach to the dynamic traffic assignment problem of reaching the optimal splitting rates, we start with a definition of the basic elements of the reverse Stackelberg game, linked to the traffic control context.

*Players and Decision Variables*

- The single **leader player** represents the road authority that aims at accomplish an optimal use of the roads of a given traffic network.
- A **follower player** represents a homogeneous group of vehicles that desire to travel according to a certain origin-destination (OD) pair $(o,d) \in \mathcal{O} \times \mathcal{D}$. The total number of OD-pairs is denoted by $N_\mathrm{OD} = |\mathcal{O}| \cdot |\mathcal{D}|$, where $|X|$ represents the cardinality of $X$.

  Further, the group of drivers should be homogeneous in the sense that they have a similar monetary value of time, as will be elaborated upon below. We denote the value-of-time-class by $h \in \mathcal{H} := \{1,\ldots,H\}$, where the set of classes of drivers with a particular OD-pair index $i \in \{1,\ldots,N_\mathrm{OD}\}$ is denoted by $\mathcal{H}_i$. Hence, the total number of follower players is represented by $N_\mathrm{F} = \sum_{i=1}^{N_\mathrm{OD}} |\mathcal{H}_i|$ where we denote the set of followers by

  $$\mathcal{F} = \{(h,i) | i \in \{1,\ldots N_\mathrm{OD}\}, h \in \mathcal{H}_i\}.$$

Further, the **decision variables** are respectively:

- A monetary incentive $\theta^{hi} \in \Omega_\mathrm{L}^{hi}$ to be paid by or received by the follower player $(h,i) \in \mathcal{F}$ where $\Omega_\mathrm{L}^{hi} := [\theta_\mathrm{min}^{hi}, \theta_\mathrm{max}^{hi}]$ denotes the range of monetary incentives that is accepted by the drivers.
- A choice of the vector of route selection variables $\zeta^{hi}(k_\mathrm{c}) \in \Omega_\mathrm{F}^{hi}$ that specifies fractions of the group of drivers that constitutes the follower player $(h,i) \in \mathcal{F}$, which take the different routes associated to OD-pair index $i \in \{1,\ldots,N_\mathrm{OD}\}$. Here,

  $$\zeta^{hi} := \left(\zeta_1^{hi} \ \ldots \ \zeta_{n_i}^{hi}\right)^\mathrm{T},$$

  with $n_i$ the number of possible routes associated with the $i$-th OD-pair.

*The Leader and Follower Objective Functions*

- The leader player aims to minimize the total travel time that the traffic spends in the system (TTS):

  $$\begin{aligned}\mathcal{J}_\mathrm{L}(k_\mathrm{c}) = T \sum_{j=1}^{N_\mathrm{p}} \sum_{i=1}^{N_\mathrm{OD}} \sum_{h \in \mathcal{H}_i} &\left(\tau^i(Mk_\mathrm{c}+j)\right)^\mathrm{T} \\ &\cdot \left(\zeta^{hi}(Mk_\mathrm{c}+j) \cdot q_\mathrm{in}^{hi}(Mk_\mathrm{c}+j)\right),\end{aligned} \tag{6}$$

  subject to consistency and capacity constraints (see Section 3.4), where $q_\mathrm{in}^{hi}(k_\mathrm{c})$ [veh/h] denotes the total demand of drivers in the value-of-time class $h \in \mathcal{H}_i$ at control time step $k_\mathrm{c}$ for the $i$-th OD-pair. Further, $\tau^i := \left(\tau_1^i \ \ldots \ \tau_{n_i}^i\right)^\mathrm{T}$ denotes the predicted travel time on each of the routes associated with the $i$-th OD-pair.
- The followers' objective is to minimize the average travel cost as a function of monetary incentives and average travel time, which is evaluated at the moment $k_\mathrm{c}$ of entering the traffic network:

  $$\begin{aligned}\mathcal{J}_\mathrm{F}^{hi}(k_\mathrm{c}) = \alpha_\mathrm{F}^h \left(\tau^i(Mk_\mathrm{c})\right)^\mathrm{T} &\left(\zeta^{hi}(Mk_\mathrm{c}) \cdot q_\mathrm{in}^{hi}(Mk_\mathrm{c})\right) \\ &+ \theta^{hi}(Mk_\mathrm{c}),\end{aligned} \tag{7}$$

  with $\alpha_\mathrm{F}^h \in \mathbb{R}_+$ the possibly time-variant [1] monetary value of time (VOT). Note that this parameter could be differentiated between given a particular car type, or it could be determined by an iterative learning process in which the value is adapted over time based on the behavior of the vehicles.

*Remark 1.* Instead of assuming a linear mapping of travel time to monetary value, (7) could be replaced by a more involved, nonlinear relation as considered in e.g., DeSerpa (1973); Blayac and Causse (2001). The consequence of a different follower objective function is in the type of leader function $\gamma_\mathrm{L}$ that is needed to reach the optimal distribution of tolls to arrive at the system optimum, as will be elaborated upon in Section 3.5.

### 3.3 The Dynamic Game

Given the classification of the road authority as a leader and a group of homogeneous drivers as a follower, while using the proposed decision variables and goal functions, a dynamic, multi-stage game can be composed. The overall process that leads to a dynamic route assignment at a minimum TTS is illustrated in the scheme of Fig. 2. Here, the following main steps are considered:

(1) Given the current traffic state and the demand for the OD-pairs as indicated by the drivers on the on-board computers, **system-optimal splitting rates** are computed for the vehicles over the available routes, with the corresponding predicted mean travel times.
(2) Given the desired distribution of vehicles and the according travel times, the road authority associates monetary incentives $\theta$ with the drivers' choice of the route selection variables $\zeta$ that specify the splitting rates for each of the nodes and outgoing links on one of the routes towards the desired destination. This results in a **leader function** $\gamma_\mathrm{L}^{hi}(k_\mathrm{c}) : \Omega_\mathrm{F} \to \Omega_\mathrm{L}$ for each of the $N_\mathrm{F}$ followers.

[1] In the literature, this parameter is often taken to be constant (Joksimovic et al., 2005; Staňková et al., 2009). However, the value could change depending on the period of the day.

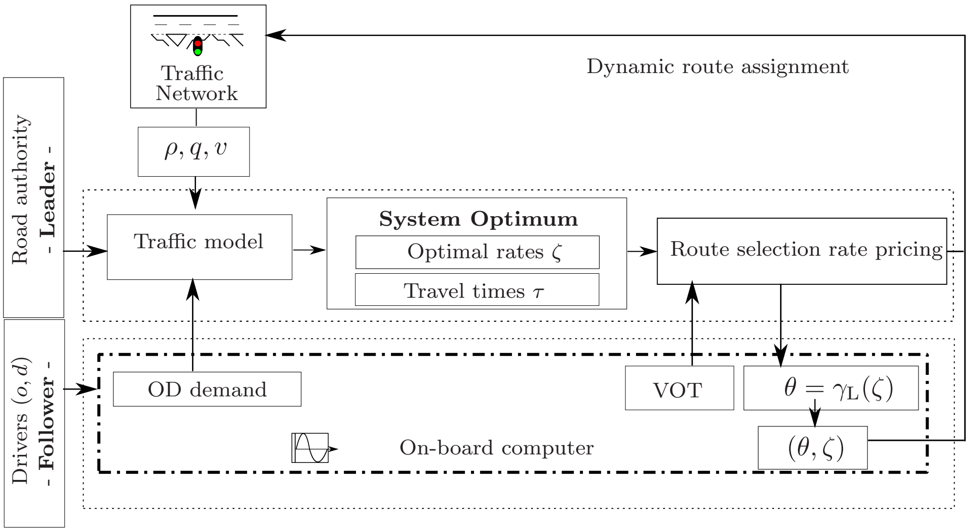



Fig. 2. Schematic framework of the splitting rate based reverse Stackelberg approach to dynamic route assignment

(3) As a response to an optimal leader function, a rational follower player will choose the desired combination of monetary incentive and route selection rates, i.e., the pair $(\theta, \zeta)$ that minimizes the follower's objective function will result in system-optimal splitting rates according which the drivers will be distributed over the traffic network.

This reverse Stackelberg approach can thus be seen as a lower-level problem of realizing certain splitting rates, as determined via dynamic traffic assignment.

The system-optimal or desired road selection rates $\zeta^{\mathrm{d},hi}$ of the homogeneous followers $(h,i) \in \mathcal{F}$ follow from the desired splitting rates $\beta^{\mathrm{d}}_{n,\ell,d}$. Hence, the distribution of traffic demand over the particular routes, leading to a flow $q_j$ on route $j \in \{1,...,n_i\}$ for OD-pair index $i \in \{1,...,N_{\mathrm{OD}}\}$ can be written:

$$q_j := \sum_{h\in\mathcal{H}_i} q^{hi}_{\mathrm{in}} \cdot \zeta^{hi}_j, \text{ with } \zeta^{hi}_j := \prod_{\substack{n\in\mathcal{P}^{\mathcal{N}}_{ij}\\ \ell\in\mathcal{P}^{\mathcal{L}}_{ij}}} \beta_{n,\ell,d(i)}, \tag{8}$$

where $\mathcal{P}^{\mathcal{N}}_{ij}$ denotes the set of nodes on the path or route $j$ and $\mathcal{P}^{\mathcal{L}}_{ij}$ denotes the set of links on the path $j$ for the $i$-th OD-pair, with $d(i)$ the associated destination. Further, for the flow on a link $\ell \in \mathcal{L}$ with destination $d \in \mathcal{D}$ holds:

$$q_{\ell,d} = \sum_{i\in I_d}\sum_{h\in\mathcal{H}_i}\sum_{j\in J_{i,\ell}} q^{hi}_{\mathrm{in}}\zeta^{hi}_j, \tag{9}$$

with $I_d$ the set of OD-pairs with destination node $d$ and $J_{i,\ell}$ the set of routes for the $i$-th OD-pair containing link $\ell$.

### *3.4 Computation of the Optimal Splitting Rates*

The system-optimal splitting rates can be obtained by solving a dynamic version of the minimum cost flow problem, through which an optimal distribution of a given traffic flow over the network is computed which was the problem described in Section 3.1:

$$\min_{q_{\ell,d}} T\sum_{j=1}^{N_{\mathrm{p}}}\sum_{d\in\mathcal{D}}\sum_{\ell\in\mathcal{L}} q_{\ell,d}(k+j)\tau_\ell(k+j). \tag{10}$$

Here, $\tau_\ell$ denotes the predicted travel time for link $\ell \in \mathcal{L}$. The following constraints are needed to link the flows through the traffic network:

$$\sum_{h\in\mathcal{H}_i} q^{hi}_{\mathrm{in}}(k) = q_{\ell_{\mathrm{out}}(o),d}(k)\ \forall i \in \{1,\ldots,N_{\mathrm{OD}}\} \tag{11}$$

$$\sum_{d\in\mathcal{D}}\sum_{\ell\in I(n)} q_{\ell,d}(k) = \sum_{d\in\mathcal{D}}\sum_{\ell\in O(n)} q_{\ell,d}(k)\ \forall n \in \mathcal{N} \tag{12}$$

$$\sum_{d\in\mathcal{D}} q_{\ell,d}(k) \le q_{\mathrm{cap},\ell}\ \forall \ell \in \mathcal{L} \tag{13}$$

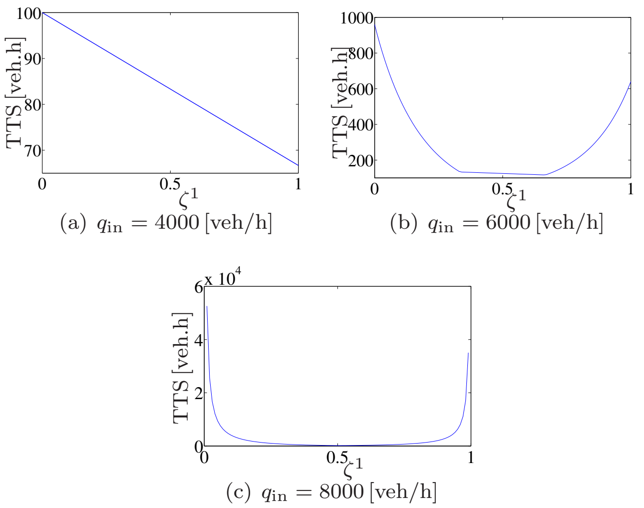
$$q_{\ell,d}(k) \ge 0\ \forall \ell \in \mathcal{L}, \forall d \in \mathcal{D}, \tag{14}$$

where, given a driver demand pattern $q^{hi}_{\mathrm{in}}(k)\ \forall\ i \in \{1,\ldots,N_{\mathrm{OD}}\}, h \in \mathcal{H}_i$, a system-optimal – with respect to the TTS – distribution of the traffic demand over the road network can be computed, as well as the associated mean travel times $\tau^i(k)$ for each of the routes $j \in \{1,\ldots,n_i\}$. The optimal splitting rates now follow straightforwardly from the optimal flows $q_{\ell,d}(k)$ for each road stretch $\ell \in \mathcal{L}$ towards the destination $d \in \mathcal{D}$, see (8)-(9).

Further, the travel time $\tau_\ell(k)$ for a particular link $\ell \in \mathcal{L}$ depends on the average velocity $\tilde{v}_\ell(k)$, i.e., $\tau_\ell(k) = L_\ell/\tilde{v}_\ell(k)$, which is again influenced by the traffic flow and density. The way in which the average speed is computed determines the complexity of the above problem. If one takes $\tilde{v}_\ell(k)$ as a constant, equal to the currently measured speed, a linear programming problem results (Ahuja et al., 1993).

However, if a prediction model like METANET (Messmer and Papageorgiou, 1990), the cell transmission model (Daganzo, 1994), or the link transmission model (LTM) (Yperman et al., 2006) is used to determine the speed at a certain time (which is again a function of the splitting rates), a more complex, nonlinear optimization problem should be solved. A less accurate but computationally more efficient alternative is to derive $\tilde{v}_\ell(k)$ from a fundamental diagram or from the nonlinear expression

$$\tilde{v}_\ell(k) = v_{\mathrm{free},\ell}\exp[-\frac{1}{a_\ell}(\frac{\rho_\ell(k)}{\rho_{\mathrm{crit},\ell}})^{a_\ell}], \tag{15}$$




(a) $q_{\text{in}} = 4000$ [veh/h] (b) $q_{\text{in}} = 6000$ [veh/h]

(c) $q_{\text{in}} = 8000$ [veh/h]

Fig. 3. The TTS resulting from different splitting rates under three levels of demand

where $v_{\text{free},\ell}$ denotes the free-flow speed [veh/h] and $\rho_{\text{crit},\ell}$ [veh/km/lane] the critical density of segment $\ell \in \mathcal{L}$, and where $a_\ell$ represents a model parameter (May, 1990).

### *3.5 Composition of an Optimal Leader Function*

The decision space of the follower is multidimensional, i.e., the decision variable of player $(h, i) \in \mathcal{F}$ represents a vector of route selection rates $\zeta^{hi}$. The leader function should therefore allocate a monetary incentive to each possible combination of the splitting rates towards one of the road stretches that is on a route to the destination $d$ corresponding to the OD-pair index $i \in \{1, ..., N_{\text{OD}}\}$.

An optimal leader function is such that the rational response of the follower that minimizes his goal function (7), brings about the system-optimal, hence desired, splitting rates $\beta^{\text{d}}_{n,\ell,d}$ $\forall n \in \mathcal{N}$, $\ell \in \mathcal{L}$, and $d \in \mathcal{D}$. As has also been illustrated in Section 2, this leader function should not contain any points on the sublevel curve for the desired splitting rate vector, or within the sublevel set of the follower. In these cases the follower could choose an alternative splitting rate at the same objective function value, or splitting rates that yield the follower a better objective function value. This concept of an optimal leader function is illustrated in the following example.

*Example 2.* (Optimal leader function). Fig. 3 shows the TTS values for three static demand scenarios under the possible splitting rates $\zeta^1$ towards route 1 for the simple network with two non-overlapping routes depicted in Fig. 4. Here, we assume both routes to have a capacity $q_{\text{cap}} = 4000$ veh/h and the lengths are respectively $L_1 = 2$ km and $L_2 = 3$ km. Whereas the TTS is minimized by routing all traffic to the shortest route 1 in case the demand $q_{\text{in}} \leq q_{\text{cap}}$, i.e., at the splitting rate value $\zeta^1 = 1$, in case of a higher demand, the flow is split over the routes.

Fig. 5 shows several level curves, i.e., combinations of splitting rates and monetary incentives that yield the same value of the follower's objective function (7) for a demand of 6000 veh/h as also applies in Fig. 3(b). Two possible optimal leader functions are indicated, i.e., an affine ($\gamma_1$) and parabolic ($\gamma_2$) mapping of splitting rates between road

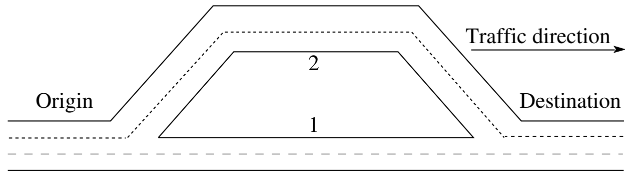



Fig. 4. Two-route traffic network

1 and 2 to a monetary value. For both functions, the optimal choice of splitting rates for the follower player will be the system-optimal $\zeta^{\text{d},1} = 0.66$, which is associated with a monetary value of 4 € in this example.

## 4. CASE STUDY

Finally, in order to give an indication of the performance of the reverse Stackelberg routing approach, we simulated the traffic behavior in the simple network depicted in Fig. 4. Here, we compared the TTS while using the optimal splitting rates as achieved by the proposed reverse Stackelberg game approach to the TTS that is yielded when using DRIPs.

Recall that Example 2 shows the composition of the optimal leader functions in the static game, i.e., for each control time step $k_\text{c}$. We now simulate the results over a horizon of 1 hour. For the simulation of the traffic behavior when using DRIP panels for route guidance, the logit model (Cramer, 1991) is adopted to obtain splitting rates as a function of the difference in predicted travel time between the alternative routes.

The results of this simulation, plotted in Fig. 6, show that in the reverse Stackelberg approach, most use is made of the shortest route. Until the capacity flow is reached, the proposed leader functions can drive the traffic towards route 1, whereas the traffic is distributed over the two alternative routes once the capacity is exceeded. In the DRIP approach however, an increasing number of vehicles choose the longer route as the flow on the shortest route reaches closer to the capacity. This results in a TTS of $6.50 \cdot 10^4$ veh.h for the DRIP approach versus $6.25 \cdot 10^4$ veh.h for the reverse Stackelberg approach. The latter number is only slightly lower, however, in larger networks that allow for multiple routes and larger differences in total travel time, this difference is expected to be more apparent.

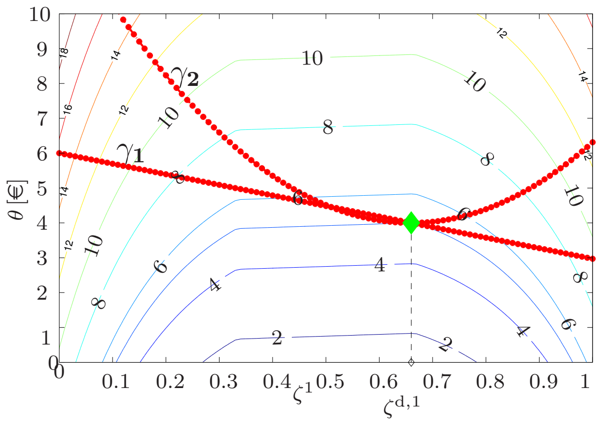



Fig. 5. Optimal affine ($\gamma_1$) and quadratic ($\gamma_2$) leader functions

## 5. DISCUSSION AND FURTHER RESEARCH

A game-theoretical method has been proposed to accomplish a system-optimal distribution of traffic over the available roads in a freeway network. The method is based on a reverse Stackelberg game in which a group of drivers, homogeneous in the value of time, chooses upon a splitting rate according to the road authority's proposed mapping of splitting rates to monetary incentives.

As compared to alternative road-tolling methods or route guidance methods that rely on travel time information to influence the drivers' route choice, this approach can achieve the desired splitting rates under the assumption that drivers make rational decisions, optimizing a combination of travel time and monetary incentives.

Here, it should be noted that the travel cost of a follower player is still a group average, i.e., individual drivers will follow different routes according to the group splitting rate, which results in different travel costs between the drivers. The actual division of the homogeneous individual drivers over the alternative routes is therefore a lower-level problem that will be addressed in future research.

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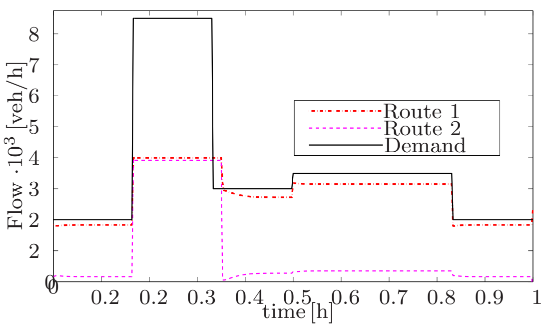
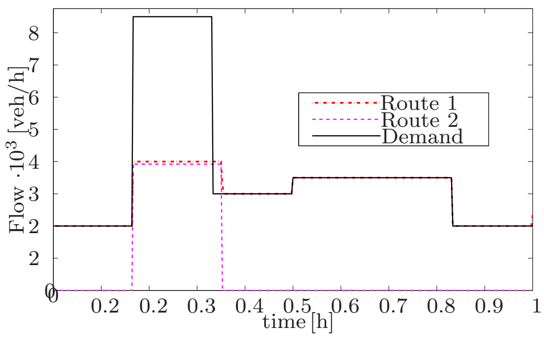
## REFERENCES

Ahuja, R.K., Magnanti, T.L., and Orlin, J.B. (1993). *Network Flows: Theory, Algorithms, and Applications*. Prentice Hall, Upper Saddle River, NJ, USA.




(a) DRIP approach




(b) Reverse Stackelberg approach

Fig. 6. A comparison of the distribution of traffic demand

Blayac, T. and Causse, A. (2001). Value of travel time: a theoretical legitimization of some nonlinear representative utility in discrete choice models. *Transportation Research Part B: Methodological*, 35(4), 391–400.

Cramer, J.S. (1991). *The Logit Model: An Introduction for Economists*. Edward Arnold, London, UK.

Daganzo, C.F. (1994). The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B*, 28(4), 269–287.

Deflorio, F.P. (2003). Evaluation of a reactive dynamic route guidance strategy. *Transportation Research Part C*, 11(5), 375–388.

DeSerpa, A.J. (1973). Microeconomic theory and the valuation of traveltime: some clarifications. *Regional Urban Economics*, 2, 401–410.

Groot, N., De Schutter, B., and Hellendoorn, H. (2012). Dynamic optimal routing based on a reverse Stackelberg approach. In *Proceedings of the 15th International IEEE Conference on Intelligent Transportation Systems*. Anchorage, AK, USA.

Joksimovic, D., Bliemer, M.C.J., Bovy, P.H.L., and Verwater-Lukszo, Z. (2005). Dynamic road pricing for optimizing network performance with heterogeneous users. In *Proceedings of the IEEE International Conference on Networking, Sensing and Control*, 407–412. Tucson, AZ, USA.

Karimi, A., Hegyi, A., De Schutter, B., Hellendoorn, J., and Middelham, F. (2004). Integrated model predictive control of dynamic route guidance information systems and ramp metering. In *Proceedings of the 7th International IEEE Conference on Intelligent Transportation Systems*, 491–496. Washington, DC, USA.

May, A.D. (1990). *Traffic Flow Fundamentals*. Prentice-Hall, Englewood Cliffs, New Jersey.

Messmer, A. and Papageorgiou, M. (1990). METANET: A macroscopic simulation program for motorway networks. *Traffic Engineering and Control*, 31(9), 466–470.

Olsder, G.J. (2009). Phenomena in inverse Stackelberg games, part 1: Static problems. *Journal of Optimization Theory and Applications*, 143(3), 589–600.

Staňková, K., Olsder, G.J., and Bliemer, M.C.J. (2009). Comparison of different toll policies in the dynamic second-best optimal toll design problem: Case study on a three-link network. *European Journal of Transport and Infrastructure Research*, 9(4), 331–346.

van den Berg, M., Bellemans, T., De Schutter, B., De Moor, B., and Hellendoorn, J. (2004). Anticipative ramp metering control using dynamic traffic assignment. In *Proceedings of the 7th International IEEE Conference on Intelligent Transportaion Systems*, 503–508. Washtingon, DC, USA.

von Stackelberg, H. (1952). *The Theory of Market Economy*. Oxford University Press, Oxford, UK.

Wardrop, J.G. (1952). Some theoretical aspects of road traffic research. *Proceedings of the Institute of Civil Engineers, PART II*, 1(2), 325–378.

Yperman, I., Logghe, S., Tampère, C., and Immers, L. (2006). The multi-commodity link transmission model for dynamic network loading. In *Proceedings of the 85th Annual Meeting of the Transportation Research Board*. Washington, DC, USA.