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Control of Traffic Networks Using the Link Transmission Model and Mixed Integer Linear Programming

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1 Introduction

With the increasing number of vehicles, highways are becoming more and more congested. This along with increasingly stringent traffic requirements necessitates the use of efficient large-scale traffic management and control methods. One particular solution to this problem is based on Model Predictive Control (MPC), where a finite-horizon constrained optimal control problem is solved in a receding horizon fashion [1]. MPC requires a traffic model that can provide accurate prediction of the traffic states while it has low computational complexity. The METANET model is a second-order model that is able to model the traffic network with good accuracy. However, this is a nonlinear model and when it is used as prediction model in the MPC framework, a nonlinear nonconvex optimization problem results. In [2] a method to transform the original nonlinear problem into a mixed integer linear optimization problem is proposed. This has been done by approximating the METANET model by piecewise affine (PWA) functions. Although this can solve the convexity problem, using this approach for larger networks is impractical and still takes large amount of computation time.

One way to overcome this problem for large-scale traffic networks is to use first-order models like the cell transmission model (CTM) [3]. However, using the CTM as prediction model in the MPC framework will result in a nonlinear optimization problem that still requires a significant computation time for large networks. One way to tackle this problem is to use the relaxed formulations proposed in [4] and to recast the problem into a linear optimization problem. Recently, the link transmission model (LTM) has been developed in [5] which has less computational complexity compared to the CTM and the METANET. This model has been basically developed for dynamic traffic assignment problems, but this paper considers using it for traffic control purposes. Although to reduce computational efforts in CTM, one could enlarge the length of the time step, such operation leads to reduction in accuracy. In the LTM, it can be proved that one can get acceptable accuracy with less computational effort. However, the LTM is still a nonlinear model and according to the delays in its equations, it requires more memory storage. These delays also make working with the model more difficult.

In this work, we aim at using a procedure to reformulate the LTM. Based on this new formulation, one can build a mixed integer linear problem that can be solved more efficiently than the original nonlinear MPC based on the original LTM model. In the next sections, we briefly describe our approach.

2 Link Transmission Model

The LTM [5] uses links to model homogeneous sections of a road and nodes to model origins, destinations, on-ramps, off-ramps, intersections, etc. The LTM is capable of determining time-dependent link volumes and route travel times in traffic networks. To this aim, the LTM uses the cumulative number of vehicles as a representation for the traffic evolution. The cumulative number of vehicles that passed the upstream and downstream boundaries of the links are tracked and their values are updated using flow functions of links and nodes. These flow functions are characterized by capacities of the links, link travel times, demand profiles, available storage space at origins, etc. First, sending and receiving flows of all links are determined and subsequently, transition flows of the nodes are obtained using the flow functions of their incoming and outgoing links. For each type of the nodes a different formulation for the transition flow is defined. The reader is referred to [5] for more details about the LTM equations.

3 Mixed Integer Linear Programming

Since the LTM is nonlinear in nature, the optimization problem that has to be solved to obtain the optimal control signals is nonlinear and nonconvex. In order to get rid of the nonlinear nonconvex original optimization problem, one can use the approach proposed in [6] to obtain a mixed-integer linear programming (MILP) problem. The main idea is to define some binary variables to describe different regions of operation of the system. Next by using the binary variables and adding some inequality constraints, the LTM model will be reformulated with a system of linear equalities and inequalities consisting of real and integer variables. After transforming the LTM model into the mixed integer linear equations, the only remaining part is to recast the traffic objective function in a linear form. The linear objective function along with the reformulated model, constructs an MILP problem that can be solved efficiently.

4 Case Study

For testing the proposed approach, a benchmark traffic network example has been selected from [7]. The network, shown in Fig. 1 consists of a mainstream freeway with a metered on-ramp and it is modeled using a modified version of the LTM that includes the metering signals. In Fig. 2, the demand profile along with densities of the links and queue lengths of the origins derived by the LTM are shown. The flows of the vehicles from the mainstream origin and the on-ramp are controlled in order to minimize the total time that vehicles spend on the road and in queues. The control signals are obtained first by using an MPC controller based on the original nonlinear LTM model and next by using the MILP approach. The performance of the two approaches is compared in Table 1 in term of total cost. According to Table 1, the MILP approach returns values that are close to the original optimization problem, while needing a shorter computation time.

5 Conclusions

Modeling and control of traffic networks using the LTM has been presented in this research. The LTM was used as prediction model in the MPC framework. Since a direct MPC implementation based on the nonlinear LTM is still computationally inefficient, a reformulation of the LTM was proposed in order to eventually obtain an MILP. This new approach give results close to the ones obtained by the nonlinear MPC while the CPU time goes significantly down. Moreover, it is expected that for larger networks the benefits of the new approach over the nonlinear MPC will become even more clear.

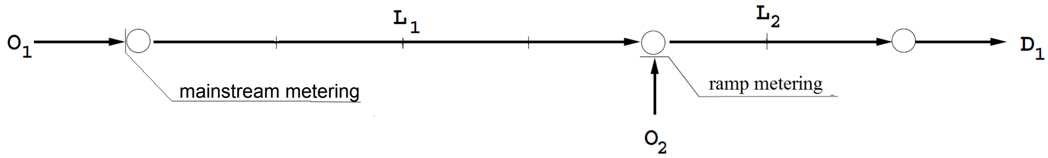
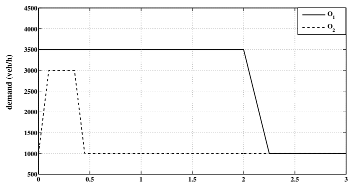
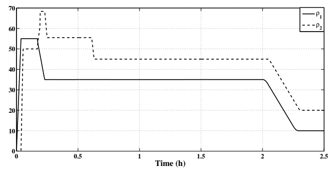


Figure 1: Set-up of the case study



(a) demand



(b) densities

(c) queues

Figure 2: Results of the LTM simulation

Table 1: Comparison of total time spent (veh.h) for the two approaches

Prediction and Control Horizon	Nonlinear MPC	MILP
$N_\mathrm{p} = N_\mathrm{c} = 7$	549.07 veh.h	551.8 veh.h
$N_\mathrm{p} = 7, N_\mathrm{c} = 3$	556.03 veh.h	561.1 veh.h

References

[1] Rawlings J., and Mayne D., *Model Predictive Control: Theory and Design.* Nob Hill Publishing, Madison, WI, U.S.A., 2009.

[2] Groot N., De Schutter B., and Hellendoorn H., “Model-based traffic and emission control using PWA models – A mixed-logical dynamic approach”, Proceedings of the

14th International IEEE Conference on Intelligent Transportation Systems (ITSC 2011), Washington, DC, 2142-2147, 2011.

[3] Daganzo C., "The cell transmission model: A simple dynamic representation of freeway traffic", *Transportation Research B* 28(4), 269-287, 1994.

[4] Ziliaskopoulos A, "A linear programming model for the single destination system optimum dynamic traffc assigment problem", *Transportation Science* 34(1), 37-49, 2000.

[5] Yperman I., *The Link Transmission Model for Dynamic Network Loading.* Ph.D. dissertation, Katholieke Universiteit Leuven, Leuven, Belguim, 2007.

[6] Bemporad A., and Morari M., "Control of systems integrating logic, dynamics, and constraints", *Automatica* 35(3), 407-427, 1999.

[7] Hegyi A., De Schutter B., and Hellendoorn H., "Model predictive control for optimal coordination of ramp metering and variable speed limits", *Transportation Research Part C* 13(3), 185-209, 2005.