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# Application of Distributed and Hierarchical Model Predictive Control in Hydro Power Valleys

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**Abstract:** We provide an overview on the application of distributed and hierarchical model predictive control (MPC) algorithms for the power reference tracking problem of the HD-MPC Hydro Power Valley (HPV) system (Savorgnan and Diehl, 2011). Serving as a case study for distributed and hierarchical MPC, the HPV benchmark has various challenging features, including nonlinear, non-smooth, and coupled cost function and nonlinear coupled subsystem dynamics. We propose different approaches to address these challenges and summarize our recently developed hierarchical and distributed MPC frameworks that could be applied to the HPV control problem. A comparison of distributed MPC based on a state-of-the-art distributed optimization method (Giselsson et al., 2012) with centralized and decentralized MPC is provided via numerical simulations. It is shown that by using a dynamic division of total power reference to deal with the coupling in the cost function and a specific formulation of the dual optimization problem, distributed MPC achieves almost the same tracking performance as centralized MPC, with the advantage of being implementable in a distributed setting.

*Keywords:* Distributed Model Predictive Control, Hydro Power Valley Control, Power Reference Tracking MPC

## 1. INTRODUCTION

Hierarchical and distributed Model Predictive Control (MPC) has been an active research area, with the aim to design suitable methods for controlling large-scale industrial processes and infrastructure systems involving interacting subsystems and critical operational constraints that must be enforced (Scattolini, 2009). The control problems in such systems are often approached using MPC (Maciejowski, 2002; Camacho and Bordons, 1999; Rawlings and Mayne, 2009; Qin and Badgwell, 2003), due to its ability to handle important process constraints explicitly. MPC relies on solving finite-time optimal control problems repeatedly online, which may become prohibitive for large-scale systems due to the computational or communication limitations, thus hierarchical and distributed MPC methods have been investigated to deal with these issues. Recent developments in the field can be found in Scattolini (2009); Alessio et al. (2011); Alvarado et al. (2011); Doan et al. (2011a); Maestre et al. (2011); Scheu and Marquardt (2011); Stewart et al. (2011) and the references therein.

In this paper we consider the HD-MPC Hydro Power Valley (HPV) system (Savorgnan and Diehl, 2011) and investigate the applicability of hierarchical and distributed MPC approaches, while providing an overview of available alternative control strategies. An HPV may contain several rivers and lakes, spanning a wide geographical area and exhibiting complex dynamics. There are several hydropower plants placed along the rivers and the control objective is to coordinate the whole plant to track a total power-production reference by regulating the water flows in the system. Decentralized MPC methods for the control of open water systems, especially irrigation canals, have been studied by Fawal et al. (1998); Georges (1994); Sawadogo et al. (1998); Gomez et al. (2002); Sahin and Morari (2010). Distributed MPC approaches based on coordination and cooperation for water delivery canals were presented by Georges (1994); Negenborn et al. (2009); Igreja et al. (2011); Anand et al. (2011). The typical control objective in these studies is to regulate water levels and to deliver the required amount of water to the right place at some time in the future, i.e., the cost function can be considered rather standard with penalties on states and inputs. In contrast, for the hydro power control problem considered in this paper, there are output penalty terms in the cost function, which represent the objective of manipulating power production. Recent literature taking into account this different type of cost function includes centralized nonlinear MPC with a parallel version of the multiple-shooting method using continuous-time nonlinear dynamics (Savorgnan et al., 2011), and a software framework that formulates a discrete-time linear MPC controller with the possibility to integrate a nonlinear prediction model and to use commercial solvers to solve the optimization problem (Petrone, 2010).

In this paper, we summarize our recently proposed hierarchical and distributed MPC frameworks that could be applied to the HPV control problem. We also present techniques to deal with the couplings in the dynamics (i.e., subsystems that dynamically interact with each other), and the coupling in the cost function due to the requirement that the whole system must track a total power reference. Our distributed MPC design approach is enabled by a state-of-the-art distributed optimization algorithm that has recently been developed together with our co-authors in Giselsson et al. (2012). This optimization algorithm is designed for a class of strongly convex problems with coupled constraints and mixed 1-norm and 2-norm terms in the cost function, which perfectly suits the power reference tracking objective in HPV control. The underlying optimization algorithm has a fast convergence rate thanks to the use of an accelerated proximal gradient algorithm with the optimal step size. We will show the advantage of the proposed distributed MPC approach by simulations in an HPV case study.

The remaining parts of the paper are organized as follows. Section 2 describes the Hydro Power Valley system and the control problem. The modeling of the HPV system is discussed in Section 3. In Section 4, the MPC control problem is formulated, then we summarize the control architectures for applying MPC in the HPV control problem, with the focus on distributed MPC and the methods to deal with the couplings in the dynamics and in the cost function. The simulation results of applying distributed MPC in the HPV case study are presented in Section 5, in comparison with centralized and decentralized MPC. Through various aspects of the comparison including performance, computational efficiency, and communication requirements, the advantages of the distributed MPC algorithm will be highlighted. Section 6 concludes the paper and outlines future work.

## 2. HYDRO POWER VALLEY CONTROL PROBLEM

In this section, we provide a summary of the HD-MPC Hydro Power Valley benchmark (Savorgnan and Diehl, 2011) and then discuss the main control challenges.

### *2.1 Hydro power valley system*

We consider a hydro power plant composed of several interconnected subsystems, as illustrated in Figure 1. The plant is divided into 8 subsystems, of which subsystem $S_1$ is composed of the lakes $L_1, L_2$, the duct $U_1$ connecting them, and the ducts $C_1, T_1$ that respectively connect $L_1$ with the reaches $R_1$, $R_2$, where a reach is a river segment between two dams. Subsystem $S_2$ is composed of the lake $L_3$ and the ducts $C_2, T_2$ that connect $L_3$ to the reaches $R_4, R_5$, respectively. There are 6 other subsystems each of which consists of a reach and the dam at the end of the reach. These six reaches $R_1$ to $R_6$ are connected in series, separated by the dams $D_1$ to $D_5$. The large lake that follows the dam $D_6$ is assumed to have a fixed water level, which will absorb all the discharge. The outside water flows enter the system at the upstream of the reach $R_1$ and at the middle of the reach $R_3$.

There are structures placed in the ducts and at the dams to control the flows. These are the turbines placed in the ducts $T_1, T_2$ and at each dam for power production. In the ducts $C_1, C_2$ there are composite structures that can either function as pumps (for transporting water to the lakes) or as turbines (when water is drained from the lakes).

The whole system has 10 manipulated variables, which are composed by six dam flows ($q_{D1}$, $q_{D2}$, $q_{D3}$, $q_{D4}$, $q_{D5}$, $q_{D6}$), two turbine flows ($q_{T1}$, $q_{T2}$), and two pump/turbine flows ($q_{C1}$, $q_{C2}$).

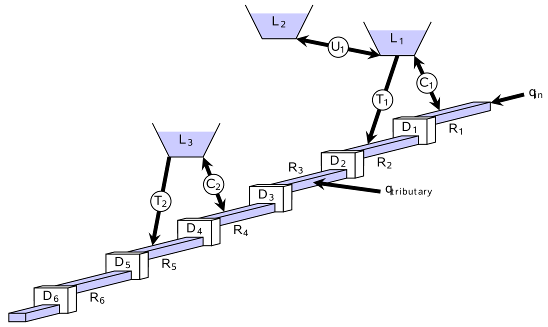



Fig. 1. Overview of the HD-MPC hydro power valley system (Savorgnan and Diehl, 2011)

The detailed setup of the HPV system is given in (Savorgnan and Diehl, 2011), with the continuous-time nonlinear model constructed by using the Saint Venant partial differential equations and employing spatial discretization for obtaining a system of ordinary differential equations.

### *2.2 Power reference tracking problem*

The control problem is to track a power production profile on a daily basis with the following cost function:

$$
\begin{aligned}
J(t) \triangleq & \int_0^T \gamma \left| p_{\mathrm{r}}(t) - \sum_{i=1}^{8} p_i(x(t), u(t)) \right| dt \\
& + \sum_{i=1}^{8} \int_0^T (x_i(t) - x_{\mathrm{ss},i})^T Q_i (x_i(t) - x_{\mathrm{ss},i}) dt \\
& + \sum_{i=1}^{8} \int_0^T u_i(t)^T R_i u_i(t) dt \qquad (1)
\end{aligned}
$$

subject to the nonlinear dynamics and the operational constraints on water levels and flows, which are denoted by the variables $x$ and $u$, respectively. The weights $Q_i, R_i, i = 1, \ldots, 8$, $\gamma$, and the testing period $T$ are the parameters of the benchmark. The first term in the cost function represents the absolute value of the power tracking error and it reflects the economical cost, with $p_{\mathrm{r}}$ the power reference and $p_i$ the power produced by subsystem $i \in \{1, \ldots, 8\}$. The quadratic terms in the cost function represent the penalties on the state deviation from the steady state $x_{\mathrm{ss}}$ and the energy used for manipulating the inputs.

*Remark 1.* There are several challenges posed by this control problem, of which the most critical ones are the relatively large size and the non-smoothness of the problem, which includes the absolute value term in the cost function and the power produced by the composite structures in the ducts $C_1$ and $C_2$. Moreover, the constraints on the control inputs and outputs complicate the problem, preventing the choice of most classical control methods, such as PID or LQR approaches.

In order to enforce the constraints while optimizing the cost function, we choose a model predictive control approach that is well-suited and efficient for control of constrained multi-input multi-output systems (Qin and Badgwell, 2003). In MPC, the controller solves an optimization problem online, which can be a computationally demanding task when the size of the problem is large. Nowadays state-of-the-art solvers are capable of solving convex problems with thousands of decision variables in a matter of seconds. Following the modeling approach described in Section 3, it turns out that in terms of problem size alone, the HPV benchmark under study falls in the class of problems that can be "managed" using centralized solver routines. However, besides computational complexity, gathering all sensor measurements and communicating actuation and coordination information in such geographically widely spread large-scale systems can be problematic. These limitations might favor a different approach, where the computations (for local control actions) are distributed over the subsystems, and only a sparsely connected communication network is utilized to share information among them. This approach presents a need for an optimization algorithm that, while being fast, is able to utilize the computational resources available at every subsystem locally. As a consequence, although our problem size alone would not necessitate a departure from a centralized approach, we focus on designing distributed MPC (DMPC) controllers for the hydro power valley. With this we aim to illustrate the possibilities of our approach for even larger-scale, sparsely connected systems (with many more than 8 subsystems), where its benefits could be even more pronounced in terms of flexible system maintenance and reliability, problem size tractability, and limited communication.

Before describing the various MPC approaches that will be investigated, in the next section we will first discuss the modeling of the HPV system for completeness, based on content from Savorgnan and Diehl (2011).

## 3. MODELING OF THE HYDRO POWER VALLEY

### 3.1 Nonlinear modeling, discretization and linearization

The model of the reaches is based on the one-dimensional Saint Venant partial differential equation, representing the mass and momentum balance:

$$\begin{cases} \dfrac{\partial q(t,z)}{\partial z} + \dfrac{\partial s(t,z)}{\partial t} = 0 \\ \dfrac{1}{g}\dfrac{\partial}{\partial t}\left(\dfrac{q(t,z)}{s(t,z)}\right) + \dfrac{1}{2g}\dfrac{\partial}{\partial z}\left(\dfrac{q^2(t,z)}{s^2(t,z)}\right) + \dfrac{\partial h(t,z)}{\partial z} \\ +I_{\mathrm{f}}(t,z) - I_0(z) = 0 \end{cases} \tag{2}$$

with $z$ the spatial variable, $t$ the time, $q$ the river flow, $s$ the cross-section surface of the river, $h$ the water level w.r.t. the river bed, $I_{\mathrm{f}}$ the friction slope, $I_0(z)$ the river bed slope, and $g$ the gravitational acceleration constant.

The partial differential equation (2) can be converted into a system of ordinary differential equations by using spatial discretization. To this aim, each reach is divided into 20 cells, yielding 20 additional states, which are the water levels at the beginning of the cells. As a consequence, a set of nonlinear dynamical equations are obtained, with a total of 249 states (Savorgnan and Diehl, 2011).

A linear time-invariant model of the hydro power valley is next obtained by linearizing the nonlinear model at a steady operating condition, followed by discretization in time. The use of the discrete-time linearized model will simplify the analysis and enable specific MPC approaches. Due to the coupling structure of the system, the discrete-time linear model of each of the eight subsystems $i = 1,\ldots,8$ can be expressed in the following form:

$$\begin{aligned} x_i(k+1) &= A_{ii}x_i(k) + \sum_{j=1}^{8} B_{ij}u_j(k) \\ y_i(k) &= C_i x_i(k) \end{aligned} \tag{3}$$

in which the variable $x$ and $u$ respectively stand for the deviation of the water levels and flows from the steady-state values, the output $y$ corresponds to the states of the last cell of the reaches, and the subscripts $i, j$ stand for the subsystem index. Note that the subsystems are coupled through the inputs only. Furthermore, there are no couplings between subsystems ($B_{ij} = 0$) that are not connected together. Based on this dynamical coupling structure, we define the *neighborhood set* $\mathcal{N}_i$ for each subsystem $i = 1,\ldots,8$ that includes all other subsystems which influence the dynamics of subsystem $i$, i.e.

$$\mathcal{N}_i = \{j | B_{ij} \neq 0\}. \tag{4}$$

### 3.2 Treatment of non-smoothness

One of the difficulties in applying a linear MPC approach to the hydro power valley is the non-smoothness of the power functions associated with the ducts $C_1$ and $C_2$. The non-smoothness is caused by the fact that the flow through $C_1$ and $C_2$ can have two directions and the powers generated or consumed do not have equal coefficients. For example, the power produced at $C_1$ can be expressed as

$$p_{C_1}(t) = k_{C_1}(q_{C_1}(t))q_{C_1}(t)\Delta h_{C_1}(t), \tag{5}$$

where $\Delta h_{C_1}(t)$ it the duct head which depends on the water level in lake $L_1$ and reach $R_1$ and

$$k_{C_1}(q_{C_1}(t)) = \begin{cases} k_{\mathrm{t}_{C_1}} & \text{when } q_{C_1}(t) \geq 0 \\ k_{\mathrm{p}_{C_1}} & \text{when } q_{C_1}(t) < 0 \end{cases} \tag{6}$$

To deal with this issue, we use the double-flow technique, which involves introducing two separate positive variables to express the flow in $C_1$:

- $q_{C_{1\mathrm{p}}}(t)$: virtual flow such that $C_1$ functions as a pump
- $q_{C_{1\mathrm{t}}}(t)$: virtual flow such that $C_1$ functions as a turbine

Using these two flows, the power function associated with $C_1$ is replaced by two continuous functions that express the power produced ($q_{C_{1\mathrm{p}}}(t)$) and consumed ($q_{C_{1\mathrm{t}}}(t)$):

$$p_{C_1}(t) = p_{C_{1t}}(t) + p_{C_{1p}}(t), \text{where} \tag{7}$$

$$p_{C_{1t}}(t) = k_{\mathrm{t}_{C_1}} q_{C_{1\mathrm{t}}}(t)\Delta h_{C_1}(t) \tag{8}$$

$$p_{C_{1p}}(t) = k_{\mathrm{p}_{C_1}} q_{C_{1\mathrm{p}}}(t)\Delta h_{C_1}(t) \tag{9}$$

This approach allows the optimization solver to deal with smooth functions only. When the solution is obtained, we combine the virtual flows to get the real flow through $C_1$:

$$q_{C_1}(t) = q_{C_{1\mathrm{t}}}(t) - q_{C_{1\mathrm{p}}}(t) \tag{10}$$

The double-flow approach is also applied for $C_2$. Consequently, the new linear model has 12 inputs. Another challenge of the linear model is the existence of a number of unobservable and uncontrollable modes due to the dependencies between states that represent adjacent water levels along the reaches. Moreover, the linear model has a large number of states, causing a significant computational burden. Therefore, we use balanced truncation for model order reduction (Gugercin and Antoulas, 2004) to remove less significant modes (all stable), including all unobservable and uncontrollable ones. This results in a model of the form (3) with 32 states in total.

## 4. HIERARCHICAL AND DISTRIBUTED MODEL PREDICTIVE CONTROLLER DESIGN

### *4.1 MPC optimization problem*

The requirement of power-reference tracking while satisfying hard constraints suggests the use of MPC. In the MPC framework, the control problem is formulated as an optimization problem to be solved online at each sampling time. The MPC optimization problem for the HPV has the following form:

$$\begin{aligned}\min_{\{\mathbf{x}_k^i\}_{k=0,\ldots,N-1}^{i=1,\ldots,8}} \quad & \sum_{k=0}^{N-1}\left\{\left|p_k^{\mathrm{ref}} - \sum_{i=1}^{8} P_i \mathbf{x}_k\right| + \sum_{i=1}^{8} {\mathbf{x}_k^i}^T \mathbf{Q}_i \mathbf{x}_k^i\right\} \\ \text{s.t.} \quad & A_1\mathbf{x}_k = B_1, \quad k = 0,\ldots,N-1 \\ & A_2\mathbf{x}_k \le B_2, \quad k = 0,\ldots,N-1 \end{aligned} \tag{11}$$

in which the cost function represents the control goal, i.e., power-reference tracking and simultaneously keeping water levels close to the steady-state values. The equality and inequality constraints respectively represent the discretized dynamics of the subsystems and the limitation of the physical values, including water levels and flows. We use $k$ as sampling time step index, $i$ as subsystem index, and $N$ as the prediction horizon. The variable $\mathbf{x}_k^i$ combines the state $x_i$ and input $u_i$ of each subsystem in the predicted step $k$, while $\mathbf{x}_k = [{\mathbf{x}_k^1}^T, \ldots, {\mathbf{x}_k^8}^T]^T$ is used for the aggregate variable at one step. In addition, $\mathbf{Q}_i$, which is properly formed from $Q_i$ and $R_i$ with minor modification to be a block-diagonal positive definite matrix, represents the penalties for each subsystem's states and inputs. The term $P_i\mathbf{x}_k$ represents the power function belonging to each subsystem $i$, which is a linearization of the bilinear power function at the steady-state condition. The matrix $P_i$ is sparse, since all of its columns that correspond to the variables of the subsystems outside $\mathcal{N}_i$ are zero.

*Remark 2.* Note that the constraint matrix $A_2$ is block-diagonal since there are only local constraints on states and inputs in the HPV problem, but the constraint matrix $A_1$ is not block-diagonal, due to the couplings in the dynamics. However, since the matrix $A_1$ is sparse, we can use dual decomposition techniques to deal with this coupled constraint in a hierarchical or distributed way (see also Section 4.2.4).

Another coupling is the objective of total power reference tracking, which leads to the coupled cost function: it cannot be separated as the sum of local costs. The coupling in the cost function prevents the dual decomposition techniques from being applied directly. In order to treat this coupling, we need to modify the cost function in a specific way that facilitates distributed and hierarchical solution approaches. Obviously, the result with the modified cost function will not be optimal for problem (11), but in some ways we can reduce the sub-optimality. In the next section, we will present different approaches to modify the cost function that lead to distributed or hierarchical MPC.

### *4.2 MPC algorithms for HPV control problem*

The structured sparse model of the Hydro Power Valley prompts us to consider designing a hierarchical or distributed MPC algorithm in which the main computing tasks are carried out by subsystems' controllers. In this section we will discuss different control architectures for MPC, including centralized MPC, hierarchical MPC, distributed MPC, decentralized MPC, together with the outcomes of each approach.

*Centralized MPC* In the centralized MPC approach, there is one centralized controller that gathers all the necessary information, i.e., measurements of the sensors, and then solves the optimization problem. Once the solution is obtained, the new control input values are sent to the actuators. The controller must use an appropriate solver that is capable of solving the optimization problem. Indeed, the optimization problem (11) can be recast as a convex quadratic program (QP) by transforming the absolute term of power-reference tracking in the overall objective function of (11) into the following equivalent formulation:

$$\min_{\mathbf{x}} |P\mathbf{x} - p| \qquad \Leftrightarrow \qquad \begin{aligned} & \min_{\mathbf{x},v} \mathbf{1}^T v \\ & \text{s.t.} - v \le P\mathbf{x} - p \le v \end{aligned} \tag{12}$$

As convex QPs can be solved efficiently by many state-of-the-art solvers, giving exact optimal solution, it is preferable to use centralized MPC when global communications are possible. However, when global communications are costly, limited, or not possible at all, centralized MPC may not be a good candidate. In that case, one can consider the other control architectures that are discussed next.

*Decentralized MPC* One extreme case that avoids the need of communications, is decentralized MPC, where there is no communication at all. The decentralized MPC setting for the HPV is defined as follows:

- There are 8 local controllers, each of them is responsible for the control of one subsystem. Each one can only measure its own output and can only control its own manipulator(s).
- The controllers use linearized local models with the double-flow technique of Section 3.2 in order to apply discrete-time linear MPC to the local problem.
- No information exchange is allowed. The steady-state inputs and states are the only common information of the local controllers, and this information is shared before the operation starts. Any subsystem interaction will be modeled by using the steady-state variables of the other models.
- The overall power reference tracking problem is thus separated into several local tracking problems, with the local power references proportional to the steady-state power of the corresponding subsystem.

We proceed by defining the local MPC cost functions in (11) as follows:

$$\min_{\mathbf{x}_k^i} \quad \sum_{k=0}^{N-1} \left|p_k^{\mathrm{ref},i} - P_i\mathbf{x}_k^i\right| + {\mathbf{x}_k^i}^T\mathbf{Q}_i\mathbf{x}_k^i, \quad i = 1,\ldots,8$$

where $p_k^{\mathrm{ref},i}$ is the local power reference at step $k$. The values of $p_k^{\mathrm{ref},i}$ are computed such that the following condition is maintained:

$$\frac{p_k^{\mathrm{ref},i}}{p_k^{\mathrm{ref}}} = \frac{p_{\mathrm{ss},i}}{p_{\mathrm{ss}}}, \quad \forall i, \forall k \tag{13}$$

in which $p_{\mathrm{ss},i}$ and $p_{\mathrm{ss}}$ respectively represent the steady-state powers of subsystem $i$ and of the whole plant. Although there are many alternative ways of computing local power references, the idea behind our choice is to ensure that the local control actions can remain unchanged if the reference matches the steady-state value. The constraints of the local MPC problems are also simplified from the constraints of the original centralized problem (11), in the sense that each subsystem considers the variables of the other subsystems as the steady-state values.

Although the implementation is straightforward and the problem size is small for every subsystem, using decentralized MPC leads to a trade-off in terms of performance and potential instability. Since the couplings in both the constraints and the cost function are neglected, the decentralized MPC in general achieves poor performance.

*Hierarchical MPC* When global communication channels exist but are limited, one can use hierarchical MPC. The hierarchical architecture aims to distribute the tasks to local controllers, while still keeping some information exchange at the upper layer, e.g., via a global coordinator, so that the couplings can be treated properly.

In order to address the coupled constraints in a hierarchical framework, the optimization problem is often decomposed using a dual decomposition method (Bertsekas, 1999). However, dual decomposition methods often result in iterative schemes that converge to the dual optimal solution only asymptotically. This may also create difficulties in obtaining a primal *feasible* solution before convergence.

In Doan et al. (2011b) we have proposed a hierarchical optimization approach for solving large-scale MPC problems with coupling in dynamics and constraints that guarantees primal feasible solutions even after only a finite number of iterations. The primal feasible solution is achieved by employing a combination of a primal averaging scheme, a distributed Jacobi iterative method, and constraint tightening. The core idea of the approach is to use constraint tightening and then solve the tightened problem using nested iterations: a projected gradient method for the outer loop to solve the tightened dual problem, and making use of the approximate solution of the primal problem that is obtained by the Jacobi iteration in the inner loop. The coordinator is needed to compute several common parameters that have to be passed to local controllers.

The advantage of the hierarchical MPC framework in Doan et al. (2011b) is that all the constraints are satisfied after a finite number of iterations, and that most of the computations are performed by the local controllers. On the other hand, by putting the highest priority for the constraint satisfaction, the performance can be significantly reduced due to the tightening of the constraints. Moreover, the hierarchical MPC framework does not handle equality constraints, thus the state variables need to be eliminated, which will enlarge the neighborhood sets of the subsystems. Therefore this method is not readily suitable for applying to the HPV control problem where the number of subsystems is limited compared to the sparsity of the interconnections.

*Distributed MPC* In case global communications are not available, but local communications between subsystems can be used, then a suitable approach is distributed MPC, which often achieves better performance than decentralized MPC, since the couplings are not neglected. In this section we discuss distributed MPC frameworks that are based on distributed optimization, i.e., the distributed nature comes from the algorithm to solve the optimization problem (11).

The two main traditional classes of techniques for solving a coupled optimization problem are primal and dual decomposition. With primal decomposition, the problem structure is exploited such that each subsystem can solve its own problem, which involves the variables of itself and its neighbors only. One typical method in this class is the Jacobi iterative algorithm (Bertsekas and Tsitsiklis, 1989, Chapter 3), which was adopted for distributed MPC in Venkat et al. (2008). The main idea is that each subsystem solves its own problem using the previous update of the others, and then uses a convex combination of the new and old values to guarantee that the new iterate is plant-wide feasible. In Venkat et al. (2008), it is proved that, similar to the Jacobi algorithm, the distributed MPC solution converges to the centralized MPC solution (assuming no coupling constraints).

In the class of dual decomposition approaches, a dual problem of (11) is formulated and taken as the main optimization problem. It is well-known by the duality theorem (Bertsekas, 1999) that the dual optimum is the same as the primal optimum when the original problem is strictly convex, which is the case for the HPV problem since the matrices $\mathbf{Q}_i$ are positive definite for all $i$. In the dual problem, the couplings in the constraints are then transformed into separable terms in the cost function. Since in most cases the dual problem is separable, we can employ a number of optimization algorithms using first-order derivatives, which then allow a distributed implementation. However, in the particular HPV problem (11), since there is also coupling in the cost function, the dual problem is not separable. To obtain a separable dual problem, we first need to reformulate the cost function of (11) in a separable form. Hereby we present two ways of power reference division for the HPV problem. For the sake of brevity, we now focus on decomposing the following problem:

$$\min_{\{\mathbf{x}^i\}_{i=1,\ldots,8}} \quad \left| p^{\mathrm{ref}} - \sum_{i=1}^{8} P_i \mathbf{x} \right| \tag{14}$$

**Static power division**

In order to avoid global communications, we can divide the total power reference into several local ones and use the following cost function instead of (14):

$$\min_{\{\mathbf{x}^i\}_{i=1,\ldots,8}} \quad \sum_{j=1}^{\bar{M}} \left| p^{\mathrm{ref},j} - \sum_{i \in \mathcal{G}_j} P_i \mathbf{x} \right| \tag{15}$$

with $\bar{M}$ the number of separate power references such that $\sum_{j=1}^{\bar{M}} p^{\mathrm{ref},j} = p_{\mathrm{ref}}$, and with $\mathcal{G}_j$ the group of subsystems that are assigned to track the power reference $p^{\text{ref},j}$. The disadvantage of this power reference division is that the total power production may not be able to track the total power reference in some cases. This issue will be discussed with the illustrative results for the case study in Section 5.

We list here alternative strategies for dividing the total power reference that will be considered in the case study:

(1) DIST–REF1: Use two sequences $p_k^{\text{ref},1}, p_k^{\text{ref},2}$ such that $p_k^{\text{ref}} = p_k^{\text{ref},1} + p_k^{\text{ref},2}$. The total power production of subsystems 1, 3, 4, and 5 will track $p_k^{\text{ref},1}$, while the sum of power produced by subsystems 2, 6, 7, and 8 will track $p_k^{\text{ref},2}$.

(2) DIST–REF2: Use 4 sequences: $p_k^{\text{ref}} = p_k^{\text{ref},1} + p_k^{\text{ref},2} + p_k^{\text{ref},3} + p_k^{\text{ref},4}$, with $p_k^{\text{ref},1}$ to be tracked by subsystems 1, 3, and 4, $p_k^{\text{ref},2}$ to be tracked by subsystem 5, $p_k^{\text{ref},3}$ to be tracked by subsystems 2, 6, and 7, and $p_k^{\text{ref},4}$ to be tracked by subsystem 8.

(3) LOCAL–REF: Use 8 sequences: $p_k^{\text{ref}} = \sum_{i=1}^{8} p_k^{\text{ref},i}$, with each local power reference $p_k^{\text{ref},i}, i = 1, \ldots, 8$ to be tracked by subsystem $i$.

In all these schemes, the local power reference sequences are computed by keeping the same proportion to the total power reference as in the steady-state operating conditions, which is similar to the formula (13). The scheme LOCAL–REF means that each subsystem tracks a local power reference, however this does not lead to a decentralized MPC setting, since the subsystems have to take into account the coupled dynamics. The ways we define the tracking tasks with DIST–REF1 and DIST–REF2 are aimed at exploiting the existing structure of couplings in dynamics between subsystems.

**Dynamic power division**

Static power division essentially means that each group of subsystems always tracks a given fraction of power reference (based on steady-state conditions). When the total power reference deviates a lot from the steady-state power, then this idea may not work well. To address this issue, we now introduce the dynamic power division, in which the subsystems have more flexibility in choosing the appropriate local power reference to be tracked. The main idea is that each subsystem will "trade" an amount of power reference with its neighbors.

Let us define for each pair $(i, j)$ with $j \in \mathcal{N}_i$ a pair of power exchange variables

$$\delta_{ij} = \delta_{ji} \tag{16}$$

and we assign either subsystem $i$ or subsystem $j$ to "lead" the exchange between them [1], then for each subsystem we form the set

$$\Delta_i = \{j | j \in \mathcal{N}_i, \delta_{ij} \text{ is managed by subsystem } i\} \tag{17}$$

Then we replace (14) by the following cost function:

$$\min_{\{\mathbf{x}^i\}_{i=1,\ldots,8}} \sum_{i=1}^{8} \left| p^{\text{ref},i} + \sum_{j \in \Delta_i} \delta_{ij} - \sum_{j \in \mathcal{N}_i \setminus \Delta_i} \delta_{ij} - P_i \mathbf{x} \right| \tag{18}$$

[1] A simple way is to let the subsystem with smaller index lead the exchange, i.e., $\Delta_i = \{j | j \in \mathcal{N}_i, j > i\}$.

with $p^{\text{ref},i}$ the nominal power reference for subsystem $i$ that is defined in the same way as (13), and subject to the constraints (16) for all pairs of $(i, j)$ with $j \in \mathcal{N}_i$. In other words, the local power reference for each subsystem $i$ may deviate from the nominal value by adding the exchange amounts of the links that $i$ manages and subtracting the exchange amounts of the links that affect $i$ but are decided upon by its neighbors. Note that problem (18) is decomposable, and the additional constraints (16) can be dualized in a similar way as the other constraints in (11), without expanding the neighbor set of each subsystem.

The advantage of this dynamic power division is that it makes use of the existing network topology to form a separable cost function, and the total power reference is preserved even if the local power references deviate from the nominal values since

$$\sum_{i=1}^{8} \left\{ p^{\text{ref},i} + \sum_{j \in \Delta_i} \delta_{ij} - \sum_{j \in \mathcal{N}_i \setminus \Delta_i} \delta_{ij} \right\} = p^{\text{ref}} \tag{19}$$

Now that we have a separable cost function by using either static or dynamic power division technique, we can form a dual problem and apply a distributed optimization algorithm for the dual problem. In the case study of Section 5, we will use the distributed *dual accelerated proximal gradient* (DAPG) algorithm that has recently been developed in Giselsson et al. (2012), as it has a fast convergence rate and was designed to treat problems with mixed absolute and quadratic terms, such as the HPV problem, in a more efficient way than translating the problem into a QP (as mentioned in Section 4.2.1). By formulating a specific dual problem that the *accelerated proximal gradient* method can handle, the DAPG algorithm does not need to treat the absolute term in the cost function as shown in (12). The fast convergence rate comes from the nature of the *accelerated proximal gradient* method and the optimal choice of step size based on the strong convexity of the cost function, which is discussed in detail in Giselsson et al. (2012).

## 5. COMPARISON OF MPC SCHEMES

We performed numerical simulations of the HPV using centralized MPC, decentralized MPC, and distributed MPC based on the DAPG algorithm (Giselsson et al., 2012) with the 3 static power division schemes: DIST–REF1, DIST–REF2, and LOCAL–REF, and the dynamic power division scheme referred to as DYN–REF. With the centralized and decentralized MPC, the optimization problems are transformed into QPs and are solved by the solver *quadprog* in the Optimization Toolbox of MATLAB. In the distributed MPC approach, we implemented our own DAPG solver in MATLAB that iteratively achieves the exact solution of the optimization problem up to a small tolerance in each MPC step. The closed-loop simulation results are obtained by applying the computed inputs to the original nonlinear continuous-time model (using the MATLAB function *ode15*). The simulations were implemented on a PC running MATLAB on Linux with an Intel(R) Core(TM)2 Duo CPU running at 2.33 GHz with 2 GB RAM.

### 5.1 Performance comparison

The power reference tracking results are plotted in Figures 2(a)–2(f). We can see the trade-off due to the division in power reference: the centralized MPC achieves the best tracking performance at the price of using global communications, while the distributed MPC with static power division schemes DIST1–REF, DIST2–REF and LOCAL–REF show deterioration of the tracking performance. Another interesting point is that the scheme DIST–REF2 achieves a better tracking performance than LOCAL–REF, while they use the same communication structure. This observation suggests to consider finding the division of power reference that resembles the structure of dynamical couplings between subsystems, i.e., defining groups for tracking power references such that any pair of subsystems in a group are neighbors of each other. The tracking performance of the decentralized MPC is very poor, due to the lack of communications. The best result, in view of both achieving performance and using local communications only, is distributed MPC with dynamic power division scheme DYN–REF, which achieves almost the same tracking performance as centralized MPC, while relying on the same communication structure as the LOCAL–REF scheme (see also the comparison of communication requirements).

Figures 4 and 5 show the input and output evolutions and the corresponding constraints with the scheme DYN–REF as an example. For the cases of DIST–REF1, DIST–REF2, and LOCAL–REF, all the constraints on the inputs and outputs are also satisfied. We note that this is guaranteed due to the fact that constraints are not relaxed, and the couplings in dynamics are taken into account.

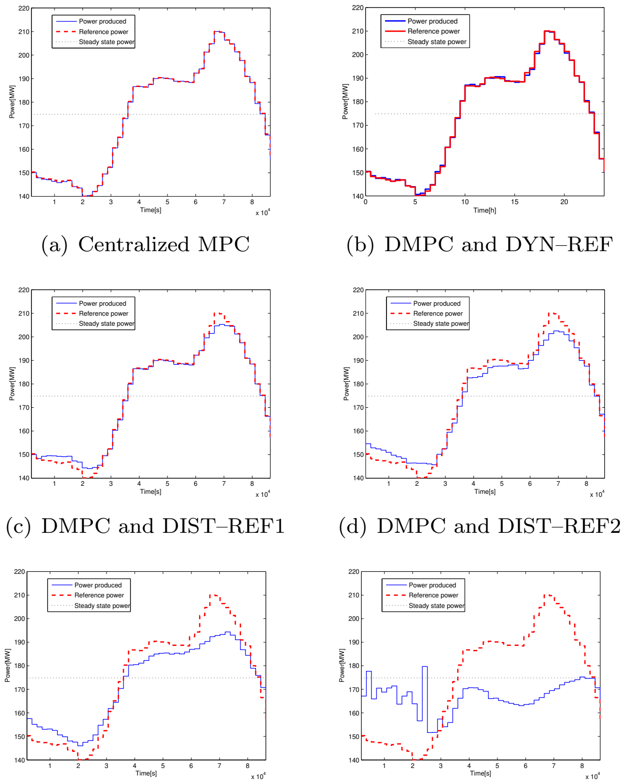
### 5.2 Computational efficiency

In Figure 3, we plot the comparison of computation time of the solvers *quadprog* and the DAPG, both are implemented in MATLAB, for solving the same optimization problem of the centralized MPC at each sampling step. Figure 3 shows that the total computation time (summed up over all subsystems) of the DAPG algorithm is much lower than the computation time of *quadprog*, which reflects the fast convergence rate of the DAPG algorithm and the efficiency of dealing with the absolute term in the cost function.

### 5.3 Communication requirements

In Table 1, we provide the comparison of the *communication neighborhood* sets between the distributed MPC schemes. The communication neighborhood set of a subsystem $i$, denoted by $\mathcal{M}_i$, indicates the need of communication links between subsystem $i$ with each of its neighbors. Notice that these communication neighborhood sets are not only the sets $\mathcal{N}_i$ defined by the coupling dynamics in (4), but can be extended due to the additional coupled constraints that are introduced by the power reference division schemes. In the scheme DIST–REF1, the sets $\mathcal{M}_i$ are larger than $\mathcal{N}_i$, since we form the power tracking groups that involve subsystems which are not dynamically coupled, e.g., between subsystems 1 and 5, 2 and 8. With the schemes DIST–REF2, LOCAL–REF, and DYN–REF, the sets $\mathcal{M}_i$ are the same as $\mathcal{N}_i$.

Combining the comparison of communications and performance, the distributed MPC with dynamic power division scheme DYN–REF would be the best choice for the total power reference tracking problem of the HPV, as it achieves almost the same optimal solution as the centralized MPC, while allowing a distributed implementation using the existing dynamical coupling structure. Among the distributed MPC with static power division schemes, the scheme LOCAL–REF is the most naive idea and its performance suffers the most. The scheme DIST–REF1 shows a fairly good tracking performance of total power reference, however it employs a more complex communication structure. The scheme DIST–REF2 has a good balance between tracking performance and limiting communication requirement, it achieves a much better result than the LOCAL–REF scheme while sharing the same communication structure. It should also be noted that decentralized MPC is not recommended unless the communication is prohibited, otherwise the performance will significantly deteriorate.




(a) Centralized MPC

(b) DMPC and DYN–REF

(c) DMPC and DIST–REF1

(d) DMPC and DIST–REF2

(e) DMPC and LOCAL–REF

(f) Decentralized MPC

Fig. 2. Comparison of power reference tracking performance using centralized MPC, DMPC, and decentralized MPC approaches.

## 6. CONCLUSIONS AND FUTURE WORK

We discussed various challenges of the HPV control problem, and provided an overview of our recently developed techniques to address them. We described several hierarchical and distributed MPC frameworks that could be applied to the HPV control problem, and adapted the optimization problem using static and dynamic power division schemes so that it is suitable for distributed MPC.

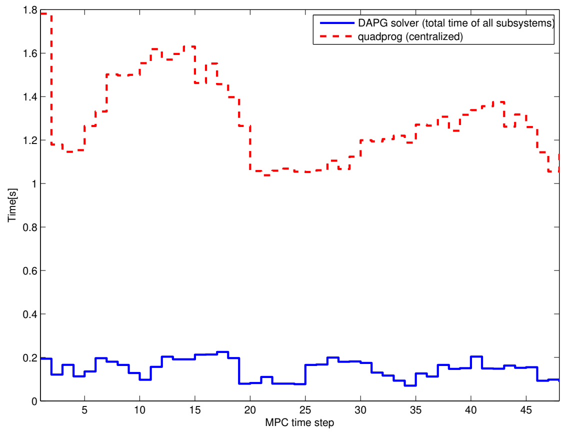



Fig. 3. Comparison of computation time to solve the MPC optimization problem per each sampling step

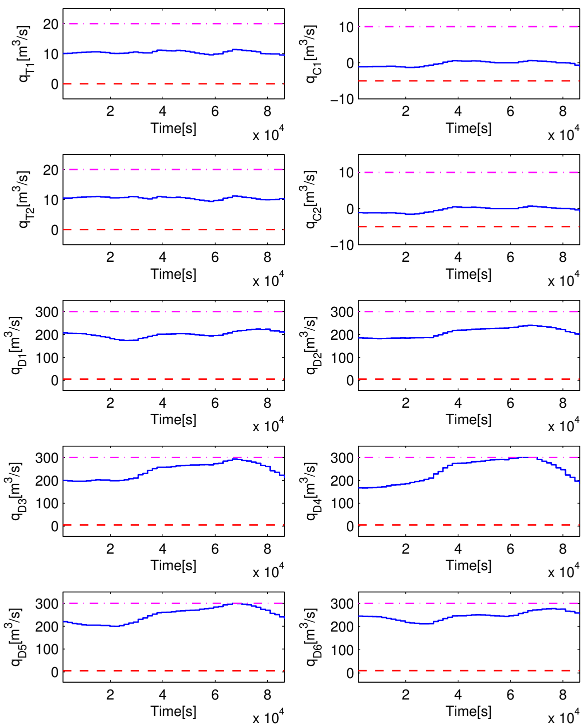



Fig. 4. Input constraint satisfaction with the DMPC based on DAPG algorithm with dynamic power division scheme. Dash-dotted lines: upper bounds, dashed lines: lower bounds.

By means of numerical simulations in a case study, we showed that our proposed distributed MPC approach outperforms the centralized and decentralized MPC in terms of computational efficiency, communication requirements, and tracking performance. Future developments will include the implementation of distributed estimation and analysis of stability and robustness of the method when there is model mismatch and measurement noise.

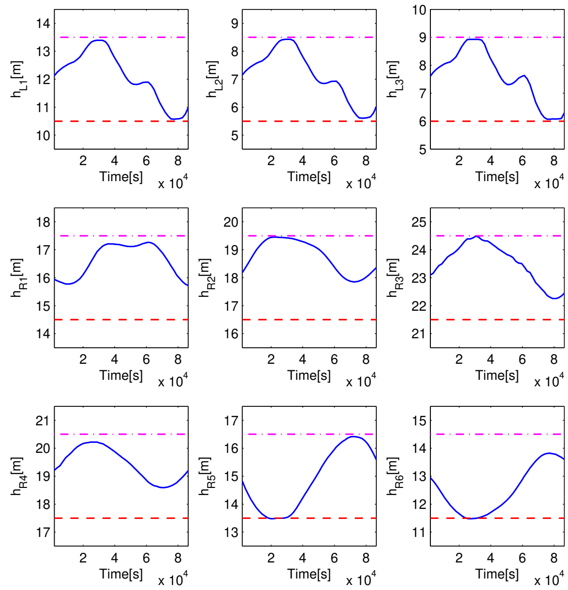



Fig. 5. Output constraint satisfaction with the DMPC based on DAPG algorithm with dynamic power division scheme. Dash-dotted lines: upper bounds, dashed lines: lower bounds.

Table 1. Communication neighborhoods of subsystems ($\mathcal{M}_i$)

| $i$ | DMPC **DIST–REF1** | DMPC **DIST–REF2** | DMPC **LOCAL–REF** | DMPC **DYN–REF** |
|---|---|---|---|---|
| 1 | $\{1,3,4,5\}$ | $\{1,3,4\}$ | $\{1,3,4\}$ | $\{1,3,4\}$ |
| 2 | $\{2,6,7,8\}$ | $\{2,6,7\}$ | $\{2,6,7\}$ | $\{2,6,7\}$ |
| 3 | $\{3,1,4,5\}$ | $\{3,1,4\}$ | $\{3,1,4\}$ | $\{3,1,4\}$ |
| 4 | $\{4,1,3,5\}$ | $\{4,1,3,5\}$ | $\{4,1,3,5\}$ | $\{4,1,3,5\}$ |
| 5 | $\{5,1,3,4,6\}$ | $\{5,4,6\}$ | $\{5,4,6\}$ | $\{5,4,6\}$ |
| 6 | $\{6,2,7,8,5\}$ | $\{6,2,7,5\}$ | $\{6,2,7,5\}$ | $\{6,2,7,5\}$ |
| 7 | $\{7,2,6,8\}$ | $\{7,2,6,8\}$ | $\{7,2,6,8\}$ | $\{7,2,6,8\}$ |
| 8 | $\{8,2,6,7\}$ | $\{8,7\}$ | $\{8,7\}$ | $\{8,7\}$ |

Centralized MPC uses global communications: $\mathcal{M}_i = \{1,\ldots,8\}, \forall i$

Decentralized MPC does not use communications: $\mathcal{M}_i = \{i\}, \forall i$

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