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A Model Predictive Control Approach for the Line Balancing in Baggage Handling Systems ⋆

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Abstract: This paper proposes an efficient solution approach for the line balancing problem in state-of-the-art baggage handling systems that are based on destination coded vehicles that transport the bags from their origins to their destinations. First, a simple event-driven model of the process is presented. Next, this model is applied within the context of model predictive control to determine the optimal number of destination coded vehicles to be dispatched from the central depot to each loading station. The performance criterion is minimizing the overall baggage waiting time as well as the energy consumption.

Keywords: model predictive control, nonlinear optimization, mixed integer programming, linear programming, logistics.

1. INTRODUCTION

Advanced automated baggage handling systems in large airports are often based on destination coded vehicles (DCVs), which are tubs moving on conveyor belts or high-speed carts moving on a network of tracks. Moving through the network of tracks, DCVs transport luggage from their origin to their destination. A DCV-based baggage handling system consists of several parts: loading stations (where the bags enter the system having cleared the check-in and security check), unloading stations (which are the final destinations of the bags from where the bags are loaded onto the planes), a network of single-direction tracks possibly with several (local) loops (for loading, unloading, and temporary storage of DCVs), and the early baggage storage, where the bags that enter the system too early can be stored. From a high-level point of view, the control problems in automated baggage handling system can be divided into three categories: (i) route choice control for DCVs, (ii) line balancing, and (iii) empty cart management. This paper focuses on the line balancing problem.

The term line balancing has also been used extensively in the context of assembly lines in production systems to refer to the problem of optimally partitioning the assembly work among the assembly stations with respect to some objective and the precedence constraints of the tasks (Becker and Scholl, 2006) . Since its first mathematical formulation by Salveson (1955), many varieties of this problem have been developed in the literature. The simple assembly line balancing problem (Baybars, 1986) is the basic version of the problem with many simplifying assumptions. Further extensions were added to this problem in later work such as Becker and Scholl (2006) in order to move towards generalized assembly line balancing problem, which integrates more practical aspects. The core problem of all these extensions is the assignment of tasks to assembly stations (Boysen et al., 2007). Depending on the type of the problem, the objective could be to minimize the number of stations, to maximize the line efficiency or to minimize the cost. The solution approach can be static, in which case the solution is pre-computed, or dynamic, in which case the optimization problem is solved online over a planning horizon and the solution is updated whenever necessary. A classification methods for assembly line balancing problem can be found in Boysen et al. (2007).

Even though dynamic line balancing is also addressed in the literature related to production systems, we cannot apply their methods to our problem in a straightforward manner. In the context of baggage handling systems, we do not deal directly with tasks. The concepts such as the precedence constraints of tasks as well as cycle times are not relevant for baggage handling systems. The control system needs to deal with dynamically changing demands within the planning horizon. This necessitates the use of dynamical models whereas in the context of production systems, static models are used and average estimate of rates (for wages, operational costs etc.) are assumed during the planning horizon. Moreover, by using our proposed model, we are able to directly penalize the baggage delay as well as energy consumption and wear and tear. This differentiates our problem with the one discussed in production systems and makes methods and solutions developed for line balancing in production systems inapplicable to our particular problem.

Tarău et al. (2009) and Tarău et al. (2010) have developed methods for predictive route choice control of DCVs by assuming there is always a sufficient number of free DCVs at the loading stations such that the bags are immediately transported upon arrival. In practice, the number of free

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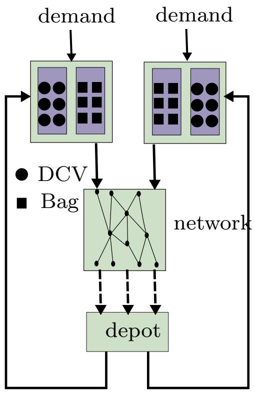


Fig. 1. A simple layout of the baggage handling system

DCVs is limited, necessitating dynamical assignment of free DCVs to the loading stations. In this context, line balancing is the problem of assigning a limited number of free DCVs in the central or local depots to the loading stations. The solution must satisfy the following requirement: the overall waiting time of the baggage in the loading stations is not too large while the number of DCVs dispatched is as small as possible. The first part of this requirement concerns the overall time delay in the loading stations and the second part concerns energy consumption and wear and tear due to DCVs moving around in the network.

In this paper, we propose one particular solution to the line balancing problem that is based on Model Predictive Control (MPC), where a finite-horizon constrained optimal control problem is solved in a receding horizon fashion using a dynamic model of the system (Rawlings and Mayne, 2009; Maciejowski, 2002). For the sake of simplicity we consider a simple configuration of the baggage handling system as depicted in Fig. 1. It will be shown that the proposed control scheme can achieve an acceptable balance between the overall baggage waiting time and the energy consumption. In general, the optimization problem based on our developed model is a nonlinear optimization problem. We will show that the problem can be recast as a mixed integer linear programming (MILP) problem and as a linear programming (LP) problem by making some approximations. The performance and computational effort of control scheme based on LP will be compared with the ones of the scheme based on nonlinear optimization, highlighting the trade-off between optimality and computational efficiency.

This paper is organized as follows: Section 2 describes the dynamical model of the system that will be used for MPC. In Section 3, the MPC optimization problem is formulated and the choice of the cost function is motivated. In Section 4, the procedure of transforming the nonlinear optimization problem into an MILP problem and an LP problem is presented. The comparison between performance of the proposed control scheme is illustrated by simulations in Section 5, followed by some remarks. Finally, Section 6 concludes the paper.

2. DYNAMICAL MODEL

Throughout this paper, a simple configuration as in Fig. 1 will be used as an illustrative example for the baggage handling system. For this configuration we derive a continuous-time event-driven model by making the following assumptions:

A1 The network of tracks that connects the loading stations to the unloading stations is modeled as a time delay system. Moreover, regardless of the assigned route for each DCV, the travel time between a given loading station and unloading station pair through the network is considered to be known and equal for all DCVs (i.e., we do not take into account the routing of DCVs).

A2 The return path of DCVs from the depot to the loading stations through the network is also considered as a time delay system with known delays.

A3 For each loading station we associate a baggage queue and a DCV queue. Moreover, it is assumed that only fully loaded DCVs can leave the loading station. This implies that the baggage queue outflow must be equal to the DCV queue outflow.

A4 Continuous variables are used for the number of DCVs in the depot and for the queue lengths.

Without making Assumptions A1 and A2, one would have to solve the routing problem and the line balancing problem simultaneously. Assumption A3 is made as dispatching of empty DCVs in not considered here. Assuming continuous variables for queue lengths is an approximation as in reality the queue lengths are integer numbers. However, these assumptions are necessary to keep the control design problem tractable. Nevertheless, they can be justified for many practical cases. For example continuous queue lengths are justified when there is a large number of DCVs (bags). Assumptions A1 and A2 can be justified when the variation of the travel times with respect to the chosen route is not significant. In other words these assumptions allow us to make a trade-off between accuracy of the model and tractability of the control problem.

Consider the following notation:

$L \in \mathbb{N}$: the number of loading stations with $\mathbb{N}$ being the set of natural numbers.
$i \in I$: an index with $I = \{1, 2, \ldots, L\}$ being the index set.
$l_{\mathrm{DCV},i}$: the DCV queue length at loading station i.
$l_{\mathrm{bag},i}$: the baggage queue length at loading station i.
$q^{\mathrm{in}}_{\mathrm{DCV},i}$: the inflow of the DCV queue i.
$q^{\mathrm{in}}_{\mathrm{bag},i}$: the inflow (baggage demand) of baggage queue i.
$q^{\mathrm{out}}_{\mathrm{DCV},i}$: the outflow of DCV queue i.
D: the number of DCVs in the central depot.
$q^{\mathrm{out},i}_{\mathrm{D}}$: the outflow of DCVs from the central depot to DCV queue i.
τ_i: the travel time from loading station i to the depot.
τ^{D}_i: the travel time from the depot to loading station i.
$Q^{\max}_{\mathrm{DCV},i}$: the maximum outflow of DCV queue i.

Consider the following regions partitioned over the space of baggage and DCV queue lengths:

$$
\begin{aligned}
R_1 &\triangleq \{(l_{\mathrm{bag},i}, l_{\mathrm{DCV},i}) | l_{\mathrm{bag},i} > 0, l_{\mathrm{DCV},i} > 0\} \\
R_2 &\triangleq \{(l_{\mathrm{bag},i}, l_{\mathrm{DCV},i}) | l_{\mathrm{bag},i} > 0, l_{\mathrm{DCV},i} = 0\} \\
R_3 &\triangleq \{(l_{\mathrm{bag},i}, l_{\mathrm{DCV},i}) | l_{\mathrm{bag},i} = 0, l_{\mathrm{DCV},i} > 0\} \\
R_4 &\triangleq \{(l_{\mathrm{bag},i}, l_{\mathrm{DCV},i}) | l_{\mathrm{bag},i} = 0, l_{\mathrm{DCV},i} = 0\}
\end{aligned} \tag{1}
$$

Then, for $i \in I$, the dynamics of the system are described by the following model:

$$\frac{d}{dt}l_{\mathrm{DCV},i}(t) = q_{\mathrm{D},i}^{\mathrm{out}}(t-\tau_i^{\mathrm{D}}) - q_{\mathrm{DCV},i}^{\mathrm{out}}(t) \tag{2a}$$

$$\frac{d}{dt}l_{\mathrm{bag},i}(t) = q_{\mathrm{bag},i}^{\mathrm{in}}(t) - q_{\mathrm{DCV},i}^{\mathrm{out}}(t) \tag{2b}$$

$$\frac{d}{dt}D(t) = \sum_{i=1}^{L}\Big(q_{\mathrm{DCV},i}^{\mathrm{out}}(t-\tau_i) - q_{\mathrm{D},i}^{\mathrm{out}}(t)\Big) \tag{2c}$$

with

$$q_{\mathrm{DCV},i}^{\mathrm{out}}(t) = \begin{cases} Q_{\mathrm{DCV},i}^{\max} \\ \qquad \text{if } (l_{\mathrm{bag},i}(t), l_{\mathrm{DCV},i}(t)) \in R_1 \\ \min\Big(q_{\mathrm{D},i}^{\mathrm{out}}(t-\tau_i^{\mathrm{D}}), Q_{\mathrm{DCV},i}^{\max}\Big) \\ \qquad \text{if } (l_{\mathrm{bag},i}(t), l_{\mathrm{DCV},i}(t)) \in R_2 \\ \min\Big(q_{\mathrm{bag},i}^{\mathrm{in}}(t), Q_{\mathrm{DCV},i}^{\max}\Big) \\ \qquad \text{if } (l_{\mathrm{bag},i}(t), l_{\mathrm{DCV},i}(t)) \in R_3 \\ \min\Big(q_{\mathrm{D},i}^{\mathrm{out}}(t-\tau_i^{\mathrm{D}}), q_{\mathrm{bag},i}^{\mathrm{in}}(t), Q_{\mathrm{DCV},i}^{\max}\Big) \\ \qquad \text{if } (l_{\mathrm{bag},i}(t), l_{\mathrm{DCV},i}(t)) \in R_4 \end{cases} \tag{2d}$$

and

$$q_{\mathrm{D},i}^{\mathrm{out}}(t) = \begin{cases} u_{\mathrm{D},i}^{\mathrm{out}}(t) & \text{if } D(t) > 0 \\ \min\Big(u_{\mathrm{D},i}^{\mathrm{out}}(t), \dfrac{1}{L}\displaystyle\sum_{i=1}^{L} q_{\mathrm{DCV},i}^{\mathrm{out}}(t-\tau_i)\Big) & \text{if } D(t) = 0 \end{cases} \tag{2e}$$

where the outflow of the depot $u_{\mathrm{D},i}^{\mathrm{out}}(t)$ is the control variable. Equations (2a-2c) describe the evolution of the DCV and baggage queue lengths at loading station i and the number of DCVs in the depot, respectively. Since the DCV and baggage queues are coupled, the outflow of DCV queue (baggage queue) i is defined in (2d) such that the queue lengths are non-negative. If at a given instant of time at loading station i, both the baggage and DCV queue lengths are non-zero, the outflow from the loading station takes its maximum value $Q_{\mathrm{DCV},i}^{\max}$. If a queue length is zero, the outflow from the loading station must be equal to the inflow of the corresponding queue or $Q_{\mathrm{DCV},i}^{\max}$, whichever is less. If both queues lengths are zero, then the outflow must be either the inflow of the DCV queue, the inflow of baggage queue, or $Q_{\mathrm{DCV},i}^{\max}$, whichever is the smallest. In the same manner, equation (2e) relates the outflow of the depot to the control variables $u_{\mathrm{D},i}^{\mathrm{out}}(t)$ such that the number of DCVs in the depot is non-negative. Hereafter, we will refer to the dynamical model of (2) as the nonlinear model.

3. OPTIMIZATION PROBLEM

In this section the MPC control problem is formulated. First, in addition to assumptions made in Section 2, we make the following assumption:

A5 The inflows (baggage demands) of baggage queues and the control signal are constant between two consecutive controller sampling time instants.

By Assumption A5, there will be a finite number of zero crossings for the baggage and DCV queue lengths between two sampling time instants. This allows us to explicitly calculate the time instants at which a queue length becomes zero and to update the corresponding outflow according to (2d). Obviously, a small sampling time yields a more accurate model at the cost of increasing the computational burden. From the practical point of view, a reasonable sampling time can be chosen based on physical specifications of the process (e.g., the distances between loading and unloading stations, average traveling speed of the DCVs, and the baggage demand profile).

At the current time $t = kT_{\mathrm{s}}$, the MPC optimization problem is then formulated as:

$$\begin{aligned} \min_{\boldsymbol{u}} \sum_{i=1}^{L}\int_0^{N_{\mathrm{p}}T_{\mathrm{s}}} & l_{\mathrm{bag},i}(t+\tau)d\tau \\ + \gamma_1 \int_0^{N_{\mathrm{p}}T_{\mathrm{s}}} & \left(N_{\mathrm{DCV}} - D(t+\tau) - \sum_{i=1}^{L} l_{\mathrm{DCV},i}(t+\tau)\right)d\tau \\ & + \gamma_2 \sum_{i=0}^{L}\sum_{l=0}^{N_{\mathrm{p}}-1}\left|u_l^i - u_{l-1}^i\right| \end{aligned} \tag{3}$$

subject to:

$$u_{\mathrm{D},i}^{\mathrm{out}}(t+\theta) = u_l^i,\ lT_{\mathrm{s}} \le \theta < (l+1)T_{\mathrm{s}} \tag{4a}$$

$$l_{\mathrm{bag},i}(t+\tau) \le l_{\mathrm{bag,i}}^{\max} \tag{4b}$$

$$l_{\mathrm{DCV},i}(t+\tau) \le l_{\mathrm{DCV,i}}^{\max} \tag{4c}$$

$$D(t+\tau) \le D^{\max} \tag{4d}$$

$$u_l^i \ge 0 \tag{4e}$$

$$\sum_{i=1}^{L} u_l^i \le Q_{\mathrm{D}}^{\max} \tag{4f}$$

for $0 < \tau \le N_{\mathrm{p}}T_{\mathrm{s}}$, $l = 0, \ldots, N_{\mathrm{p}}-1$, and for each $i \in I$. In the above N_{p} is the prediction horizon, N_{DCV} is the total number of DCVs in the system, and $l_{\mathrm{bag},i}^{\max}$, $l_{\mathrm{DCV},i}^{\max}$, and $D^{\max}$ are the maximum queue levels for the baggage and DCV queue i, and for the depot, respectively. In addition, $Q_{\mathrm{D,max}}$ is the maximum possible outflow of the depot. We have also used the following notation:

$$\boldsymbol{u}_l = [u_l^1, u_l^2, \ldots, u_l^L]^{\mathrm{T}}$$
$$\boldsymbol{u} = [\boldsymbol{u}_0^{\mathrm{T}}, \boldsymbol{u}_1^{\mathrm{T}}, \ldots, \boldsymbol{u}_{N_{\mathrm{p}}-1}^{\mathrm{T}}]^{\mathrm{T}}$$

where $\boldsymbol{u}$ is the decision variable. In (3), γ_1 and γ_2 are weight factors indicating the relative priority of different terms in the objective function.

Constraint (4a) expresses that the controls are piecewise constant (i.e., they are constant within the sampling interval). Constraints (4b-4d) impose bounds on the queue levels and (4e-4f) impose bounds on the decision variable. Based on measurements at $t = kT$s, this optimization problem can be solved using multi-start sequential quadratic programming (Antoniou and Lu, 2007) or a global optimization method. According to the receding horizon policy only the first element of the decision vector, $\boldsymbol{u}_0$, is applied to the system and a new optimization problem is solved at the next sampling time using new measurements.

Now we will motivate our choice of the objective function (3). The first term in the objective function penalizes the sum of the integral over time of the baggage queues in the loading stations, which is an indirect way of penalizing the overall baggage waiting time. The second term in the cost function penalizes the number of DCVs running around in the network, which is an indication of energy consumption as well as an indication of wear and tear. The last term in the cost function penalizes the sum of variations in the control sequence, which affects the cost due to maintenance of the actuators.

4. ALTERNATIVE SOLUTION APPROACHES

The optimization problem as stated in Section 3 is a nonlinear optimization problem, which cannot be solved efficiently for a large number of loading stations or a large prediction horizon. In this section, we will derive the discrete-time evolution equations of the queue lengths that are next used to transform the optimization problem into a mixed integer linear programming (MILP) problem and a linear programming (LP) problem.

4.1 Mixed Integer Linear Programming

As discussed in De Schutter (2002), this sort of problems can be recast as a MILP problem by making some approximations including the assumptions of Section 2 as well as the delays being integer multiples of the sampling time T_s, which then should be sufficiently small.

At the current time kT_s, consider the regions as defined in (1). It is now assumed that no region transition happens between time step k and time step $k+1$. This assumption is necessary to make the discrete-time representation feasible. The outflow of the baggage and DCV queues is determined based on the state of $l_{\mathrm{DCV},i}(k)$ and $l_{\mathrm{bag},i}(k)$, and it is assumed constant until time step $k+1$. To ensure the queue lengths are non-negative at time-step $k+1$, we explicitly saturate the computed queue lengths at zero. For the sake of brevity, we only present the discrete-time evolution of the DCV queues. Similar equations can be derived for the baggage queues and the depot. Consider the following shorthand notation:

$$q^{\mathrm{in}}_{\mathrm{DCV},i}(k) \triangleq q^{\mathrm{out}}_{\mathrm{D},i}(k-n^{\mathrm{D}}_i)$$
$$n^{\mathrm{D}}_i \triangleq \tau^{\mathrm{D}}_i/T_\mathrm{s}$$
$$n_i \triangleq \tau_i/T_\mathrm{s}$$

Now we have

$$\begin{aligned}
q^{R_1}_{\mathrm{DCV},i}(k) &\triangleq q^{\mathrm{in}}_{\mathrm{DCV},i}(k) - Q^{\max}_{\mathrm{DCV},i}\\
q^{R_2}_{\mathrm{DCV},i}(k) &\triangleq q^{\mathrm{in}}_{\mathrm{DCV},i}(k) - \min\Big(Q^{\max}_{\mathrm{DCV},i}, q^{\mathrm{in}}_{\mathrm{DCV},i}(k)\Big)\\
q^{R_3}_{\mathrm{DCV},i}(k) &\triangleq q^{\mathrm{in}}_{\mathrm{DCV},i}(k) - \min\Big(Q^{\max}_{\mathrm{DCV},i}, q^{\mathrm{in}}_{\mathrm{bag},i}(k)\Big)\\
q^{R_4}_{\mathrm{DCV},i}(k) &\triangleq q^{\mathrm{in}}_{\mathrm{DCV},i}(k) - \\
&\qquad \min\Big(Q^{\max}_{\mathrm{DCV},i}, q^{\mathrm{in}}_{\mathrm{bag},i}(k), q^{\mathrm{in}}_{\mathrm{DCV},i}(k)\Big)
\end{aligned}$$

Then, the discrete-time evolution of the DCV queue lengths is given by:

$$l_{\mathrm{DCV},i}(k+1) = \max\left(l_{\mathrm{DCV},i}(k) + q^{R_j}_{DCV,i}(k)T_\mathrm{s}, 0\right)$$
$$\text{if } \Big(l_{\mathrm{DCV},i}(k),\ l_{\mathrm{bag},i}(k)\Big) \in R_j \qquad (5)$$

for each $i \in I$. Note that (5) is an approximation of the nonlinear model as the transitions between the regions, within one sampling interval, are taken into account in the nonlinear model. For a small sampling time, inaccuracies due to this approximation would be negligible at the cost of increased computational complexity of the model. To illustrate the difference between (5) and the original nonlinear model, consider the scenario shown in Fig. 2, where the DCV and baggage queues of Fig. 2(a) are approximated with the ones of Fig. 2(b).

The system described by (5) is piecewise affine. The first two terms of objective function (3) can also be approximated by affine expressions after discretization. By introducing some dummy variables, the third term in (3) can be recast as a linear objective term subject to linear constraints. The constraints of (4) are linear in the states of the model and the input. Therefore, the optimization problem can be solved within the MILP framework (Bemporad and Morari, 1999), which yields a MILP. However, MILP is NP-hard as the number of binary variables and thus the computation time increases with the number of loading stations. Therefore, we will not focus further on the MILP approach here.

4.2 Linear Programming

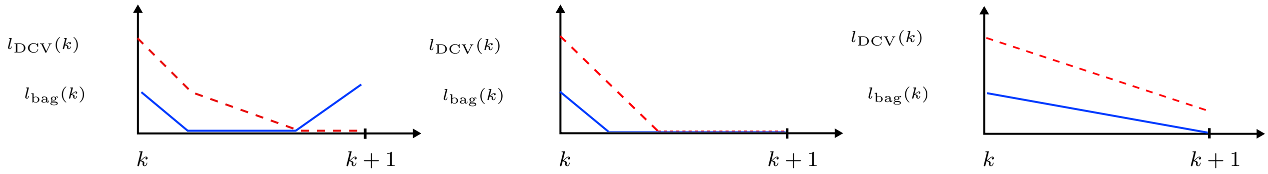
We will now show that the optimization problem of Section 3 can be approximated as an LP problem. In the dynamical model of Section 2, the non-negativity requirement on the queue lengths is integrated in the model. This along with the coupling between DCV and baggage queues [1], yields the regions of (2d). When the non-negativity requirements are not integrated in the model but imposed as optimization constraints, these regions will not appear, enabling us to formulate the optimization problem as an LP problem. To achieve this, we introduce the additional control variable $q^{\mathrm{out}}_{\mathrm{DCV},i}(k)$, $i \in I$, which is the DCV (baggage) outflow from loading station i at time step k. We constrain the controls $q^{\mathrm{out}}_{\mathrm{DCV},i}(k)$ and $q^{\mathrm{out}}_{\mathrm{D},i}(k)$ such that the computed queue lengths at time step $k+1$ are non-negative. This is illustrated in Fig. 2(c), where the queue lengths cannot be zero within the sampling time interval. Hence, the discrete-time dynamics of the system are described by the following constrained linear difference equations for each $i \in I$:

$$l_{\mathrm{DCV},i}(k+1) = l_{\mathrm{DCV},i}(k) + T_\mathrm{s}\Big(q^{\mathrm{out}}_{\mathrm{D},i}(k-n^{\mathrm{D}}_i) - q^{\mathrm{out}}_{\mathrm{DCV},i}(k)\Big) \qquad (6a)$$

$$l_{\mathrm{bag},i}(k+1) = l_{\mathrm{bag},i}(k) + T_\mathrm{s}\Big(q^{\mathrm{in}}_{\mathrm{bag},i}(k) - q^{\mathrm{out}}_{\mathrm{DCV},i}(k)\Big) \qquad (6b)$$

$$D(k+1) = D(k) + T_\mathrm{s}\sum_{i=1}^{L}\Big(q^{\mathrm{out}}_{\mathrm{DCV},i}(k-n_i) - q^{\mathrm{out}}_{\mathrm{D},i}(k)\Big) \qquad (6c)$$

[1] Recall that the baggage and DCV queues have the same outflow.



(a) Nonlinear model (b) MILP model (c) LP model

Fig. 2. Evolution of baggage (solid line) and DCV (dashed-line) queue lengths between time-instants k and $k+1$, for a particular scenario. One can see that the slope of the queue lengths can change several times for the nonlinear model within the sampling time interval. For the MILP model, the queues start with the initial slope and saturate at zero, and for the LP model, the queue lengths will not hit zero in between two sampling time instants.

$$l_{\mathrm{bag},i}(k+1) \geq 0 \tag{6d}$$

$$l_{\mathrm{DCV},i}(k+1) \geq 0 \tag{6e}$$

$$D(k+1) \geq 0 \tag{6f}$$

Using (6), the MPC optimization problem can be recast as an LP problem. Consider the following notation:

$$\boldsymbol{u}_1(k) = [q^{\mathrm{out}}_{\mathrm{D},1}(k), \ldots, q^{\mathrm{out}}_{\mathrm{D},L}(k)]^{\mathrm{T}}$$
$$\boldsymbol{u}_2(k) = [q^{\mathrm{out}}_{\mathrm{DCV},1}(k), \ldots, q^{\mathrm{out}}_{\mathrm{DCV},L}(k)]^{\mathrm{T}}$$
$$\boldsymbol{u}_1^{N_{\mathrm{p}}}(k) = [\boldsymbol{u}_1^{\mathrm{T}}(k), \boldsymbol{u}_1^{\mathrm{T}}(k+1), \ldots, \boldsymbol{u}_1^{\mathrm{T}}(k+N_{\mathrm{p}}-1)]^{\mathrm{T}}$$
$$\boldsymbol{u}_2^{N_{\mathrm{p}}}(k) = [\boldsymbol{u}_2^{\mathrm{T}}(k), \boldsymbol{u}_2^{\mathrm{T}}(k+1), \ldots, \boldsymbol{u}_2^{\mathrm{T}}(k+N_{\mathrm{p}}-1)]^{\mathrm{T}}$$
$$\boldsymbol{l}^{\max}_{\mathrm{DCV}} = [l^{\max}_{\mathrm{DCV},1}, \ldots, l^{\max}_{\mathrm{DCV},L}]^{\mathrm{T}}$$
$$\boldsymbol{l}^{\max}_{\mathrm{bag}} = [l^{\max}_{\mathrm{bag},1}, \ldots, l^{\max}_{\mathrm{bag},L}]^{\mathrm{T}}$$
$$\boldsymbol{Q}^{\max}_{\mathrm{DCV}} = [Q^{\max}_{\mathrm{DCV},1}, \ldots, Q^{\max}_{\mathrm{DCV},L}]^{\mathrm{T}}$$

The optimization problem is then formulated as:

$$\min_{\boldsymbol{u}_1^{N_{\mathrm{p}}}(k), \boldsymbol{u}_2^{N_{\mathrm{p}}}(k)} \frac{T_{\mathrm{s}}}{2} \sum_{i=1}^{L} \sum_{j=1}^{N_{\mathrm{p}}} \Big(l_{\mathrm{bag},i}(k+j) + l_{\mathrm{bag},i}(k+j-1) \Big)$$
$$+ \gamma_1 \frac{T_{\mathrm{s}}}{2} \sum_{i=1}^{L} \sum_{j=1}^{N_{\mathrm{p}}} \Big(2N_{\mathrm{DCV}} - l_{\mathrm{DCV},i}(k+j) - l_{\mathrm{DCV},i}(k+j-1)$$
$$- D(k+j) - D(k+j-1) \Big) + \gamma_2 \left\| \boldsymbol{u}_1^{N_{\mathrm{p}}}(k-1) - \boldsymbol{u}_1^{N_{\mathrm{p}}}(k) \right\|_1 \tag{7}$$

subject to the following constraints:

$$0 \leq l_{\mathrm{DCV}}(k+j) \leq \boldsymbol{l}^{\max}_{\mathrm{DCV}} \tag{8a}$$
$$0 \leq l_{\mathrm{bag}}(k+j) \leq \boldsymbol{l}^{\max}_{\mathrm{bag}} \tag{8b}$$
$$0 \leq D(k+j) \leq D_{\max} \tag{8c}$$
$$0 \leq \boldsymbol{u}_2(k+j-1) \leq \boldsymbol{Q}^{\max}_{\mathrm{DCV}} \tag{8d}$$
$$\boldsymbol{u}_1(k+j-1) \geq 0 \tag{8e}$$
$$\|\boldsymbol{u}_1(k+j-1)\|_1 \leq Q^{\max}_{\mathrm{D}} \tag{8f}$$

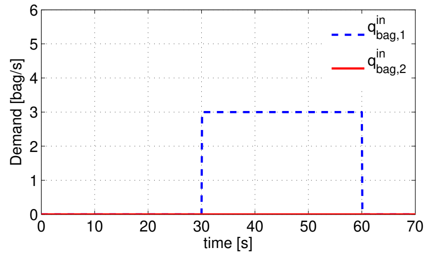
for $j = 1, \ldots, N_{\mathrm{p}}$. One should note that the LP formulation is an approximation of nonlinear one. The difference between (6) and the original nonlinear model can be realized by comparing Fig. 2(a) with Fig. 2(c), where the evolution of the baggage queue length is plotted for a particular scenario. It is obvious that the linear model is less accurate than the nonlinear model. Therefore, the solution of the LP may be suboptimal with respect to the one of the nonlinear optimization problem. However, the LP can be solved much more efficiently than the nonlinear optimization problem. Therefore, the LP approximation can provide a balanced trade-off between computational effort and optimal cost.

5. SIMULATION RESULTS

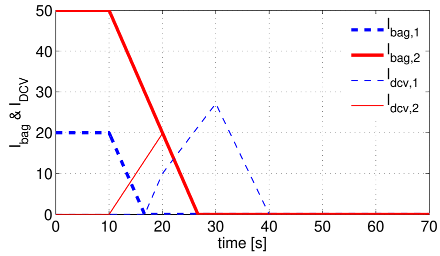
In order to illustrate the performance of the proposed control approach, we consider the baggage handling system of Fig. 1 with two loading stations. The simulation parameters are given in Table 1. We consider the following scenario: There are initially 20 and 50 bags in the baggage queues 1 and 2 respectively, and 120 DCVs in the depot. The DCV queues are initially empty. A pulse-shaped demand of 60 s duration arrives at loading station 1 at $t = 30$ s. Fig. 3(c) depicts the optimal control sequence computed for the demand of Fig. 3(a). The resulting queue lengths are illustrated by Fig. 3(b). It is observed that for loading station 2, the controller dispatches just enough DCVs as needed to transfer all bags. For loading station 1, the controller dispatches DCVs in advance such that there are enough DCVs in the DCV queue when the demand pulse arrives and it continues to dispatch DCVs at the same rate as the demand during the presence of demand. The number of DCVs in the depot is depicted in Fig. 3(d). In Table 2, we compare computation time and total closed-loop cost of three different optimization approaches for a particular scenario and for different sampling times. These methods are sequential quadratic programming (SQP), SQP initialized by the solution of the LP problem (LP-SQP), and LP. For the SQP approach, a multi-start algorithm with 10 random feasible initial points is used. For the LP-SQP approach, the LP problem is solved and its solution $\boldsymbol{u}^*_{\mathrm{LP}}$ is used as the initial point for the SQP algorithm. The LP problem is solved using the simplex method. The last column of Table 2 shows the relative cost and computation time of the LP approach with respect to the LP-SQP approach. The relative cost and relative CPU time are obtained by dividing the cost and computation time of the LP-SQP approach by the corresponding values of the LP approach. It is obvious that in most cases the LP-SQP approach achieves the global optimum as obtained by the SQP approach with less computational effort. One can observe that for a given sampling time, the closed-loop cost of the LP approach is higher than the ones of the SQP and LP-SQP approaches while its computation time is significantly less. Furthermore, for decreasing values of the sampling time, the performance of the LP approach gets closer to the one of the LP-SQP approach while its relative computation burden is low.

Table 1. Simulation parameters

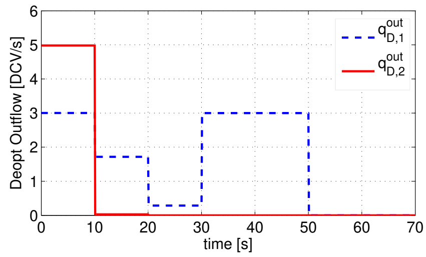
$Q^{\max}_{\mathrm{DCV},i}$ [DCV/s]	$Q^{\max}_{\mathrm{D}}$ [DCV/s]	n_i	n^{D}_i	N_{DCV}	T_{s} [s]	N_{p}
3	8	1	1	120	10	5



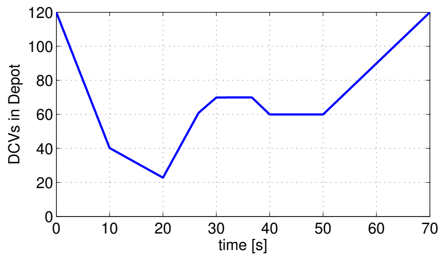
(a) demand



(b) baggage and DCV queue lengths



(c) control signal



(d) number of DCVs in depot

Fig. 3. closed-loop simulation results using SQP method.

6. CONCLUSIONS

A continuous-time event-driven model was developed for the baggage handling system. Based on this model, a solution for the line balancing problem was proposed within the context of model predictive control that aims at minimizing the overall baggage delay as well as energy consumption and wear and tear. The underlying optimization problem is a nonlinear optimization problem. We showed that by making some simplifying assumptions, the problem can be recast as a linear programming problem. Our results show that the nonlinear optimization approach based on multi-start sequential quadratic programming achieves the lowest closed-loop cost but it is computationally very expensive. The linear programming approach is the fastest but yields suboptimal solutions. However, for a sufficiently small sampling time, it can achieve a performance very close to the one of the multi-start sequential quadratic programming method with significantly less computational burden. We also showed that for large sampling times, the LP solution can still be used in combination with the SQP algorithm to yield fast yet near optimal solution.

Table 2. Optimal closed-loop cost and computation time for different methods.

Ts [s]	opt. method	cost	cpu time [s]	rel. cost / rel. cpu
5	SQP	674.167	313.866	
	LP-SQP	674.170	6.403	98.78%
	LP	682.5	0.848	7.55
10	SQP	926.667	101.496	
	LP-SQP	925.993	2.823	96.53%
	LP	960	0.448	6.30
20	SQP	1438.667	34.589	
	LP-SQP	1438.667	1.698	94.65%
	LP	1520	0.240	7.07
40	SQP	2484.670	19.935	
	LP-SQP	2484.670	1.186	81.73%
	LP	3040	0.146	8.12
60	SQP	3554.667	10.891	
	LP-SQP	3554.667	0.753	77.95%
	LP	4560	0.118	6.38

Further steps will include relaxing some of the assumptions we made to obtain a more realistic model as well as investigating other solution approaches by using methods from explicit model predictive control and methods from classical control of time-delay systems.

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