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Model Predictive Traffic Control: A Mixed-Logical Dynamic Approach Based on the Link Transmission Model

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Abstract: In this paper, model predictive control of traffic networks using first-order macroscopic link transmission model (LTM) is considered. The LTM model provides fast yet accurate predictions for traffic networks compared to other models. In order to use this model for traffic control, it is extended to include ramp metering. Using the extended LTM model as prediction model in a model predictive control framework, one can determine optimal control signals for metered on-ramps. However, the optimization problem is still nonlinear and nonconvex, and in general it is not tractable to find its global optimum, as global or multi-start local optimization techniques take considerable time. Therefore, in this paper the extended LTM model is transformed into a mixed logical dynamic model. The resulting optimization problem can be recast as a mixed integer linear program (MILP) that can be solved much more efficiently than the nonlinear optimization problem, and it allows to determine a global optimum efficiently. A simple case study is selected, first to test the modeling performance of the extended LTM and next to compare the control performance of the MILP approach and the original nonlinear formulation in terms of computational efficiency and total cost.

Keywords: Traffic control, link transmission model, predictive control, mixed logical dynamics, mixed integer linear programming.

1. INTRODUCTION

With the increasing number of vehicles, highways are becoming more and more congested. This along with increasingly stringent traffic requirements necessitates the use of efficient large-scale traffic management and control methods. One particular solution to this problem is based on Model Predictive Control (MPC), where a finite-horizon constrained optimal control problem is solved in a receding horizon fashion (Rawlings and Mayne, 2009; Maciejowski, 2002).

In the MPC framework a model of the process is required to predict its behavior over a prediction window. For traffic networks, a wide range of traffic flow models have been developed (Hoogendoorn and Bovy, 2001). MPC requires a traffic model that can provide accurate predictions of the traffic states while it has low computational complexity. The METANET model (Messmer and Papageorgiou, 1990) is a second-order model that is able to model the traffic network with good accuracy. However, this is a nonlinear model and when it is used as prediction model in the MPC framework, a nonlinear nonconvex optimization problem results. This approach has been considered in Kostsialos et al. (2002), Hegyi et al. (2005), and Bellemans et al. (2006); for a simple case study, it has been shown that solving the nonlinear optimization problem based on the METANET model takes considerable time and in fact there is no guarantee to have a unique global optimal solution. Groot et al. (2011) proposed a method to transform the original nonlinear problem into a mixed integer linear optimization problem. This has been done by approximating the METANET model by piecewise affine (PWA) functions. Although this can solve the aforementioned computational complexity problem, using this approach for larger networks is impractical and still takes large amount of computation time.

One way to overcome this problem for large-scale traffic networks is to use first-order models like the cell transmission model (Daganzo, 1994) and the link transmission model (Yperman, 2007). These models are mostly used for dynamic traffic assignment problems (Peeta and Ziliaskopoulos, 2001). However, this paper considers using them for other control purposes. The Cell Transmission Model (CTM) has been widely used to model traffic evolution.

However, using the CTM as prediction model in the MPC framework will result in a nonlinear optimization problem. One way to tackle this problem is using the relaxed formulations proposed by Ziliaskopoulos (2000), Ukkusuri and Waller (2008) to obtain linear problems. This is however an approximation for the original optimization problem. Moreover, according to Lo (2001) this approximation leads to a phenomenon called *vehicle holding*. This means that vehicles are unnecessarily held in cells for some time period despite there being spare capacity at the next downstream cell. Therefore, Lin and Wang (2004) and Lo (2001) have proposed a new formulation based on the CTM that leads to a mixed integer linear optimization problem. Besides not using approximation, reaching a global optimum is also guaranteed.

Recently the link transmission model (LTM) has been developed by Yperman (2007). The LTM has a lower computational complexity than the CTM and METANET. This is due to the fact that the LTM calculates the traffic variables for only the boundaries of the links. Moreover, to reduce computational efforts in CTM, one could enlarge the length of the time step, but such an operation leads to reduction in accuracy. In the LTM, it can be proved (Yperman et al., 2005) that one can get acceptable accuracy with less computational effort. For the first time, we aim at using the LTM for prediction in the MPC framework. However, the LTM is still a nonlinear model.

In this work, we aim at using a procedure for transforming the nonlinear LTM into a system of linear equations and inequalities with real and integer variables. Based on this new formulation, one can build a mixed integer linear problem, which can be solved more efficiently than the original nonlinear optimization problem based on the original LTM.

The remainder of this paper is organized as follows. In Section 2, first the original LTM is briefly introduced and next the model is extended to include ramp metering. Next, in Section 3 MPC for traffic networks is presented and traffic performance functions are briefly reviewed. In Section 4, rules for transforming the LTM into a system of linear equalities and inequalities are presented. At the end, the final mixed integer linear optimization problem based on the new formulation of the LTM is established. A case study in Section 5 is presented to test the nonlinear MPC based on the original LTM and the new approach based on the new formulation of the LTM. The performance of the two approaches is compared in terms of computational efficiency and total cost. Conclusions and topics for further research are given in Section 6.

2. THE LINK TRANSMISSION MODEL

The Link Transmission Model (LTM) is a model originally developed for dynamic traffic assignment (Yperman, 2007). The role of dynamic traffic assignment (DTA) (Peeta and Ziliaskopoulos, 2001) is to first assign optimal routes to the travelers using a route choice model and then simulate and evaluate the assigned routes using a traffic model. The traffic model should be capable of describing the traffic evolution of a transportation network (e.g. freeway or urban networks). In the following, the LTM is presented briefly. The reader is referred to Yperman (2007) and Yperman et al. (2005) for an in-depth description of the LTM. Right after this part, an extension to the original model for including ramp metering signals is proposed.

In the LTM framework, traffic networks consist of homogeneous links i, that start at an upstream boundary denoted by x_i^0 and end at a downstream boundary denoted by x_i^{L}. The links have a length L_i and they are connected to each other via nodes. A node can represent a change in the characteristics of a road such as capacity, speed limits, etc. (inhomogeneous nodes), merging lanes and/or on-ramps (merge nodes), or diverging lanes and/or off-ramps (diverge nodes). There are also origin and destination nodes, which can be included in the inhomogeneous nodes category. Moreover, cross nodes are defined for modeling urban intersections.

The LTM is capable of determining time-dependent link volumes, link travel times, and route travel times in traffic networks. To this aim, the LTM uses the cumulative number of vehicles as a representation for the traffic evolution. The cumulative number of vehicles $N(x,t)$ is defined for the upstream and downstream boundaries of the links. The values of $N(x_i^0,t)$ and $N(x_i^{\mathrm{L}},t)$ are updated using flow functions of links and nodes defined in the following.

The sending number of vehicles [1] $S_i(t)$ of link i at time t is defined as the maximum amount of vehicles that could leave the downstream end of this link during the time interval $[t,t+\Delta t]$, where Δt is the simulation time step. It is constrained by the link's maximum flow $q_{\mathrm{M},i}$ and is formulated as

$$S_i(t) = \min\left[N\left(x_i^0, t+\Delta t - \frac{L_i}{\upsilon_{\mathrm{free},i}}\right) - N(x_i^{\mathrm{L}},t),\ q_{\mathrm{M},i}\Delta t\right] \quad (1)$$

where $\upsilon_{\mathrm{free},i}$ and L_i are the free-flow speed and the length of link i.

Similarly, the receiving number of vehicles $R_i(t)$ of link i at time t is defined as the maximum amount of vehicles that could enter the upstream end of this link during the time interval $[t,t+\Delta t]$, and it is also limited by the link's maximum flow. It is formulated as follows

$$R_i(t) = \min\left[N\left(x_i^{\mathrm{L}}, t+\Delta t + \frac{L_i}{w_i}\right) + \rho_{\max}L_i - N(x_i^0,t),\ q_{\mathrm{M},i}\Delta t\right] \quad (2)$$

where w_i and $\rho_{\max}$ are the congestion speed and the jam density, respectively.

2.1 Node models

For each of the nodes, a transition number of vehicles $G_{ij}(t)$ is defined and determined by using the sending and receiving numbers of vehicles of the connected links. In fact, the transmission flow determines the maximum number of vehicles that can be transferred from incoming links to outgoing links of a node during the time interval $[t,t+\Delta t]$.

For the inhomogeneous nodes, the transition number $G_{ij}(t)$ is formulated as

$$G_{ij}(t) = \min\left[S_i(t), R_j(t)\right] \quad (3)$$

where i is the incoming link and j is the outgoing link.

For origin nodes, the transition number of vehicles is determined as follows:

$$G_j(t) = \min\left[N_o(t+\Delta t) - N(x_j^0,t),\ R_j(t)\right] \quad (4)$$

[1] Yperman (2007) uses the term *"sending flow"* for this purpose, but since it is not a flow, we prefer to use term *"number of vehicles"*. The same holds for other model variables that will be defined later.

where j is the index of the first link connected to the origin and N_o denotes the traffic demand in origins in terms of the cumulative number of vehicles. A simple queue model for origins is defined as:

$$\omega_o(t) = N_o(t) - N(x_j^0, t) \quad (5)$$

where $\omega_o(t)$ and $N(x_j^0, t)$ denote the number of vehicles standing in the queue and the cumulative number of vehicles that already entered the network at time t, respectively. It should be noted that this a Point-Queue(P-Q) model.

The transition number of vehicles for destination nodes is equal to the sending number of vehicles of the last link connected to the destination node:

$$G_j(t) = S_i(t) \quad (6)$$

with j and i the destination and the last link. Merge nodes can represent merging of links and/or on-ramps in traffic networks. To model a merging node n with predefined priorities for the incoming links, Lebacque (1996) has proposed the following equation for the transition number of vehicles of the incoming links of the merge node

$$G_{ij}(t) = \min\left[S_i(t), p_{ij}R_j(t)\right] \quad \text{for all} \quad i \in I_n \quad (7)$$

where p_{ij} is the priority parameter associated with incoming link i connected to the only outgoing link j via the merge node, and I_n is the set of incoming links to node n. The priority parameter is determined for each link based on the characteristics of the link (e.g. capacity, number of lanes,...) and it should be noted that $\sum_i p_{ij} = 1$.

Jin and Zhang (2003) proposed another model that does not have fixed priority parameters. The priority proportions are equal to the proportions of $S_i(t)$ of the incoming links. The transition number of vehicles is formulated as

$$G_{ij}(t) = \min\left[\frac{R_j(t)S_i(t)}{\sum\limits_{i' \in I_n} S_{i'}(t)}, S_i(t)\right] \quad \text{for all} \quad i \in I_n \quad (8)$$

where j is the only outgoing link of merge node n. Finally, the third model for a merge node proposed by Daganzo (1995), is formulated as follows:

$$G_{ij}(t) = \text{median}\Bigg[S_i(t), R_j(t) - \bigg(\Big(\sum_{i' \in I_n} S_{i'}(t)\Big) - S_i(t)\bigg), p_{ij}R_j(t)\Bigg] \quad \text{for all} \quad i \in I_n \quad (9)$$

G_{ij} is determined for each link i from the set of incoming links I_n connected to outgoing link j via the merge node n.

For diverge nodes that are used to model diverging links and/or off-ramps in traffic networks, two types of equations have been proposed in the literature. The first one was proposed by Daganzo (1995):

$$G_{ij}(t) = q_{ij} \min\left[S_i(t), \min_{j' \in J_n}\left(\frac{R_{j'}(t)}{q_{ij'}}\right)\right] \text{ for all } j \in J_n \quad (10)$$

where J_n is the set of outgoing links, q_{ij} is the split factor and i is the unique incoming link of node n. The second type of equation for a diverge node has been proposed by Lebacque (1996):

$$G_{ij}(t) = \min\left[p_{ij}S_i(t), R_j(t)\right] \quad \text{for all} \quad j \in J_n \quad (11)$$

In general, for intersections with two or more upstream and downstream links, we can combine the merge and diverge models. As in Yperman et al. (2005), we combine (8) and (10). The resultant equation becomes:

$$G_{ij}(t) = p_{ij} \min\left[\min_{j' \in J_n}\left(\frac{R_j(t)S_i(t)}{\sum\limits_{i' \in I_n} p_{i'j'}S_{i'}(t)}\right), S_i(t)\right] \quad \text{for all} \quad i \in I_n \quad \text{and for all} \quad j \in J_n \quad (12)$$

Having determined the transition number of vehicles of all nodes, the cumulative number of vehicles for the upstream and downstream boundaries of links can be updated using the following equations:

$$N(x_i^{\mathrm{L}}, t + \Delta t) = N(x_i^{\mathrm{L}}) + \sum_{j \in J_n} G_{ij}(t) \quad \text{for all } i \in I_n \quad (13)$$

$$N(x_j^0, t + \Delta t) = N(x_j^0) + \sum_{i \in I_n} G_{ij}(t) \quad \text{for all } j \in J_n \quad (14)$$

For each node $n \in N$ where N is the set of all nodes in the traffic network.

2.2 Extension of the LTM model for ramp metering

In this section the LTM model is extended to include control signals. In order to implement the action of ramp metering in the LTM framework, one can add a constraint on the transition number of vehicles from the on-ramp to the mainstream. Recall from (7), the modified equation is as follows:

$$G_{ij}(t) = \min\left[S_{ij}(t), p_{ij}R_j(t), C_i r_{\mathrm{i}}(t)\right] \quad (15)$$

where i is the incoming link (the on-ramp), j is the outgoing link, C_i is the capacity of the on-ramp (veh/h), and $r_{\mathrm{i}}(t) \in [0, 1]$ is the metering signal. A similar modification can be applied to either (8) or (9).

3. MODEL PREDICTIVE CONTROL FOR TRAFFIC NETWORKS

Model Predictive Control (MPC) (Maciejowski (2002), Rawlings and Mayne (2009)) is an advanced control method for industrial processes and traffic networks. The main idea is to use a prediction model of the process (in our case: the traffic network) and an objective function assessing the desired performance of the process, and to find the optimal control inputs by means of an optimization algorithm. In our case, the LTM is used to predict the behavior of a traffic network over a prediction horizon. The optimization algorithm minimizes the objective function and finds a sequence of optimal control inputs for the whole prediction horizon,but only the first control input sample is applied to the traffic network and the procedure is repeated in the next control step but in a rolling horizon style. In other words, the prediction horizon is shifted one step forward, and the prediction and optimization procedure over the shifted horizon are repeated using new measurements.

For a traffic network, one can define different objective functions based on travel time, fuel consumption of vehicles, emissions, etc. The objective function we chose is the total time spent in the traffic network, consisting of the time vehicles spend in queues at mainstream origins and on-ramps and the travel time on the freeway. The

Total Time Spent (TTS) objective function for the MPC controller is formulated as follows

$$J_{\mathrm{TTS}}(k_{\mathrm{c}}) = T \sum_{k=Mk_{\mathrm{c}}}^{M(k_{\mathrm{c}}+N_{\mathrm{p}})-1} \left(\sum_{i \in I_{\mathrm{all}}} \rho_i(k) L_i \lambda_i + \sum_{o \in O_{\mathrm{all}}} \omega_o(k) \right) \tag{16}$$

where T is the simulation time step length, k_{c} is the controller time step counter, and k is the model time step counter. In fact, we assume that the controller time step length is an integer multiple of the simulation time step length: $T_{\mathrm{c}} = MT$. Moreover, N_{p} is the control horizon, ρ_i is the density of link i, ω_o is the queue length at origin o, and I_{all} and O_{all} are the set of all links and the set of all origins, respectively. In order to apply the objective function (16) to our continuous-time LTM model, we have to discretize the model and take care of the delay in the sending/receiving number of vehicles. Further, we have to estimate the density of links using the following:

$$\rho_i(k) = \frac{N(x_i^{\mathrm{L}}, k) - N(x_i^0, k)}{L_i} \tag{17}$$

However, we are not able to apply control inputs that have high fluctuations. This is due to the fact that in reality traffic signals cannot vary with high frequency over time. Further, high fluctuations in control inputs may cause instability in some cases. Therefore, a penalty term on control input deviations is usually added to the objective function. In our case, the control inputs are the metering signals of on-ramps. The penalty term is formulated as

$$\zeta \sum_{l=k_{\mathrm{c}}}^{k_{\mathrm{c}}+N_{\mathrm{p}}-1} \sum_{o \in O_{\mathrm{ramp}}} \left| r_o(l) - r_o(l-1) \right| \tag{18}$$

where r_o is the metering signal and O_{ramp} is the set of indices of metered ramps [2]. The ζ is a weighting coefficient. Also, to reduce the complexity, control variables are sometimes taken constant after passing a predefined control horizon N_{c}. Taking this into account, N_{p} in (18) should be replaced by N_{c}. Finally, The penalty term is added to the TTS objective function (16). However, the total objective function is nonlinear [3]. This along with using the LTM model for prediction leads to a nonlinear nonconvex optimization problem that has to be solved in the MPC framework to find the optimal control signals. At every control time step, there is no guarantee to be able to find a unique global solution. Furthermore, the nonlinear optimization may take considerable time to find a (local) optimum. In the next section, a solution to this problem is proposed. In fact, we aim to transform the nonlinear nonconvex optimization problem into a Mixed Integer Linear Problem (MILP).

4. MLD-MPC FORMULATION FOR THE LTM

Using the methods proposed by Williams (1993) and adopted in Bemporad and Morari (1999), one can transform the model and the objective function into a system of linear equalities and inequalities consisting of real and integer variables and end up with an MILP. The MILP can be efficiently solved using existing MILP solvers like CPLEX, GLPK, or lp_solve (see Atamturk and Savelsbergh (2005)). The MILP solvers can find the global optimum and this is a significant advantage over the nonlinear optimization problem solvers.

4.1 Mixed Logical Dynamic Models

In order to get an MILP, we first have to transform the model of the system into a Mixed Logical Dynamic (MLD) form. An MLD model is described by the following system of equations (Bemporad and Morari (1999))

$$\begin{aligned} x(k+1) =& Ax(k) + B_1u(k) + B_2\delta(k) + B_3z(k) + f \\ y(k) =& Cx(k) + D_1u(k) + D_2\delta(k) + D_3z(k) + g \\ & E_1x(k) + E_2u(k) + E_3\delta(k) + E_4z(k) \leq h, \end{aligned}$$

where $\delta(k) \in \{0,1\}^{n_{\mathrm{b}}}$ denotes the vector of binary variables used to indicate which region of operation the system is in, and $z(k) \in \mathbb{R}^{n_z}$ is the vector of auxiliary variables. In the following, we will elaborate more on how to get the MLD form by introducing some basic transformation rules. Consider the statement $f(x) \leq 0$, where f is an affine function over a bounded set X of the input variable x. Moreover, assume that the constants m and M are lower and upper bounds of the function f over X. By defining $\delta \in \{0,1\}$, the following holds

$$[f(x) \leq 0] \Leftrightarrow [\delta = 1], \quad \text{iff} \begin{cases} f(x) \leq M(1-\delta) \\ f(x) \geq \epsilon + (m-\epsilon)\delta \end{cases} \tag{19}$$

where ϵ is a small tolerance, typically the machine precision [4]. Moreover, $\delta f(x)$ can be replaced by the auxiliary real variable $z = \delta f(x)$. In fact, $z = \delta f(x)$ is equivalent to

$$\begin{cases} z \leq M\delta \\ z \geq m\delta \\ z \leq f(x) - m(1-\delta) \\ z \geq f(x) - M(1-\delta) \end{cases} \tag{20}$$

Now, using the basic rules, we can transform the LTM model into an MLD form.

4.2 MLD transformation of the LTM

The LTM model is continuous-time in nature. Thus, in order to apply the MLD transformations, first the equations should be discretized. To this aim, we assume that $\left(\frac{L_i}{v_{\mathrm{free},i}}\right)$ is a multiple integer of the sample time T. Further, time index t is replaced by k.

- Recalling the sending number of vehicles equation from (1), it can be rewritten as follows

$$S_i(k) = f_{i,1}(k) + \underbrace{[f_{i,2}(k) - f_{i,1}(k)]\delta_i(k)}_{z_i(k)}, \tag{21}$$

with

$$f_{i,1}(k) = N\left(x_i^0, k+1-\frac{L_i}{v_{\mathrm{free},i}}\right) - N(x_i^{\mathrm{L}}, k) \tag{22}$$

[2] It should be noted that mainstream origins' outflows can also be controlled in some cases, so in that case they can also be included in the set O_{ramp}.

[3] But in fact it is piecewise-affine (PWA), a property that will be used later on in the next section.

[4] The reason for introducing ϵ is that an equation like $f(x) > 0$ does not fit the MLD framework, in which only nonstrict inequalities are allowed. Therefore, the equation $f(x) > 0$ is replaced by the equation $f(x) \geq \epsilon$.

$$f_{i,2}(k) = q_{\mathrm{M},i}T \tag{23}$$

and $\delta_i(k) = 1 \Leftrightarrow [f_{i,2}(k) - f_{i,1}(k)] \le 0$. Following the MLD rules, we reach the following linear equation

$$S_i(k) = f_{i,1}(k) + z_i(k), \tag{24}$$

subject to the following constraints,

$$\begin{cases} \{f_{i,2}(k) - f_{i,1}(k)\} \le M(1-\delta_i(k)) \\ \{f_{i,2}(k) - f_{i,1}(k)\} \ge \epsilon + (m-\epsilon)\delta_i(k) \\ z_i(k) \le M\delta_i(k) \\ z_i(k) \ge m\delta_i(k) \\ z_i(k) \le [f_{i,2}(k) - f_{i,1}(k)] - m(1-\delta_i(k)) \\ z_i(k) \ge [f_{i,2}(k) - f_{i,1}(k)] - M(1-\delta_i(k)) \end{cases}$$

where M and m are upper and lower bounds for $\{f_{i,2}(k)-f_{i,1}(k)\}$. These constraints along with (22), (23), and (24) are equivalent to (1).

For the receiving number of vehicles (2), the transformation procedure is similar.

- The transition number of vehicles for merging nodes can also be transformed into the MLD form. For (7), one can assume that the priority parameter p_{ij} is constant, which is not far from reality (since this parameter is mostly related to physical properties of the links). By this assumption, the transformation will be similar to the sending number case. For (8), one can use simple approximations for the multiplication of the sending numbers like assuming that they can be taken as constant over a certain period of time, or trying to approximate the function with piecewise-affine (PWA) functions Groot et al. (2011). Next, the PWA approximation can be transformed into the MLD form (Bemporad and Morari, 1999). For (9), recall

$$G_{ij}(k) = \mathrm{median}\Big[S_i(k), R_j(k) - \Big(\big(\sum_{i'\in I_n} S_{i'}(k)\big) - S_i(k)\Big), p_{ij}R_j(k)\Big] \text{ for all } i \in I_n \tag{25}$$

The *median* function is equal to the following conditions:

$$G = \mathrm{median}(a,b,c) = \begin{cases} a & \text{if } [(a \le b \text{ and } c \le a) \text{ or } (a \le c \text{ and } b \le a)] \\ b & \text{if } [(a \le b \text{ and } b \le c) \text{ or } (c \le b \text{ and } b \le a)] \\ c & \text{if } [(a \le c \text{ and } c \le b) \text{ or } (b \le c \text{ and } c \le a)] \end{cases}$$

Thus, following the MLD rules, these conditions can also be transformed into the MLD form, yielding an expression of the form

$$G = \underbrace{a\delta_1}_{z_1} + \underbrace{b\delta_2}_{z_2} + \underbrace{c\delta_3}_{z_3} \tag{26}$$

along with a set of 34 inequalities.

- Transformation of other types of node models is straightforward, since they contain the *min* function and the transformation procedure for that was explained in the $S_i(k)$ case.

4.3 Final MILP problem

After transforming the LTM model into the MLD form, one can recast the original nonlinear optimization problem into an MILP. However, to this aim a linear objective function is needed. Recall from Section 3, the penalty term (18) that has been added to the objective function is piecewise affine. Thus it can also be transformed into a mixed-integer linear form by defining additional binary and auxiliary variables. However, there is another approach to recast (18) as a linear problem that does not need any binary variable. It can be easily proved that the following optimization problems have the same optimal solution:

$$\min_{\theta} \sum |\theta_i| \Longleftrightarrow \begin{cases} \min\limits_{\theta,\beta} \sum \beta_i \\ \beta_i \ge \theta_i \\ \beta_i \ge -\theta_i \end{cases}$$

Hence, instead of reformulating (18) into an MLD form too, one can use the above linear problem.

Finally, using the total linear objective function and the MLD-LTM model, the final MILP problem can be constructed. The MILP can be solved using efficient solvers like CPLEX.

5. CASE STUDY

In order to test the proposed approach, a benchmark traffic network example has been selected from Hegyi et al. (2005). As shown in Fig. 1 the network consists of a two-lane freeway with an on-ramp. Both the mainstream origin and the on-ramp are controlled. The network is modeled using the modified version of the LTM that includes the ramp metering signals. We take the standard parameter settings used by Hegyi et al. (2005): $\upsilon_{\mathrm{free}} = 102$ km/h, $T_{\mathrm{s}} = 12$ s, $\rho_{\max} = 180$ veh/km/lane, $\rho_{\mathrm{crit}} = 33.5$ veh/km/lane, $L_1 = 4$ km, $L_2 = 2$ km and simulate over a time horizon of 2.5 hours. Simulation results for the no-control case are shown in Fig. 2

The flows of the vehicles from the mainstream origin and the on-ramp are controlled in order to minimize the sum of the TTS objective function (16) and the penalty term (18) with $\zeta = 0.4$. The control signals are obtained first by using an MPC controller based on the nonlinear LTM and next by using the MLD-MPC approach. The performance of the two approaches is compared in terms of computational efficiency and total cost. The results are shown in Table 1. Based on the prediction horizon and the control horizon, different scenarios have been defined. The simulation time step T and the control time step T_{c} are both 12 s. For each scenario, the total time spent over the full 2.5 hours simulation period is compared for both approaches. Also the computation time for one run of the optimization step is presented, averaged over the number of simulation steps. As can be seen in the table, the MLD-MPC approach returns values that are close to the original TTS, while needing a shorter computation time. It should be noted that for the nonlinear optimization algorithm, a multi-start optimization approach with several initial points should be used (In our case we called the nonlinear optimization algorithm 6 times within each MPC optimization step). It means that the CPU times for nonlinear MPC can in fact be a multiple of the current values presented in Table 1. With the increase in the prediction horizon and the control horizon, the mean computation time of one optimization step over the simulation horizon for the MLD-MPC approach increases only a little while in the

Table 1. Comparison of TTS (veh.h) and CPU Time (s) for two approaches

Scenario	TTS (nonlinear MPC)	TTS (MLD-MPC)	CPU time (nonlinear MPC)	CPU time (MLD-MPC)
$N_\mathrm{p} = 7, N_\mathrm{c} = 3$	893.4 veh.h	897.5 veh.h	0.8306 s	0.0886 s
$N_\mathrm{p} = 7, N_\mathrm{c} = 5$	893 veh.h	896.8 veh.h	1.0450 s	0.1473 s
$N_\mathrm{p} = 10, N_\mathrm{c} = 9$	891.4 veh.h	893.7 veh.h	2.7230 s	0.2452 s

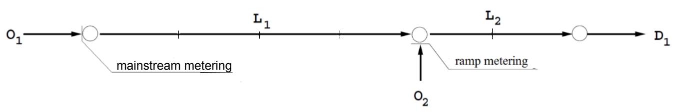

Fig. 1. Set-up of the case study

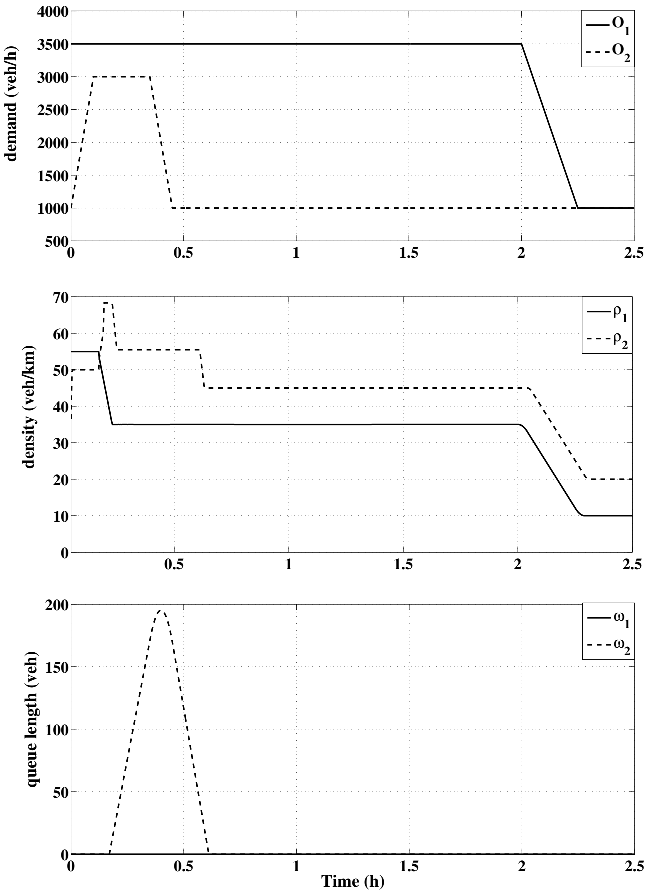

Fig. 2. Simulation results for the '*no control*' case

nonlinear MPC, the amount of change is considerable. Therefore, from the increasing values in computation time for larger prediction and control horizons, the MLD-MPC approach is expected to even perform significantly better, when it is applied to larger traffic networks.

6. CONCLUSIONS AND FURTHER RESEARCH

Modeling and control of traffic networks using the Link Transmission Model has been presented in this research. The LTM model was extended to include ramp metering and then was used to model a section of a freeway. Simulation results showed fast yet accurate modeling using the LTM. For the first time, the LTM was used as prediction model in the MPC framework in order to minimize a traffic objective function. Since a direct MPC implementation based on the nonlinear LTM was still computationally inefficient, a reformulation of the LTM was proposed in order to eventually obtain an mixed integer linear problem. For the given case study, this new approach gives results close to the ones obtained by the nonlinear MPC while the CPU time goes down significantly. Moreover, it is expected that for larger networks the benefits of the new approach over the nonlinear MPC will become even more clear.

As an extension to this work, the LTM could be modified in order to include the effects of variable speed limits. Once the new modifications are evaluated and approved on a case study with real data, one can use the basic rules to transform the new model into an MLD form too. With this, full control of traffic networks using ramp metering and variable speed limits will become possible using a fast traffic model (extended LTM) and an efficient control approach (MLD-MPC).

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REFERENCES

Atamturk, A. and Savelsbergh, M.W.P. (2005). Integer-programming software systems. *Annals of Operation Research*, 140(1), 67–124.

Bellemans, T., De Schutter, B., and De Moor, B. (2006). Model predictive control for ramp metering of motorway traffic: A case study. *Control Engineering Practice*, 14(7), 757–767.

Bemporad, A. and Morari, M. (1999). Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3), 407–427.

Daganzo, C. (1994). The cell transmission model: A simple dynamic representation of freeway traffic. *Transportation Research B*, 28(4), 269–287.

Daganzo, C. (1995). The cell transmission model, part ii: network traffic. *Transportation Research B*, 29(1), 79–94.

Groot, N., De Schutter, B., Zegeye, S., and Hellendoorn, H. (2011). Model-based traffic and emission control using PWA models – A mixed-logical dynamic approach. In *Proceedings of the 14th International IEEE Conference on Intelligent Transportation Systems (ITSC 2011)*, 2142–2147. Washington, DC.

Hegyi, A., De Schutter, B., and Hellendoorn, H. (2005). Model predictive control for optimal coordination of ramp metering and variable speed limits. *Transportation Research Part C*, 13(3), 185–209.

Hoogendoorn, S. and Bovy, P. (2001). State-of-the-art of vehicular traffic flow modelling. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 215(4), 283–303.

Jin, W. and Zhang, H. (2003). On the distribution schemes for determining flows through a merge. *Transportation Research B*, 37(6), 521–540.

Kostsialos, A., Paageorgiou, M., Mangeas, M., and Haj-Salem, H. (2002). Coordinated and integrated control of motorway networks via nonlinear optimal control. *Transportation Research C*, 10(1), 65–84.

Lebacque, J. (1996). The Godunov scheme and what it means for first order traffic flow models. In *Proceedings of the 13th International Symposium of Transportation and Traffic Theory (ISTTT)*, 621–644. Lyon, France.

Lin, W. and Wang, C. (2004). An enhanced 0-1 mixed-integer LP formulation for traffic signal control. *IEEE Transactions on Intelligent Transportation Systems*, 5(4), 238–245.

Lo, H.K. (2001). A cell-based traffic control formulation: Strategies and benefits of dynamic timing plans. *Transportation Science*, 35(2), 148–164.

Maciejowski, J. (2002). *Predictive Control with Constraints*. Prentice Hall, Harlow, England.

Messmer, A. and Papageorgiou, M. (1990). Metanet: A macroscopic simulation program for motorway networks. *Traffic Engineering and Control*, 31(9), 466–470.

Peeta, S. and Ziliaskopoulos, A. (2001). Foundations of dynamic traffic assignment: The past, the present and the future. *Networks and Spatial Economics*, 1(3/4), 233–265.

Rawlings, J. and Mayne, D. (2009). *Model Predictive Control: Theory and Design*. Nob Hill Publishing, Madison, WI, U.S.A.

Ukkusuri, S.V. and Waller, S.T. (2008). Linear programming models for the user and system optimal dynamic network design problem: Formulations, comparisons and extensions. *Networks and Spatial Economics*, 8(4), 383–406.

Williams, H. (1993). *Model Building in Mathematical Programming*. Wileyl, New York, U.S.A.

Yperman, I. (2007). *The Link Trasmission Model for Dynamic Network Loading*. Ph.D. dissertation, Katholieke Universiteit Leuven, Leuven, Belgium.

Yperman, I., Logghe, S., and Immers, L. (2005). The link transmission model: An efficient implementation of the kinematic wave theory in traffic networks. In *Proceedings of 10^{th} EWGT Meeting and 16^{th} Mini-EURO Conference*, 122–127. Poznan, Poland.

Ziliaskopoulos, A. (2000). A linear programming model for the single destination system optimum dynamic traffic assignment problem. *Transportation Science*, 34(1), 37–49.