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Distributed Identification of Fuzzy Confidence Intervals for Traffic Measurements

Alfredo Núñez, Bart De Schutter

Abstract— **A distributed fuzzy confidence interval identification method for traffic measurements is considered. Using historical data of the density measured on different sections of the freeway, the idea is to find fuzzy confidence intervals that define the bands that contain almost all the density measurements. The purpose of the proposed approach is twofold. First, to obtain a band as narrow as possible for each of the sections of the freeway. And second, to have a high percentage of the data contained in the bands. The method we propose in this paper is completely distributed, and can be used not only to describe any uncertain nonlinear distributed parameter system but also as a key element in a robust controller.**

I. INTRODUCTION

Traffic congestion and the inefficient operation of traffic networks are critical problems due to the important costs produced by travel time delays along with their negative impact on the environment. Nowadays, those problems are becoming even more critical, not only because their effects are continuously increasing, but because of the general awareness of people and authorities in topics like pollution, waste of fuel, health problems, noise, stress, and the deterioration of the quality of life in general, produced by traffic congestion in both urban and freeway networks. In this sense, a more efficient use of the existing infrastructure with the use of intelligent traffic management and control [3], [8]–[10], [19], [24], seems a good alternative to obtain sustainable mobility of the people, especially in cases that the construction of an alternative road is just not feasible, etc.

In this paper, we will focus on the topic of robust traffic monitoring systems, which is an essential element in any robust traffic control design. The purpose is to provide a distributed methodology to estimate fuzzy confidence interval models for traffic, defining a band that contains the measurement values with certain confidence. In general, a good confidence interval model identification method should have a two-fold objective. On the one hand, to obtain parameters of the confidence interval model that generate a confidence band that is as narrow as possible. On the other hand, the parameters should also generate a band that contains a high percentage of the data. In the literature, the use of interval-valued fuzzy sets and numbers is quite well-known and they have been applied to different fields of approximate inference, signal processing, and controllers [17], [20], [21], [26].

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Regarding the classes of models used to model traffic [18], we will consider first order macroscopic models. that describe the dynamics of the densities in different segments of the freeway called “cells”, and are based on the fundamental assumption that the relation between the density and the flows in each cell is given by a pre-specified traffic model, see [1], [5], [6], [11]. To illustrate the proposed method we will consider fuzzy macroscopic models, first-order in time, second-order in space. Then, for example, to model the density in the section i, $\rho_i(t)$, we will use the regressors $\rho_{i-1}(t-1)$, $\rho_i(t-1)$, and $\rho_{i+1}(t-1)$, which are the densities in the sections $i-1$, i and $i+1$ in the previous time instant.

It is relevant to highlight that in this paper we will focus on the distributed identification problem of interval fuzzy models. Even if we validate the procedure reference to a specific class of non-linear systems, the methodology we propose is generalizable and can be used for the modeling and monitoring of other classes of distributed parameter system like the ones presented in [16], [27].

By relying on experimental data measured on a portion of the A12 freeway of The Netherlands, we apply the method in a practical case, showing how the proposed procedure enables to attain a satisfactory accuracy of the distributed interval fuzzy model.

II. TAKAGI-SUGENO FUZZY MODELING AND FUZZY CONFIDENCE INTERVAL MODELING

The Takagi-Sugeno (TS) fuzzy model approximates nonlinear systems by smoothly interpolating affine local models. Each local model contributes to the global model according to a membership function [23]. The problem setting is as follows. Assume that we have s sensors located in different locations, in a distributed parameter system. Each sensor provides the measurement of the output $y_i(t)$ that in our case will be the density of a segment i in a highway. The structure of the TS fuzzy model for the variable $y_i(t)$, $i = 1,...,s$, is described as:

$$\begin{aligned}
&y_i(t) = f_i^{\mathrm{TS}}(\mathbf{z}_i^{t-1},\mathbf{x}_i^{t-1},\mathbf{u}_i^{t-1}) = \sum_{j=1}^{R_i} \beta_{ij}(\mathbf{z}_i^{t-1}) y_{ij}(\mathbf{x}_i^{t-1},\mathbf{u}_i^{t-1}),\\
&y_{ij}(\mathbf{x}_i^{t-1},\mathbf{u}_i^{t-1}) = (\mathbf{a}_{ij})^T\mathbf{x}_i^{t-1} + (\mathbf{b}_{ij})^T\mathbf{u}_i^{t-1} + r_{ij},\\
&\beta_{ij}(\mathbf{z}_i^{t-1}) = \frac{\prod_{r=1}^{p_i} A_{ij,r}(z_r(t-1))}{\sum_{j=1}^{R_i}\prod_{r=1}^{p_i} A_{ij,r}(z_r(t-1))},
\end{aligned} \tag{1}$$

where $y_i(t) \in \mathbb{R}$ is the current output, $\mathbf{x}_i^{t-1} \in \mathbb{R}^{n_i}$ are past outputs or other exogenous variables, $\mathbf{u}_i^{t-1} \in \mathbb{R}^{m_i}$

are past inputs, the vector of the premises is $\mathbf{z}_i^{t-1} = [z_1(t-1),\ldots,z_{p_i}(t-1)]^T$, p_i is the number of inputs at the premises. The variables $\mathbf{x}_i^{t-1}$, $\mathbf{u}_i^{t-1}$ and $\mathbf{z}_i^{t-1}$ are the important variables to properly model the output i of the vector $\mathbf{y}^t = [y_1(t),\ldots,y_s(t)]^T$. The premises variables are permitted to be inputs, outputs or other exogenous variables, and in this paper we will assume $\mathbf{z}_i^{t-1} = \left[(\mathbf{x}_i^{t-1})^T,(\mathbf{u}_i^{t-1})^T\right]^T$, so $p_i = n_i + m_i$. Moreover, $(\mathbf{a}_{ij})^T$, $(\mathbf{b}_{ij})^T$, r_{ij} are the parameters of the fuzzy model $f_i^{\mathrm{TS}}(\cdot)$ for the output y_i on rule j, R_i is the number of rules of the fuzzy model, $A_{ij,r}(z_r(t-1)) : \mathbb{R} \to [0,1]$ is the membership degree for the input in premise r, i.e., $z_r(t-1)$, for the variable $y_i(t)$ and rule j, and $\beta_{ij}(\mathbf{z}_i^{t-1})$ is the activation degree of the jth rule that belongs to the fuzzy model. In this paper we will use Gaussians to model the membership degrees.

Let us assume that input-output and exogenous data is available, $(\mathbf{y}^t,\mathbf{x}^{t-1},\mathbf{u}^{t-1})$, $t = 1,\ldots,N_{\mathrm{d}}$. By only using a finite data set of the process, the identification problem of a TS fuzzy model given by (1) consists of estimating the following parameters for each output $y_i(t)$, $i = 1,\ldots s$: the number of rules R_i, the parameters of the membership functions $A_{ij,r}(\cdot)$, and the parameters $(\mathbf{a}_{ij})^T,(\mathbf{b}_{ij})^T,r_{ij}$. Usually the identification procedure consists of minimizing a cost function with respect to the unknown parameters:

$$V_{N_{\mathrm{d}}} = \frac{1}{N_{\mathrm{d}}}\sum_{t=1}^{N_{\mathrm{d}}}\sum_{i=1}^{s} V\left(y_i(t) - f_i^{\mathrm{TS}}(\mathbf{z}_i^{t-1},\mathbf{x}_i^{t-1},\mathbf{u}_i^{t-1})\right) \quad (2)$$

where V is a cost function for the error, typically a quadratic function. The minimization of (2) is in general a non-linear optimization problem. In the literature, it is standard to split the identification of TS fuzzy models in two parts. First, to determine the premises by using a specific clustering method. And second, to obtain the remaining parameters in the consequents by using least squares. This hierarchical procedure provides meaningful rules as the clusters can be used to determine which operation point the system is currently working in. Once the TS models are obtained, a lower and upper confidence interval of the local linear models can be defined for each output as in [22]:

$$\begin{aligned}
&\overline{y}_i(t) = \overline{f}_i^{\mathrm{TS}}(\mathbf{z}^{t-1},\mathbf{x}^{t-1},\mathbf{u}^{t-1}) = \\
&\sum_{j=1}^{R_i} \beta_{ij}(\mathbf{z}^{t-1})(y_{ij}(\mathbf{x}^{t-1},\mathbf{u}^{t-1}) + \overline{\alpha}_{ij}\Delta_{ij}(\mathbf{x}^{t-1},\mathbf{u}^{t-1})),\\
&\underline{y}_i(t) = \underline{f}_i^{\mathrm{TS}}(\mathbf{z}^{t-1},\mathbf{x}^{t-1},\mathbf{u}^{t-1}) = \\
&\sum_{j=1}^{R_i} \beta_{ij}(\mathbf{z}^{t-1})(y_{ij}(\mathbf{x}^{t-1},\mathbf{u}^{t-1}) - \underline{\alpha}_{ij}\Delta_{ij}(\mathbf{x}^{t-1},\mathbf{u}^{t-1})),\\
&y_{ij}(\mathbf{x}^{t-1},\mathbf{u}^{t-1}) = (\mathbf{a}_{ij})^T\mathbf{x}^{t-1} + (\mathbf{b}_{ij})^T\mathbf{u}^{t-1} + r_{ij},\\
&\psi_{ij} = \left[(\mathbf{x}_i^{t-1})^T,(\mathbf{u}_i^{t-1})^T,1\right]^T,\\
&\Delta_{ij}(\mathbf{x}^{t-1},\mathbf{u}^{t-1}) = \sigma_{ij}\left(1 + \psi_{ij}^T(\Psi_{ij}\Psi_{ij}^T)^{-1}\psi_{ij}\right)^{0.5}\\
&\beta_{ij}(\mathbf{z}^{t-1}) = \frac{\prod_{r=1}^{p_i} A_{ij,r}(z_r(t-1))}{\sum_{j=1}^{R_i}\prod_{r=1}^{p_i} A_{ij,r}(z_r(t-1))},
\end{aligned} \quad (3)$$

where $\overline{f}_i^{\mathrm{TS}}$ is the upper bound, and $\underline{f}_i^{\mathrm{TS}}$ is the lower bound, $\Psi_{ij}\Psi_{ij}^T$ is a covariance matrix of the local model, σ_{ij} is the variance of the local noise signal. The parameters $\underline{\alpha}_{ij}$ and $\overline{\alpha}_{ij}$ are tuning parameters for the lower and upper fuzzy bounds. The function $\Delta_{ij}(\mathbf{x}^{t-1},\mathbf{u}^{t-1})$ is the square root of the covariance of the residual between the model output and the new set of data in each local domain. In other words, it represents a kind of time-variant standard deviation, or a function which decides the variations with respect a nominal value considering the effect of different inputs.

In the next section, we describe the distributed identification procedure for TS fuzzy systems and the fuzzy confidence intervals for traffic measurements.

III. Distributed Fuzzy Confidence Interval Model Identification Procedure for Traffic Measurements

Throughout this paper we assume that N_{d} input/output data pairs have been collected:

$$\Phi = \begin{bmatrix} (\mathbf{x}^1)^T & (\mathbf{u}^1)^T \\ (\mathbf{x}^2)^T & (\mathbf{u}^2)^T \\ \vdots & \vdots \\ (\mathbf{x}^{N_{\mathrm{d}}})^T & (\mathbf{u}^{N_{\mathrm{d}}})^T \end{bmatrix}, \quad (4)$$

where N_{d} denotes the number of data samples, $(\mathbf{x}^t)^T = (\mathbf{y}^t)^T = [\rho_1(t),\rho_2(t),\ldots,\rho_s(t)] \in \mathbb{R}^s$ are the outputs, and $(\mathbf{u}^t)^T = [\rho_0(t),\rho_{N+1}(t)] \in \mathbb{R}^2$ are the inputs. All those variables were measured at time steps $t = 0,\ldots,N_{\mathrm{d}}$ and represent the densities in different s sections of the highway. The density in the section i is denoted by $\rho_i(t)$, and the density in the next section of the highway is denoted by $\rho_{i+1}(t)$, and so on. The procedure we will use can be easily extended to model not only densities, but also other traffic signals such as the average velocities, flows, weather conditions, effects of on/off ramps, etc. The details of each step are explained next.

For the model of the density $\rho_i(t)$, $i = 1,\ldots,s$, we will use the available data in the following way:

$$\Phi_i = \begin{bmatrix} \rho_i(1) & \rho_{i-1}(0) & \rho_i(0) & \rho_{i+1}(0) \\ \rho_i(2) & \rho_{i-1}(1) & \rho_i(1) & \rho_{i+1}(1) \\ \vdots & \vdots & \vdots & \vdots \\ \rho_i(N_{\mathrm{d}}) & \rho_{i-1}(N_{\mathrm{d}}-1) & \rho_i(N_{\mathrm{d}}-1) & \rho_{i+1}(N_{\mathrm{d}}-1) \end{bmatrix}, \quad (5)$$

Note that for the first and the last sections, we will use the outputs $\rho_0(t)$ and $\rho_{N+1}(t)$ respectively.

***Step 1*: Clustering**. Determine the fuzzy clusters over the data Φ_i, using the G-K algorithm [7]. This algorithm includes fuzzy covariances in an n-dimensional space, and closely resembles maximum likelihood estimation on mixture densities. The algorithm will cluster the data given a specified number of clusters c, the parameters for the cluster fuzziness, and the stopping criterion. The G-K algorithm provides the centers of the clusters $\mathbf{v}_j$, and the covariance matrix for each fuzzy cluster l.

For the G-K algorithm, the number of clusters c is needed. A large number of clusters results in a complicated rule-base model, while a small number of clusters in a not good model. To preserve the small clusters is sometimes important, especially when we are not sure of the correct experiment design (by the proper excitation of the system). To obtain the optimum number of clusters it is possible, after using the G-K algorithm, to use the compatible cluster merging method, [2], [14]. This method works as follows: let the centers of two clusters be $\mathbf{v}_{j_1}$ and $\mathbf{v}_{j_2}$, with φ_{1,j_1} and φ_{1,j_2} the eigenvectors associated with the minimum eigenvalue λ_{1,j_1} and λ_{1,j_2} respectively (from the covariance matrix of the cluster). The criteria to merge the clusters proposed in [14] consider that the nearly-parallel major axes of consecutive clusters should be merged ($|\varphi_{1,j_1} \cdot \varphi_{1,j_2}| \geq k_1$, with k_1 close to 1) and also the cluster centers should be sufficiently close for merging ($\|\mathbf{v}_{j_1} - \mathbf{v}_{j_2}\|_2 \leq k_2$, with k_2 close to 0). Finally, after this step, the number of rules R_i and the membership functions $A_{ij,r}(\cdot)$ are obtained.

***Step 2*: Local fuzzy identification**. For each segment of the highway i, The next step is to identify the consequent parameters of each rule of the TS model. An identification procedure is carried out by minimizing the following cost function with respect to the unknown parameters Θ_{ij} of the fuzzy rule j of the output $y_i(t) = \rho_i(t)$:

$$V_{N_\mathrm{d}} = \frac{1}{N_\mathrm{d}} \sum_{t=1}^{N_\mathrm{d}} \left(\beta_{ij}(\mathbf{z}^{t-1})\right)^2 \left(y(t) - \left[(\mathbf{y}^{t-1})^T \;\; (\mathbf{u}^{t-1})^T \;\; 1\right] \Theta_{ij}\right)^2 \tag{6}$$

where N_d is the number of input-output data. Let us write the consequent parameters for the fuzzy rule j for the output i as follows:

$$\Theta_{ij} = \begin{bmatrix} \mathbf{a}_{ij} \\ \mathbf{b}_{ij} \\ r_{ij} \end{bmatrix}, \tag{7}$$

The model parameters for the rule j of region i can be obtained using the least squares identification method as follows:

$$\Theta_{ij} = (\Psi_{ij}^T \Psi_{ij})^{-1} \Psi_{ij}^T Y_{ij} \tag{8}$$

where the matrices Ψ_{ij} and Y_{ij} are the following:

$$\Psi_{ij} = \begin{bmatrix} \beta_{ij}(\mathbf{z}^0)[(\mathbf{x}^0)^T \quad (\mathbf{u}^0)^T \quad 1] \\ \beta_{ij}(\mathbf{z}^1)[(\mathbf{x}^1)^T \quad (\mathbf{u}^1)^T \quad 1] \\ \vdots \\ \beta_{ij}(\mathbf{z}^{N_\mathrm{d}-1})[(\mathbf{x}^{N_\mathrm{d}-1})^T \quad (\mathbf{u}^{N_\mathrm{d}-1})^T \quad 1] \end{bmatrix}, \tag{9}$$

$$Y_{ij} = \begin{bmatrix} \beta_{ij}(\mathbf{z}^0) y(1) \\ \beta_{ij}(\mathbf{z}^1) y(2) \\ \vdots \\ \beta_{ij}(\mathbf{z}^{N_\mathrm{d}-1}) y(N_\mathrm{d}) \end{bmatrix}, \tag{10}$$

By the identification of each rule (not the overall model), and also by weighting the data for the corresponding activation degree of each rule β_{ij}, a better conditioning of the matrices is obtained, compared to the conditioning of the whole data matrix. This approach leads to a better estimation of the TS fuzzy model parameters as the data close to the center of the cluster will be more important to minimize than the data at the borders (see [12], [13] and [15]).

***Step 3*: Confidence interval fuzzy models**. To determine the fuzzy lower and upper confidence interval from (3), the variables to be determined are: $\Psi_{ij}\Psi_{ij}^T$ the covariance matrix of the local model, σ_{ij}^2 is the variance of the local noise signal, and $\underline{\alpha}_{ij}$ and $\overline{\alpha}_{ij}$. For the covariance matrix it possible to use the covariance obtained in (9). The variance of the local noise can be estimated easily by using the identification data. The crucial parameters in this step are $\underline{\alpha}_{ij}$ and $\overline{\alpha}_{ij}$. Depending on how those parameters are tuned, we will have more narrow bands but with a higher number of data points outside the band. In one of the axis in this front we will have the number of data points outside the band, and in the other the area covered by the band (which is a way to analyze how narrow is the band). The multi-objective optimization problem to solve in this step is the following:

$$\begin{aligned}
&\min_{\{\underline{\alpha}_{ij},\overline{\alpha}_{ij}: i=1,\ldots,s, j=1,\ldots R_i,\}} \{J_1, J_2\} \\
&J_1 = \frac{1}{N_\mathrm{v} s} \sum_{t=1}^{N_\mathrm{v}} \sum_{i=1}^{s} \overline{\delta}_i(\mathbf{x}^{t-1}, \mathbf{u}^{t-1}) + \underline{\delta}_i(\mathbf{x}^{t-1}, \mathbf{u}^{t-1}) \\
&\overline{\delta}_i(\mathbf{x}^{t-1}, \mathbf{u}^{t-1}) = \begin{cases} 1 & \text{if} \quad \overline{f}_i^{\mathrm{TS}}(\mathbf{x}^{t-1}, \mathbf{u}^{t-1}) - y_i(t) < 0 \\ 0 & \qquad \text{otherwise} \end{cases} \\
&\underline{\delta}_i(\mathbf{x}^{t-1}, \mathbf{u}^{t-1}) = \begin{cases} 1 & \text{if} \quad \underline{f}_i^{\mathrm{TS}}(\mathbf{x}^{t-1}, \mathbf{u}^{t-1}) - y_i(t) > 0 \\ 0 & \qquad \text{otherwise} \end{cases} \\
&J_2 = \frac{1}{N_\mathrm{v} s} \sum_{t=1}^{N_\mathrm{v}} \sum_{i=1}^{s} \overline{f}_i^{\mathrm{TS}}(\mathbf{x}^{t-1}, \mathbf{u}^{t-1}) - \underline{f}_i^{\mathrm{TS}}(\mathbf{x}^{t-1}, \mathbf{u}^{t-1})
\end{aligned} \tag{11}$$

where J_1 considers the number of data points outside the band, and J_2 and approximation of the area of the band.

In this step, using validation data, if the intervals are not good enough according to a performance criterion, we can go back to *Step 1* and to consider the inclusion of more regressors (structural optimization), or to increase the number of clusters by changing the sensibility of the clustering merge procedure. This step determine the convergence of the algorithm.

Finally, as the three steps are in a distributed scheme, with no connections between optimization problems of neighbors, all those problems can be solved in parallel for $i = 1, \ldots, s$. The computational time of this case is the maximum of all the times spent in each problem, and the number of variables obtained in each problem depends on the number of clusters and the simplifications we assume to obtain a reasonably good confidence band.

IV. EXPERIMENTAL RESULTS

In this section we summarize the simulation tests conducted to show the application of the identification procedure to a real scenario. A 1.915 *km* long stretch of the A12 freeway, in The Netherlands has been used as test field to validate the identification method. This road connects the city of The Hague, with the German border, near Zevenaar. The stretch we use is in the segment that crosses the Dutch province of South Holland. In Fig. 1 the scheme of the stretch is depicted. In this paper, the identification procedure will be explained with a single link case study, with no on-ramps or off-ramps.

A period of eight hours (4:00-12:00) representative of typical working Monday will be modeled. The data from the 21 and 28 of September 2009, was used for identification, see Figures 2(a) and 2(b). For validation, data from Monday 12 of October was used.

Next, we present the experimental results relying on the real-life data. In the first section, the results of the distributed identification of the TS models for traffic. In the second, the interval fuzzy models for different cases are illustrated. Finally, in the last section, a fault tolerant scheme is evaluated using the interval fuzzy models obtained.

1) Distributed interval fuzzy model identification: First the *Step 1* and *Step 2* (clustering and local fuzzy identification) of the procedure are performed. In Figure 3, the one-step ahead prediction results for the four cells of the case-study using the validation data are shown. The number of clusters after the process of merge were 10 for each TS fuzzy model.

The sum of the Root Mean Square (RMS) of the prediction error of densities in the cells is 7.8181, and the standard deviation of all the signals 1.9927. This step is important to perform and to verify, as the TS model is one of the components of the interval fuzzy model.

For simplicity of the *Step 3* (determination of the confidence interval fuzzy models), let us assume that the tuning parameters for the lower and upper fuzzy bounds to be the same $\underline{\alpha}_{ij} = \overline{\alpha}_{ij} = \alpha$. Figure 4 shows the Pareto front of the normalized area of the band, and the normalized number of data points outside the band, when the parameter α changes from 0 to 400.

Each point in the Pareto front is related with a value of the optimization variable α. For the normalization, the maximum values of J_1 and J_2 are 11204 and 276000 respectively. Just for illustrative purposes, we have selected the distributed interval fuzzy models generated with the values $\alpha = 40$ and $\alpha = 120$. In the Figures 5 and 6 we can see the fuzzy confidence intervals for the densities, for the cases $\alpha = 40$ and $\alpha = 120$ respectively. As expected, the fuzzy confidence intervals for the case $\alpha = 120$ contains a higher amount of data in the band; however, the area of the band is much higher than in the case $\alpha = 40$.

2) Virtual Sensor: failure in the sensor i: In this subsection, we analyze the effect of a failure in the sensor *i*. We assume the failure is detected immediately, and we replace the sensor *i* with the TS fuzzy model identified in the previous section, one sensor at the time. In Fig. 7 we show the estimated values with the fuzzy models, the interval fuzzy models and the real measurement, when the sensor in segment *i* has failed. The Root Mean Square (RMS) for each case of virtual sensors prediction is 2.5825, 1.8428, 1.8745, 1.5182 respectively. From the figure, we can see when the sensor 1 fails, the most important errors happen.

V. CONCLUSIONS AND FUTURE WORK

In this paper, a distributed identification of fuzzy confidence interval for a chosen freeway has been analyzed. The method proposed in this paper is completely distributed. To decide how to properly split the fuzzy interval identification problem, by changing the interactions between different hierarchical and distributed optimization problems, is a topic for further research.

By relying on experimental data measured on a portion of the A12 freeway of The Netherlands, we have shown that it is possible to find fuzzy intervals considering the compromise between the number of data points outside the bands and area of the bands. Finally, to show the good properties of the identified model, an evaluation of the performance of the identified model used as a set of virtual sensors in different scenarios was made. More sophisticated schemes of failures and detection can also be analyzed in the future, as well as on-line identification, and the inclusion of other traffic measurements.

VI. ACKNOWLEDGMENTS

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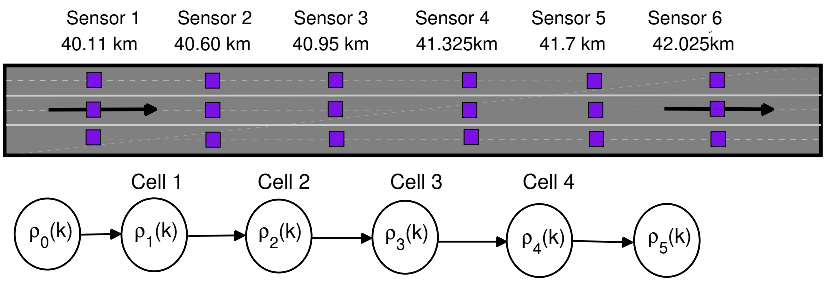
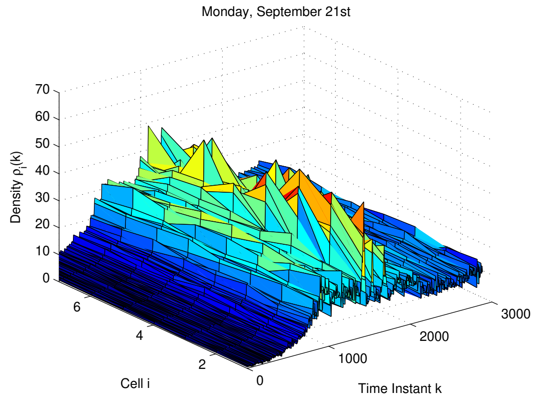
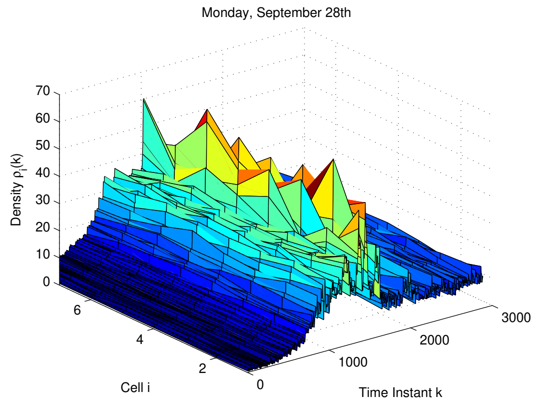


Fig. 1. Schematic sensor positions and cells of the part of the A12 freeway in The Netherlands considered in the case study.



(a)



(b)

Fig. 2. Identification data measured: (a) Sept. 21, 2009. (b) Sept. 28, 2009.

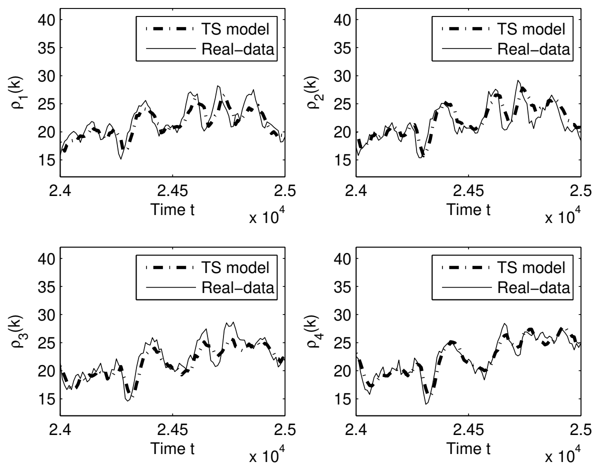


Fig. 3. One-step ahead prediction cells 1, 2, 3 and 4.

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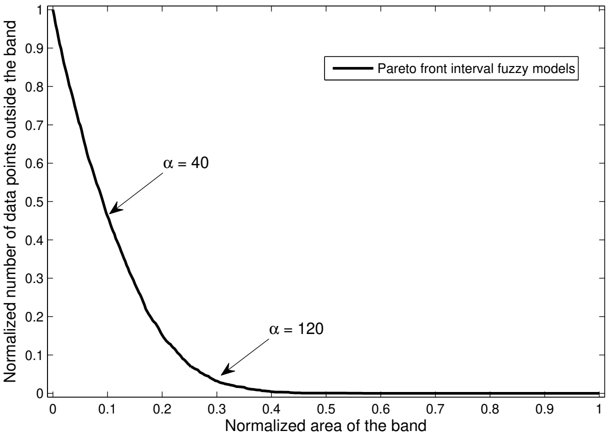

Fig. 4. Pareto front.

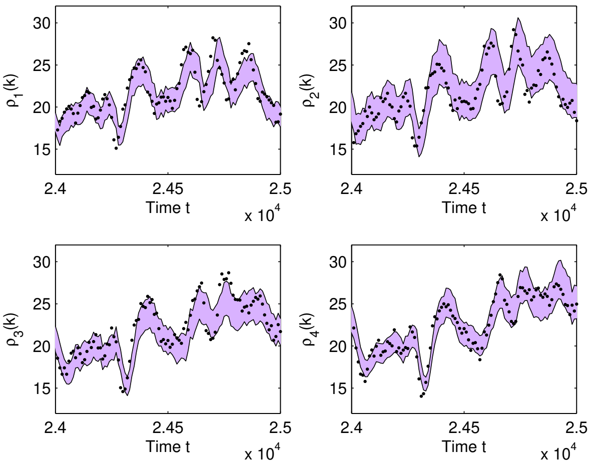

Fig. 5. Distributed fuzzy confidence interval, $\alpha = 40$.

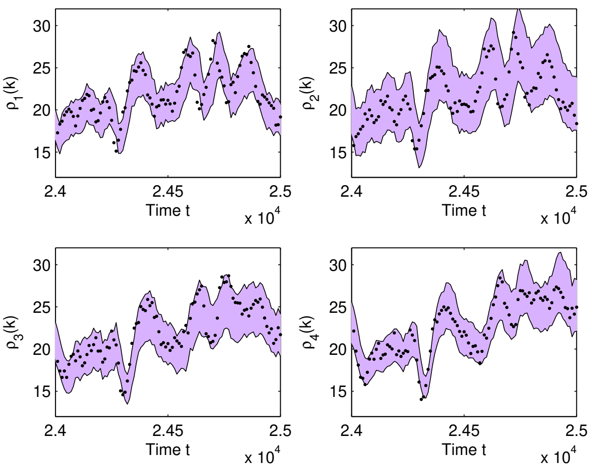

Fig. 6. Distributed fuzzy confidence interval, $\alpha = 120$.

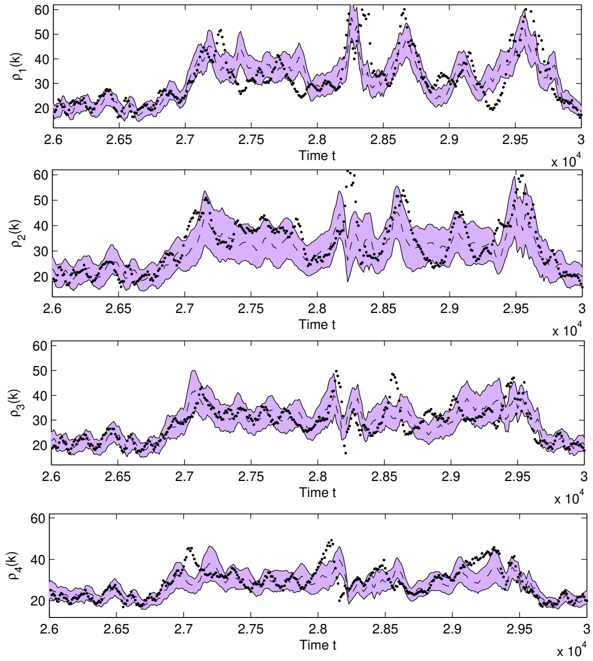

Fig. 7. Interval fuzzy models in case of failure of sensor i.

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