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# Decentralized model-based predictive control for urban traffic control

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## 1 Introduction

In urban areas, as a way to tackle the effects of congestion and the continuous increase in the number of vehicles, solutions based on the improvement of public transportation, dynamic pricing, and strategies for regulating the urban traffic flows have emerged during the last decades. The research oriented to reduce congestion on urban roads is very important because of its elevated costs and impact on the quality of life of the people (Kaparias et al., 2010). In this paper, we aim to increase mobility and to reduce congestion on an urban traffic arterial by improving the operation of traffic signals by applying the appropriate retiming strategy, using a decentralized model predictive control approach. This approach has not been applied before in the literature in the context of the urban traffic model we use, and in this paper, we establish its benefits and performance. The approach is tested by simulation and compared with a centralized version of the controller, an optimized fixed-time retiming methodology, the adaptive controller SCOOT (Robertson and Bretherton, 1991), and a decentralized non-linear state-feedback controller.

## 2 Decentralized Model Predictive Control for Urban Traffic

To represent urban traffic, the S model is considered (Lin et al., 2011). The main advantage of the S model over other urban traffic models is its reduced computational burden. In the S model, a link  $(u, d)$  represents a road between intersections  $u$  and  $d$ . Two states variables in each link  $(u, d)$  are defined at every time step  $k_d$ : the number of vehicles  $\eta_{u,d}(k_d)$ , and the number of vehicles  $q_{u,d,o}(k_d)$  waiting in queue turning to the direction  $o$ .

Using this model, in recent years some optimization-based strategies have been proposed to address traffic control in urban areas (Lin et al., 2012). Despite the advantages of model predictive control (MPC), its application in large-scale systems presents challenges for real-life implementation due to the computational efforts and communication needed to solve the related optimization problems.

In order to overcome the problems of centralized MPC, we propose a decentralized MPC scheme, resulting in algorithms with a low computational cost, and a reduced vulnerability to communication failures, and that are able to reach near-optimal solutions. In this paper, the system is decoupled according to the intersections. Then, each controller will handle the states related to each link in the intersection and will obtain the control actions for the corresponding traffic signal. Each intersection can have its own step-size, hence, its own step counter  $k_m$ . The nonlinear decentralized MPC optimization problem for each intersection  $m$  at the time step  $k_m$  is as follows:

$$\begin{aligned} & \min_{U_{k_m}^m} J_m \left( U_{k_m}^m, U_{k_m}^{\hat{m}}, x_m(k_m) \right) & (1) \\ \text{s.t.} \quad & x_m(k_m + t + 1) = f_m(x_m(k_m + t), u_m(k_m + t), u_{\hat{m}}(k_m + t)), \quad x_m(k_m) = x_{k_m}^m, \\ & 0 \leq \eta_{u,m}(k_m + t) \leq C_{u,m}, \quad 0 \leq q_{u,m,o}(k_m + t), \quad 0 \leq g_{u,m,o}(k_m + t - 1) \leq c_m, \\ & \text{for } t = 0, \dots, N_p - 1, \quad (u, m) \in L_m, \quad o \in O_m \end{aligned}$$

where  $J_m$  is the objective function (in the case study we use the total time spent),  $x_m(k_m)$  the state vector at time step  $k_m$ ,  $U_{k_m}^m = [u_m(k_m)^T, \dots, u_m(k_m + N_p - 1)^T]^T$  is the control sequence for the traffic signal at intersection  $m$ ,  $U_{k_m}^{\hat{m}}$  is the sequence of inputs and states  $u_{\hat{m}}(k_m + t)$  from other subsystems affecting intersection  $m$ . The inputs and states of the remaining subsystems (denoted as  $\hat{m}$ ) are constants as for system  $m$  and nominal values are used (we assume no communication between the different MPC). The state vector is composed of the number of vehicles in each link  $\eta_{u,m}(k_m)$  (with a capacity  $C_{u,m}$ ) and the number of vehicles  $q_{u,m,o}(k_m)$  waiting in queue  $o$  (left-turn, right-turn or straight through queue) in the intersection  $m$ , and the inputs  $g_{u,m,o}(k_m)$  are the green times for each phase of the traffic signal, and  $c_m$  is the sampling time. The function  $f_m(\cdot)$  is given by the S model,  $L_m$  and  $O_m$  the set of links and origins relevant for the intersection  $m$ . Once (1) is solved, from the control sequence only the first control action  $u_m(k_m)$  is applied at intersection  $m$ , and the same procedure is repeated in the next instant step  $k_m + 1$  considering the new measurements (rolling horizon procedure).

## 2.1 Decentralized Non-Linear State-Feedback Controller

In the design of this control scheme, the length of the queues are considered to compute the control actions at each time step  $k$ . Such control actions are determined by the control

law (2).

$$u_m(k+1) = u_m(k) + K_{m,f}q_{\Gamma m,f}(k) + K_{m,\hat{f}}q_{\Gamma m,\hat{f}}(k) \quad (2)$$

where  $K_{m,f}$  and  $K_{m,\hat{f}}$  are the gain matrices of the controller at intersection  $m$  and the rest of the network different than  $m$  ( $\hat{f}$  denotes all elements different than  $f$ , i.e., the complement of  $f$ ).  $q_{\Gamma m,f}(k)$  is the sum of the queues at intersection  $m$  associated with the control action  $f$ , i.e.,  $q_{\Gamma m,f}(k) = \sum_{o \in O_{u,d}^f} q_{u,d,o}(k)$ , and  $q_{\Gamma m,\hat{f}}(k)$  is the sum of the queues at intersection  $m$  associated with the remaining control actions at such intersection, i.e.,  $q_{\Gamma m,\hat{f}}(k) = \sum_{o \in O_{u,d}^{\hat{f}}} q_{u,d,o}(k)$ .

Let  $\Delta u_m(k+1)$  be the changes on the control actions from step time  $k$  to step time  $k+1$ , i.e.,  $\Delta u_m(k+1) = u_m(k+1) - u_m(k)$ . Then the control law (2) can be rewritten as

$$\Delta u_m(k) = K_m Q_m(k) \quad (3)$$

where  $K_{mT} = [K_{m,f}, K_{m,\hat{f}}]$ , and  $Q_m(k) = [q_{\Gamma m,f}(k), q_{\Gamma m,\hat{f}}(k)]^T$ . Note that  $Q_m(k) = C_m x_m(k)$ , with  $C_m$  a selection matrix whose entries are ones or zeros depending on the queues involved on the computation of  $Q_m(k)$ , and  $x_m(k)$  the states associated with the intersection  $m$ . Thus, (3) becomes

$$\Delta u_m(k) = K_{mT} C_m x_m(k) \quad (4)$$

Clearly, (4) is a non-linear state-feedback control law where the evolution of  $x(k)$  is determined by the equations of the S model. In order to implement this control scheme, three steps are considered:

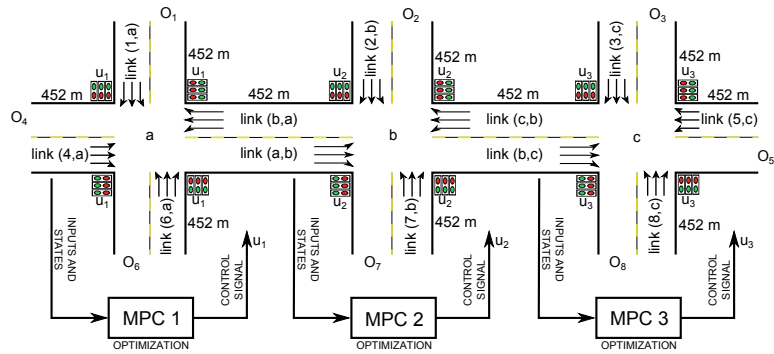
1. The local states are measured.
2. The local control actions are computed based on the information received from the remaining controllers.
3. Back to step 1.

The gains of each one controller are equal for each subsystem along the time, this is because the dynamic of queues in the network determine these values. If the time of green signal for the phase 1 in the  $m$ -intersection increases, this is a consequence for the increase in the number of vehicles in the links associated to this phase, and vice versa.

### 3 Simulation results

The benchmark system consists of three connected intersections, as shown in Figure 1(a). The motivation for the benchmark is the future implementation of these strategies in an important corridor of the city of Medellín, Colombia, as shown in Figure 1(b) (specifically

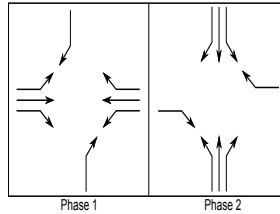
in San Juan avenue, which crosses the city east to west). So far, a fixed-time signal strategy is implemented in the real system, with values that were not optimized, so we aim to propose a feasible and practical solution to the traffic authorities in the city. The simulation model and the prediction model are the same. The sampling time of the system is  $c_m = 30$  s for all intersections, and to run the simulation the initial states are 20 vehicles in each link and 5 vehicles at each input queue. Each link has a length of 450 m, the length of each vehicle is 7 m, all links were designed with 3 lanes and the free flow velocity 50 km/h. The capacity of each link is 192 vehicles.



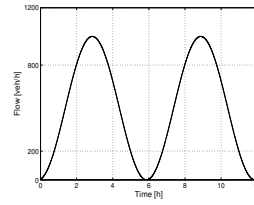
(a) Three intersections interconnected and decentralized MPC



(b) Urban traffic network of Medellín, Colombia



(c) Traffic signal phases



(d) Inflow rate for the link (4, a)

In each intersection traffic signals are located with two operation modes or phases, as shown in Figure 1(c). The inflow rates for the links (1, a), (6, a), (2, b), (7, b), (3, c) and (8, c) are 900 veh/h each, and for the link (5, c) the inflow rate is 980 veh/h. The inflow rate for the link (4, a) is shown in Figure 1(d). Table 1 presents results of total time spent (TTS) and total computation time in (s) during the complete simulation time, using the different control strategies. Both MPC based controllers improve the TTS with respect to the other methods. The centralized control approach has a slightly better control performance with respect to the decentralized one. However, the advantages over the centralized controller in terms of computational time are confirmed.

Table 1: Comparison total time spent (TTS) and computation times.

Configuration	TTS [(veh)(h)]	Computational time (s)
Fixed Time	3719.8	-
Centralized MPC	3318.8	704.8
Decentralized MPC	3328.4	341.7
State-Feedback	3457.9	-
SCOOT	3359.7	117.6

## 4 Conclusion and further research

In this paper, a centralized and a decentralized MPC scheme for the control of an urban traffic network are presented, using the S model as prediction model. We claim that with a decentralized MPC scheme it would be possible to control larger urban traffic networks, especially where due the growth of the number of variables and non-linearities of the urban traffic model, it would not be feasible to implement the centralized scheme in real-time. As future work, different hierarchical and distributed MPC schemes can be analyzed, as well as the multimodal traffic signal control, so to include at each intersection the effects of cars, buses, pedestrians, and bicycles interacting with each other.

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