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Energy-Efficient Transportation over Flowing Water

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***Abstract*—Transportation of water is necessary to manage the water levels in rivers to ensure minimum depths for freight, to buffer water to avoid flooding downstream during wet periods, or to save water for anticipated dry periods. This manipulation of water flows influences the speed of the water in the rivers and canals, and thereby it affects the speed and energy consumption of vessels used for transportation over water. When considering the problem of scheduling micro-ferries in a harbour, the influence of fluctuating water speeds (due to the tide or river flows) can be taken into account in the aim of providing energy-efficient schedules for pick-ups and deliveries of passengers. This paper introduces the effect of flowing water on energy-efficient scheduling, and it proposes a mixed-integer linear programming formulation for solving the problem.**

I. Introduction

This paper discusses the micro-ferry scheduling problem as introduced in [1], with the extension of taking into account the effect of flowing water. With micro-ferries we mean small, autonomous water-taxis with a limited amount of energy that can recharge (or refuel) at stations located throughout a harbour. The problem consists in finding an optimal schedule for a fleet of micro-ferries that ensures that all transportation requests from one station to another are handled. The aim is to obtain a schedule for the micro-ferries that minimises the energy consumption, ensures the micro-ferries do not run out of energy, and tries to serve the passengers within their desired pick-up time-window as much as possible.

The main focus is on minimising energy consumption, and the problem can be seen as a variant of the travelling salesman problem (TSP) [2], [3]. Traditionally, variants of the TSP — such as the pick-up and delivery problem [4] and the vehicle routing problem [5], [6]— focus on minimising the travel distance or travel time. However, these problems do not take into account the energy needed to fulfil a route or schedule.

Recently some work on energy consumption in routing and scheduling has appeared in the literature. Energy consumption is considered for the vehicle routing problem in [7], where the energy consumption is defined by multiplying the load of the vehicle by the travelled distance, independent of the speed of the vehicle. In [8] the pollution routing problem is proposed, where a trade-off is made in minimising the travel distances, travel times, transport costs, and greenhouse emissions. The pollution is related to the energy consumption and dependent on both the speed and the load of the vehicle. A vehicle routing problem where the fuel cost is minimised is proposed in [9]. Fuel costs are defined as unit fuel cost $\times$ fuel consumption rate $\times$ road length; minimising the fuel costs equals minimising the fuel consumption.

In the micro-ferry scheduling problem as proposed in [1] the aim is to minimise the total energy consumption, while picking up the passengers from a station within their desired time-window. Charging of the micro-ferries has been taken into account in [10], ensuring that all requests can be handled without running out of energy. For still water the problem can be modelled as a non-linear programming problem that can easily and efficiently be approximated by a mixed-integer linear program (MILP) [11]. In these papers the speed of the micro-ferries has been taken as one of the optimisation variables, thereby influencing both the energy consumption and the travel times. Note that the term *speed* represents *the amplitude of the vessel velocity*. In the current paper the previous work is extended by taking into account the effect of flowing water on the travel time of the micro-ferries, and its influence on the energy consumption. Taking into account the water flows is important for the scheduling problem, otherwise

- the micro-ferries could run out of energy on the water,
- the communicated pick-up times become inaccurate.

As opposed to the authors' previous work [1], [10], [11], in this paper the speed of the vessels is constant; combining flowing water and variable speeds is considered for future work.

II. Effects of Flowing Water

When considering still water the time it takes to travel from one station to another is simply the along-path distance[1] between the stations divided by the speed of the vessel. For still water the speed of the vessel relative to the water equals the speed of the vessel relative to the land. When the water is flowing, the speed relative to the water no longer equals the speed relative to the land, as will be discussed in this section.

A. *Definition of velocities*

Velocities can conveniently be described using vectors. Throughout this paper three different velocity vectors are used:

$\vec{v}_\mathrm{b}$	vessel velocity relative to the water,
$\vec{v}_\mathrm{l}$	vessel velocity relative to the land,
$\vec{v}_\mathrm{r}$	water velocity relative to the land.

[1]With the along-path distance we mean the total distance along the curve describing the path; it can be a straight line but also a non-linear path.

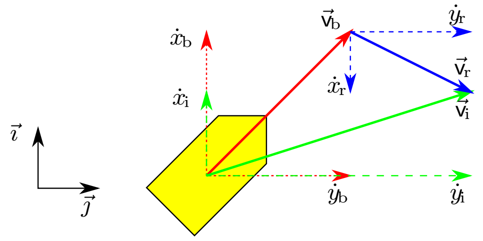

Fig. 1. *The velocity $\vec{\mathrm{v}}_{\mathrm{i}}$ with respect to the land is the sum of the velocity $\vec{\mathrm{v}}_{\mathrm{b}}$ of the micro-ferry plus the velocity $\vec{\mathrm{v}}_{\mathrm{r}}$ of the water.*

The three velocities relate to each other as (see Figure 1)

$$\vec{\mathrm{v}}_{\mathrm{i}} = \vec{\mathrm{v}}_{\mathrm{r}} + \vec{\mathrm{v}}_{\mathrm{b}}, \tag{1}$$

and they also have the following meaning. The velocity $\vec{\mathrm{v}}_{\mathrm{b}}$ describes the motion of the body-fixed reference frame (B-frame) of the vessel with respect to the water, the velocity $\vec{\mathrm{v}}_{\mathrm{i}}$ represents the movement of the vessel in the inertial reference frame (I-frame), whereas $\vec{\mathrm{v}}_{\mathrm{r}}$ gives the velocity of the B-frame relative to the I-frame.

Each of the velocity vectors can be decomposed in speed components in the x and y direction of the inertial reference frame, for each $* \in \{\mathrm{b}, \mathrm{i}, \mathrm{w}\}$ written as

$$\vec{\mathrm{v}}_{*} = \dot{x}_{*}\vec{\imath} + \dot{y}_{*}\vec{\jmath}, \tag{2}$$

where $\dot{x}_{*}$ and $\dot{y}_{*}$ denote the speeds in the x_{i} and y_{i} direction respectively, whereas $\vec{\imath}$ and $\vec{\jmath}$ denote the unit vector in the x_{i} and y_{i} direction of the I-frame respectively. The speed u_{*} associated with a velocity $\vec{\mathrm{v}}_{*}$ can be determined as

$$u_{*} = |\vec{\mathrm{v}}_{*}| = \sqrt{\dot{x}_{*}^{2} + \dot{y}_{*}^{2}}. \tag{3}$$

B. Assumptions on water flows

To obtain schedules that take into account the effects of different water flows within reasonable computation times, some assumptions are made for modelling the problem:

A1 The water flow is uniform and time-invariant,
A2 Side-slip of the micro-ferries can be neglected,
A3 The acceleration and deceleration close to the stations can be neglected, as well as the changes of the water flows near the stations.

The first item ensures that $\vec{\mathrm{v}}_{\mathrm{r}}$ is constant, no matter where in the network the micro-ferries are, and at which time instant. The second assumption justifies modelling the energy consumption using only the vessel speed (hence it is orientation-independent). The final assumption is valid if the travelled distances are long enough to neglect differences in vessel speeds and water flow at the start and end of a travelled path.

C. Paths of the micro-ferries

Paths are defined as displacements over time in a reference frame. In the following derivations we will assume that all micro-ferries travel with the same speed u_{b} relative to the water and in a straight line. Hence the displacement over time of a micro-ferry can be represented by a vector. We define the vectors $\vec{\mathrm{p}}_{\mathrm{i}}$, $\vec{\mathrm{p}}_{\mathrm{b}}$, and $\vec{\mathrm{p}}_{\mathrm{r}}$ associated with the velocities discussed above. A displacement $\vec{\mathrm{p}}_{*}$ can be decomposed as

$$\vec{\mathrm{p}}_{*} = x_{*}\vec{\imath} + y_{*}\vec{\jmath}, \tag{4}$$

with the associated path length

$$l_{*} = |\vec{\mathrm{p}}_{*}| = \sqrt{x_{*}^{2} + y_{*}^{2}}. \tag{5}$$

The path $\vec{\mathrm{p}}_{\mathrm{i}}$ that a micro-ferry should travel in the I-frame between e.g. the pick-up location and the delivery location for a transportation request is known beforehand. It is a straight-line path between both stations, as depicted in Figure 2. The length of the path $\vec{\mathrm{p}}_{\mathrm{r}}$ —representing the contribution of the water flow to the displacement of the micro-ferry— is proportional to the travel time T, as $\vec{\mathrm{p}}_{\mathrm{r}} = T\vec{\mathrm{v}}_{\mathrm{r}}$. Figure 2 shows that in order to travel the same path in the I-frame, the travelled path $\vec{\mathrm{p}}_{\mathrm{b}}$ of the vessel on the water will change as the travel time (and hence the velocity $\vec{\mathrm{v}}_{\mathrm{b}}$) changes.

III. Scheduling in Flowing Water

With respect to the scheduling problem for micro-ferries, the velocity of the micro-ferry has an effect on two distinct properties; both the **travel time** and the **energy consumption** of the micro-ferries change when changing the velocity. Which velocity is needed to correctly determine the value of these two quantities is discussed next.

A. Effect of flowing water on time

Due to the flowing water the time it takes to travel from one location to another will be affected. This section explains how the currents affect the travel times, and it introduces the concept of request times.

1) Travel time: The travel time of a micro-ferry equals the distance travelled divided by the travel speed; more specifically it is the time it takes to travel from one station to the next. Due to the current, the path $\vec{\mathrm{p}}_{\mathrm{b}}$ of the micro-ferry relative to the water will be dependent on the velocity $\vec{\mathrm{v}}_{\mathrm{r}}$ of the water relative to the land, and the travel time T. Note that for a micro-ferry travelling with a velocity $\vec{\mathrm{v}}_{\mathrm{i}}$ in the I-frame, the travel time T and the travelled path $\vec{\mathrm{p}}_{\mathrm{i}}$ are related as

$$\vec{\mathrm{p}}_{\mathrm{i}} = T\vec{\mathrm{v}}_{\mathrm{i}}. \tag{6}$$

Combined with the relation between the different velocities as given in (1), the velocity of the micro-ferry relative to the water can be written as

$$\vec{\mathrm{v}}_{\mathrm{b}} = \frac{1}{T}\vec{\mathrm{p}}_{\mathrm{b}} = \frac{1}{T}\vec{\mathrm{p}}_{\mathrm{i}} - \vec{\mathrm{v}}_{\mathrm{r}}. \tag{7}$$

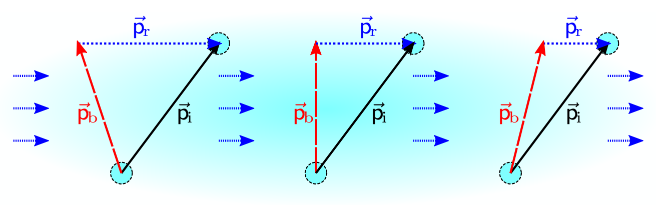

Fig. 2. *The same path $\vec{\mathrm{p}}_{\mathrm{i}}$ in the inertial frame can be accomplished at different velocities, resulting in different paths $\vec{\mathrm{p}}_{\mathrm{b}}$ in the body frame*

Since both $\vec{p}_i$ and $\vec{v}_r$ are constants, the velocity of the micro-ferry through the water only varies with the travel time T. The speed u_b is related to the water flow and the displacement by

$$\begin{aligned} u_b^2 &= |\vec{v}_b|^2 = \dot{x}_b^2 + \dot{y}_b^2 = \left(\frac{x_i}{T} - \dot{x}_r\right)^2 + \left(\frac{y_i}{T} - \dot{y}_r\right)^2 \\ &= \left(\dot{x}_r^2 + \dot{y}_r^2\right) - 2\left(x_i\dot{x}_r + y_i\dot{y}_r\right)\frac{1}{T} + \left(x_i^2 + y_i^2\right)\frac{1}{T^2} \\ &= u_r^2 - 2\left(x_i\dot{x}_r + y_i\dot{y}_r\right)\frac{1}{T} + l_i^2\frac{1}{T^2}. \end{aligned} \tag{8}$$

Since $l_b = u_b T$, we have

$$l_b^2 = u_b^2 T^2 = u_r^2 T^2 - 2\left(x_i\dot{x}_r + y_i\dot{y}_r\right)T + l_i^2. \tag{9}$$

This equation has the form $\alpha T^2 + \beta T + \gamma = 0$ with

$$\alpha = u_r^2 - u_b^2, \quad \beta = -2\left(x_i\dot{x}_r + y_i\dot{y}_r\right), \quad \gamma = l_i^2, \tag{10}$$

and hence T can be determined using the quadratic formula. To ensure that the micro-ferries can move forward against the current, the speed of the micro-ferry u_b should be larger than the speed of the water u_r, resulting in $\alpha < 0$. Furthermore, $\gamma > 0$ since it represents the distance between two stations. Therefore, using the variables defined in (10) the discriminant of Equation (9) satisfies

$$\Delta = \beta^2 - 4\alpha\gamma > \beta^2 > 0. \tag{11}$$

The second inequality shows that there are two distinct real-valued solutions for T, whereas the first inequality shows that

$$-\beta + \sqrt{\Delta} > -\beta + |\beta| \geq 0, \quad -\beta - \sqrt{\Delta} < -\beta - |\beta| \leq 0. \tag{12}$$

Since $\alpha < 0$, positive travel times T can be found by

$$\begin{aligned} T &= \frac{-\beta - \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} = \frac{-\frac{1}{2}\beta - \sqrt{\frac{1}{4}\beta^2 - \alpha\gamma}}{\alpha} \\ &= \frac{\left(x_i\dot{x}_r + y_i\dot{y}_r\right) - \sqrt{\left(x_i\dot{x}_r + y_i\dot{y}_r\right)^2 - (u_r^2 - u_b^2)l_i^2}}{u_r^2 - u_b^2}. \end{aligned} \tag{13}$$

Note that the water flow-related coefficients $\dot{x}_r, \dot{x}_r, u_r$ are constant for all possible trajectories between the stations, whereas the coefficients x_i, y_i, l_i depend on the start and end location a and z for a certain transportation. The speed u_b of the micro-ferry in the water could be varied (as was done in e.g. [11]), but since the introduction of currents in the scheduling problem results in highly non-linear energy-consumption terms (as will be shown next) in the speed variable u_b, it is chosen to be constant in this paper. Therefore, the travel time T_{az} from location a to z can be determined *a priori* for solving the optimisation problem, but it will vary with the current velocity.

2) Request time: A transportation of passengers associated with request j consists of the following steps (see Figure 3):

- a micro-ferry should relocate from some station towards the pick-up location for request j,
- the customer(s) should embark the micro-ferry,
- the micro-ferry should transport the customer(s) to the delivery location for request j,
- the customer(s) should disembark the micro-ferry.

Each of these four steps take time, which should be accounted for in the scheduling of pick-up times t_j.

For embarking and disembarking the micro-ferries a (combined) duration t_d can be chosen by the operator, which will be a trade-off between giving the customers enough time to safely enter and exit the micro-ferry, and not wasting time at the station. The duration for the relocation of the micro-ferry from the delivery station of request i towards the pick-up location of station j is given by the travel time $T_{d_i p_j}$ calculated using (13), where d_i and p_j denote the index number of the delivery station of request i and the index number of the pick-up station of request j respectively. The duration for the transportation of the customers from pick-up station p_j to delivery station d_j is given by the travel time $T_{p_j d_j}$ in (13), where p_j and d_j denote the index number of the pick-up and delivery station of request j respectively. We define the *request time* θ_{ij} as the time it *would take* to handle request j after request i given by

$$\theta_{ij} = T_{d_i p_j} + T_{p_j d_j} + \mathsf{t}_d, \tag{14}$$

which is constant for a steady water flow; the value of this constant depends on the travel time that depends on the amplitude and direction of the water flow through (13).

Note: The request time θ_{ij} does not provide the actual time it takes to handle request j, but it represents the time it takes to handle request j *if it would be preceded by request* i.

B. Effect of flowing water on energy consumption

Due to the flowing water the micro-ferries might need more or less energy to travel from one location to another as compared to still water, depending on whether or not they are travelling against the current. This section explains how the currents affect the energy consumption, and it introduces the concept of request consumption.

1) Energy consumption: In [1] it was argued that the instantaneous power of a micro-ferry can be described by

$$P = \pi_2 u^2 + \pi_1 u + \pi_0, \tag{15}$$

such that —when the vessel speed u is constant and T is the travel time— the energy consumption becomes

$$E = PT = \left(\pi_2 u^2 + \pi_1 u + \pi_0\right)T. \tag{16}$$

The speed u used in these equations should be the speed of the vessel relative to the water (i.e. u_b), since this is the speed that determines the energy consumption. The time T in (16) equals the travel time $T = T_{az}$ between stations a and z given by (13). Therefore, the energy consumption from a to z is

$$E_{az} = PT_{az} = \left(\pi_2 u_b^2 + \pi_1 u_b + \pi_0\right)T_{az} \tag{17}$$

which is a non-linear equation in the variables u_b representing vessel speed; the travel time T_{az} is a non-linear function in the vessel speed u_b as given by (13). By choosing a constant speed u_b, the travel time will be constant for a certain current, and hence the energy consumption E_{az} needed to travel from a to z is constant, and can be determined *a priori* for the optimisation problem using (17).

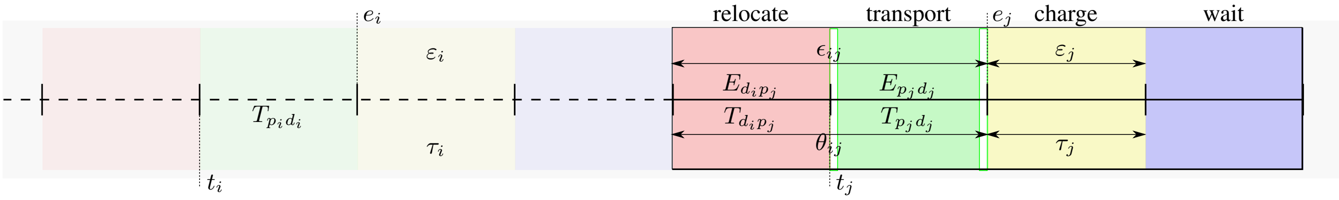

Fig. 3. *Overview of the four phases of a request, with the associated energy (top) and time (bottom) variables*

2) Request consumption: Similar to the request time defined in (14), we define the *request consumption* ϵ_{ij} as the consumed energy when request j would succeed request i — combining the energy consumption during the relocation phase and transportation phase (see Figure 3)— given by

$$\epsilon_{ij} = E_{d_i p_j} + E_{p_j d_j}, \tag{18}$$

where the terms $E_{d_i p_j}$ and $E_{p_j d_j}$ can be obtained using (17).

Note: The term ϵ_{ij} represents the energy that is needed to handle request j *if it would be preceded by request* i.

IV. Flowing-Water Micro-Ferry Scheduling

A. Description of the problem

The scheduling problem for micro-ferries over water can be described as follows. Using a fleet of M micro-ferries one wants to handle N requests to transport customers between several locations. Representing the micro-ferries by a set of nodes $\mathcal{M} = \{1, \ldots, M\}$, and the new requests by a set of nodes $\mathcal{N} = \{M+1, \ldots, R\}$ with $R = M + N$, the problem can be stated as (a variant of) a multi-depot travelling salesman problem [2]. The micro-ferry nodes in $\mathcal{M}$ represent the depots, whereas the request nodes in $\mathcal{N}$ represent the cities. Nodes in $\mathcal{R} = \{1, \ldots, R\}$ are associated with transportations of customers between pick-up and delivery locations, whereas arcs between the nodes are associated with relocations between the delivery and pick-up locations without customers aboard.

B. Optimisation variables

To describe the micro-ferry scheduling problem for flowing water, the following variables are used:

- $x_{ij} \in \{0, 1\}$: binary variable representing whether ($x_{ij} = 1$) or not ($x_{ij} = 0$) request j succeeds request i,
- $y_j \in \{0, 1\}$: binary variable representing whether ($y_j = 1$) or not ($y_j = 0$) the micro-ferry is recharged after request j,
- $t_j \in \mathbb{R}_+$: pick-up time for the passengers of request j,
- $s_j \in \mathbb{R}_+$: time window mismatch for request j,
- $\tau_j \in \mathbb{R}_+$: charging time after handling request j,
- $\theta_{ij} \in \mathbb{R}_+$: time for handling request j if preceded by i,
- $k_j \in \{1, \ldots, M\}$: micro-ferry number handling request j,
- $e_j \in [0, \mathsf{E}]$: energy level after completion of request j,
- $\xi_j \in [0, \mathsf{E}]$: energy charged after handling request j,
- $\epsilon_{ij} \in [0, \mathsf{E}]$: energy consumed during request j if preceded by request i.

The charging time τ_j is defined as the fixed time $\mathsf{t_c}$ it takes to couple and decouple the micro-ferry to the charger, plus the time $\frac{1}{\mathsf{r_c}}\xi_j$ it takes to increase the energy level by ξ_j. This gives

$$\tau_j = \mathsf{t_c} y_j + \frac{1}{\mathsf{r_c}} \xi_j. \tag{19}$$

Due to the relation (19) it possible to use either the charging time τ_j or the charged energy ξ_j as optimisation variable; both are used in the description of the problem for ease of presentation and understanding. For the actual optimisation only the variable τ_j is used, where ξ_j is substituted using (19) to represent it in terms of τ_j and y_j. For the optimisation variables τ_j and y_j it should hold that

$$\tau_j = 0 \quad \Leftrightarrow \quad y_j = 0, \tag{20}$$

meaning that the charging time will be zero *if and only if* no charging is scheduled ($y_j = 0$). Notice that through (19) this also means that the charged energy $\xi_j = 0$ if $\tau_j = 0$ and $y_j = 0$. The equivalence (20) will be assured by the constraints of the optimisation problem presented next (see Appendix A).

C. Mixed-integer linear programming formulation

In previous work the micro-ferry scheduling problem has been developed for still water, using the micro-ferry speed as an optimisation variable [1], [10], [11]. For flowing water the energy consumption term becomes a highly non-linear function in the speed u_b as discussed in Section III-B. To keep the focus on the effect of flowing water the speed is fixed in this paper.

Using the variables defined above, the micro-ferry scheduling problem for flowing water can be stated as the MILP

$$\min.\ \alpha_{\text{ec}} \sum_{i\in\mathcal{R}} \sum_{j\in\mathcal{R}} \epsilon_{ij} x_{ij} + \alpha_{\text{tw}} \sum_{j\in\mathcal{R}} s_j \tag{21a}$$

$$\text{s.t.}\ \sum_{i\in\mathcal{R}} x_{ij} = 1; \quad \sum_{i\in\mathcal{R}} x_{ji} = 1 \qquad \forall\, j\in\mathcal{R} \tag{21b}$$

$$t_i - t_j + \theta_{ij} + \tau_i \le \mathsf{T}(1 - x_{ij}) \qquad \forall\, i\in\mathcal{R}, j\in\mathcal{N} \tag{21c}$$

$$\mathsf{t}_{\text{c}} y_j \le \tau_j \qquad \forall\, j\in\mathcal{R} \tag{21d}$$

$$t_{\text{a},j} - s_j \le t_j \le t_{\text{b},j} + s_j \qquad \forall\, j\in\mathcal{R} \tag{21e}$$

$$|e_i - e_j + \xi_i - \epsilon_{ij}| \le \mathsf{E}(1 - x_{ij}) \qquad \forall\, i\in\mathcal{R}, j\in\mathcal{N} \tag{21f}$$

$$\xi_j \le \mathsf{E} y_j \qquad \forall\, j\in\mathcal{R} \tag{21g}$$

$$e_j + \xi_j \le \mathsf{E} \qquad \forall\, j\in\mathcal{R} \tag{21h}$$

$$\xi_j = \mathsf{r}_{\text{c}} \tau_j - \mathsf{r}_{\text{c}} \mathsf{t}_{\text{c}} y_j \qquad \forall\, j\in\mathcal{R} \tag{21i}$$

$$k_i - k_j \le (M-1)(1 - x_{ij} - x_{ji}) \qquad \forall\, i\in\mathcal{R}, j\in\mathcal{N} \tag{21j}$$

$$t_j = t_{\text{o},j}; \quad e_j = e_{\text{o},j}; \quad k_j = k_{\text{o},j} \ \forall\, j\in\mathcal{M} \tag{21k}$$

$$x_{ij}, y_j \in \{0,1\} \qquad \forall\, i,j\in\mathcal{R} \tag{21l}$$

where E is the upper bound on the energy levels e_j, and T should be chosen larger than the latest expected pick-up time (conform the big-M method [12]). The constants r_{c}, t_{c}, and t_{d} represent the charging rate, the fixed charging time, and the disembarking plus embarking time respectively. The initial conditions for the pick-up times, the energy levels, and the index numbers of the micro-ferries are represented by $t_{\text{o},j}$, $e_{\text{o},j}$, and $k_{\text{o},j}$ respectively.

The objective function (21a) consists of the total energy consumption (first term) and the total time-window misfit (second term). A trade-off between using less energy and assigning less pick-up times outside the desired time windows can be made by changing the weights α_{ec} and α_{tw}.

Equalities (21b) are the assignment constraints ensuring that every request is handled once and only once, (21c) ensures consistency in the pick-up times, (21d) ensures that the charging time is larger than the fixed charging time (needed to connect the micro-ferry to the charger) when charging, and (21e) assigns values to the slack variables s_j representing the misfit to the desired time-window $[t_{\text{a},j}, t_{\text{b},j}]$.

Inequalities (21f) set the energy levels after delivery for request j equal to the energy level after delivery for request i, plus the charged energy during request i, minus the energy consumption during request j, when $x_{ij} = 1$ (see Figure 3). Inequality (21g) ensures that the energy increase ξ_j is zero when no charging is scheduled, whereas (21h) avoids over-charging by limiting the charged energy to the maximum level E minus the level e_j before charging. Either the equality constraint (21i) should be used to enforce (19), or ξ_j can be eliminated from (21f–21h) by using the equivalence (21i), and removing this equality constraint from the MILP.

Due to (21j) each request is assigned a unique index number, and the initial conditions for the micro-ferries are set by (21k). The variables x_{ij} and y_j should be treated as binary variables, as stated by (21l).

V. Case Study

As a case study the current-dependent program (21) is used to schedule micro-ferries in a part of the Rotterdam harbour. All computations are performed on a desktop computer with an Intel Core2 Quad Q8400 processor and 4 GB of RAM, running 64-bit versions of SUSE Linux Enterprise Desktop 11, Matlab R2012a, and Tomlab 7.8 using CPLEX 12.2.

A schematic drawing of the harbour is shown in Figure 4. The dotted line indicates the centre of the river Maas passing through the city, flowing downstream (from the east to the west) in the direction of the North Sea. Several fictive docking stations have been added to the map, where passengers may enter and exit the micro-ferries.

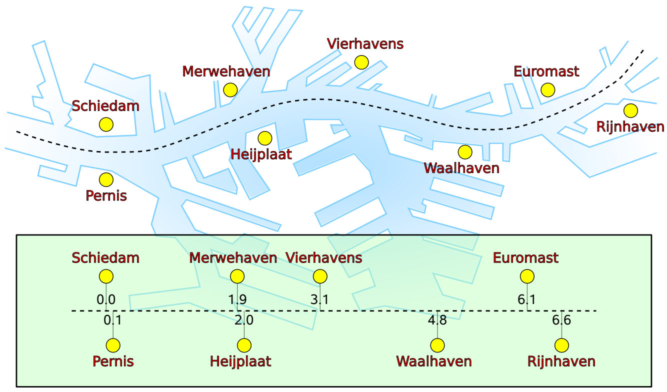

Fig. 4. Schematic drawing of the Rotterdam harbour, including docking locations for the micro-ferries

As the main flow of the current will follow the shape of the river, the dotted line can be considered the axis along which the water moves in parallel. Therefore, the scheduling problem along the curved river as shown at the top of Figure 4 can be transformed into the equivalent problem of solving the scheduling problem in the network shown at the bottom of Figure 4 with a uniform current in parallel to the dotted line. The along-path distances (in $[\text{km}]$) are shown along the horizontal axis, whereas the vertical distance to the centre axis is 250 $[\text{m}]$ for all locations. The locations are labelled as s_1 to s_8, with their index number increasing from left to right.

Since the current flows in parallel with the x_{i}-axis (the dotted line), the relative speed $\dot{y}_{\text{r}} \equiv 0$ whereas $\dot{x}_{\text{r}}$ can have a non-zero value. Currents in the range from -5 to +5 $[\text{m/s}]$ are used to investigate the effect of a current on the energy consumption and time-window mismatch for the micro-ferry scheduling problem. We choose $u_{\text{b}} = 10\ [\text{m/s}]$ as the relative speed of the micro-ferries, and the energy level $e_j \in [0, 100]$ represents the percentage of energy left. The instantaneous power is chosen such that a fully charged micro-ferry ($e_j = 100$) can travel for 20 $[\text{km}]$, resulting in $P = \frac{100}{20000} \cdot 10 \left[\frac{\%}{\text{m}} \cdot \frac{\text{m}}{\text{s}}\right] = 0.05 \left[\frac{\%}{\text{s}}\right]$. A test case with $M = 8$ micro-ferries and $N = 40$ new request is randomly generated. The resulting energy consumption J_{ec} in $[\%]$, time-window mismatch J_{tw} in $[\text{s}]$, and empty travel time[2] J_{tw} in $[\text{s}]$ are given in Table I for different current speeds $\dot{x}_{\text{r}}$.

[2]The time a micro-ferry relocates without a passenger aboard.

TABLE I
RESULTS FOR FLOW-DEPENDENT SCHEDULING

$\dot{x}_\mathrm{r}$	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5
J_ec	651	594	547	526	517	506	517	532	560	618	694
J_tw	97	42	30	22	19	18	13	10	9	10	13
J_et	983	906	828	786	763	743	751	804	847	858	959

The results show that both the energy consumption and the empty travel time are minimal when no current is present, and their values increase with an increasing amplitude of the current. Even though the problem is a multi-objective optimisation problem, this result can be accounted for by the fact that the *average travel time* increases when the strength of the current increases. Therefore, for a random set of pick-up and delivery locations it is to be expected to see an increase in the energy consumption (which equals a constant instantaneous power times the travel time) and the empty travel distance (which equals the constant micro-ferry speed times the travel time) when the amplitude of the current increases. See Appendix B for more details.

The minimum of the time-window mismatch is not obtained when no current exists, but when the current has a speed of $\dot{x}_\mathrm{r} = 3$ [m/s]. Although there are as many transportations from west to east as from east to west, due to the desired time windows, it is easier to schedule the pick-ups on time when a small current exists flowing from west to east. Since these time windows are obtained randomly, it is expected that the minimum of J_tw can be different for every set of transportation requests.

VI. CONCLUSION

This paper has introduced the effect of currents in a micro-ferry scheduling problem. To avoid schedules where micro-ferries run out of energy on the water, and to provide accurate pick-up times, it is important to take the effect of the flowing water into account. The influence of the current on the travel time is determined, and it was shown in theory and by example that the energy consumption will increase when the amplitude of the current will increase, irrespective of the direction. Although the energy consumption will always increase with larger currents, a different schedule using less energy may be obtained for a certain current compared to the obtained schedule without taking into account the currents.

This paper has discussed the effect of constant currents that flow perpendicular to the paths of the vessels; extensions to time-varying currents and flows from different directions are considered as future research. Furthermore, the vehicle speed is fixed in this paper while results exist for variable speeds and still water [1], [10], [11]; energy-efficient scheduling in flowing water with variable speeds is considered as future research.

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APPENDIX

A. No charging means zero charging time

The MILP (21) ensures that the relationship (20) holds. Indeed, by inequality (21d) and the integrality condition (21l) we have for $\tau_j = 0$ and $\mathsf{t}_\mathrm{c} > 0$ that

$$\mathsf{t}_\mathrm{c} y_j \leq \tau_j = 0 \quad \Rightarrow y_j = 0.$$

Also, by (19) and (21g) we have for $y_j = 0$ and $\mathsf{r}_\mathrm{c} > 0$ that

$$\xi_j = \mathsf{r}_\mathrm{c}(\tau_j - \mathsf{t}_\mathrm{c} y_j) = \mathsf{r}_\mathrm{c}\tau_j \leq \mathsf{E} y_j = 0 \quad \Rightarrow \tau_j = 0.$$

Therefore, we conclude that $\tau_j = 0 \;\Leftrightarrow\; y_j = 0$.

B. Current versus travel times

In the example —where the current flows in parallel to the riverbed such that $\dot{y}_\mathrm{r} \equiv 0$— the travel times defined in (6) only change with the current speed $\dot{x}_\mathrm{r}$. This gives

$$T = \frac{\sqrt{(x_\mathrm{i}\dot{x}_\mathrm{r})^2 + (u_\mathrm{b}^2 - u_\mathrm{r}^2) l_\mathrm{i}^2} - x_\mathrm{i}\dot{x}_\mathrm{r}}{u_\mathrm{b}^2 - u_\mathrm{r}^2}.$$

The travel time T_{az} from location a to z can be found by considering the displacement $\vec{\mathsf{p}}_{\mathrm{i},az} = (x_{\mathrm{i},az}, y_{\mathrm{i},az})$, which is a fixed vector in the I-frame. The displacement from location z to a is given by $\vec{\mathsf{p}}_{\mathrm{i},za} = -\vec{\mathsf{p}}_{\mathrm{i},az} = (-x_{\mathrm{i},az}, -y_{\mathrm{i},az})$, such that $l_{\mathrm{i},az} = l_{\mathrm{i},za}$. When $u_\mathrm{b}^2 - u_\mathrm{r}^2 > 0$[3] the difference in travel time of going one way or the other between a and z is

$$\Delta_T = T_{az} - T_{za} = \frac{-2x_\mathrm{i}\dot{x}_\mathrm{r}}{\sqrt{u_\mathrm{b}^2 - u_\mathrm{r}^2}}$$

which shows that the travel times are different if there is a current ($\dot{x}_\mathrm{r} \neq 0$) as expected. Perhaps less obvious is the fact that the travel time for a round trip (from a to z and back to a) has a larger travel time when the current's amplitude increases:

$$\Sigma_T = T_{az} + T_{za} = \frac{2\sqrt{(x_\mathrm{i}\dot{x}_\mathrm{r})^2 + (u_\mathrm{b}^2 - u_\mathrm{r}^2)\, l_\mathrm{i}^2}}{u_\mathrm{b}^2 - u_\mathrm{r}^2},$$

which has a minimum of

$$\Sigma_{T,\min} = 2\frac{l_\mathrm{i}}{\sqrt{u_\mathrm{b}^2 - u_\mathrm{r}^2}} \tag{22}$$

for $\dot{x}_\mathrm{r} = 0$, and Σ_T increases for larger currents. Hence, the larger $|\dot{x}_\mathrm{r}|$, the larger the travel times within the micro-ferry network, and —by (17)— the larger the energy consumption needed to handle the requests.

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[3]This holds for $|u_\mathrm{b}| > |u_\mathrm{r}|$, which is a necessary condition to be able to move forwards under all circumstances, as desired under normal operations.

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