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A Suboptimal Control Scheme of Multiple Trains Based on Mode Vector Constraints

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Abstract
The optimal control problem of multiple trains under a moving block signaling (MBS) system is considered in this paper. We reformulate this problem as a mixed integer linear programming (MILP) problem. With the increasing number of integer variables or the increasing number of trains taken into consideration, the computation time needed for solving the MILP problem increases quickly. A suboptimal control scheme with mode vector constraints is introduced to reduce the computation time but without a considerable effect on optimality. The good performance of the proposed MILP approach with mode vector constraints is demonstrated via a case study.

Keywords
Multiple trains operation, MILP, Mode vector constraints

1 Introduction

Rail traffic plays a key role in public transportation. More specifically, a safe, fast, punctual, energy-efficient, and comfortable railway system is important for the sustainability of the overall transportation system. Due to the increasing energy prices and environmental concerns, energy efficiency in railway systems is becoming more and more important.

Railway operations include two basic elements, viz. timetabling and traffic management [1], which are inherently linked. However, in practice the preplanned timetable cannot be executed precisely due to disturbances. Hence, real-time rescheduling is proposed to improve punctuality, to increase the network capacity, and to reduce energy consumption by identifying and resolving conflicts arising during actual operations [2]. Advanced train control systems, such as the European Train Control System, enable real-time rescheduling and energy-efficient driving [1], where the decisions made in control centers can be transmitted through GSM-R to on-board equipment, such as the driver machine interface and the automatic train operation system. The train driver or automatic operation system can then operate the train according to the decision given by the control center.

The optimization of speed profiles for trains is essential for the real-time rescheduling process since the speed profile has a significant impact on driving behavior, energy efficiency, and achievable accuracy, especially after the rescheduling. There are two important strategies to calculate the speed profiles for trains [3]. One possibility is that the control center only specifies operational constraints (e.g. reference passing time at particular points) during the rescheduling process and the on-board equipment, i.e. the driver machine interface or automatic train operation system, calculates the speed profile for each train. For more information on the speed profile optimization with operational constraints, see [4]. The other possibility is to calculate the speed profiles for multiple trains in the control center as part of the rescheduling process and next transmit speed profiles to on-board equipment. In this paper, we select the second option and then focus on the calculation of speed profiles for multiple trains.

The research on optimal control for a single train's operation began in the 1960s and various methods were proposed for the problem. These methods can be grouped into two main categories: analytical solutions and numerical optimization. For analytical solutions, the maximum principle is applied and it results in four optimal regimes (maximum traction, cruising, coasting, and maximum braking) [5–7]. It is difficult to obtain the analytical solution if more realistic conditions are considered, since these introduce more complex nonlinear terms into the model equations and the constraints [8]. Numerical optimization approaches are applied more and more for optimal control for trains due to the comparable high computing power available nowadays. A number of advanced techniques such as fuzzy and genetic algorithms have been proposed to calculate the optimal speed profile for trains, see e.g. [9–11]. However, in these approaches, the optimal solution is not always guaranteed. Therefore, we have formulated this problem as a mixed logical linear programming (MILP) problem in [12], which can be solved efficiently using existing commercial and free solvers.

The approaches mentioned above ignore the impact caused by the signaling systems, e.g. a fixed-block signaling system or a moving block signaling (MBS) system. Especially in busy railway systems, the train's operation will be adjusted by the signaling system. Lu and Feng [13] consider the constraints caused by the leader train in a four-aspect fixed block signaling system when optimizing the trajectory of the follower train. More specifically, a parallel genetic algorithm is used to optimize the trajectories for the leader train and the follower train and it results a lower energy consumption [13]. Gu et al. [14] apply a nonlinear programming method to optimize the trajectory for the follower train. Ding et al. [15] take the constraints caused by the MBS system into account and develop an energy-efficient multi-train control simulator that is used to calculate the optimal speed profiles. Three optimal control regimes, i.e. maximum traction, coasting, and maximum braking, are adopted in their simulator and the sequences of the optimal control regimes are determined by a predefined logic. In [16], we have extended the MILP approach that we have introduced in [12] to obtain the speed profiles for two trains under an MBS system.

The MILP problem can be solved efficiently for small to medium sized problems. However, the computation time grows quickly with the number of trains considered since the worst case computation time of an MILP problem grows exponentially with the number of integer variables. For sewer networks, an alternative suboptimal approach is proposed in [17] to reduce the complexity of the MILP computations by suitably constraining the mode vector. Therefore, in this paper we will apply mode vector constraints to the MILP approach for the optimization of speed profiles for multiple trains in order to reduce the computation time.

The remainder of this paper is structured as follows. In Section 2, the train model and the MILP approach for a single train are summarized based on [12]. Section 3 introduces the constraints for the follower train caused by the leader train under the MBS system. Two approaches are proposed to solve the optimal control problem for multiple trains: the greedy approach and the simultaneous approach. In Section 4, the mode vector constraints are introduced and the issues involved in the selection of an appropriate reference mode vector is discussed. Section 5 illustrates the performance of the MILP approach with mode vector constraints using data from the Beijing Yizhuang subway line. We conclude with a short discussion on some topics for future work in Section 6.

2 Train model and the MILP approach for a single train

The dynamics of a train can be described by the following simple continuous-time model [7]:

$$\begin{aligned} &m\rho\frac{\mathrm{d}v}{\mathrm{d}t} = u(t) - R_{\mathrm{b}}(v) - R_{\mathrm{l}}(s,v), \\ &\frac{\mathrm{d}s}{\mathrm{d}t} = v, \end{aligned} \tag{1}$$

where m is the mass of the train, ρ is a factor to consider the rotating mass [1], v is the velocity of the train, s is the position of the train, u is the control variable (i.e., the traction or braking force), which is bounded by the maximum traction force $u_{\max}$ and the maximum braking force $u_{\min}$, $u_{\min} \leq u \leq u_{\max}$, $R_{\mathrm{b}}(v)$ is the basic resistance including the roll resistance and air resistance, and $R_{\mathrm{l}}(s,v)$ is the line resistance caused by track slope, curves, and tunnels. We refer the interested reader to [12] for more details.

Franke et al. [18] choose the kinetic energy per mass unit $\tilde{E} = 0.5v^2$ and time t as states, and the position s as the independent variable. The optimal control problem for a single train is then formulated as [12]:

$$J = \int_{s_{\mathrm{start}}}^{s_{\mathrm{end}}} \max(u(s), 0)\mathrm{d}s \tag{2}$$

subject to the model (1) and the following constraints:

$$\begin{aligned} &u_{\min} \leq u(s) \leq u_{\max}, \\ &E_{\min}(s) \leq E(s) \leq E_{\max}(s), \\ &E(s_{\mathrm{start}}) = E_{\mathrm{start}}, \;\; E(s_{\mathrm{end}}) = E_{\mathrm{end}}, \\ &t(s_{\mathrm{start}}) = 0, \;\; t(s_{\mathrm{end}}) = T, \end{aligned} \tag{3}$$

where the objective function J represents energy consumption of the train without regenerative braking; $\tilde{E}_{\min}(s)$ is the minimum kinetic energy per mass, which is assumed larger than 0 (i.e., the train travels nonstop) [6]; $\tilde{E}_{\max}(s)$ is equal to $0.5V_{\max}^2(s)$ where $V_{\max}(s)$ is the maximum allowable velocity, which depends on the train characteristics and line conditions, and as such it is usually a piecewise constant function of the coordinate s [6, 7]; s_{start}, $\tilde{E}(s_{\mathrm{start}})$, and $t(s_{\mathrm{start}})$ are the position, the kinetic energy per mass, and the time at the beginning of the route; s_{end}, $\tilde{E}(s_{\mathrm{end}})$, and t_{end} are the position, the kinetic energy per mass, and the time at the end of the route; the scheduled running time T is given by the timetable or the rescheduling process.

The nonlinear continuous-time train model (1) is first discretized in the space interval $[s_{\mathrm{start}}, s_{\mathrm{end}}]$ by considering N subintervals, $[s_k, s_{k+1}]$ for $k = 1, 2, \ldots, N$, where $s_1 = s_{\mathrm{start}}$ and $s_{N+1} = s_{\mathrm{end}}$. It is assumed that the track and train parameters as well as the traction or braking force are constant in each subinterval. The dynamics of train operations can then be transformed into the following mixed logical dynamic (MLD) model (see [12] for details):

$$\begin{aligned} x(k+1) = &A_k x(k) + B_k u(k) + C_{1,k}\delta(k) + C_{2,k}\delta(k+1) \\ &+ D_{1,k} z(k) + D_{2,k} z(k+1) + e_k, \end{aligned} \tag{4}$$

$$\begin{aligned} &R_{1,k}\delta(k) + R_{2,k}\delta(k+1) + R_{3,k} z(k) + R_{4,k} z(k+1) \\ &\leq R_{5,k} u(k) + R_{6,k} x(k) + R_{7,k}. \end{aligned} \tag{5}$$

Based on the MLD model, the optimal control problem can be recast as the following MILP problem:

$$\min_{\tilde{V}} \quad C_J^T \tilde{V}, \tag{6}$$

subject to

$$F_1 \tilde{V} \leq F_2 x(1) + f_3 \tag{7}$$

$$F_4 \tilde{V} = F_5 x(1) + f_6 \tag{8}$$

where $C_J = \begin{bmatrix} 0 & \cdots & 0 & \Delta s_1 & \cdots & \Delta s_N \end{bmatrix}^T$, $\tilde{V} = \begin{bmatrix} \tilde{u}^T & \tilde{\delta}^T & \tilde{z}^T & \tilde{\omega}^T \end{bmatrix}^T$,

$$\tilde{u} = \begin{bmatrix} u(1) \\ u(2) \\ \vdots \\ u(N) \end{bmatrix}, \tilde{\delta} = \begin{bmatrix} \delta(1) \\ \delta(2) \\ \vdots \\ \delta(N+1) \end{bmatrix},$$

$$\tilde{z} = \begin{bmatrix} z(1) \\ z(2) \\ \vdots \\ z(N+1) \end{bmatrix}, \tilde{\omega} = \begin{bmatrix} \omega(1) \\ \omega(2) \\ \vdots \\ \omega(N) \end{bmatrix},$$

for properly defined F_1, F_2, f_3, F_4, F_5, and f_6. The MILP problem (6)-(8) can be solved by existing commercial and free solvers, such as CPLEX, Xpress-MP, GLPK (see e.g. [19,20]).

3 Optimal control problem for multiple trains in the moving block signaling system

In this paper, we consider the moving block signaling (MBS) system [21, 22]. An MBS system relies on the continuous bidirectional communication connections between trains and zone controllers. The zone controller makes sure that trains within the zone it controls will be running with a safe distance with respect to other trains by calculating the limit-of-movement-authority for trains. In the MBS system, the minimum distance between two successive trains is basically the instantaneous braking distance required by the follower train plus a safety margin. A constraint on this minimum distance is equivalent to a constraint on the minimum time difference of two successive trains:

$$t^{\mathrm{F}}(s) - t^{\mathrm{L}}(s) \geq t_{\mathrm{r}}^{\mathrm{F}} + t_{\mathrm{b}}^{\mathrm{F}}(s) + t_{\mathrm{safe}}^{\mathrm{F}}(s), \tag{9}$$

where $t^{\mathrm{L}}(s)$ and $t^{\mathrm{F}}(s)$ are the time instants at which of the front of respectively the leader train and the follower train pass position s, $t_{\mathrm{b}}^{\mathrm{F}}(s)$ is the braking time of the follower train, and $t_{\mathrm{safe}}^{\mathrm{F}}(s)$ is the time margin due to the safe margin distance and the train length. We refer to [16] for more details. The constraint (9) due to the MBS system can then be discretized at the grid points s_k as

$$t^{\mathrm{F}}(k) \geq t^{\mathrm{L}}(k) + t_{\mathrm{r}}^{\mathrm{F}} + t_{\mathrm{b}}^{\mathrm{F}}(k) + t_{\mathrm{safe}}^{\mathrm{F}}(k), \quad \text{for} \quad k = 1, 2, \ldots, N-1, \tag{10}$$

$$t^{\mathrm{F}}(k) \geq t^{\mathrm{L}}(k) + t_{\mathrm{r}}^{\mathrm{F}} + t_{\mathrm{b,max}}^{\mathrm{F}} + t_{\mathrm{d}}^{\mathrm{L}} + t_{\mathrm{safe}}^{\mathrm{L}}, \quad \text{for} \quad k = N. \tag{11}$$

In addition, some intermediate constraints are introduced to ensure that the points in between the grid points also satisfy the constraints due to the MBS system. According to (9), we obtain the following constraint for each $s \in [s_k, s_{k+1}]$ as:

$$t^{\mathrm{F}}(s) - t_{\mathrm{r}}^{\mathrm{F}} - t_{\mathrm{b}}^{\mathrm{F}}(s) - t_{\mathrm{safe}}^{\mathrm{F}}(s) \geq t^{\mathrm{L}}(s). \tag{12}$$

If we assume the left-hand side of (12) to be an affine function in the interval $[s_k, s_{k+1}]$, then we can add the following constraints:

$$\begin{aligned}(1-\alpha)(t^{\mathrm{F}}(k) - t_{\mathrm{r}}^{\mathrm{F}} - t_{\mathrm{b}}^{\mathrm{F}}(k) - t_{\mathrm{safe}}^{\mathrm{F}}(k)) + \alpha(t^{\mathrm{F}}(k+1) - t_{\mathrm{r}}^{\mathrm{F}} \\ - t_{\mathrm{b}}^{\mathrm{F}}(k+1) - t_{\mathrm{safe}}^{\mathrm{F}}(k+1)) \geq t^{\mathrm{L}}(s + \alpha \Delta s_k),\end{aligned} \tag{13}$$

for some real values α in a finite subset $S_\alpha \in [0,1)$, e.g. $S_\alpha = \{0.1, 0.2, \ldots, 0.9\}$, where $t^{\mathrm{L}}(s + \alpha\Delta s_k)$ is known in the greedy approach since the speed profile of the leader train is optimized first. Note that for $\alpha = 0$ and $\alpha = 1$ (10) is retrieved (except if $k = N-1$). In the simultaneous MILP approach, we need to optimize both trajectories simultaneously. In this case, the term $t^{\mathrm{L}}(s + \alpha\Delta s_k)$ is unknown. If we assume the right-hand side of (12) is also an affine function, i.e. $t^{\mathrm{L}}(s + \alpha\Delta s_k) = (1-\alpha)t^{\mathrm{L}}(k) + \alpha t^{\mathrm{L}}(k+1)$, then it is sufficient to check (12) in the points k and $k+1$ (i.e., for $\alpha = 0$ and $\alpha = 1$), since due to linearity (12) will then also automatically be satisfied for all intermediary points. The constraints (10), (11), and (13) can be approximated as linear constraints [16], which can be easily included in the MILP approach.

There exist two solution approaches for the optimal control problem for two trains in the MBS system. One is a greedy approach, where we first optimize the speed profile of the leader train, and next optimize the follower train based on the results of the leader train. The other solution approach consists in optimizing speed profiles of the two trains simultaneously. Furthermore, these two approaches proposed here can be extended to multiple trains.

4 Mode vector constraints

The computation time needed for solving an MILP problem for a single train is usually small if we take a small value for N. However, the computation time increases quickly with the value of N and the number of trains considered in the MILP problem for multiple trains. In the worst case, the computation time grows exponentially with the number of integer variables. In order to solve the MILP problem for multiple trains in a reasonable time, we introduce the mode vector constraints which has been applied to the sewer networks [17].

A mode of the MLD model for a single train is referred to a specific value of the binary vector $\delta^i = \begin{bmatrix}\delta_1^i & \delta_2^i & \ldots & \delta_{M_i}^i\end{bmatrix}^T$, where i represents the ith train and M_i denotes the dimension of the binary vector of the ith train (see the MILP formulation (6)-(8)). Furthermore, a mode vector is defined as a tuple of binary vectors for each train considered in the MILP problem, i.e. $\Delta = (\delta^{1T}, \delta^{2T}, \ldots, \delta^{IT})^T$, where I is the number of trains considered in the problem. Let $\bar{\Delta} = (\bar{\delta}^{1T}, \bar{\delta}^{2T}, \ldots, \bar{\delta}^{IT})^T$ be a reference mode vector of the MILP problem for multiple trains. Such a vector can be generated by solving the optimal control problem for each train sequentially, since the computation time of the solving I single-train MILP problems will be much less than solving the full I-trains MILP problem all at once. Note however that this comes at a cost of reduced optimality; the optimality can next be improved again by solving the full I-trains MILP problem with the mode constraints. The mode vector constraints can then be defined as

$$\sum_{m=1}^{M_i} |\bar{\delta}_m^i - \delta_m^i| \leq D_i \quad \text{for} \quad i = 1, 2, \ldots, I, \tag{14}$$

or

$$\sum_{i=1}^{I} \sum_{m=1}^{M_i} |\bar{\delta}_m^i - \delta_m^i| \leq D, \tag{15}$$

where D_i and D are preselected bounds (a nonnegative integer value) on the number of switches from the reference mode vector. Note that the mode vector constraints (14) and (15) can be recast as linear constraints by introducing some auxiliary variables to deal with the absolute values $|\bar{\delta}_m^i - \delta_m^i|$ for $i = 1, 2, \ldots, I$ and $m = 1, 2, \ldots, M_i$. Here, we only consider mode vector constraint (15). Furthermore, the mode vector constraints can be seen as the Hamming distance between Δ and $\bar{\Delta}$ if we would expand Δ and $\bar{\Delta}$ into binary strings [17].

An important practical problem for the mode vector constraints is to find $\bar{\Delta}$ such that the MILP problem for multiple trains including the mode vector constraints is still feasible. It is stated in [23] that physics or heuristic knowledge of the system can often be used to find a feasible solution that fulfills the physical constraints of the system. As we mentioned before, a good candidate for the reference mode vector can be obtained by solving the optimal control problem for multiple trains sequentially.

5 Case study

In order to assess the control performance and computation time of the MILP approach with mode vector constraints and to compare it with the original MILP approach, the optimal control problem for two trains under the MBS system is considered. The line data and train characteristics of the Beijing Yizhuang subway line are used as a case study. In this paper, we consider the last two stations in the Yizhuang subway line: Ciqu and Yizhuang. The track length between these two stations is 2610 m and the speed limits and grade profile are shown in Figure 1. The parameters of the train and the line path are listed in Table 1. The rotating mass factor is often chosen as 1.06 in the literature [1] and therefore we also adopt this value. According to the assumption made in [12], the unit kinetic energy should be larger than zero. In this test case, the minimum kinetic energy $E_{\min}$ is chosen as 0.1 J. The maximum traction force of the train in the Yizhuang line is a nonlinear function of the train's velocity and the maximum value of this function is 315 kN. The objective function of the optimal train control problem considered here is the energy consumption of the train operation without regenerative braking (cf. (2)).

In the case study, the headway between the leader train and the follower train is assumed as 70 s. In addition, we assume that there exists one extra constraint on the leader train due to

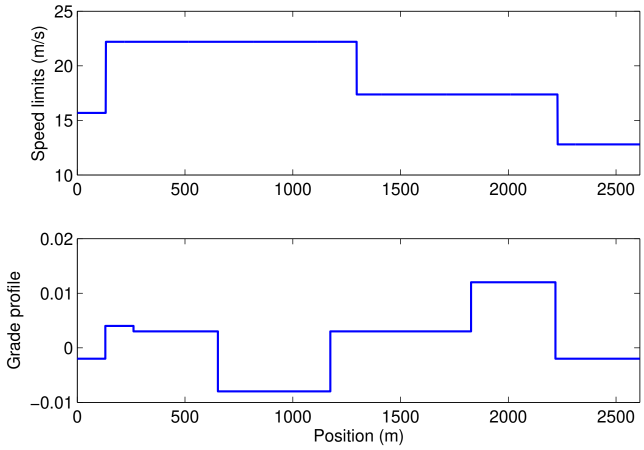

Figure 1: The speed limits and the grade profile between Ciqu station and Yizhuang station

Table 1: Parameters of train and line path

Property	Symbol	Value
Train mass [kg]	m	$4.15 \cdot 10^5$
Basic resistance [N/kg]	R_b	$0.0142 + 4.1844 \cdot 10^{-5} v^2$
Mass factor	ρ	1.06
Maximum velocity [m/s]	V_max	22.2
Line length [m]	s_end	2610
Minimum kinetic energy [J]	E_min	0.1
Maximum traction force [N]	u_max	$3.15 \cdot 10^5$
Maximum braking force [N]	u_min	$-3.32 \cdot 10^5$

technical reasons: the speed after 1300 m is limited to 40 km/h, i.e. 11.1 m/s. Note that this constraints only apply to the leader train, not to the follower train. Furthermore, we assume that the leader train and the follower train will arrive to different platforms in Yizhuang station. The running times for the leader train and the follower train are 216 s and 194 s, respectively.

The length Δs_k for interval $[s_k, s_{k+1}]$ depends on the speed limits, gradient profile, and so on. In addition, if the number of the space intervals N is larger, then the accuracy will be better, but the computation time of the MILP approach will be longer. We choose 40 space intervals here, which implies that the length of each interval is 65.25 m, i.e. $\Delta s_k = 65.25$ m for $k = 1, 2, \ldots, 40$. Furthermore, the nonlinear terms are approximated by using PWA approximations with 3 subfunctions. The calculation time of the MILP greedy approach is 55.34 s as shown in Table 2 and the optimal control inputs obtained are shown in Figure 2. The solid line and the dash-dotted line represent the optimal inputs of the leader train and the follower train. The speed profiles of the leader train and the follower train shown in Figure 2 are obtained by applying these inputs to the nonlinear train model (1). The calculation time of the optimal control problem of two trains using the simultaneous MILP approach

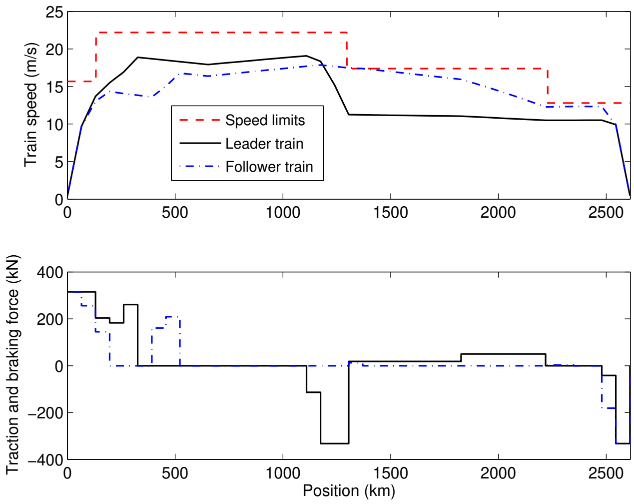

Figure 2: The optimal control inputs obtained by the greedy MILP approach and the trajectories generated by the nonlinear continuous-time train model (1) using optimal control inputs

with $N = 40$ is much longer: it is equal to 653.80 s. The optimal control inputs produced by the simultaneous MILP approach and the corresponding speed profiles are illustrated in Figure 3.

The optimal results of the greedy MILP approach yield the value of the reference mode vector $\bar{\Delta}$. The MILP problem with mode vector constraints using the simultaneous approach can then be solved. In this case study, we choose the value of D in (15) in the set $\{0, 1, 2, \ldots, 6, 10, 11, 12\}$. The performance of the simultaneous MILP approach with mode vector constraints are given in Table 2. It can be observed that there is a considerable reduction in calculation time for $D = 0, 1, 2$ compared to the other cases, where the calculation time of the MILP problem with mode vector constraints is less than 15 s if the reference mode vector is given. The total CPU time for $D = 0, 1, 2$ is less than 70 s when the computation time of the reference mode vector, i.e. the computation time of the MILP problem for multiple trains using the greedy approach, is included. The calculation time grows quickly when D increases. When D is larger than or equal to 10, adding mode vector constraints does not improve the calculation time but rather deteriorates it. When $D = 0$, the total energy consumption of the MILP approach with mode vector constraints is 1% larger than that of the original simultaneous MILP approach and the total end time violation is doubled. So there is only a limited decrease in the optimality. It is seen that with the increase of D, the total energy consumption and the total end time violation decrease, but this happens at the cost of a longer computation time.

Table 2: Performance comparison of the greedy approach, the simultaneous approach, and the simultaneous approach with mode vector constraints

Approach	D	Total energy consumption [MJ]	Total end time violation [s]	CPU time for given $\bar{\Delta}$	Total CPU time [s]
Greedy	-	185.20	2.03	-	55.34
Simultaneous	-	181.86	1.91	-	653.80
Simultaneous with mode vector constraints	0	183.69	3.97	1.59	56.93
	1	183.51	3.42	3.79	59.13
	2	183.18	3.38	13.39	68.73
	3	183.00	3.34	53.34	108.68
	4	182.59	3.31	56.85	112.19
	5	182.41	3.34	79.62	134.96
	6	182.21	3.17	95.91	151.25
	10	181.63	2.92	637.39	692.73
	11	181.60	2.88	697.13	752.47
	12	181.59	2.86	982.52	1037.86

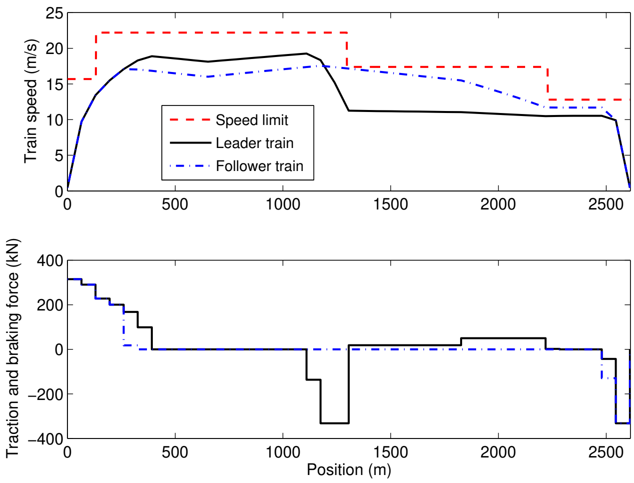

Figure 3: The optimal control inputs obtained by the simultaneous MILP approach and the trajectories generated by the nonlinear continuous-time train model (1) using optimal control inputs

6 Conclusions and future work

A suboptimal control scheme for the optimal control problem of multiple trains under a moving block signaling (MBS) system has been proposed in this paper. By using the MILP approach with mode vector constraints, the computation time can be reduced remarkably by choosing the appropriate value of switches from the reference mode vector but without a considerable effect on the optimality. As future work, we will investigate other approaches to solve the optimal control problem of multiple trains and compare them with the MILP approach with mode vector constraints indicated in this paper.

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