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Integrated urban traffic control for the reduction of travel delays and emissions

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Abstract—Refining transportation mobility and improving the living environment are two important issues that need to be addressed in urban traffic. In order to reduce traffic delays as well as traffic emissions for urban traffic networks, this paper first proposes an integrated macroscopic traffic model that integrates a macroscopic urban traffic flow model with a microscopic traffic emission model for individual vehicles. This integrated model is able to predict the traffic flow states, as well as the emissions released by every vehicle at different operational conditions, i.e. the speed and the acceleration. Then, Model Predictive Control is applied to control urban traffic networks based on this integrated traffic model, aiming at reducing both travel delays and traffic emissions of different gases. Finally, simulations are performed to assess this multi-objective control approach. The obtained simulation results illustrate the control effects of the model predictive controller.

Index Terms—Urban traffic network control, Model predictive control, Urban traffic modeling.

I. INTRODUCTION

Traffic emissions are one of the biggest sources of the environmental pollution in cities. The emissions of vehicles contain several harmful substances, such as nitrogen oxides ($\text{NO}_x$, i.e. nitrogen monoxide, nitrogen dioxide), hydrocarbons (HC), carbon monoxide (CO), carbon dioxide ($\text{CO}_2$), and fine particulate matter. Around the world, approximately 50% of the $\text{NO}_x$ emission, and 90% of the CO emission come from traffic [1]. These emissions can pollute the air that people breathing, cause smog for cities, do harm to soil, water, buildings, etc. In general, traffic pollution deteriorates our living environment, and increases the risk for people who have heart or lung diseases. Therefore, it is necessary to integrate traffic emission control into the urban traffic management system, so as to provide a healthier, safer, and more comfortable living environment for people in urban areas.

For most of the existing traffic control strategies, the control objectives are economy-oriented: they are mainly focusing on reducing traffic delays, travel time, and traffic congestion, and on improving the traffic flow throughput. For instance, SCATS [2] aims at reducing the traffic occupancy in front of the traffic signals; SCOOT [3] mainly focuses on controlling the length of queues; The total time spent or total travel delays is usually taken as the control objective in many optimization-based traffic control strategies [4]–[12]. Since traffic emissions have a significant influence on air, climate, and human health, it is not a sustainable policy to only consider economy-oriented performance criteria for traffic control strategies. Consequently, traffic emissions, as an environment-oriented traffic control objective, draws more attention [13]–[19].

In the aforementioned research works taking traffic emissions as control objectives, traffic emissions are controlled from two points of view: intelligent-vehicle-based methods and traffic-management-based methods. Intelligent-vehicle-based methods implement cruise control function to vehicles by adopting vehicle-to-vehicle or infrastructure-to-vehicle communication technologies, so as to get a fuel efficient and reduced emission driving behavior. Adaptive cruise control algorithms were proposed adopting upcoming road infrastructure information (e.g. traffic signals) so as to smooth traffic flows, and thus to reduce fuel consumptions [13], [14]. A vehicular network with virtual traffic lights was proposed in [20], which can improve the utilization efficiency of an intersection, and was proved to have good effect on reducing $\text{CO}_2$ emission [15]. Due to the use of information of neighboring vehicles and infrastructures (e.g. traffic lights), intelligent-vehicle-based method opens a way to help drivers have more optimal driving behaviors, and consequently to decrease the emission level of a traffic network. However, the intelligent-vehicle-based method focuses more on the individual vehicles and is often under a decentralized control structure, therefore it lacks a global regulation and control of entire traffic networks. On the contrary, traffic-management-based method can control and optimize traffic control measures to smooth and schedule traffic flows, and balance traffic emissions within traffic networks. Traffic control strategies were proposed for traffic emission reduction based on the traffic flow information collected from in-vehicle information broadcasting devices [16]. Stevanovic et al. [17] integrated a microscopic traffic simulator, a microscopic emission model, and a signal optimization tool together to search for the best traffic signal timings for fuel consumption and emission reduction. This research established an effective tool for off-line optimization of the traffic signals for vehicular emission mitigation. Zegeye et al. [18], [19] designed model predictive control strategies to online control both the total travel time and the total emissions, based on an integrated model of a macroscopic traffic flow model and a microscopic traffic emission model. But this control strategy is only for freeway traffic networks. For urban traffic networks, there is still lacking of efficient and effective traffic control strategy for reduction of both travel delays and emissions.

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Therefore, in this paper, a model predictive control (MPC) control strategy is proposed considering both traffic delays and emissions of traffic networks. As a model-based optimization control approach, MPC [21] is an advanced control method that can globally coordinate traffic network flows, and can easily combine multiple objectives into one control problem. An integrated traffic model is established for the MPC controller to predict/estimate both the travel delays and the traffic emissions of traffic networks. Since an MPC controller needs to optimize the control performance in a receding horizon way, it has a high requirement for online computation efficiency. Thereafter, the prediction model of the MPC controller has to keep efficient to satisfy online computing requirement. The proposed integrated traffic flow and emission model combines a macroscopic urban traffic flow model (i.e. the S model [22]) with a microscopic traffic emission model (i.e. the VT-micro model [23]). The VT-micro model can give a comparatively accurate estimate of the vehicular emissions based on the operational conditions of the vehicles, i.e. velocity and acceleration. The S model is proved to be a fast and accurate macroscopic traffic flow model for MPC control purposes. The integrated traffic flow and emission model keeps the advantages of the two models, and is able to provide reasonable estimates of both travel delays and emissions for the MPC controller.

This paper is organized as follows. Section II and III briefly introduce a macroscopic urban traffic flow model (S model) and a microscopic vehicle emission model (VT-micro). Section IV describes the integrated traffic flow and emission model. The MPC controller design is given in Section V. Section VI presents the results, and Section VII concludes the paper.

II. MACROSCOPIC URBAN TRAFFIC MODEL (S MODEL)

In the macroscopic urban traffic model of [22], called the S model, we define \( J \) the set of nodes (intersections), and \( L \) the set of links (roads) in the urban traffic network. A link \((u, d)\) is marked by its upstream node \( u \) \((u \in J)\) and downstream node \( d \) \((d \in J)\). The input and output links of link \((u, d)\) can be also specified by these upstream and downstream nodes. The sets of input and output nodes for link \((u, d)\) are \( I_{u,d} \subset J \) and \( O_{u,d} \subset J \). Some variables are defined as (see Fig. 1):

\[
\begin{align*}
I_{u,d} & : \text{set of input nodes of link } (u,d), \\
O_{u,d} & : \text{set of output nodes of link } (u,d), \\
k_d & : \text{simulation step counter}, \\
c_d & : \text{cycle time for intersection } d, \\
n_{u,d}(k_d) & : \text{number of vehicles in link } (u,d) \text{ at step } k_d, \\
q_{u,d}(k_d) & : \text{queue length at step } k_d \text{ in link } (u,d) \text{ (in number of vehicles)}; q_{u,d,o} \text{ is the queue length of the sub-stream turning to link } o, \\
\alpha^\text{leave}_{u,d}(k_d) & : \text{flow rate leaving link } (u,d) \text{ at step } k_d; \alpha^\text{leave}_{u,d,o}(k_d) \text{ is the leaving flow rate of the sub-stream towards } o, \\
\alpha^\text{leave,cont}_{u,d,o}(t) & : \text{the continuous-time leaving flow rate from the upstream link } i \text{ of link } (u,d) \text{ (} i \in I_{u,d} \text{)}, \\
\alpha^\text{arriv}_{u,d}(k_d) & : \text{flow rate arriving at the end of the queue in link } (u,d) \text{ at step } k_d; \alpha^\text{arriv}_{u,d,o}(k_d) \text{ is the arriving flow rate of the sub-stream towards } o, \\
\beta_{u,d,o}(k_d) & : \text{relative fraction of the traffic on link } (u,d) \text{ turning to } o \text{ at step } k_d, \\
\mu_{u,d} & : \text{saturated flow rate leaving link } (u,d), \\
g_{u,d,o}(k_d) & : \text{green time length during step } k_d \text{ for the traffic stream towards } o \text{ in link } (u,d), \\
\gamma & : \text{relative fraction of the traffic on link } (u,d) \text{ turning to } o \text{ at step } k_d, \\
C_{u,d} & : \text{capacity of link } (u,d) \text{ expressed in number of vehicles}, \\
l^\text{lane}_{u,d} & : \text{number of lanes in link } (u,d), \\
\Delta_c & : \text{cycle time offset between node } u \text{ and node } d, \\
\ell_{\text{veh}} & : \text{average vehicle length}.
\end{align*}
\]

In the S model, every intersection takes the cycle time as its simulation time interval. The cycle times for intersections \( u \) and \( d \), which are denoted by \( c_u \) and \( c_d \) respectively, can be different from each other. In this situation, the simulation step counters of different intersections are not same. As cycle times are the simulation time intervals of the S model, the input and output flow rates of the link are averaged over the cycle times in the S model.

Taking the cycle time \( c_d \) as the length of the simulation time interval for link \((u,d)\) and \( k_d \) as the corresponding time step counter, the number of the vehicles in link \((u,d)\) is updated according to the input and output flow rate over \( c_d \):

\[
n_{u,d}(k_d + 1) = n_{u,d}(k_d) + \left( \alpha^\text{enter}_{u,d}(k_d) - \alpha^\text{leave}_{u,d}(k_d) \right) \cdot c_d . \tag{1}
\]

The leaving flow rate \( \alpha^\text{leave}_{u,d,o}(k_d) \) is the sum of the leaving flow rates \( \alpha^\text{leave}_{u,d}(k_d) \) turning to each output link \( o \in O_{u,d} \).

The average leaving flow rate over \( c_d \) is determined by the capacity of the intersection, the number of cars waiting and arriving, and the available space in the downstream link:

\[
\alpha^\text{leave}_{u,d,o}(k_d) = \min \left( \beta_{u,d,o}(k_d) \cdot \mu_{u,d} \cdot g_{u,d,o}(k_d) / c_d, \quad q_{u,d,o}(k_d) / c_d + \alpha^\text{arriv}_{u,d,o}(k_d), \quad \beta_{u,d,o}(k_d) \cdot (C_d - n_{u,d,o}(k_d)) / c_d \right) . \tag{2}
\]

The number of vehicles waiting in the queue turning to link \( o \) is updated as

\[
q_{u,d,o}(k_d + 1) = q_{u,d,o}(k_d) + \left( \alpha^\text{arriv}_{u,d,o}(k_d) - \alpha^\text{leave}_{u,d,o}(k_d) \right) \cdot c_d . \tag{3}
\]

The flow of vehicles that entered link \((u,d)\) will arrive at the end of the queues after a time delay \( \tau(k_d) \cdot c_d + \gamma(k_d) \):

\[
\alpha^\text{arriv}_{u,d}(k_d) = c_d \cdot (\gamma(k_d) / c_d) \cdot \alpha^\text{enter}_{u,d}(k_d - \tau(k_d) - 1) + \gamma(k_d) / c_d, \tag{4}
\]

with

\[
\tau(k_d) = \min \left\{ \left( C_{u,d} - q_{u,d}(k_d) \right) / \ell_{\text{veh}} \right\} + \min \left\{ L^\text{lane}_{u,d} \cdot \gamma_{\text{free},u,d} \cdot c_d \right\} . \tag{5}
\]
When reaching the end of the link, the arriving flow rate is separated into sub-streams by multiplying it with the turning rate $\beta_{d,a}(k_d)$.

The flow rate entering link $(u,d)$ is the sum of the flow rates entering from all the upstream links:

$$\alpha_{\text{enter},u,d}(k_d) = \sum_{i \in I_{u,d}} \alpha_{\text{enter},u,d}(k_{iu,d}) = \sum_{i \in I_{u,d}} \alpha_{\text{leave},u,d}(k_{iu}) \cdot \left( t \leq (k_u + 1) \cdot c_u \right).$$  

(6)

Some operations need to be carried out to synchronize the leaving and entering flow rates. This goes as follows: A common control time interval is adopted by all the intersections in the network, as

$$T_{\text{cont}} = a_j \cdot c_j, \text{ for all } j \in J, \text{ with } a_j \text{ an integer},$$

(7)

where $T_{\text{cont}}$ is the least common multiple of all the intersection cycle times in the traffic network.

The leaving flow rates in the timing of intersection $u$ can be recast into the entering flow rates in the timing of intersection $d$ as follows. First, we transform the discrete-time leaving flow rates from the upstream links into continuous time using a zero-order hold strategy, as

$$\alpha_{\text{leave},u,d}^\text{cont}(t) = \alpha_{\text{leave},u,d}(k_u), \quad k_u \cdot c_u \leq t < (k_u + 1) \cdot c_u.$$  

(8)

Then, we convert the result again to obtain the average entering flow rates in time step $k_d$,

$$\alpha_{\text{enter},u,d}(k_d) = \int_{k_u \cdot c_u + \Delta \alpha_{\text{leave},u,d}(t)}^{(k_u + 1) \cdot c_u + \Delta \alpha_{\text{leave},u,d}(t)} c_d \, dt.$$  

(9)

IV. INTEGRATED TRAFFIC FLOW AND TRAFFIC EMISSION MODEL

A. Urban traffic behaviors for individual vehicles

As a microscopic model, the VT-micro model provides the emissions of an individual vehicle at a certain location and a certain time instant. But, as a macroscopic model, the S model only provides information of traffic flows instead of every detail of every individual vehicle. However, the S model can capture the main behavior of the vehicles, when they are running along a road. The time period spent by a vehicle running along a road can be divided into several parts, in each of which the behavior of the vehicle is assumed to be uniform. Define the set of the behaviors as $B = \{\text{free, idling, dec, acc, nonstop, sns}\}$, where dec stands for deceleration, acc stands for acceleration, and sns stands for start-and-stop behavior. Fig. 2 shows how the velocity of a vehicle could vary in different behavior regions, when it travels along an urban road.
emissions for the nonstop vehicles are (see Fig. 2(b))
\[
E_{\text{nonstop}}(k) = \begin{cases} 
\frac{a_{\text{acc}}}{v_{\text{agg}} - v_{\text{free}}} \int_{v_{\text{free}}}^{v_{\text{agg}}} E_{\theta, i}(v, a_{\text{acc}}) \, dv & \text{if } O \leq \lambda, \\
- \frac{a_{\text{dec}}}{v_{\text{con}} - v_{\text{free}}} \int_{v_{\text{free}}}^{v_{\text{con}}} E_{\theta, i}(v, a_{\text{dec}}) \, dv & \text{if } O > \lambda, 
\end{cases}
\]
(15)
where the less conservative drivers will speed up to \(v_{\text{agg}}(> v_{\text{free}})\), the more conservative drivers will decrease their speed to \(v_{\text{con}}(< v_{\text{free}})\), \(O = \alpha_{\text{arrv}}(\lambda_d) \mu_{u,d}(kT \in [c_d, k_d, c_d, (k_d + 1)])\) is defined as the capacity occupancy rate of a link (i.e. the arriving traffic flow rate divides the saturation flow rate of the link), and \(\lambda \in [0, 1]\) is the threshold.

When the traffic in a link is saturated, there are vehicles arriving at the link, but that cannot leave the link within the same cycle. These vehicles have to accelerate and then decelerate to keep on waiting in queues, which we call start-and-stop behavior. The emissions for the start-and-stop vehicles can be estimate as
\[
E_{\text{stop}}(k) = \frac{a_{\text{acc}} a_{\text{dec}}}{v_{\text{free}} - v_{\text{agg}}} \int_{v_{\text{free}}}^{v_{\text{agg}}} E_{\theta, i}(v, a_{\text{acc}}) \, dv + \frac{a_{\text{acc}} a_{\text{dec}}}{v_{\text{con}} - v_{\text{low}}} \int_{v_{\text{low}}}^{v_{\text{con}}} E_{\theta, i}(v, a_{\text{dec}}) \, dv,
\]
(16)
where \(v_{\text{ls}}\) is the speed that a vehicle reaches when it is subject to a start-and-stop behavior in waiting queues.

Remark 1: In this subsection, for the sake of simplicity of the explanation, all the variables are assumed to be the same for a vehicle on any link. If the locations of vehicles are considered, then the emission \(E_{\text{stop}}^b(\theta, M)\) of vehicle \(i\) on link \((u, d)\) in behavior \(b\) should be remarked as \(E_{\text{stop}}^b(u, d, i)\).

B. Integrated VT-S traffic emission model

The VT-S model provides macroscopic traffic states for each link \((u, d) \in L\) in each simulation time interval (cycle time). The traffic states include the number of vehicles traveling with free-flow speed, the number of vehicles decelerating and accelerating, and the number of vehicle waiting and idling in queues. The vehicles idling in front of the stop-line in link \((u, d)\) can be classified into four groups:

- **Idling 1**: Vehicles idling for the entire cycle time after deceleration;  
- **Idling 2**: Vehicles idling between deceleration and acceleration;  
- **Idling 3**: Vehicles idling for the rest of the cycle time after idling;  
- **Idling 4**: Vehicles idling from the start of the cycle time until acceleration.

Correspondingly, the VT-micro model can provide an estimate of the emissions for the vehicles under these traffic states as Section IV-A shows. By summing up the emissions of vehicles in different traffic states provided by the S model, the total emission of a road network can be calculated.

Based on the traffic flow state information and the vehicle emission information, a macroscopic traffic emission model can be obtained by combining the macroscopic S model and the VT-micro model together, which results in a macroscopic integrated traffic flow and emission model, which we call the VT-S model.
The VT-S model for emission $\theta \in M$ in link $(u,d) \in L$ during time period $[c_d \cdot k_d, c_d \cdot (k_d + 1)]$ is

$$E_{\theta,u,d}(k_d) = \sum_{b \in B} E_{\theta,u,d}^b(k_d) = \sum_{t \in T} E_{\theta,u,d}^b(k_d) = \sum_{b \in B} E_{\theta,u,d}^b(k_d) \cdot N_{u,d}^b(k_d) \cdot I_{u,d}^b(k_d), \quad (17)$$

where $\mathcal{V}(b,u,d,k)$ is the set of vehicles that have behavior $b \in B$ at time step $k$ in link $(u,d)$, $\mathcal{X}(d,k_d)$ is the set of time steps $k$ such that $kT \in [c_d \cdot k_d, c_d \cdot (k_d + 1)]$ at which the vehicles are in behavior $b$ in link $(u,d)$, $E_{\theta,u,d}^b(k_d)$ is the constant traffic emission for emission $\theta$ of a vehicle on link $(u,d)$ with behavior $b$ during time period $[c_d \cdot k_d, c_d \cdot (k_d + 1)]$, $N_{u,d}^b(k_d)$ is the number of vehicles that have behavior $b$ in link $(u,d)$ during time period $[c_d \cdot k_d, c_d \cdot (k_d + 1)]$, and $I_{u,d}^b(k_d)$ is the length of the time period that the vehicles keep having this behavior.

Urban traffic states on a link can be separated into different scenarios according to the level of the traffic density. In the saturated traffic scenario, the queues of vehicles resulting from the red phase cannot be dissolved completely at the following green phase, i.e. all the arriving vehicles have to stop and wait once for the next green light to leave the link. For the over-saturated traffic scenario, the vehicles need to wait for even more cycle times in the queues than in saturated scenario. On the contrary, in the under-saturated traffic scenario, all the accumulated vehicles during the red phase are able to leave the link in the following green phase, some vehicles can even leave the link without any stop. Since the traffic behaviors could differ between these scenarios, the VT-S model can be further illustrated for the three scenarios.

1) Saturated scenario: In the saturated scenario, not all the vehicles waiting and arriving in the queues could leave the link in the current green phase, some vehicles have to wait until the next green phase, i.e. the number of vehicles waiting and arriving to leave the link exceeds the maximum number of vehicles that could leave at most in one cycle time, but the queues can be dissolved in the current green phase. This is characterized by the following condition:

$$q_{u,d}(k_d) \leq \sum_{c \in O_{u,d}} \beta_{u,d,o}(k_d) \cdot \mu_{u,d} \cdot g_{u,d,o}(k_d) \leq c_d \cdot g_{u,d}(k_d) \cdot q_{u,d}(k_d). \quad (18)$$

So, all the vehicles have to wait once for a red traffic signal in the queues before leaving the link, i.e. no vehicle can leave the link without a stop. For the saturated scenario, the number of vehicles that have behavior $b \in B$ in link $(u,d)$ during time period $[c_d \cdot k_d, c_d \cdot (k_d + 1)]$ is given by

$$N_{u,d}^{\text{free}}(k_d) = n_{u,d}(k_d) - c_d \cdot g_{u,d}(k_d) + q_{u,d}(k_d) \quad (19)$$

$$N_{u,d}^{\text{idling,1}}(k_d) = c_d \cdot g_{u,d}(k_d) - q_{u,d}(k_d) \quad (20)$$

$$N_{u,d}^{\text{idling,2}}(k_d) = \sum_{c \in O_{u,d}} \beta_{u,d,o}(k_d) \cdot \mu_{u,d} \cdot g_{u,d,o}(k_d) - q_{u,d}(k_d) \quad (21)$$

$$N_{u,d}^{\text{idling,3}}(k_d) = 0 \quad (22)$$

and the length of the time periods that the vehicles keep having this behavior during time period $[c_d \cdot k_d, c_d \cdot (k_d + 1)]$ are

$$t_{u,d}^{\text{idling,1}}(k_d) = c_d \quad (23)$$

$$t_{u,d}^{\text{idling,2}}(k_d) = 0 \quad (24)$$

and the length of the time periods that the vehicles keep having this behavior during time period $[c_d \cdot k_d, c_d \cdot (k_d + 1)]$ are

$$N_{u,d}^{\text{free}}(k_d) = n_{u,d}(k_d) - c_d \cdot g_{u,d}(k_d) = q_{u,d}(k_d) \quad (25)$$

$$N_{u,d}^{\text{idling,1}}(k_d) = c_d \cdot g_{u,d}(k_d) - q_{u,d}(k_d) \quad (26)$$

$$N_{u,d}^{\text{idling,2}}(k_d) = \sum_{c \in O_{u,d}} \beta_{u,d,o}(k_d) \cdot \mu_{u,d} \cdot g_{u,d,o}(k_d), \quad (27)$$

and the length of the time periods that the vehicles keep having this behavior during time period $[c_d \cdot k_d, c_d \cdot (k_d + 1)]$ are

$$t_{u,d}^{\text{idling,1}}(k_d) = c_d \quad (28)$$

$$t_{u,d}^{\text{idling,2}}(k_d) = 0 \quad (29)$$

$$N_{u,d}^{\text{free}}(k_d) = n_{u,d}(k_d) - c_d \cdot g_{u,d}(k_d) = q_{u,d}(k_d) \quad (30)$$

$$N_{u,d}^{\text{idling,1}}(k_d) = c_d \cdot g_{u,d}(k_d) - q_{u,d}(k_d) \quad (31)$$

$$N_{u,d}^{\text{idling,2}}(k_d) = \sum_{c \in O_{u,d}} \beta_{u,d,o}(k_d) \cdot \mu_{u,d} \cdot g_{u,d,o}(k_d) \quad (32)$$

In the saturated scenario, equation (19) gives the number of vehicles that are running on link $(u,d)$ with free-flow speed during the time period shown in (28). Equation (20) gives the number of vehicles that arrive at the end of the queues and decelerate to a low speed in link $(u,d)$, and then keep idling for the time period as in (29). Equation (21) gives the number of vehicles that decelerate to arrive at the end of the queues, keep idling for time period in (30), and then accelerate to leave link $(u,d)$. Equation (22) gives the number of vehicles in the queues that keep idling for the time period as in (32), and finally accelerate and leave link $(u,d)$. All the vehicles arriving at the end of the queues need to decelerate as (24) shows, and all the vehicles leaving link $(u,d)$ will accelerate as (25) shows. The waiting vehicles in (27) that cannot leave the link in the current time interval will start and then stop again to keep on waiting in queues.

2) Over-saturated scenario: In the over-saturated scenario, the vehicles waiting in the queues could not leave the link in the current green phase. Hence, the number of vehicles waiting in the queues to leave the link exceeds the maximum number of vehicles that could leave at most in one cycle time:

$$\sum_{c \in O_{u,d}} \beta_{u,d,o}(k_d) \cdot \mu_{u,d} \cdot g_{u,d,o}(k_d) < q_{u,d}(k_d). \quad (33)$$

For the over-saturated scenario, the number of vehicles having behavior $b \in B$ during $[c_d \cdot k_d, c_d \cdot (k_d + 1)]$ is

$$N_{u,d}^{\text{free}}(k_d) = n_{u,d}(k_d) - c_d \cdot g_{u,d}(k_d) \quad (34)$$

$$N_{u,d}^{\text{idling,1}}(k_d) = c_d \cdot g_{u,d}(k_d) \quad (35)$$

$$N_{u,d}^{\text{idling,2}}(k_d) = \sum_{c \in O_{u,d}} \beta_{u,d,o}(k_d) \cdot \mu_{u,d} \cdot g_{u,d,o}(k_d) \quad (36)$$

$$N_{u,d}^{\text{idling,3}}(k_d) = 0 \quad (37)$$
and leave link (42) will be idling for time period (51), and then accelerate idling for the entire cycle time. All the vehicles as shown in maximum number of vehicles that could leave in one cycle vehicles waiting and arriving to leave the link is less than the nario, the queues can be dissolved before the current green end of the queues as shown in (39) will decelerate and the same as in the saturated scenario. All the vehicles arriv ing this behavior in link (42) approximately separated into two parts, one is the green time time for link , , , , \( d \cdot d \cdot d \cdot d \cdot d \), and then, after the traffic leaves the link with the demand flow rate. The quantities \( g_{u,d}(k_d) \) and \( g_{u,d}^{s}(k_d) \) satisfy

\[
\begin{align*}
\forall \alpha \in \{d - d, 0\} \\
\forall \alpha \in \{d - d, 0\}
\end{align*}
\]

Hence, we have

\[
\begin{align*}
g_{u,d}^{s}(k_d) &= \frac{c_d \cdot \alpha_{u,d}^{\text{arr}}(k_d) + q_{u,d}(k_d) - g_{u,d}(k_d) \cdot (d - d) \cdot d \cdot d}{\mu_d - \alpha_{u,d}^{\text{arr}}(k_d)} \\
g_{u,d}^{d}(k_d) &= \frac{g_{u,d}(k_d) \cdot \mu_d - c_d \cdot \alpha_{u,d}^{\text{arr}}(k_d) - q_{u,d}(k_d)}{\mu_d - \alpha_{u,d}^{\text{arr}}(k_d)}.
\end{align*}
\]

For the under-saturated scenario, the number of vehicles that have behavior \( b \in \mathcal{B} \) in link \( (u, d) \) \( c_d \cdot (k_d + 1) \) is

\[
\begin{align*}
N_{u,d}^{\text{free}}(k_d) &= n_{u,d}(k_d) - c_d \cdot \alpha_{u,d}^{\text{arr}}(k_d) - q_{u,d}(k_d) \\
N_{u,d}^{\text{idling,1}}(k_d) &= 0 \\
N_{u,d}^{\text{idling,2}}(k_d) &= (c_d - g_{u,d}^{d}(k_d)) \cdot \alpha_{u,d}^{\text{arr}}(k_d) \\
N_{u,d}^{\text{idling,3}}(k_d) &= 0 \\
N_{u,d}^{\text{idling,4}}(k_d) &= (c_d - g_{u,d}^{d}(k_d)) \cdot \alpha_{u,d}^{\text{arr}}(k_d) \\
N_{u,d}^{\text{dec}}(k_d) &= g_{u,d}(k_d) \cdot \mu_d \\
N_{u,d}^{\text{acc}}(k_d) &= g_{u,d}(k_d) \cdot \mu_d \\
N_{u,d}^{\text{nonstop}}(k_d) &= 0, \\
N_{u,d}^{\text{sms}}(k_d) &= 0,
\end{align*}
\]

and the time periods that the vehicles keep having this behavior in link \( (u, d) \) are given by

\[
\begin{align*}
t_{u,d}^{\text{free}}(k_d) &= c_d \\
t_{u,d}^{\text{idling,1}}(k_d) &= c_d - (\nu_{\text{low}} - \nu_{\text{free}}) / \alpha_{\text{dec}} \\
t_{u,d}^{\text{idling,2}}(k_d) &= 0 \\
t_{u,d}^{\text{idling,3}}(k_d) &= c_d \\
t_{u,d}^{\text{idling,4}}(k_d) &= c_d - (\nu_{\text{free}} - \nu_{\text{low}}) / \alpha_{\text{acc}} \\
t_{u,d}^{\text{dec}}(k_d) &= (\nu_{\text{low}} - \nu_{\text{free}}) / \alpha_{\text{dec}} \\
t_{u,d}^{\text{acc}}(k_d) &= (\nu_{\text{free}} - \nu_{\text{low}}) / \alpha_{\text{acc}} \\
t_{u,d}^{\text{nonstop}}(k_d) &= 0 \\
t_{u,d}^{\text{sms}}(k_d) &= (\nu_{\text{low}} - \nu_{\text{free}}) / \alpha_{\text{dec}} + (\nu_{\text{free}} - \nu_{\text{low}}) / \alpha_{\text{acc}}.
\end{align*}
\]

Except for the “idling” behavior, all the above formulas are the same as in the saturated scenario. All the vehicles arriving at the end of the queues as shown in (39) will decelerate and be idling for time period (48). A part of the vehicles waiting in the queues as in (41) cannot leave link \( (u, d) \), and will be idling for the entire cycle time. All the vehicles as shown in (42) will be idling for time period (51), and then accelerate and leave link \( (u, d) \).

3) Under-saturated scenario: In the under-saturated scenario, the queues can be dissolved before the current green phase ends. Thus, the traffic demand, i.e. the number of vehicles waiting and arriving to leave the link is less than the maximum number of vehicles that could leave in one cycle time, which is characterized as

\[
c_d \cdot \alpha_{u,d}^{\text{arr}}(k_d) + q_{u,d}(k_d) < \sum_{o \in \mathcal{O}_{u,d}} \beta_{u,d,o}(k_d) \cdot \mu_{u,d} \cdot g_{u,d,o}(k_d).
\]

Therefore, during a green phase, the vehicles waiting in the queues can be considered to first leave the link according to the saturated flow rate of the link \( \mu_{u,d} \), and then, after the queues are dissolved, the arriving vehicles will leave the link without a stop according to the arriving (or demand) flow rate \( \alpha_{u,d}^{\text{arr}}(k_d) \) in the rest of the green time. Hereafter, the green time for link \( (u, d) \) in the \( k_d \)th cycle time, \( g_{u,d}(k_d) \), can be approximately separated into two parts, one is the green time \( g_{u,d}^{s}(k_d) \) in which the traffic leaves the link with the saturated flow rate, the other is the green time \( g_{u,d}^{d}(k_d) \) during which the traffic leaves the link with the demand flow rate. The quantities \( g_{u,d}(k_d) \) and \( g_{u,d}^{d}(k_d) \) satisfy

\[
\begin{align*}
c_d \cdot \alpha_{u,d}^{\text{arr}}(k_d) + q_{u,d}(k_d) &= g_{u,d}^{s}(k_d) \cdot \mu_{u,d} + g_{u,d}^{d}(k_d) \cdot \alpha_{u,d}^{\text{arr}}(k_d) \\
g_{u,d}^{s}(k_d) + g_{u,d}^{d}(k_d) &= g_{u,d}(k_d).
\end{align*}
\]

In the under-saturated scenario, no vehicle will be held at the stop-line for more than one cycle time, i.e. all the queues will be dissolved in the following green time. Thus, only “idling,2” and “idling,4” vehicles exist. All the arriving vehicles except the “nonstop” vehicles (as in (62)) will experience deceleration and acceleration, and be idling for the time period in (72). All the waiting vehicles in the queues in (64) will be idling for the time period (74), and then accelerate to leave the link. Only the arriving vehicles except the vehicles that do not need to stop will decelerate and wait in queues as in (65). All the vehicles leaving at the saturation flow rate have to accelerate to leave the link as (66) shows. The arriving vehicles as shown
in (67) will leave link \((u, d)\) with acceleration or deceleration (depending on the capacity occupancy rate of the link) without stops.

V. MPC FOR URBAN TRAFFIC NETWORKS

Model Predictive Control [21] is a methodology that implements and repeatedly applies Optimal Control in a rolling horizon way. The aforementioned integrated VT-S model is able to efficiently predict both the traffic flows and the traffic emissions, which can be used to compute the control performance, and thus the VT-S model can be used as the prediction model of the MPC controller.

Based on the prediction model, at every control time step, an optimization problem needs to be solved on-line over a prediction horizon to derive the sequence of the optimal future control decisions. Solving this optimization problem is the key part of the MPC methodology, and is also the step that costs most of the on-line computation. Given the control time interval \(T_{\text{ctrl}}\) and the simulation time interval \(c_d\) of node \(d \in J\), there exists an integer \(N_d\) such that

\[
T_{\text{ctrl}} = N_d c_d,
\]

(79)

according to the definition of the cycle times for the nodes in a traffic network, as shown in (7). For a given \(k_d\), the corresponding value of \(k_{\text{ctrl}}(k_d)\) is given by

\[
k_{\text{ctrl}}(k_d) = \left\lfloor \frac{k_d}{N_d} \right\rfloor,
\]

(80)

where \([x]\) for \(x\) a real number denotes the largest integer less than or equal to \(x\). On the other hand, a given value \(k_{\text{ctrl}}\) of the control time step corresponds to the set \(\{k_{\text{ctrl}}N_d, k_{\text{ctrl}}N_d + 1, \cdots, (k_{\text{ctrl}} + 1)N_d - 1\}\) of simulation time steps.

When the prediction horizon is \(N_p\), the optimization problem of MPC can be expressed as

\[
\begin{align*}
\min_{\mathbf{g}(k_{\text{ctrl}})} & \quad J(k_{\text{ctrl}}) \\
\text{s.t.} & \quad \text{VT-S model} \\
& \quad \Phi(\mathbf{g}(k_{\text{ctrl}})) = 0 \\
& \quad g_{\text{min}} \leq \mathbf{g}(k_{\text{ctrl}}) \leq g_{\text{max}}
\end{align*}
\]

(81)

where \(\mathbf{g}(k_{\text{ctrl}})\) is the future control input at control step \(k_{\text{ctrl}}\) (e.g. the green times), \(i\), i.e. \(\mathbf{g}(k_{\text{ctrl}}) = [g^T(k_{\text{ctrl}}|k_{\text{ctrl}}), g^T(k_{\text{ctrl}} + 1|k_{\text{ctrl}}), \cdots, g^T(k_{\text{ctrl}} + N_p - 1|k_{\text{ctrl}})]^T\), and the vector \(g(k_{\text{ctrl}} + j|k_{\text{ctrl}})\) denotes the control input at the \(j\)th control step in the future based on information at the current control time step \(k_{\text{ctrl}}\). The equality constraint in (81) is the cycle time constraint, i.e. the sum of the green times of all the phases equals to the cycle time in an intersection. To decrease the on-line computational complexity, a control horizon \(N_c\) \((N_c < N_p)\) can be defined, such that \(g(k_{\text{ctrl}} + i|k_{\text{ctrl}}) = g(k_{\text{ctrl}} + N_c - 1|k_{\text{ctrl}})\) for \(i = N_c, \cdots, N_p - 1\). This nonlinear optimization problem can be solved by e.g. multi-start Sequential Quadratic Programming (SQP) algorithm [25, Chapter 5].

The objective function of the integrated urban control problem at control time step \(k_{\text{ctrl}}\) is

\[
J(k_{\text{ctrl}}) = \sum_{\theta \in \Theta} \frac{\lambda_{\theta}}{E_{\theta, \text{nomin}}(u, d)} \sum_{(u, d) \in L} \sum_{k_d = k_{\text{ctrl}} + 1}^{N_d} E_{\theta, u, d}(k_d),
\]

(82)

where \(E_{\theta, u, d}(k_d)\) denotes the estimated partial criterion for \(\theta\) in link \((u, d)\) at simulation time step \(k_d\), \(\Theta = \{\text{TTS}, \text{CO}, \text{NO}_x, \text{HC}\}\) is the set of the control objectives, \(E_{\theta, \text{nomin}}\) is the nominal performance for objective \(\theta \in \Theta\) to normalize the partial objective of \(\theta\), and \(\lambda_{\theta}\) is the weight parameter for objective \(\theta\). For the Total Time Spent (TTS), we have

\[
E_{\text{TTS}, u, d}(k_d) = T_{\text{sim}} \cdot n_{u, d}(k_d),
\]

(83)

where \(n_{u, d}(k_d)\) is derived by (1), \(T_{\text{sim}}\) is the simulation time step of the VT-S model, and (17) will be used for computing emissions. The goal of the control problem is to reduce the combined performance of the Total Time Spent and the variety of traffic emissions (i.e. CO, NO\(_x\), and HC) of the whole urban traffic network over the entire prediction horizon. Hence, it turns out to be a multiple objective control problem. By changing the weights of the objective function, a different emphasis can be assigned for different kinds of control purposes.

Once the optimal control input sequence \(\mathbf{g}^*(k_{\text{ctrl}})\) is determined by the optimization, the first sample of the optimal results, \(\mathbf{g}^*(k_{\text{ctrl}} | k_{\text{ctrl}})\), is implemented in the urban traffic network. When arriving to the next control step, the prediction model is fed with the newly measured traffic states (i.e. the number of vehicles on a link [26]), the whole prediction horizon is shifted one step forward, and the optimization starts over again. By operating the on-line optimization in the receding horizon way, the MPC controller closes the control loop, and enables the system to get feedback from the real traffic network, which makes the controller adaptive to the uncertainties and disturbances caused by model mismatches and errors in the external demand prediction.

VI. SIMULATIONS

We use CORSIM [27] to simulate the real traffic environment, and design MPC controllers to decide control inputs for the traffic signals in CORSIM. The simulated urban road subnetwork is shown in Fig. 3. Nodes marked as “S” are the source nodes where traffic flows enter and leave the network.

![Fig. 3. An urban traffic network](image)

MPC controllers for urban traffic are designed to reduce both TTS and TE (Total Emissions for CO, NO\(_x\), and HC) for this urban traffic network. MPC controllers are designed
based on the weights specified for the different objectives, as shown in Table I.

<table>
<thead>
<tr>
<th>MPC</th>
<th>$\lambda_{TTS}$</th>
<th>$\lambda_{CO}$</th>
<th>$\lambda_{NO_x}$</th>
<th>$\lambda_{HC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTS</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TE</td>
<td>0</td>
<td>0.33</td>
<td>0.33</td>
<td>0.34</td>
</tr>
<tr>
<td>TTS+TE</td>
<td>0.5</td>
<td>0.16</td>
<td>0.17</td>
<td>0.17</td>
</tr>
</tbody>
</table>

In the traffic network shown in Fig. 3, the lengths of the roads are indicated in meter. Each of the roads in the traffic network has 3 lanes, and the turning rates for each link are all the same, i.e. left turn 33%, through turn 34%, right turn 33%. The storage capacities of the links are fixed according to the link lengths, the number of lanes, and the average vehicle length (7 m). The free-flow speed is 50 km/h. The simulations run for 80 min, and the traffic demands of all the source nodes (i.e. network inflows) are given in Fig. 4.

For the set-up of the traffic controllers of this network, the cycle time is set to 60 s for all the intersections, except 30 s for Intersection 8. During the experiments, the simulation time intervals are set to equal to the cycle times of every intersection. For the MPC controllers, the control time interval $T_{cd}$ is 120 s, the prediction horizon $N_p$ is 5, and the control horizon is set to $N_c = N_p$. The results of MPC controllers are compared with that of a fixed-time strategy. The fixed-time control strategy is defined having constant phases, cycle times, green time durations, and the offsets. The fixed-time signals are determined based on the data for the saturated scenario, i.e. the green times are proportional to the traffic demands from each direction, which depend on the saturated flow rates and the turning rates under the saturated scenario [22], [28].

The performance indicators that CORSIM provides to evaluate the effect of the controllers include the TTS, the TE for CO, NO$_x$, and HC respectively, the mean speed, the number of stops, etc. The results for each control performance are illustrated in Table II for the different control strategies. As Table II shows, the MPC controllers are able to reduce the objectives, including TTS and the TE for CO, NO$_x$, and HC, compared to the FT controller. The TTS-based MPC and the TE-based MPC are MPC controllers taking only the TTS or only the TE of the whole network as control objective respectively. For the TTS-based MPC, the TTS is reduced obviously, but its ability for reduction of the TE is quite limited, where the emissions for HC even increase. For the TE-based MPC, the TE for each of the gases is reduced, but the TTS becomes higher than the TTS-based MPC. When both the TTS and the TE are considered for the control objective as in the TTS+TE-based MPC, a trade-off is made to balance the TTS and the TE. The TE-based MPC is able to increase the mean speed and to reduce the number of stops for the traffic network, so as to obtain smoother traffic flows. The TTS-based MPC can also improve the mean speed of the traffic network, but at the cost of increasing the stop-and-go behavior and the vehicular emissions. The improvements made by the MPC controllers also illustrate the adaptiveness of MPC to the uncertainties and disturbances caused by model mismatches and errors in the external demand prediction.

In urban traffic networks, traffic flows with fewer stops, shorter delays, and moderate speed will release less emissions [17]. In Fig. 4, we demonstrate the evolution of some traffic states for the entire traffic network in Fig. 3, including the number of vehicles, the emissions for CO, NO$_x$, and HC, the mean speed, and the number of stops per minute. As the figure shows, the TE-based MPC controller obtains higher network mean speeds and fewer number of stops compared to the TTS-based MPC controller, which results in smoother traffic flows within the traffic network. Consequently, TE-based MPC is able to keep the total emissions from increasing when the network inflow grows. On the contrary, TTS-based MPC successfully reduces the number of vehicles in the traffic network, but it is not good at regulating emissions.

In Fig. 5 and 6 illustrate the evolution of the traffic states for Link 2 and Link 3 in the network of Fig. 3. The results of the Fixed-time controller (FT), the TTS-based MPC, and the TE-based MPC are compared in the figures. At the start of the simulation, all the emissions are high in both links, because at the beginning of the simulation, the traffic network is empty and the network inflows are low, so vehicles tend to accelerate more and made fewer stops until the traffic flows reach an equilibrium or the network inflows increase. When the number of vehicles grows with the network inflow in the links (i.e. the links become more crowded), we can see that the mean speed decreases and the number of stops increases, thereafter, more emissions are released for the different kinds of gases. In Fig. 5, the TE-based MPC has a good effect on reducing...
TABLE II
PERFORMANCE COMPARISON OF TTS AND TE FOR A FIXED-TIME CONTROLLER (FT) AND FOR THE MPC CONTROLLERS WITH VARIOUS OBJECTIVE FUNCTIONS (SEE TABLE I)

<table>
<thead>
<tr>
<th>Controller</th>
<th>TTS (veh·h)</th>
<th>TE (kg)</th>
<th>Mean Speed (veh/h)</th>
<th>Stops (veh/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FT</td>
<td>$1.58 \times 10^7$</td>
<td>18.5668</td>
<td>1.3605</td>
<td>0.2674</td>
</tr>
<tr>
<td>TE</td>
<td>$1.34 \times 10^7 (-15.19%)$</td>
<td>17.7603 (-4.34%)</td>
<td>1.2774 (-6.11%)</td>
<td>0.2511 (-6.11%)</td>
</tr>
<tr>
<td>TTS</td>
<td>$1.26 \times 10^7 (-20.25%)$</td>
<td>18.5332 (-0.18%)</td>
<td>1.3600 (-0.04%)</td>
<td>0.2690 (0.60%)</td>
</tr>
<tr>
<td>TTS+TE</td>
<td>$1.31 \times 10^7 (-17.09%)$</td>
<td>18.1040 (-2.49%)</td>
<td>1.2932 (-4.95%)</td>
<td>0.2539 (-5.05%)</td>
</tr>
</tbody>
</table>

The data in parenthesis are the relative change for each performance indicator compared with that of the FT controller.

As Table II illustrates, since the MPC controllers focus on different control objectives, TTS-based MPC can reduce the TTS more than TE-based MPC; and on the contrary, TE-based MPC is able to reduce the TE more than TTS-based MPC. A trade-off between the TTS and TE can be obtained by integrating the two control objectives. For freeway traffic, the speed range of vehicles is normally from 0 km/h to 120 km/h, in which range the vehicles will release comparatively high emissions when the speed of vehicles is either very low or very high. Thus, the TTS performance conflicts with the TE performance, when the speed of vehicles grows high [19]. However, for urban traffic, due to the speed limit (normally 60 km/h), a high speed is not an important cause for high emissions anymore, but the stop-and-go driving behavior is the main cause of generating emissions. Therefore, for urban traffic emission reduction, we need to focus on smoothing traffic flows and decreasing the number of accelerations and

Fig. 5. Evolution of the traffic states for Link 2

Fig. 6. Evolution of the traffic states for Link 3

As Table II illustrates, since the MPC controllers focus on different control objectives, TTS-based MPC can reduce the
more traffic emissions; the TE-based MPC decreases the total control objectives respectively. The TTS-based MPC performs MPC take the total time spent and the total emissions as the performance indicator. The TTS-based MPC and the TE-based travel delays and traffic emissions.

This model enables the MPC controller to address states, as well as the emissions released by every vehicle controller. The VT-S model is able to predict the traffic flow controller. TTS-based MPC obtained higher number of stops than the FT with the aid of traffic signals. To achieve a lower TTS, the reducing the TTS of traffic network by regulating the stop-and-go density on the links, smoother traffic flows and mean speeds, and at the same time, to keep a more steady vehicle only traveling efficiency, but also considers the stop-and-go emission model, the TE control objective focuses on not time, is a performance aiming at controlling the traveling based mainstream traffic flow control on motorways using variable speed limits, " IEEE Transactions on Intelligent Transportation Systems, vol. 13, no. 2, pp. 838–847, 2012.

In the future, we will extend the VT-S model into a multi-class model to describe the emissions of different types of vehicles, and also consider the surrounding traffic dynamics, then further evaluate the proposed model-based controller for reducing fuel consumption and CO2 emission for urban traffic networks. Moreover, we will calibrate the model and implement the proposed model-based controller in a real-life traffic network, and further assess its control effect.

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VII. CONCLUSION

An integrated macroscopic urban traffic flow and emission model, the VT-S model, is proposed for urban traffic MPC controller. The VT-S model is able to predict the traffic flow states, as well as the emissions released by every vehicle at different operational conditions, i.e. the speed and the acceleration. This model enables the MPC controller to address control problems with multiple objectives, i.e. reducing both travel delays and traffic emissions.

Based on the VT-S model, the MPC controller can successfully reduce both the total emissions and the total time spent of urban traffic networks by specifying the control performance indicator. The TTS-based MPC and the TE-based MPC take the total time spent and the total emissions as the control objectives respectively. The TTS-based MPC performs better in decreasing the total time spent at a cost of releasing more traffic emissions; the TE-based MPC decreases the total emission level obviously, but its ability in reducing the total time spent is restricted. The MPC based on both control objectives achieves a trade-off between the total time spent and the total emissions of the traffic network. By applying the total emissions as the control objective, the MPC controller can not only reduce the traffic emissions of urban traffic networks, but also keep a comparatively smoother traffic flow mean speed and less vehicle stops.

![Fig. 7](image-url) The mean speed on the links for (a) the FT controller, (b) the TE-based MPC controller, and (c) the TTS-based MPC controller.


