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# Multi-Class Traffic Flow and Emission Control for Freeway Networks

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**Abstract**—Multi-class emission models aim at capturing the heterogeneous nature of traffic flows. These models are helpful for improving the efficiency of the reduction of emissions. We develop a macroscopic emission model that takes into account the multi-class nature of traffic. This results in an integrated macroscopic multi-class traffic flow and emission model called the multi-class VT-Macro model. Model predictive control is adopted to reduce the total time spent and the total emissions. End point penalties are included in the objective function, taking into account the different distances to the destination of different vehicles. A case study based on a benchmark network is implemented. The simulation results show that better performance is obtained by using the multi-class VT-macro model and end point penalties.

**Keywords:** multi-class traffic, reduction of emissions, multi-class emission model, MPC, end point penalties.

## I. INTRODUCTION

The growing requirements on transportation and the increasing number of vehicles result in a considerable amount of traffic jams and accidents. In addition, the emissions of exhaust fumes and particle matters are damaging people's health. As a result, reduction of the emissions is an important issue in both the current and the long-term perspective.

In order to implement traffic management for reducing traffic jams and accidents, appropriate models are needed to describe and predict the traffic flow phenomena. In practice, macroscopic traffic flow models are easy to adopt in on-line model-based traffic control, due to the good trade-off between the accuracy and the computation speed. Nevertheless, most macroscopic traffic flow models are still single-class, and as a result they are in general not appropriate for control of multi-class traffic. Indeed, real traffic flows are usually multi-class, i.e. they include different types of vehicles such as cars, trucks, buses, mini-vans, and trucks with trailers. Currently, most multi-class traffic models are microscopic, but such models are not appropriate for real-time model-based control due to the slow computation speed. Therefore, fast multi-class prediction models are needed for efficient traffic management. A few macroscopic multi-class traffic flow models have been proposed for use in traffic control such as the multi-class LWR model [1], the FASTLANE model [2], and the multi-class METANET model [3]. The corresponding simulation results show that considering the heterogeneous nature of traffic flow can significantly improve the performance of traffic network.

The efficiency of emission control depends on the emission models used. In principle, microscopic emission

models provide more accurate descriptions than macroscopic models. However, the high computational demands make it impossible to use microscopic emission models in on-line model-based traffic control. Macroscopic emission models can be used to reduce the computation time, and to make on-line traffic management feasible for real traffic networks. The VT-macro model is a macroscopic emission model proposed by Zegeye et al. [4]. In particular, it is developed based on the integration of the single-class METANET traffic flow model [5] and the VT-micro emission model [6]. Currently, the VT-macro model is still single-class. To improve the model accuracy and the control performance, we propose a multi-class VT-macro model in this paper.

This paper is organized as follows. In Section II, we introduce the basics of the original METANET model, the multi-class METANET model, and the (single-class) VT-macro model. We propose the new multi-class VT-macro model in Section III. Afterwards, a multi-class traffic flow and emission control approach is developed in Section IV. Next, a case study is implemented in Section V.

## II. TRAFFIC FLOW AND EMISSION MODELS

### A. Original METANET Model

The METANET model [5] describes a traffic network with uniform links corresponding to freeway stretches. These links have the same features, without any on-ramps, off-ramps, or changes in geometry. Otherwise, a node is placed to capture the changes. Each link is divided into several segments with the same length. These segments are characterized by traffic density ( $\rho_{m,i}(k)$ ), space mean speed ( $v_{m,i}(k)$ ), and traffic outflow ( $q_{m,i}(k)$ ) in each segment  $i$  of each link  $m$  at time step  $k$ . The equations describe the evolution of the traffic

variables are

$$q_{m,i}(k) = \rho_{m,i}(k)v_{m,i}(k)\lambda_m \quad (1)$$

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_m\lambda_m}(q_{m,i-1}(k) - q_{m,i}(k)) \quad (2)$$

$$v_{m,i}(k+1) = v_{m,i}(k) + \frac{T}{\tau}(V(\rho_{m,i}(k)) - v_{m,i}(k)) + \frac{T}{L_m}v_{m,i}(k)(v_{m,i-1}(k) - v_{m,i}(k)) - \frac{T\eta}{L_m\tau} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa} \quad (3)$$

$$V(\rho_{m,i}(k)) = v_{\text{free},m} \exp \left[ -\frac{1}{a_m} \left( \frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}} \right)^{a_m} \right] \quad (4)$$

where  $T$  is the simulation time interval,  $k$  is the time step counter,  $\lambda_m$  is the number of lanes in link  $m$ ,  $\tau, \eta, \kappa$ , and  $a_m$  are model parameters,  $V(\rho)$  is the desired speed at density  $\rho$ ,  $v_{\text{free},m}$  is the average speed in free flow, and  $\rho_{\text{crit},m}$  is the critical density. If there is an on-ramp, the following term is added to (3) to consider the merging phenomena:

$$-\frac{\delta T q_o(k) v_{m,1}(k)}{L_m \lambda_m (\rho_{m,1}(k) + \kappa)} \quad (5)$$

where  $v_{m,1}(k)$  and  $\rho_{m,1}(k)$  are the velocity and density of the first segment of the link  $m$  connected to the on-ramp,  $q_o$  is the ramp flow, and  $\delta$  is a model parameter.

In the evolution of speed, the desired speed  $V(\rho)$  is used. According to Hegyi et al. [7], a dynamic speed limit can be incorporated in the computation of desired speed as follows:

$$V(\rho_{m,i}(k)) = \min \left( v_{\text{free},m} \exp \left[ -\frac{1}{a_m} \left( \frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}} \right)^{a_m} \right], (1 + \alpha) v_{\text{control},m,i}(k) \right) \quad (6)$$

where  $v_{\text{control},m,i}(k)$  is the speed limit imposed in segment  $i$  of link  $m$ , and  $1 + \alpha$  is a non-compliance factor.

In addition, mainstream and on-ramp origins are described with the length of the queue at the corresponding origins:

$$w_o(k+1) = w_o(k) + T(d_o(k) - q_o(k)) \quad (7)$$

where  $w_o(k)$  is the queue length at the mainstream origin or on-ramp origin  $o$ ,  $d_o(k)$  is the origin demand, and  $q_o(k)$  is the origin outflow, which in case of an on-ramp is determined by the following equation:

$$q_o(k) = \min \left[ d_o(k) + \frac{w_o(k)}{T}, C_o r_o(k), C_o \left( \frac{\rho_{\text{max},m} - \rho_{m,1}(k)}{\rho_{\text{max},m} - \rho_{\text{crit},m}} \right) \right] \quad (8)$$

where  $C_o$  is the capacity of origin  $o$ , and  $\rho_{\text{max},m}$  is the maximum density of the link  $m$  to which the on-ramp connects. For a mainstream origin, the outflow is

$$q_o(k) = \min \left[ d_o(k) + \frac{w_o(k)}{T}, q_{\text{lim},m,1}(k) \right] \quad (9)$$

where  $q_{\text{lim},m,1}$  is the maximal inflow in the first segment of link  $m$  connected to the origin:

$$q_{\text{lim},m,1}(k) = \begin{cases} \lambda_m v_{\text{lim},m,1}(k) \rho_{\text{crit},m} \left[ -a_m \ln \left( \frac{v_{\text{control},m,1}(k)}{v_{\text{free},m}} \right) \right]^{\frac{1}{a_m}}, & \text{if } v_{\text{lim},m,1}(k) < V(\rho_{\text{crit},m}) \\ \lambda_m V(\rho_{\text{crit},m}) \rho_{\text{crit},m}, & \text{if } v_{\text{lim},m,1}(k) \geq V(\rho_{\text{crit},m}) \end{cases} \quad (10)$$

with  $v_{\text{lim},m,1}(k) = \min(v_{\text{control},m,1}(k), v_{m,1}(k))$ .

## B. Multi-Class METANET Model

The multi-class METANET model includes the heterogeneous nature of traffic flows [3]. More specifically, it is assumed that several different classes of vehicles are in the traffic network. Each class is represented by its own state variables (traffic density, space mean speed, and traffic outflow). All variables are expressed in equivalent vehicles to account for the typical lengths of various classes of vehicles. In particular, the network state is represented by the following variables of every class of vehicles: the equivalent density fraction  $\theta_{m,i,c}(k)$ , the equivalent density  $\rho_{m,i,c}(k)$ , the space-mean speed  $v_{m,i,c}(k)$ , and equivalent partial outflow  $q_{m,i,c}(k)$ , where  $c$  is the index of the vehicle class. These variables are defined as

$$\rho_{m,i,c}(k) = \frac{L_c^{\text{veh}}}{L_1^{\text{veh}}} \rho_{m,i,c}^{\text{actual}}(k), \quad \theta_{m,i,c}(k) = \frac{\rho_{m,i,c}(k)}{\rho_{m,i,\text{tot}}(k)}, \quad \rho_{m,i,\text{tot}}(k) = \sum_{c=1}^C \rho_{m,i,c}(k) \quad (11)$$

where  $L_c^{\text{veh}}$  denotes the typical vehicle length for class  $c$ ,  $\rho_{m,i,c}^{\text{actual}}(k)$  is the actual density of a vehicle of class  $c$  in segment  $i$  of link  $m$  at time step  $k$ ,  $\rho_{m,i,\text{tot}}(k)$  is the total equivalent density, and  $\rho_{m,i,c}(k)$  and  $q_{m,i,c}(k)$  are computed through (1) and (2) with  $v_{m,i,c}(k)$  instead of  $v_{m,i}(k)$ . The speed  $v_{m,i,c}(k)$  is updated as follows:

$$v_{m,i,c}(k+1) = v_{m,i,c}(k) + \frac{T}{\tau_c} \left( \tilde{V}(\rho_{m,i,\text{tot}}(k), \theta_{m,i,1}(k), \dots, \theta_{m,i,C}(k), c) - v_{m,i,c}(k) \right) + \frac{T}{L_m} v_{m,i,c}(k) (v_{m,i-1,c}(k) - v_{m,i,c}(k)) - \frac{\eta_c T}{\tau_c L_m} \frac{\rho_{m,i+1,\text{tot}}(k) - \rho_{m,i,\text{tot}}(k)}{\rho_{m,i,\text{tot}}(k) + \kappa_c} \quad (12)$$

where  $\tilde{V}(\rho_{m,i,\text{tot}}(k), \theta_{m,i,1}(k), \dots, \theta_{m,i,C}(k), c)$  is the desired speed for vehicles of class  $c$  for total equivalent density  $\rho_{m,i,\text{tot}}(k)$  and density fractions  $\theta_{m,i,1}(k), \dots, \theta_{m,i,C}(k)$ . One way to determine  $\tilde{V}$  is [3]:

$$\tilde{V}(\rho_{m,i,\text{tot}}(k), \theta_{m,i,1}(k), \dots, \theta_{m,i,C}(k), c) = \min \left( V_c(\rho_{m,i,\text{tot}}(k)), \sum_{\gamma=1}^C \theta_{m,i,\gamma}(k) V_\gamma(\rho_{m,i,\text{tot}}(k)) \right) \quad (13)$$

with

$$V_c(\rho_{m,i,\text{tot}}(k)) = v_{\text{free},m,c} \exp \left[ -\frac{1}{a_{m,c}} \left( \frac{\rho_{m,i,\text{tot}}(k)}{\rho_{\text{crit},m,\text{total}}} \right)^{a_{m,c}} \right] \quad (14)$$

In (12), (13), and (14), class-dependent parameters ( $\tau_c$ ,  $\eta_c$ ,  $\kappa_c$ ,  $v_{\text{free},m,c}$ ,  $a_{m,c}$ ) are included. For the origin queue length and outflow computation, (7) and (10) still hold for the equivalent variables  $w_{o,c}$ ,  $d_{o,c}$ , and  $q_{o,c}$ . Here,  $w_{o,c}$  is the equivalent partial queue length,  $d_{o,c}$  is the equivalent partial origin demand, and  $q_{o,c}$  is the equivalent partial outflow. However, in (9) and (8) the capacity  $C_o$  and  $q_{\text{lim},m,1}$  should both be multiplied by a fraction

$$\frac{q_{o,c}^{\text{des}}(k)}{\sum_{\gamma=1}^C q_{o,\gamma}^{\text{des}}(k)} \quad (15)$$

where  $q_{o,c}^{\text{des}}(k) = d_{o,c}(k) + \frac{w_{o,c}(k)}{T}$ .

Traffic management aims at improving network performance, which can be interpreted in many ways. A common objective function that is used to describe the network performance is the total time spent (TTS). The TTS is the total time that all vehicles spend in the traffic network. In MPC, we denote the control time interval by  $T_c$ , the control time step counter by  $k_c$ , and we assume that  $M = \frac{T_c}{T}$  is an integer. Then the TTS over a period  $[T_c k_c, T_c(k_c + N_p)]$  for the multi-class METANET model is given as

$$J_{\text{TTS}}(k_c) = T \sum_{j=k_c M}^{(k_c+N_p)M-1} \sum_{c=1}^C \frac{L_1^{\text{veh}}}{L_c^{\text{veh}}} \left( \sum_{(m,i) \in I_{\text{all}}} \rho_{m,i,c}(j) L_m \lambda_m + \sum_{o \in O_{\text{all}}} w_{o,c}(j) \right) \quad (16)$$

where  $N_p$  is the prediction horizon,  $I_{\text{all}}$  is the set of indices of all pairs of segments and links, and  $O_{\text{all}}$  is the set of indices of all origins.

### C. VT-Macro Model

The VT-macro model [4] is a macroscopic model that describes emissions and fuel consumption in traffic networks. It is an integration of the VT-micro model [6] and the METANET model [5, 8]. VT-micro is a microscopic emissions and fuel consumption model that yields the emissions and fuel consumption rate of an individual vehicle based on the speed and the acceleration of that vehicle. So the estimate of the emissions and fuel consumption needs speeds and accelerations. However, the METANET model only yields space-mean speeds. The accelerations can be generated from the METANET model as follows [4]. Two acceleration components (segmental acceleration and cross-segmental acceleration) are defined:

$$a_{m,i}^{\text{seg}}(k) = \frac{v_{m,i}(k+1) - v_{m,i}(k)}{T} \quad (17)$$

$$a_{\alpha,\beta}^{\text{cross}}(k) = \frac{v_{\beta}(k+1) - v_{\alpha}(k)}{T} \quad (18)$$

where  $\alpha$  and  $\beta$  represent different segments, on-ramps, or off-ramps, with  $\beta$  being adjacent to  $\alpha$ . The numbers of

vehicles subject to these two accelerations are

$$n_{m,i}^{\text{seg}}(k) = (L_m \lambda_m \rho_{m,i}(k) - T q_{m,i}(k)) \quad (19)$$

$$n_{\alpha,\beta}^{\text{cross}}(k) = T q_{\alpha}(k) \quad (20)$$

The VT-macro model then provides the estimates of emissions per time unit:

$$J_{y,m,i}^{\text{seg}}(k) = n_{m,i}^{\text{seg}}(k) \exp(\tilde{v}_{m,i}^T(k) P_y \tilde{a}_{\text{seg},m,i}(k)) \quad (21)$$

$$J_{y,\alpha,\beta}^{\text{cross}}(k) = n_{\alpha,\beta}^{\text{cross}}(k) \exp(\tilde{v}_{\alpha}^T(k) P_y \tilde{a}_{\alpha,\beta}^{\text{cross}}(k)) \quad (22)$$

where  $y \in Y = \{\text{CO}, \text{NO}_x, \text{HC}\}$ ,  $P_y$  is a model parameter matrix [4], and  $\tilde{x} = [1 \ x \ x^2 \ x^3]^T$ . The sum of the estimates of (25) and (26) over all segments of all links and all pairs of adjacent segments yields the total emissions (TE).

### III. MULTI-CLASS VT-MACRO MODEL

At present, the multi-class case has not been considered in the VT-macro model. To reduce the emissions more efficiently, it is necessary to explore a multi-class VT-macro model. Here, we propose such a model.

For multi-class traffic flow, the equivalent variables are used in the computation of the emission estimates. The accelerations for each class  $c$  are then given by

$$a_{m,i,c}^{\text{seg}}(k) = \frac{v_{m,i,c}(k+1) - v_{m,i,c}(k)}{T} \quad (23)$$

$$a_{\alpha,\beta,c}^{\text{cross}}(k) = \frac{v_{\beta,c}(k+1) - v_{\alpha,c}(k)}{T} \quad (24)$$

The corresponding actual numbers of vehicles are

$$n_{m,i,c}^{\text{seg}}(k) = \frac{L_1^{\text{veh}}}{L_c^{\text{veh}}} (L_m \lambda_m \rho_{m,i,c}(k) - T q_{m,i,c}(k)) \quad (25)$$

$$n_{\alpha,\beta,c}^{\text{cross}}(k) = \frac{L_1^{\text{veh}}}{L_c^{\text{veh}}} T q_{\alpha,c}(k) \quad (26)$$

The emission estimates for each class are

$$J_{y,m,i,c}^{\text{seg}}(k) = n_{m,i,c}^{\text{seg}}(k) \exp(\tilde{v}_{m,i,c}^T(k) P_{y,c} \tilde{a}_{m,i,c}^{\text{seg}}(k)) \quad (27)$$

$$J_{y,\alpha,\beta,c}^{\text{cross}}(k) = n_{\alpha,\beta,c}^{\text{cross}}(k) \exp(\tilde{v}_{\alpha,c}^T(k) P_{y,c} \tilde{a}_{\alpha,\beta,c}^{\text{cross}}(k)) \quad (28)$$

where the parameter matrices  $P_{y,c}$  are class-dependent. The sum of the estimates of all classes of vehicles in all segments of all links and all pairs of adjacent segments yields the total emissions (TE) of  $y$ :

$$J_{\text{TE},y}(k_c) = T \sum_{j=k_c M}^{(k_c+N_p)M-1} \sum_{c=1}^C \left[ \sum_{(m,i) \in I_{\text{all}}} J_{y,m,i,c}^{\text{seg}}(j) + \sum_{\alpha,\beta \in P_{\text{all}}} J_{y,\alpha,\beta,c}^{\text{cross}}(j) \right] \quad (29)$$

where  $I_{\text{all}}$  is the set of all pairs  $(m, i)$  of segments and links in the network, and  $P_{\text{all}}$  is the set of all pairs of adjacent segments.

#### IV. MULTI-CLASS TRAFFIC FLOW AND EMISSION CONTROL

We adapt on-line Model Predictive Control (MPC) for the multi-class emission control. MPC [9] is a control methodology that is based on a prediction model and a receding horizon approach. In MPC, the performance is evaluated based on an objective function that captures the predicted performance of the traffic network over some prediction horizon. The controller determines the control inputs sequence that optimize the objective function, and the first element of the control sequence is applied to the control system. MPC can be adopted in nonlinear systems, and deal with multi-criteria optimization and constraints.

Here, the new multi-class VT-macro model is used as prediction model. The objective function is

$$\begin{aligned}
J(k_c) = & \xi_{\text{TTS}} \frac{J_{\text{TTS}}(k_c)}{J_{\text{TTS}}^{\text{nominal}}} + \sum_{y \in Y} \xi_{\text{TE},y} \frac{J_{\text{TE},y}(k_c)}{J_{\text{TE},y}^{\text{nominal}}} \\
& + \xi_{\text{ramp}} \sum_{j=k_c}^{k_c+N_c-1} \sum_{o \in O_{\text{ramp}}} (r_o(j) - r_o(j-1))^2 \\
& + \xi_{\text{speed}} \sum_{j=k_c}^{k_c+N_c-1} \sum_{(m,i) \in I_{\text{speed}}} \left( \frac{v_{m,i}^{\text{ctrl}}(j) - v_{m,i}^{\text{ctrl}}(j-1)}{v_{\text{free},m,\text{max}}} \right)^2 \\
& + \xi_{\text{TTS}}^{\text{end point}} J_{\text{TTS}}^{\text{end point}}(k_c) + \sum_{y \in Y} \xi_{\text{TE},y}^{\text{end point}} J_{\text{TE},y}^{\text{end point}}(k_c)
\end{aligned} \quad (30)$$

Because the orders of magnitude are different for the TTS and the TE, we use normalized terms. The first term in (34) is the TTS divided by the nominal TTS, and the second term is the TE divided by the nominal TE. The nominal TTS is value of TTS for some nominal control profile, and the nominal TE is defined in a similar way. The parameters  $\xi_{\text{TTS}}$  and  $\xi_{\text{TE},y}$  are the weights for the normalized TTS and TE. The third term and the fourth term in (34) penalize variations of the control inputs. Besides,  $\xi_{\text{ramp}}$  and  $\xi_{\text{speed}}$  are the weight parameters,  $O_{\text{ramp}}$  represents all metered origins,  $v_{\text{free},m,\text{max}} = \max_c v_{\text{free},m,c}$ , and  $I_{\text{speed}}$  represents all the segments with speed limits. The fifth term and sixth term in (34) are the end point terms for TTS and TE, which are defined below, and  $\xi_{\text{TTS}}^{\text{end point}}$  and  $\xi_{\text{TE},y}^{\text{end point}}$  are the weight parameters for the end point penalties. The end point penalty  $J_{\text{TTS}}^{\text{end point}}(k_c)$  represents an estimate of the total time spent for the remaining vehicles at time step  $(k_c + N_p)M$  to leave the network, and  $J_{\text{TE},y}^{\text{end point}}(k_c)$  represents an estimate of the total emissions  $y$  that the remaining vehicles at time step  $(k_c + N_p)M$  will generate before they leave the network.

The idea behind end point penalties is that we want the control to bring as many vehicles as close to their destination. Without an end point term we cannot capture the difference between vehicles that are almost at their destination and vehicles that are still in the origin queues. To represent the differences, the numbers of vehicles in each segment will be multiplied by the time  $t_{m,i,c}^{\text{rem}}((k_c + N_p)M)$  that a vehicle that is present in that segment at time step  $(k_c + N_p)M$  would on the average need to get to the destination. Similarly, the number of

vehicles in each queue is multiplied by the time  $t_{o,c}^{\text{rem}}((k_c + N_p)M)$  that a vehicle present in that queue at time step  $(k_c + N_p)M$  would on the average need to get to the destination. This then yields an estimate of the TTS needed for all vehicles that are still in the network/queues at time step  $(k_c + N_p)M$ :

$$\begin{aligned}
J_{\text{TTS}}^{\text{end point}}(k_c) = & \sum_{c=1}^C \frac{L_1^{\text{veh}}}{L_c^{\text{veh}}} \left[ \sum_{(m,i) \in I_{\text{all}}} L_m \rho_{m,i,c}((k_c + N_p)M) \lambda_m t_{m,i,c}^{\text{rem}}((k_c + N_p)M) \right. \\
& \left. + \sum_{o \in O_{\text{all}}} w_{o,c}((k_c + N_p)M) t_{o,c}^{\text{rem}}((k_c + N_p)M) \right]
\end{aligned} \quad (31)$$

The end point term  $J_{\text{TE},y}^{\text{end point}}$  can be formulated in a similar way. The number of vehicles in each segment at time step  $(k_c + N_p)M$  is multiplied by the emissions  $\text{TE}_{y,m,i,c}^{\text{rem}}((k_c + N_p)M)$  that a vehicle that is present in that segment at time step  $(k_c + N_p)M$  will generate on the average before leaving the network. Likewise, the number for vehicles in each queue at time step  $(k_c + N_p)M$  is multiplied by the emissions  $\text{TE}_{y,o,c}^{\text{rem}}((k_c + N_p)M)$  that a vehicle that is present in that queue at time step  $(k_c + N_p)M$  will generate on the average before leaving the network. This yields the following estimate of the total emissions that the remaining vehicles at time step  $(k_c + N_p)M$  generate before they leave the network:

$$\begin{aligned}
J_{\text{TE},y}^{\text{end point}}(k_c) = & \sum_{c=1}^C \frac{L_1^{\text{veh}}}{L_c^{\text{veh}}} \left[ \sum_{(m,i) \in I_{\text{all}}} L_m \rho_{m,i,c}((k_c + N_p)M) \lambda_m \text{TE}_{y,m,i,c}^{\text{rem}}((k_c + N_p)M) \right. \\
& \left. + w_{o,c}((k_c + N_p)M) \text{TE}_{y,o,c}^{\text{rem}}((k_c + N_p)M) \right]
\end{aligned} \quad (32)$$

#### V. CASE STUDY

##### A. Benchmark Network

In this paper a benchmark network [3, 7] with two links is used as case study. The first link has four segments, and the second link has two segments. The mainstream origin is  $O_1$ , and it connects to a main road with two lanes. An on-ramp  $O_2$  is located in between link 1 and link 2, and this on-ramp has one lane. There are two speed limits imposed on segments 3 and 4 of link 1. The destination  $D_1$  has unrestricted outflow. The network schematic is shown in Fig. 1.

The parameters are taken from [3, 7, 10]. Two vehicle classes are adopted with the following parameters:  $v_{\text{free},m,1} = 106.34$  km/h,  $a_{m,1} = 1.6761$ ,  $\alpha_1 = 0.12$ ,  $\rho_{\text{crit},m,1} = 34.7349$  veh/km/lane,  $\rho_{\text{max},1} = 175$  veh/km/lane,  $C_{\text{mainstream},1} = 2034$  veh/h/lane;  $v_{\text{free},m,2} = 82.80$  km/h,  $a_{m,2} = 2.1774$ ,  $\alpha_2 = 0.0533$ ,  $\rho_{\text{crit},m,2} = 18.9261$  veh/km/lane,  $\rho_{\text{max},2} = 75$  veh/km/lane,  $C_{\text{mainstream},2} = 990$  veh/h/lane.

We have  $L_1^{\text{veh}} = 7/3 L_2^{\text{veh}}$ . The destination  $D_1$  has an unrestricted outflow. The queue length at  $O_2$  is assumed to be limited to 100 pce (passenger car equivalents) to avoid spill-back to a surface street intersection. The

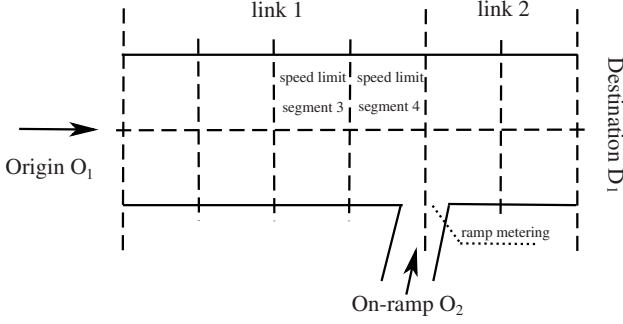


Fig. 1. The benchmark network schematic with two speed limits and one ramp metering installation

parameters for single-class model are obtained by convex combination of class 1 and class 2:  $\text{Parameters}_{\text{nominal}} = \theta_{\text{nominal}}^1 \text{Parameters}_1 + (1 - \theta_{\text{nominal}}^1) \text{Parameters}_2$ . Here we set  $\theta_{\text{nominal}}^1 = 0.7$ . The capacity of on-ramp is obtained through  $C_{\text{on ramp}}^{\text{nominal}} - 100 = C_{\text{mainstream}}^{\text{nominal}} - 100$ . Other model parameters are:  $L = 1$  km,  $\tau = 18$  s,  $\kappa = 40$  veh/h/km,  $\eta = 60$  km<sup>2</sup>/h,  $\delta = 0.0122$ . As for the control parameters, we select  $\xi_{\text{TTS}} = 1$ ,  $\xi_{\text{TE},y} = 0.1$ ,  $\xi_{\text{ramp}} = \xi_{\text{speed}} = 0.01$ ,  $\xi_{\text{TTS}}^{\text{end point}} = 0.1$ ,  $\xi_{\text{TE},y}^{\text{end point}} = 0.01$ ,  $T = 10$  s,  $T_c = 60$  s,  $N_p = 7$ ,  $N_c = 5$ .

The nominal model parameter matrices  $P_{\text{CO}}^{\text{nominal}}$ ,  $P_{\text{HC}}^{\text{nominal}}$ , and  $P_{\text{NO}_x}^{\text{nominal}}$  for the emissions are given by [6, 11]. For vehicles of class 1, we assume:

$$P_{\text{CO}}^1 = 1.1P_{\text{CO}}^{\text{nominal}} \quad P_{\text{HC}}^1 = 1.1P_{\text{HC}}^{\text{nominal}} \quad P_{\text{NO}_x}^1 = 1.1P_{\text{NO}_x}^{\text{nominal}}$$

The parameter matrices for class 2 are chosen so that the nominal parameters correspond to 70% vehicles of class 1.

The total simulation time is 2.5 h. A typical demand is applied, as shown in Fig. 2. The mainstream demand is 3500 veh/h from  $t = 0$  h up to  $t = 2$  h, and then drops to 1000 veh/h in 15 minutes. The on-ramp demand starts at 500 veh/h at  $t = 0$  h and then immediately increases to 1500 veh/h in 6 minutes and keeps this value for 15 minutes. Next, the on-ramp demand decreases to 500 veh/h in 6 minutes and stays constant for the remainder of the simulation.

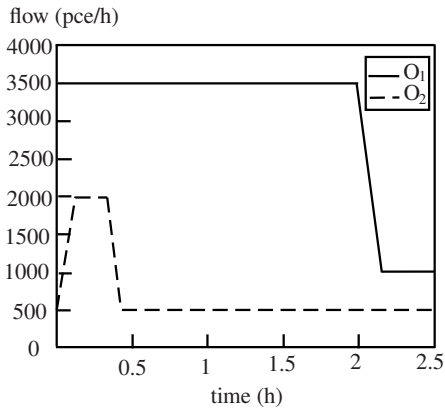


Fig. 2. The demand scenario applied in the case study

## B. The Computation of End Point Penalties

The end point penalty term  $J_{\text{TTS}}^{\text{end point}}$  is computed by estimating the TTS for the vehicles remaining in the network at control step  $k_c + N_p$ . For the layout of Fig. 1, we have

$$t_{m,i,c}^{\text{rem}}((k_c + N_p)M) = -\frac{0.5L}{v_{m,i,c}((k_c + N_p)M)} + \sum_{l=m}^{N_{\text{link}}} \sum_{j=i}^{N_{\text{seg},m}} \frac{L}{v_{l,j,c}((k_c + N_p)M)} \quad (33)$$

where  $N_{\text{link}} = 2$  is the number of links, and  $N_{\text{seg},m}$  is the number of segments in link  $m$ , with  $N_{\text{seg},1} = 4$  and  $N_{\text{seg},2} = 2$ .

For a vehicle in the queue at the origin  $O_1$ , the time that is needed to get to the destination is

$$t_{O_1,c}^{\text{rem}}((k_c + N_p)M) = \sum_{l=1}^2 \sum_{j=1}^{N_{\text{seg},l}} \frac{L}{v_{l,j,c}((k_c + N_p)M)} \quad (34)$$

For a vehicle in the queue at the on-ramp  $O_2$ , the time that is needed to get to the destination is

$$t_{O_2,c}^{\text{rem}}((k_c + N_p)M) = \sum_{j=1}^2 \frac{L}{v_{2,j,c}((k_c + N_p)M)} \quad (35)$$

The total amount of emissions that are generated on the average before leaving the network by a vehicle of class  $c$  that are in segment  $(m,i)$  at time step  $(k_c + N_p)M$  is

$$\begin{aligned} \text{TE}_{y,m,i,c}^{\text{rem}}((k_c + N_p)M) = & T \left[ \sum_{(l,j) \in S_{(m,i)}} \exp\left(\tilde{v}_{l,j,c}^T((k_c + N_p)M) P_{y,c} \tilde{a}_{l,j,c}^{\text{seg}}((k_c + N_p)M)\right) + \right. \\ & \left. \sum_{(\alpha,\beta) \in P_{(m,i)}} \exp\left(\tilde{v}_{\alpha,c}^T((k_c + N_p)M) P_{y,c} a_{\alpha,\beta,c}^{\text{cross}}((k_c + N_p)M)\right) \right] \quad (36) \end{aligned}$$

where  $S_{(m,i)}$  is the set of segments  $(l,j)$  that vehicles will travel through starting from the current segment  $(m,i)$  to the destination,  $P_{(m,i)}$  is the set of pairs of adjacent segments  $(\alpha,\beta)$  over which the vehicles will cross when traveling from the current segment  $(m,i)$  to the destination.

The total emissions generated on the average before leaving the network by a vehicle of class  $c$  that are at origin  $o \in O_1, O_2$  at time step  $(k_c + N_p)M$  are estimated as follows:

$$\begin{aligned} \text{TE}_{y,o,c}^{\text{rem}}((k_c + N_p)M) = & T \left[ \sum_{(l,j) \in S_o} \exp\left(\tilde{v}_{l,j,c}^T((k_c + N_p)M) P_{y,c} \tilde{a}_{l,j,c}^{\text{seg}}((k_c + N_p)M)\right) + \right. \\ & \left. \sum_{(\alpha,\beta) \in P_o} \exp\left(\tilde{v}_{\alpha,c}^T((k_c + N_p)M) P_{y,c} a_{\alpha,\beta,c}^{\text{cross}}((k_c + N_p)M)\right) \right] \quad (37) \end{aligned}$$

where  $S_o$  and  $P_o$  are defined in a similar way to  $S_{(m,i)}$  and  $P_{(m,i)}$ . Hence, we have the following sets:  $S_{(2,2)} = \{(2,2)\}$ ,  $S_{(2,1)} = \{(2,1)\}$ ,  $S_{(1,i)} = \{(1,i), (1,i+1), \dots, (1,4)\} \cup S_{(2,1)}$ ,  $S_{O_2} = S_{(2,1)}$ ,  $S_{O_1} = S_{1,1}$ ;  $P_{(2,2)} = \{((2,2), D_1)\}$ ,  $P_{(2,1)} = \{((2,1), (2,2))\} \cup P_{(2,2)}$ ,  $P_{(1,4)} = \{((1,4), (2,1))\} \cup P_{(2,1)}$ ,  $P_{(1,i)} = \{((1,i),$

$(1, i+1)\} \cup P_{(1,i)}$   $i = 1, 2, 3$ ,  $P_{O_2} = \{(O_2, (2, 1))\} \cup P_{(2,1)}$ ,  $P_{O_1} = \{(O_1, (1, 1))\} \cup P_{(1,1)}$ . Moreover, we assume  $v_{O_1,c}(k) = v_{(1,1,c)}(k)$ ,  $v_{D_1,c}(k) = v_{(2,2,c)}(k)$ , and  $v_{O_2,c}(k) = 30$  km/h for all  $k$ .

### C. Simulation Results

The multi-class METANET model and the multi-class VT-macro model are used as simulation models. As for the prediction models, two cases are considered. One case uses the single-class models with the nominal parameters. The other case uses the multi-class models with the real ratio of class 1. Besides, multi-class simulations with and without end point penalties are both implemented as comparison. So the simulation scenarios are

- No control,
- Single-class MPC,
- Multi-class MPC without end point penalties,
- Multi-class MPC with end point penalties.

We set the equivalent density fraction in the demand as  $\theta_1 = 0.3$ ,  $\theta_2 = 0.7$ . The results are shown in the Table 1. The TTS and TE listed in Table 1 are calculated for the entire simulation period of 2.5 h. The results show that taking into account the multi-class nature and end point penalties leads to better performance (the reduction of  $J$  is bigger).

Table 1 Simulation results

| Scenario                                 | TTS (veh-h) | TE (kg)  | $J$     |
|--|-------------|----------|---------|
| No control                               | 1427.4      | 403.4045 | 62.2896 |
| Single-class MPC                         | 1426.6      | 403.3047 | 62.2548 |
| Multi-class MPC                          | 1393.2      | 402.8921 | 60.8451 |
| Multi-class MPC with end point penalties | 1358.5      | 401.5989 | 59.3788 |

## VI. CONCLUSIONS

We have extended the single-class VT-macro emission model to a multi-class version. More specifically, equations from the multi-class METANET model are used to estimate accelerations, which are next used in a multi-class version of the VT-macro model. Moreover, we have included end point penalties in the objective function, to take into account the different distances to the destination for vehicles in different segments. By including end point penalties we aim to bring as many vehicles as close to their destinations. A case study was implemented to illustrate the efficiency of the multi-class VT-macro model and the end point penalties. Based on the simulation results, we can conclude that taking into account the multi-class nature and including end point penalties leads to better performance (for the given set-up and demand scenarios).

## ACKNOWLEDGMENTS

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