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Abstract.
To reduce the operation cost and then improve the operational efficiency, people are paying more and more attention to the energy-efficient operation of subway systems. In this paper, we present and compare to two algorithms to optimize the energy-efficient speed profile for trains of subway systems, which can reduce the energy consumption of train operations. Firstly, we formulate a mixed integer linear programming (MILP) model to get the optimal trajectory for trains. Secondly, we present an integrated algorithm for optimizing the timetable for the entire route together with the speed profiles between successive stations, which is called as integrated timetable. Finally, we give some numerical examples to illustrate the validity of the algorithms based on the data from the Beijing YiZhuang subway line in China.

Keywords: Energy-efficient operation; Optimal train control; Timetable;

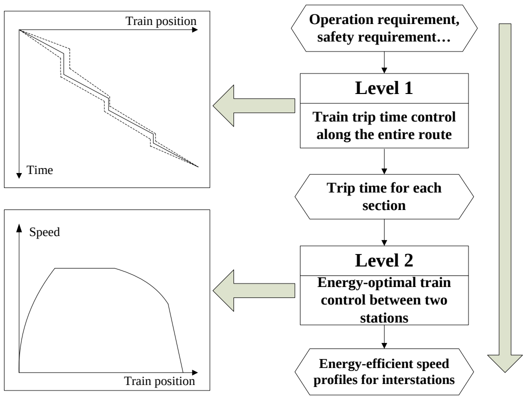

FIGURE 1 A two level approach to energy-efficient operation (*18*)

1 INTRODUCTION

Train energy-efficient operation aims to optimize the timetable and driving strategy such that the energy consumption for towing the train is minimized. Considering the process of the train operation, a two-level model was presented by Albrecht et al. (*18*) (see Figure 1). The first level of the energy-efficient operation is to distribute the total trip time over the various sections of the route, i.e., the trip time between stations and dwell time in stations, because the distribution of the running time among different sections of a line strongly influence the train's traction energy consumption during its journey. The second level is to choose the way among different driving strategies to drive a train with a given running time such that the energy consumption is minimized. During the optimizing process, the safety demands from the control system should be satisfied.

Within a period of time, the trains servicing stations in the subway system will follow each other with a fixed cycle time, which is called as the cyclic railway timetable. A Periodic Event Scheduling Problem (PESP) model, introduced by Serafini and Ukovich (*17*), lays the foundation for solving the cyclic railway timetable problem, which considers the problem of scheduling a set of periodically recurring events under periodic time window constraints. Odijk (*13*) used a cycle periodicity formulation model to formulate the cyclic behavior of the railway timetables. In the following years, the cycle periodicity formulation model was extended to consider different periods (*12*), variable travel times (*9*), and safety and frequency constraints (*14*). Other recent developments in this field mainly concentrated on the design of robust cyclic timetables to cope with stochastic delays (*8, 10*).

In literature, there are many papers focusing on how to obtain the energy-efficient speed profile between successive stations. An optimal control model was first formulated in the 1960s (*4*), which assumed that the train runs on a flat track with constant gradient and traction efficiency. By using the Pontryagin maximum principle, the optimal driving strategy was proved to consist of maximum acceleration, cruising, coasting, and maximum braking. As extension of the optimal model, variable gradients, variable speed limits, and traction efficiency were gradually considered (*2, 6, 15*). To reduce the computation time, Khmelnitsky (*6*), Howlett et al. (*3*), Liu and Golovitcher (*15*) presented more detailed and faster approaches to the train energy-efficient operation problem, in which variable gradients, variable traction efficiency, and arbitrary speed limits were all considered, and they gave an analytical iterative method to calculate the driving trajectory for each small part of the route. Besides, some evolutionary algorithms, such as generic algorithms (*1*) and ant colony optimization algorithms (*5*) were applied to the train control problem to generate an optimal speed profile.

In this paper, we distribute the route into small sections in which the gradient is constant, then use a mixed integer linear programming (MILP) method to generate the speed profile in between consecutive stations. Scheduling the timetable and optimizing the speed profile are studied separately, and we present an integrated algorithm to achieve a better performance on energy saving by optimizing the speed profile between successive stations as well as the timetable for an entire route.

The rest of this paper is organized as follows. In Section 2, we formulate a mixed integer linear programming (MILP) model and an integrated timetable model to solve the optimal train control problem. In Section 3, we present the numerical example based on the infrastructure data and the operation data from Beijing YiZhuang subway line in China, which illustrates that the proposed algorithms can yield a good performance for energy saving. A short discussion and future work are included in Section 4.

2 METHOD 1: THE MILP APPROACH

In this section, the train operation model is described as a nonlinear optimal model, and transformed into a mixed logical dynamic model by using piecewise affine approximations. Then the MILP method is proposed to solve the problem.

The mass-point model of train is widely used in the literature on optimal control of trains (*15*). The continuous-time model of train operation is described as:

$$\begin{cases} m\rho \frac{dv}{dt} = u(t) - R_{\mathrm{b}}(v) - R_{\mathrm{l}}(s,v) \\ \frac{ds}{dt} = v \end{cases} \tag{1}$$

where m is the mass of the train, ρ is a factor to consider the rotating mass, v is the velocity of the train, s is the position of the train, u is the control variable, i.e. the traction or braking force, which is bounded by the maximum traction force $u_{\max}$ and the maximum braking force $u_{\min}$: $u_{\min} \leq u \leq u_{\max}$, $R_{\mathrm{b}}(v)$ is the basic resistance including roll resistance and air resistance, and $R_{\mathrm{l}}(s,v)$ is the line resistance caused by track grade, curves, and tunnels. In practice, according to the Strahl formula (*16*) the basic resistance $R_{\mathrm{b}}(v)$ can be described as

$$R_{\mathrm{b}}(v) = m(a_1 + a_2 v^2), \tag{2}$$

where the coefficients a_1 and a_2 depend on the train characteristics and the wind speed, which can be calculated from the data known about the train. The line resistance $R_{\mathrm{l}}(s,v)$ caused by track slope, curves, and tunnels can be described by (*11*)

$$R_{\mathrm{l}}(s,v) = mg\sin\alpha(s) + f_{\mathrm{c}}(r(s)) + f_{\mathrm{t}}(l_{\mathrm{t}}(s),v), \tag{3}$$

where g is the gravitational acceleration, $\alpha(s)$, $r(s)$, and $l_{\mathrm{t}}(s)$ are the slope, the radius of the curve, and the length of the tunnel along the track, respectively. The curve resistance $f_{\mathrm{c}}(\cdot)$ and the tunnel resistance $f_{\mathrm{t}}(\cdot)$ are given by empirical formulas, see (*22, 23*) for more information.

Based on the literature (*6*), we choose the kinetic energy per mass unit $\tilde{E} = 0.5v^2$ and the time t as states, and formulate the optimal train operation problem as:

$$\begin{cases} J = \int_{s_{\mathrm{start}}}^{s_{\mathrm{end}}} max(0, u(s))ds \\ u_{\min} \leq u(s) \leq u_{\max}(\tilde{E}),\ 0 < \tilde{E}(s) \leq \tilde{E}_{\max}(s) \\ \tilde{E}(s_{\mathrm{start}}) = \tilde{E}_{\mathrm{start}},\ \tilde{E}(s_{\mathrm{end}}) = \tilde{E}_{\mathrm{end}} \\ t(s_{\mathrm{start}}) = 0,\ \ t(s_{\mathrm{end}}) = T. \end{cases} \tag{4}$$

where the objective function J is the energy consumption of the train operation, s_{start} and s_{end} are the positions at the beginning and the end of the route, respectively; and the scheduled running time T is given by the timetable or the rescheduling process. It is assumed that $\tilde{E}(s) > 0$, which means that the train's speed is always strictly larger than zero, i.e. the train travels nonstop (*6*).

We first split the position interval $[s_{\text{start}}, s_{\text{end}}]$ into N intervals and assume that the track and train parameters as well as traction or breaking force can be considered as constant in each interval $[s_k, s_{k+1}]$ with length $\Delta s_k = s_{k+1} - s_k$, for $k = 1, 2, \ldots, N$. Note that $s_1 = s_{\text{start}}$ and $s_{N+1} = s_{\text{end}}$. The continuous problem is formulated into a discrete problem. Moreover, the nonlinear terms in the state equation and the nonlinear function of the maximum traction force are approximated using piecewise affine functions (*20*). Then, the dynamics of the train operation can be transformed into a so-called mixed logical dynamic model of the following form:

$$\begin{aligned} x(k+1) = A_k x(k) + B_k u(k) + C_{1,k}\delta(k) + C_{2,k}\delta(k+1) \\ + D_{1,k}z(k) + D_{2,k}z(k+1) + e_k, \end{aligned} \tag{5}$$

$$\begin{aligned} R_{1,k}\delta(k) + R_{2,k}\delta(k+1) + R_{3,k}z(k) + R_{4,k}z(k+1) \\ \leq R_{5,k}u(k) + R_{6,k}x(k) + R_{7,k}, \end{aligned} \tag{6}$$

where $x(k) = \begin{bmatrix} E(k) & t(k) \end{bmatrix}^T$ and $\delta(k)$ and $z(k)$ are a binary variables vector and a real-valued auxiliary variables vector respectively. For the sake of simplicity, we use $E(k)$ as a short-hand notation for $\tilde{E}(s_k)$ in the rest of the paper.

A new variable $\omega(k)$ is introduced to deal with the function $max(0, u)$ in the objective function, therefore the following linear inequalities are added:

$$\omega(k) \geq u(k), \omega(k) \geq 0. \tag{7}$$

The optimal control problem can be transformed into a mixed integer linear programming (MILP) problem (*20*) at the following form:

$$\min_{\tilde{V}} \quad c_{\text{J}}^T \tilde{V}, \tag{8}$$

subject to

$$F_1\tilde{V} \leq F_2 x(1) + f_3 \tag{9}$$

$$F_4\tilde{V} = F_5 x(1) + f_6 \tag{10}$$

where $c_{\text{J}} = \begin{bmatrix} 0 & \cdots & 0 & \Delta s_1 & \cdots & \Delta s_N \end{bmatrix}^T$,
$\tilde{V} = \begin{bmatrix} \tilde{u}^T & \tilde{\delta}^T & \tilde{z}^T & \tilde{\omega}^T \end{bmatrix}^T$,

$$\tilde{u} = \begin{bmatrix} u(1) \\ u(2) \\ \vdots \\ u(N) \end{bmatrix}, \tilde{\delta} = \begin{bmatrix} \delta(1) \\ \delta(2) \\ \vdots \\ \delta(N+1) \end{bmatrix}, \tilde{\omega} = \begin{bmatrix} \omega(1) \\ \omega(2) \\ \vdots \\ \omega(N) \end{bmatrix},$$

and $\tilde{z}$ is defined in a similar way as $\tilde{\delta}$. This MILP problem can be solved by several existing commercial and free solvers, such as CPLEX, Xpress-MP, GLPK.

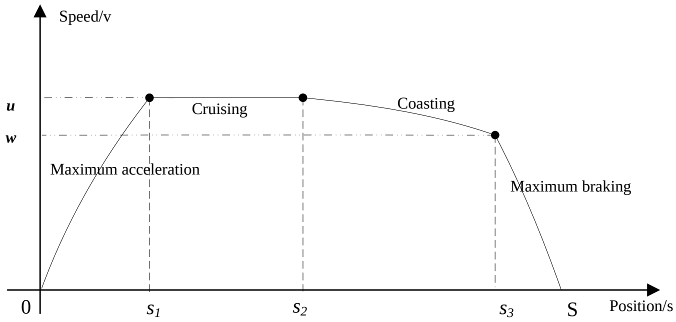

FIGURE 2 Optimal strategy in between stations

3 METHOD 2: THE INTEGRATED TIMETABLE APPROACH

Energy-optimal train control between consecutive stations has been studied for many years and there are different considerations in many papers. In (*3, 6, 15*) analytical solutions were presented to this problem. By applying the Pontryagin maximum principle, it was proved that the optimal driving strategy essentially consists of maximum acceleration with traction, cruising, coasting and maximum deceleration with the brake (See Figure 2). Considering the characteristics of a subway system, we simplify the acceleration, the braking deceleration and the running resistance as constants (*7*) and get the optimal control strategy in between two stations as follows (*21*):

$$\begin{cases} T = \sqrt{\frac{2FE}{F-r} + (\frac{Fv_0}{F-r})^2}/r - \sqrt{\frac{2B(E-rS)}{r^2(B+r)} + [\frac{Bv_T}{(B+r)r}]^2} \\ -\frac{v_0}{F-r} - \frac{v_T}{B+r} \\ u = \sqrt{2E(F-r)/F + v_0^2} \\ w = \sqrt{2(B+r)(E-r\times S + 0.5v_0^2 - 0.5v_T^2)/B + v_T^2} \\ s_1 = F(u^2 - v_0^2)/2(F-r) \\ s_2 = s_1 + E/r - F(u^2 - v_0^2)/2r(F-r) \\ s_3 = S - (E - Sr + 0.5v_0^2 - 0.5v_T^2)/B, \end{cases}$$

where v_0, v_T, u and w are respectively the initial, end, cruising and braking speed; F, B, and r are corresponding traction force, braking force and the running resistance; T, E, and S denote the running time, the energy consumption, and the trip distance; s_1, s_2, and s_3 are the switch points in Figure 2.

For dealing with different speed limits, we design an algorithm to distribute the total running time to different sections which has the constant speed limit. First, the minimum trip time is calculated with the characteristic of the train' traction and braking. Then based on the total trip time, we can get the reserve time T_r which is the difference between the minimum trip time T_{min} and the scheduled trip time T_t according to the equation as follows,

$$T_r = T_t - T_{min}. \tag{11}$$

For obtaining the running time for each section, we should distribute the reserve time to different sections, during which we keep the principle that the increase of trip time $\triangle T$ must be added to the section which has the largest ratio between energy saving $\triangle E$ and increase in trip time $\triangle T$. Taking the Figure 3 for example, we can get more energy reduction in section 2 and distribute the recovery time to this section, since the ratio $\triangle E_2/\triangle T_2$ is larger than $\triangle E_1/\triangle T_1$. Thus, we can get the trip time for each section of the route after distributing the reserve time. In addition, the function between the energy consumption and the trip time is

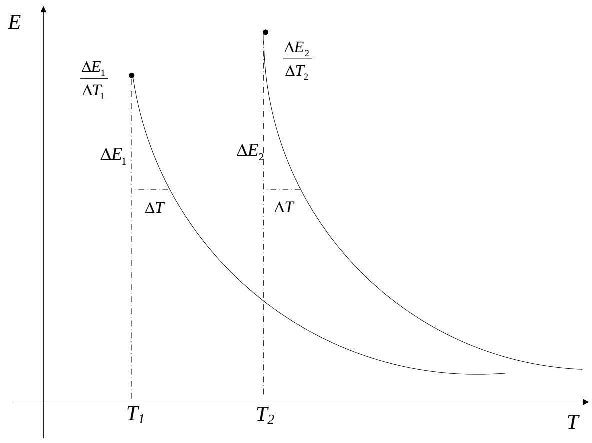

FIGURE 3 Distribution of reserve time

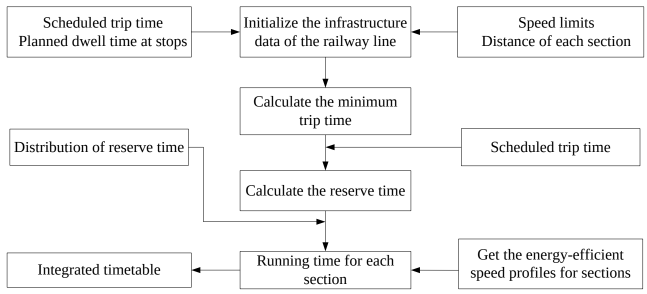

FIGURE 4 Flow chart of the final algorithm of the integrated approach

concave (*19, 21*), so the distribution of the reserve time is globally optimal.

The algorithm formulated above can be used to generate the timetable and the optimal speed profile together, and therefor it is called as integrated timetable algorithm. Firstly, we calculate the minimum trip time as well as the reserve time for the train traveling between the successive stations. Secondly, according to the principle of distribution of the reserve time, we obtain the trip time for each section with speed limits so that we can obtain the energy-efficient speed profile for each section. Finally, we can obtain the energy-efficient speed profile for each section according to the optimal trip time. The flow chart of the final algorithm is shown in Figure 4.

4 COMPARISON USING A CASE STUDY

In order to compare the performance of the two approaches for the optimal train control problem, a case study based on the data from the Beijing Yizhuang subway line in China is studied. The layout of the Beijing Yizhuang Line is illustrated in Figure 5. Furthermore, the nonlinear function of the maximum traction force is based on the data from (*7*). In addition, the speed limits for each section along the entire route and their length, as well as the average mass of the train in each section are listed in Table 1. The units of $v_{\max}$, Section, and M are m/s, m and ton respectively. Some assumptions of the parameters for the integrated approach are made as follows: the maximum acceleration per unit mass F is 0.8 $\mathrm{m/s^2}$, the maximum braking per unit mass B is 0.8 $\mathrm{m/s^2}$ (*7*), and the running resistance per unit mass r is simplified as 0.02 $\mathrm{m/s^2}$.

We apply the two methods to optimize the speed profile for the Beijing YiZhuang subway line. The calculation results are shown in Table 2, which illustrates the good energy-efficient performance of the two

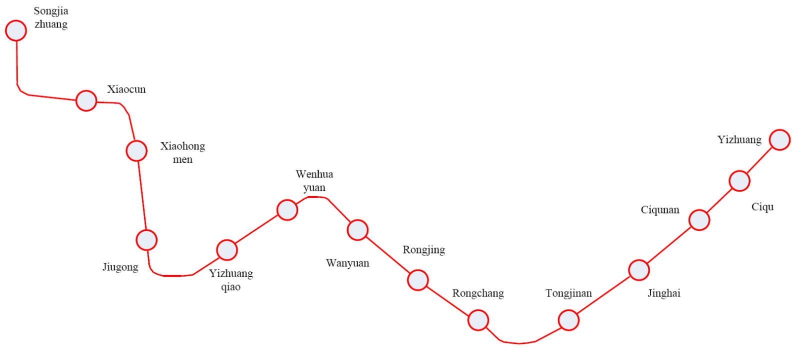

FIGURE 5 The layout of the Beijing Yizhuang Line

TABLE 1 Infrastructure data of the YiZhuang subway line

Station	v_{max}/Section	v_{max}/Section	v_{max}/Section	M
1	10/0-128	20/128-1147	14/1147-1332	331
2	13/0-130	20/130-1085	13/1085-1286	278
3	13/0-129	20/129-1897	11/1897-2086	406
4	13/0-128	20/128-2073	13/2073-2265	380
5	13/0-120	20/120-2143	12/2143-2331	355
6	14/0-130	20/130-1168	13/1168-1354	187
7	14/0-130	20/130-1087	13/1087-1280	218
8	14/0-130	20/130-1350	13/1350-1544	169
9	13/0-130	20/130-796	13/796-992	248
10	14/0-123	20/123-1781	13/1781-1975	302
11	14/0-128	20/128-2165	13/2165-2369	327
12	14/0-130	20/130-1151	13/1151-1349	357
13	13/0-133	20/133-2300	10/2300-2610	188

TABLE 2 Energy-efficient performance by using the MILP and integrated method

	Practical data		MILP	Integrated	
Destination	T_p	E_p	E_{MILP}	T_{IT}	E_{IT}
Xiaocun	102	14.52	10.05	107	8.89
Xiaohongm	99	21.58	15.26	106	11.52
Jiugong	137	48.67	37.99	146	34.93
Yizhuangq	146	27.63	23.23	147	23.42
Wenhua	158	16.76	10.84	150	14.30
Wanyuan	102	15.02	10.95	102	10.31
Rongjing	99	10.80	7.81	98	7.70
Rongchang	112	19.03	12.64	112	14.53
Tongjinan	84	17.53	10.77	90	8.12
Jinghai	132	24.56	19.17	133	19.37
Ciqunan	153	22.02	16.98	151	19.80
Ciqu	102	15.61	10.40	102	11.78
Yizhuang	194	41.80	19.10	176	32.05
Total	1620	295.53	205.19	1620	216.67
Energy saving(%)	-	-	30.57	-	26.68

approaches; E_p and T_p are the actual energy consumption and trip time in Yizhuang line, which are recorded by the on-board equipment; E_{MILP} is the energy consumption optimized by the MILP approach; E_{IT} and T_{IT} denote the energy consumption and the optimal trip time obtained by the integrated timetable method.

The optimal speed profiles obtained by these two approaches are showed in Figure 6, in which the red dashed line represents the speed limits of the subway line, the black doted line and the blue dash-doted line are the optimal speed profiles calculated by the MILP and integrated timetable method, respectively. It

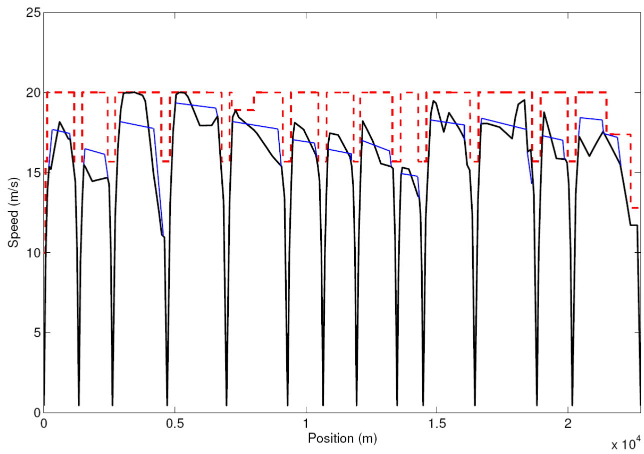

FIGURE 6 Optimal speed profiles for YiZhuang subway line of the two methods

can be seen from the Table 2 that the total energy savings of the integrated timetable method and the MILP approach are 26.68% and 30.57%, respectively. The MILP approach has a better performance because it formulates a more realistic model, which includes the variable gradient, variable maximum traction force and running resistance. However, the computation time of the MILP method is longer than the integrated timetable method. The computation time of the MILP approach is 5.54 s for the entire route and around 0.43 s for a section and the calculation time of the integrated timetable approach is 0.15 s for entire route and about 0.01 s for each section. Therefore, there exists a trade-off between the control performance and the computation time.

5 CONCLUSIONS AND FUTURE RESEARCH

The mixed integer linear programming (MILP) approach and the integrated timetable approach for solving the train optimal control problem are compared in this paper. In the MILP approach, the nonlinear terms of the train model are approximated using piecewise affine functions and the optimal control problem is then recast as an MILP problem, which can be solved using existing solvers. The integrated timetable approach is an analytical method, which optimizes the timetable of the entire route and the speed profiles together.

Based on the simulation results from Beijing's YiZhuang subway line, the integrated timetable method gets a good energy-efficient performance with about 27% energy saving. The MILP method gets a better performance with about 31% energy saving at the cost of an longer computation time. As the future work, the combination of these two methods will be explored and be applied on automatic train operation system in the subway systems.

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