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## **Co-optimization of topology design and parameterized control in a traffic network**

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**ABSTRACT**

In this paper, we introduce a co-optimization approach to jointly optimize the traffic network topology and traffic controllers in a traffic network. Since the network topology design has a long-term goal, while the traffic control is usually based on a minute-to-minute basis traffic situation, we use a so-called parameterized traffic control method to bring them to the same time scale. We apply the proposed co-optimization approach in a simulation-based case study, and compare it with a pure topology design and an iterative optimization method. The results show that our method can improve the interaction between topology design and traffic control design, and result in a better overall performance.

## 1 INTRODUCTION

In the near future the amount of traffic on the roads will keep increasing significantly. In order to match this increase, traffic authorities and policy makers seek to improve the performance of the traffic network at the same pace as the growing demand. Historically, this issue has been addressed in two different ways: (re)design of the network topology or development of traffic control strategies. Network topology design involves making decisions on changing the structure of a traffic network. The advantage of this approach is that it can effectively solve the problem of capacity limitations of infrastructure in a direct way. On the other hand, the goal of traffic control strategies is to make more efficient use of the available roads by various traffic control measures, such as traffic signals, speed limits, and route guidance. In this paper, we will consider both approaches jointly in order to improve the interaction and cooperation between them, and to get better overall performance.

Network topology design is usually posed in two different forms: a discrete form that deals with creating or removing links, or adding new lanes in existing links, and a continuous form that represents the capacity enhancement of existing links. Billheimer and Gray (1) proposed one of the first discrete methods, in which they considered the design of transportation networks with fixed construction costs and variable user costs. Leblanc (2) considered adding or removing the links in the network to minimize the total congestion, by using a mixed integer programming model. The continuous form was discussed in several papers, e.g., Abdulaal and Leblanc (3) formulated the network design problem with continuous investment variables subject to an equilibrium assignment, and Suwansirikul (4) suggested a heuristic method to find appropriate solutions to the continuous equilibrium design problem. The network topology design problem is often mentioned together with the well-known Braess paradox (5), which illustrates a counter-intuitive fact that addition of new links or capacity expansion to some links may in some cases deteriorate the performance of the traffic network. Therefore, the designer has to take such paradoxical effects into account when dealing with network topology design.

However, changing the topology of a traffic network is an expensive and time-consuming project, and sometimes the required free space may be not available. Therefore, due to financial and physical limitations, many recent studies focus on the development of advanced traffic management and control strategies (see e.g. (6–8)) to improve traffic flows. From the traffic management point of view, the objectives of traffic control can vary, from avoiding traffic congestion to increasing network safety and reliability, and to decreasing fuel consumption and pollution, etc. Therefore, an ‘appropriate’ traffic control strategy is in general defined in terms of optimality according to (multiple) specific objectives.

Traditionally, network topology and control strategies for traffic systems are designed separately. The designers either (1) design the network topology without considering any control strategy, only assuming that drivers make their own decision in a user-optimal manner, or (2) develop control strategies for an existing traffic network topology. The problem with the first approach is that the impact of the control strategies on the performance of the whole system is not taken into account during the optimization of the topology design, which could easily lead to a suboptimal system design. The main problem with the second approach is that there might be no feasible traffic control strategy if, e.g., the total demand exceeds the capacity of the network. However, the co-optimization approach can solve the problem by optimizing the network topology and traffic controllers as a whole so that they can work cooperatively. To the best knowledge of the authors, very few systematic studies have been conducted to investigate the combined design of topology and control strategies in traffic networks. One relevant topic is introduced by Cantarella et al. (9), presenting an iterative procedure with two steps: the first step dealing with integer variables (topology design), and the second step dealing with continuous variables (control signals). In the first step, the control signals are fixed so that only integer variables are optimized, and in the second step, the network topology is fixed so that only continuous variables are optimized. The algorithm keeps running the two-step loop until a stopping criterion is satisfied. A problem with this approach is that it can get stuck in a cycle around the

optimum, never reaching the optimum itself.

One challenge for the co-optimization method is that these two subproblems involve different time scales. More specifically, the solutions of the network topology design could be effectively unchanged for a decade or even longer, while the solutions of the traffic control strategy design are usually dynamic control signals that vary on a minute to minute basis according to the time-varying traffic situations. However, determining a sequence of traffic control signals over an extremely long period may induce a lack of robustness in the system, because there might be uncertainty about the typical future traffic demands. To tackle these problems, we will use a feedback control method, and optimize the *parameters* of the controllers instead of the control signals. This approach — albeit in a different setting that only focuses on traffic management — is also adopted in e.g., the ALINEA method (10). Through finding the optimal parameters of the controller, the control signals are accordingly determined via the feedback control law at each control step. This method is called parameterized traffic control (11). In such a way, the solutions of the traffic controllers design can be brought to the same time scale as the network topology design, since the parameters can be determined and applied for a long-term purpose, using forecast demands and considering different sub-periods (see Section 3.2.)

The rest of the paper is organized as follows. In Section 2, we introduce an evolutionary algorithm that will be used for the co-optimization of the network topology and traffic control strategies. Section 3 includes the formulations of the network topology design, parameterized traffic control, and the overall objective function. In Section 4, a simulation-based case study is performed on an artificial network, comparing the co-optimization method with the pure network topology design and the iterative algorithm introduced in (9). Finally, we conclude our work and offer some future research directions in Section 5.

## 2 COVARIANCE MATRIX ADAPTATION EVOLUTION STRATEGY

### 2.1 Basic algorithm

In this paper we use the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) (12) to solve the co-optimization problem of the network topology and the traffic control. The CMA-ES algorithm belongs to the class of Evolutionary Algorithms, and it is a state-of-the-art continuous optimization algorithm. We in particular select CMA-ES since in a benchmark including 24 test problems for comparison of real-parameter global optimizers it resulted to yield the best overall performance (13). Moreover, CMA-ES is a non-gradient-based optimization method that is good at solving problems with high dimensionality and complexity, such as the problem considered in this paper, and it performs a parallel search in the search space that can effectively escape from local optima that the co-optimization problem most likely has. Due to the reasons above, CMA-ES is chosen in this paper. We begin by recapitulating the algorithm.

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#### Algorithm 1 Basic structure of the CMA-ES algorithm

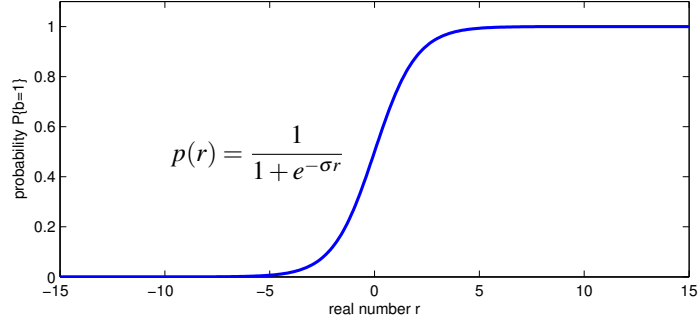
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**Input:** size of offspring population  $\lambda$ , size of parent population  $\mu$ , initial parent generation  $\mathbf{x}_\mu^{(0)}$ , initial parameters  $\gamma^{(0)}$ , maximum number of generations  $G$

- 1: **for**  $g = 0, 1, \dots, G - 1$  **do**
- 2:   generate a new generation population via the sample distribution:  $\mathbf{x}_\lambda^{(g+1)} = N(\mathbf{x}_\mu^{(g)}, \gamma^{(g)})$
- 3:   select new parents  $\mathbf{x}_\mu^{(g+1)}$  from the offspring  $\mathbf{x}_\lambda^{(g+1)}$  based on a fitness function  $F$
- 4:   update parameters  $\gamma^{(g+1)} = U(\mathbf{x}_\mu^{(g+1)}, \gamma^{(g)})$
- 5: **end for**

**Output:** The best offspring solution at generation  $G$ :  $\mathbf{x}_{\lambda, \text{best}}^{(G)}$

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**FIGURE 1** Mapping real number  $r$  into binary number  $b$

A description of basic CMA-ES structure is presented in Algorithm 1. The constants  $\lambda$  and  $\mu$  are population sizes of each offspring and parent generation, with  $\lambda \geq \mu$ . The constant  $\gamma^{(0)}$  is the initial set of all the optimization parameters in the CMA-ES algorithm, such as covariance matrix of the distribution, search step size, and so on. At each generation,  $\lambda$  children are stochastically generated via the normal multivariate distribution  $N$ , and the  $\mu$  best out of the  $\lambda$  children are selected as the parents of the next generation according to the fitness function  $F$ . Afterwards, the parameter  $\gamma$  is updated based on a so-called cumulative step-size adaptation rule  $U$  to adapt the distribution of the candidate solutions. More details about the CMA-ES algorithm can be found in (12).

## 2.2 Modification for binary form

Although CMA-ES has variants to handle integer optimization (14) and constrained optimization (15), these variants, especially the integer optimization version, have not been developed to such a degree as the unconstrained optimization version in a continuous search space. Since the optimization problem in this paper has a mixed-integer form (see Section 3.1), we propose another variant of CMA-ES dealing with the binary form in this section.

This modification is inspired by (16). The goal is to map a real number  $r$  into a binary space. The mapping has two steps:

1. For  $-\infty < r < +\infty$ , the mapping monotonously goes into the interval  $[0, 1]$  via a scaling function  $p(r) = 1/(1 + e^{-\sigma r})$ , with  $\sigma > 0$  a weight parameter;
2. The value of  $p$  is considered as the probability of a binary variable  $b$  taking the value 1.

In this way, any real number  $r$  can be mapped to a binary number  $b \in \{0, 1\}$  by (as shown in Figure 1):

$$\begin{cases} P\{b = 1\} = p(r) \\ P\{b = 0\} = 1 - p(r) \end{cases} \quad (1)$$

where  $P\{\cdot\}$  is the probability of  $b$  taking the corresponding value. It is easy to verify that when  $r = 0$ ,  $P\{b = 1\} = P\{b = 0\} = 1/2$ , and moreover, the value of  $b$  is more likely to be equal to 1 when  $r$  has a larger positive value, and equal to 0 when  $r$  has a larger negative value.

Accordingly, the steps of the basic CMA-ES algorithm are modified as follows when solving a mixed-integer optimization problem. At each generation  $g$ ,

**Step 1** divide the new offspring solution  $\mathbf{x}_\lambda^{(g)}$  into  $\mathbf{x}_{\lambda,r}^{(g)}$  and  $\mathbf{x}_{\lambda,r'}^{(g)}$ , with  $\mathbf{x}_{\lambda,r}^{(g)}$  containing the variables to be mapped into the binary space, and  $\mathbf{x}_{\lambda,r'}^{(g)}$  containing the variables kept for continuous optimization;

**Step 2** map  $\mathbf{x}_{\lambda,r}^{(g)}$  to a binary variable  $\mathbf{x}_{\lambda,b}^{(g)}$  by using (1);

**Step 3** evaluate  $\mathbf{x}_{\lambda,b}^{(g)}$  and  $\mathbf{x}_{\lambda,r'}^{(g)}$  by using the fitness function  $F$ ;

**Step 4** select the best  $\mu$  solutions from  $\mathbf{x}_{\lambda}^{(g)}$ , with  $\mathbf{x}_{\lambda,r}^{(g)}$  corresponding to  $\mathbf{x}_{\lambda,b}^{(g)}$ , as the parents for generation  $g + 1$ ;

**Step 5** update the parameters, and start the next generation.

The output is the best offspring solution at generation  $G$ :  $\mathbf{x}_{\lambda,b,\text{best}}^{(G)}$  and  $\mathbf{x}_{\lambda,r',\text{best}}^{(G)}$ .

### 3 CO-OPTIMIZATION OF TOPOLOGY DESIGN AND TRAFFIC CONTROL

#### 3.1 Network Topology Design

As the first step, we only consider the discrete form of network topology design in this paper, i.e. link addition or removal in a traffic network, or adding new lanes in existing links. A binary vector  $\delta$  is defined as the construction decision variable. The  $m$ th element  $\delta_m$  corresponds to the construction decision for link  $m$ , where  $\delta_m = 1$  means that link  $m$  is constructed, or a new lane is added to link  $m$ , and  $\delta_m = 0$  that no construction. In general, improvement of the traffic network characteristics will definitely induce changes in the traffic flow patterns, which means that the traffic state  $x$  (usually traffic flow, density, and speed) depends on the construction decision variable  $\delta$ . The network topology design is traditionally solved in a bi-level framework (17), where the lower-level optimization aims at finding a user equilibrium flow pattern in the traffic network, and the upper-level optimization then determines the decision variables based on the equilibrium flow resulting from the lower level.

In this paper the traffic control strategy is optimized jointly with the topology design, which will result in a better performance of the traffic network.

#### 3.2 Parameterized Traffic Control

In general, traffic evolution can be described by a difference equation:

$$x(k+1) = f(x(k), u(k), d(k), \delta), \quad (2)$$

where  $f$  is the traffic update function,  $x(k) \in \mathbb{R}^{n_x}$  denotes the dynamic traffic state vector at simulation step  $k$ ,  $u(k) \in \mathbb{R}^{n_u}$  denotes the control input vector at simulation step  $k$ , and  $d(k) \in \mathbb{R}^{n_d}$  denotes the disturbance (typically, the traffic demand) at simulation step  $k$ . Usually, the control interval  $T_c$  is larger than the simulation interval  $T$ :  $T_c > T$ , because traffic evolution is a comparatively fast process, while the control actions are not necessarily updated as fast as the traffic evolution. However, for the sake of simplicity, we assume that the control time interval  $T_c$  is equal to the simulation time interval  $T$  in this paper. Therefore, a parameterized traffic control law is generally formulated as:

$$u(k) = h(x(k), d(k), \theta), \quad (3)$$

where  $h$  denotes a control law, and  $\theta \in \mathbb{R}^{n_\theta}$  contains all the parameters of the traffic controllers. In the conventional optimization approach the control signal  $u(k)$  is optimized directly, e.g. in Model Predictive Control, this results in a sequence of future control signals  $u(k|k), u(k+1|k), \dots, u(k+N_p-1|k)$  over a prediction period  $[kT, (k+N_p)T]$ , with  $u(l|k)$  the control signal for simulation step  $l$  based on information at simulation step  $k$ , and  $N_p$  a prediction horizon. However, in the parameterized optimization approach the control signal  $u(k)$  is indirectly optimized according to the control law  $h$ .

		Period		
		sub-period 1	sub-period 2	
time slot 1		$\theta_1$	$\theta_1$	...
time slot 2		$\theta_2$	$\theta_2$	...
		$\vdots$		

**FIGURE 2** quasi-dynamic parameterized traffic control

Although one can design a parameterized traffic controller for any traffic model, or for any type of control measure, in this paper we first consider the dynamic speed limit control for macroscopic traffic flow models, where a link  $m$  is divided into segments with a length  $L_m$ . The control law for dynamic speed limit  $v_{m,i}^{\text{lim}}(k+1)$  of segment  $i$  on link  $m$  at simulation step  $k+1$  is defined as (11):

$$v_{m,i}^{\text{lim}}(k+1) = \theta_{m,0} v_{m,i}^{\text{lim}}(k) + \theta_{m,1} \frac{v_{m,i+1}(k) - v_{m,i}(k)}{v_{m,i+1}(k) + \kappa_{m,v}} + \theta_{m,2} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i+1}(k) + \kappa_{m,\rho}} \quad (4)$$

where  $v_{m,i}^{\text{lim}}(k)$  denotes the maximum allowed speed,  $v_{m,i}(k)$  and  $\rho_{m,i}(k)$  denote the mean speed and density on segment  $i$  of link  $m$  at the simulation step  $k$  respectively,  $\theta_{m,0}$ ,  $\theta_{m,1}$ , and  $\theta_{m,2}$  denote speed-limit-control parameters, and  $\kappa_{m,v}$  and  $\kappa_{m,\rho}$  are parameters preventing denominators from becoming zero. The speed limit controller (4) can be considered as the state feedback functions based on the relative speed difference and relative density difference of a segment w.r.t. the speed and density in the downstream segment respectively. In this parameterized traffic control setting,  $\theta_{m,0}$ ,  $\theta_{m,1}$ , and  $\theta_{m,2}$  are target parameters to be optimized.

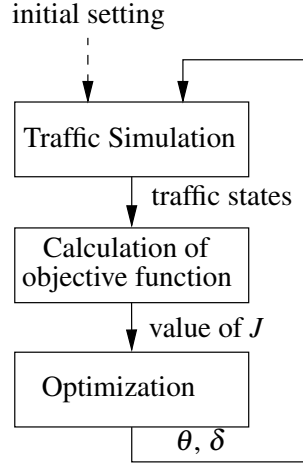
**Remark 1** The parameter  $\theta$  should be in principle constant for a relatively long time, i.e., at the same time scale as the topology design. However, simple static traffic control using the same parameters over a long period may be inadequate for modern traffic systems. In order to improve the performance of the traffic controllers, we consider a so-called quasi-dynamic parameterized traffic control approach. As shown in Figure 2, a period is first divided into several sub-periods, and each sub-period is then divided into several time slots. For the same sub-period, the parameters in different time slots have different values, while for the same time slot, the parameters in different sub-period have the same value. For instance, a month can be divided into days, and a day can be divided into different time slots, e.g., morning rush hours, midday non-rush hours, and afternoon rush hours. In this way, we only need to optimize the parameters for one day, but can apply them for a month. Similarly, a sub-period can be divided into sub-sub-periods or even further if necessary, e.g., in reality a year can be divided into seasons, a season can be divided into months, and so on. In such a way, for complicated traffic situations, the parameters can be defined much more accurate than the static traffic control.

### 3.3 Objective Function

The objective of the co-optimization method is to minimize the total construction and maintenance cost, as well as the total travel cost. The total construction and maintenance cost is formulated as :

$$J_1 = \sum_{m \in M} c_m^c, \quad (5)$$





**FIGURE 3 Illustration of simulation-based CMA-ES co-optimization**

where  $c_m^c$  is the average yearly construction and maintenance cost (\$/year) of link  $m$ , which is based on the construction decision variable  $\delta$ . The cost  $c_m^c$  includes two parts: constructing and maintaining the links, and installing the controllers on the links. It is formulated linearly as  $c_m^c = (\alpha_1 N_m L_m + \alpha_2 N_m^c) \delta_m$  with  $\alpha_1$  the unit construction and maintenance cost of the link,  $\alpha_2$  the installation cost of each controller,  $N_m$  the number of segments on link  $m$ ,  $L_m$  the length of segments on link  $m$ , and  $N_m^c$  the number of controllers on link  $m$ .

The total travel cost is formulated as :

$$J_2 = D \sum_{k \in K} \left( \sum_{m \in M} \sum_{i \in I_m} c_m^t \rho_{m,i}(k) L_m \lambda_m + \sum_{o \in O} c_o^w w_o(k) \right), \quad (6)$$

where  $c_m^t$  (\$/day) denotes the travel cost on link  $m$ ,  $c_o^w$  (\$/day) denotes the waiting cost on origin  $o$ ,  $\rho_{m,i}(k)$  (veh/h) denotes the density on segment  $i$  of link  $m$  at simulation step  $k$ ,  $\lambda_m$  denotes the number of lanes on link  $m$ ,  $w_o(k)$  (veh) denotes the number of vehicles waiting on origin  $o$  at simulation step  $k$ ,  $I_m$  denotes the set of segments on link  $m$ ,  $O$  denotes the set of origins,  $K$  denotes the set of simulation steps in a day, and  $D$  denotes the total number of days in a year so that the costs  $c_m^t$ ,  $c_o^w$ , and  $c_m^c$  are taken into the same period. Both travel time and travel distance can be evaluated monetarily. In particular, according to (18), the travel cost  $c_m^t$  can be expressed as:

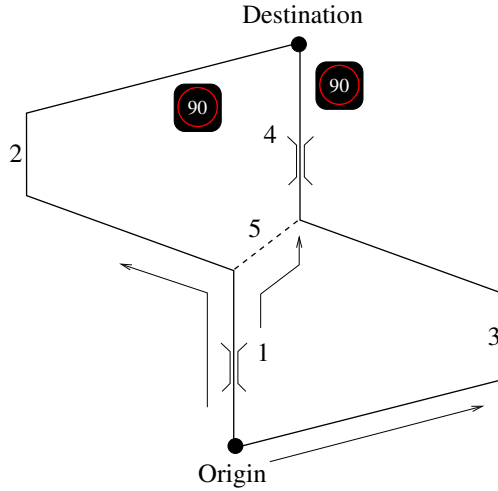
$$c_m^t = (\alpha_3 T + \alpha_4 v_m T) \delta_m, \quad (7)$$

where  $v_m$  (km/h) is the average speed on link  $m$  during a day,  $T$  (h) denotes simulation time step length,  $\alpha_3$  is the value of unit travel time, and  $\alpha_4$  is the monetary cost per unit distance. For the waiting cost  $c_o^w$ , since there is no travel distance for vehicles waiting at an origin, only the travel time cost is considered,  $c_o^w = \alpha_3$ .

From the system optimum point of view, the resulting overall optimization problem can be formulated as:

$$\begin{aligned} \min_{\theta, \delta} J &= \xi_1 J_1 + \xi_2 J_2 \\ \text{subject to } y(\theta, \delta) &= 0, \\ z(\theta, \delta) &\leq 0 \end{aligned} \quad (8)$$

where  $\xi_1, \xi_2 > 0$  are weight parameters,  $y(\cdot)$  includes all the equality constraints, and  $z(\cdot)$  captures all the inequality constraints. We solve (8) using a simulation-based optimization approach. The simulation-based



**FIGURE 4** Traffic network with road construction and speed limit controllers.

optimization means that we embed the traffic simulation into the CMA-ES algorithm, as shown in Figure 3. The advantage of this approach is that all equality constraints  $y$ , e.g., conservation law of traffic flow, are included in the traffic simulation model, and thus eliminated. The inequality constraints are added into the objective function via a penalty term, i.e.,  $J = \xi_1 J_1 + \xi_2 J_2 + \xi_3 J_{\text{pen}}$ , with  $\xi_3 > 0$  and  $\xi_3 \ll \xi_1, \xi_2$ , and  $J_{\text{pen}}$  given by, e.g.,  $\sum_{i=1}^{n_z} [\max(z_i(\theta, \delta), 0)]^2$ , with  $n_z$  the number of inequality constraints.

## 4 CASE STUDY

### 4.1 Set-up

We illustrate the effects of the co-optimization approach in a benchmark network taken from (19) where the Braess paradox may occur, depending on the level of the traffic demand. As shown in Figure 4, the network consists of one origin, one destination, and five links, with each link having only one lane. The lengths of the links are  $l_1 = l_4 = 20$  km,  $l_2 = l_3 = 40$  km, and  $l_5 = 10$  km, and the length of each segment is  $L_m = 0.5$  km. The construction decision is to determine whether link 5 (as indicated by the dashed line) should be constructed or not, and whether a new lane should be added to links 2 and 3 (we only consider adding one lane because it is sufficient for the case study network, and adding more lanes will not significantly improve the performance). Moreover, we assume that links 1 and 4 cannot be expanded due to physical limitations. The variable speed limits are considered to be applied on the segments of second half of links 2 and 4 by using control law (4).

### 4.2 Optimization and model parameters

The cost of construction of highway projects could be affected by several factors, such as terrain type (mountainous or flat), material type (concrete or asphalt), development type (rural or urban), and so on. According to a report by the U.S. General Accounting Office (20), the unit cost of building a stretch of highway ranged from about \$2.5 million per kilometer to \$16 million per kilometer in 25 U.S. states in 2002. In this case study, we set the total highway construction and maintenance cost as \$10 million per kilometer, and we assume that the traffic network is constructed to be used for twenty years. Therefore, the average yearly construction and maintenance cost is  $\alpha_1 = \$0.5$  million per kilometer per year. For other parameters, the installation cost of each controller is \$1 million, so  $\alpha_2 = \$0.05$  million per year. The travel time cost is set as  $\alpha_3 = \$10$  per hour per vehicle, and the monetary cost of driving a car is set as  $\alpha_4 = \$1$  per

**TABLE 1 Simulation results**

	total cost $J$ (dollars)	construction decision $\delta$	control parameters $\theta$
Pure topology design	$1.096 \times 10^9$	$\delta_2 = 1, \delta_3 = 1, \delta_5 = 0$	Not apply
Co-optimization	$9.139 \times 10^7$	$\delta_2 = 1, \delta_3 = 0, \delta_5 = 0$	$\theta_{2,0} = 7963.4, \theta_{2,1} = -4922.2,$ $\theta_{2,2} = -999.0, \theta_{4,0} = 6633.6,$ $\theta_{4,1} = 22666.8, \theta_{4,2} = 2830.7$
Iterative optimization	$9.423 \times 10^7$	$\delta_2 = 1, \delta_3 = 0, \delta_5 = 0$	$\theta_{2,0} = 0.9986, \theta_{2,1} = 1.2157,$ $\theta_{2,2} = 0.4330, \theta_{4,0} = 1.0033,$ $\theta_{4,1} = 10.0816, \theta_{4,2} = 0.0214$

kilometer per vehicle. The weight parameters in (8) are set as  $\xi_1 = 1$  and  $\xi_2 = 0.1$ . Since we do not have inequality constraints in this case study,  $z(\cdot)$  is void.

The METANET model (21) is used to simulate the traffic flow evolution. The model parameters are defined as follows: simulation period  $t_{\text{sim}} = 24$  h, simulation time step length  $T = 10$  s, free flow speed  $v_{\text{free}} = 102$  km/h, minimum speed  $v_{\text{min}} = 7.4$  km/h, lower bound of speed limit  $v_{\text{min}}^{\text{lim}} = 50$  km/h, critical density  $\rho_{\text{crit}} = 33.5$  veh/km/lane, maximum density  $\rho_{\text{max}} = 180$  veh/km/lane, flow capacity  $q_{\text{cap}} = 2000$  veh/h/lane. Other parameters can be found in (22).

### 4.3 Scenario

We consider a traffic demand profile during a two-hour representative design period in the simulation: the demand starts with a flow of 4000 veh/h at 6.30 a.m., decreases to 2000 veh/h at 8.15 a.m., and maintains this level until 8.30 a.m.. Moreover, we add a large pulse with a value of 60 veh/km/lane to the downstream density at the virtual downstream link of the network so that a back-propagating wave appears in the network. In this way, the network becomes more dense in the peak hours, which results in a longer travel time.

We compare the co-optimization approach with the pure network topology design and the iterative optimization approach (9). In the pure network topology design, no control strategy is introduced, but only construction/extension decisions for links 2, 3, and 5 are considered. CMA-ES is used for the binary optimization of the pure network topology design. The iterative optimization approach keeps optimizing the construction decision variables and the controller parameters in a loop until the following stopping criterion is satisfied: the binary variables between two consecutive iteration steps are the same, and the difference of the continuous variables between two consecutive iteration steps is smaller than a tolerance  $10^{-3}$ . For the real-valued optimization, we use a quasi-Newton method implemented by `fminunc` from the Matlab Optimization Toolbox, and for the binary optimization, we use a genetic algorithm implemented by `ga` from the Matlab Global Optimization Toolbox.

### 4.4 Simulation results

The results of the simulations of the three approaches are displayed in Table 1. We can see that the co-optimization approach achieves the best performance among these three methods. The construction decision determined by the co-optimization approach is the same as that of the iterative approach, but the resulting control parameters lead to a lower total cost. In the pure topology design, the total cost is much higher than both the co-optimization approach and the iterative approach.

## 5 CONCLUSIONS AND FUTURE WORK

We have introduced a co-optimization method to jointly optimize the network topology and the traffic controllers. The advantage of co-optimization is that it can improve the interaction and coordination between these two design components, and yield a better overall performance than optimizing them separately. In a simulation-based case study, we showed that the co-optimization resulted in a lower total cost than the pure topology design and the iterative approach.

In the future, we will consider a more general case: for the topology design, we will not only consider binary construction variables, but also integer variables (e.g., the number of lanes), and continuous variables (e.g., the capacity of the link), and for the traffic control measures, we will include multiple control strategies in the network, such as ramp metering and dynamic route guidance. Moreover, we will also consider more extensive case studies involving large-scale networks, implement the quasi-dynamic parameterized control as introduced in this paper, as well as perform sensitivity analysis.

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