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Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: https://www.dcsc.tudelft.nl

Congestion Management in Motorways and Urban Networks Through a Bargaining-Game-Based Coordination Mechanism

Felipe Valencia, José D. López, Alfredo Núñez, Christian Portilla, Luis G. Cortes, Jairo Espinosa, Bart De Schutter

Abstract Road traffic networks are large-scale systems that demand distributed control strategies. Distributed model predictive control (DMPC) arises as a feasible alternative for traffic control. Distributed strategies decompose the whole traffic network into different subnetworks with local optimal controllers that make decisions on actions to be taken by the actuators responsible for traffic control (traffic lights, routing signals, variable speed limits, among others). However, subnetworks are interacting elements of the whole traffic network. Hence, local control decisions made for one sub-network affect and are influenced by the decisions taken for the other subnetworks. Under these circumstances, the DMPC traffic problem can be treated as a game where the rules are provided by the physical system, the players are the local optimal controllers, their strategies are the control sequences, and the payoffs are the local performance indices (such as the total time spent by the users in the network). This configuration allows the achievement of a computational burden reduction, with a compromise between local and global performance. Since DMPC local controllers are able to communicate with each other, the control of the traf-

Felipe Valencia
Solar Energy Research Center SERC-Chile, Faculty of Mathematical and Physical Sciences Universidad de Chile, Santiago, Chile, e-mail: felipe.valencia@sercchile.cl

José D. López
SISTEMIC, Department of Electronic Engineering, Engineering Faculty, Universidad de Antioquia UdeA, Medellin, Colombia, e-mail: josedavid@udea.edu.co

Alfredo Núñez
Section of Road and Railway Engineering, Delft University of Technology, Delft, The Netherlands, e-mail: A.A.NunezVicencio@tudelft.nl

Christian Portilla, Luis G. Cortes, Jairo Espinosa
Departamento de Energía Eléctrica y Automática, Facultad de Minas, Universidad Nacional de Colombia, Medellin, Colombia, e-mail: {crportil,lgcortes,jjespino}@unal.edu.co

Bart De Schutter
Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands, e-mail: b.deschutter@tudelft.nl

fic network corresponds to a cooperative game. In this chapter, game-theory-based DMPC is developed and tested for control of urban and motorway networks.

Key words: Game Theory; Distributed Model Predictive Control; Large-Scale Systems; Motorway Control; Urban Traffic Control; Bargaining Games

1 Introduction

Sustainable mobility of people is a key issue in modern society. However, nowadays many traffic networks are operating in an inefficient way, producing several negative impacts on the environment and leading to a deterioration in quality of life for the users. Solutions such as building new roads or improving the existing infrastructure are not always feasible because of environmental and budgetary regulations. Thus the development of efficient management and control systems for traffic and transportation to satisfy the ever-increasing demand for mobility has become a crucial area of research.

Several control strategies for traffic control have been reported in the literature. Often, they are simulation based. That is, traffic models are used to determine the impact of different control strategies and the sensitivity of the performance with respect to the tuning parameters (e.g., the Adaptive Split Cycle Offset Optimisation Technique method). Among the different simulation-based strategies reported in the literature, those based on model predictive control (MPC) have been quite commonly proposed to solve traffic problems. These techniques are focused on the optimal use of the information provided by the infrastructure already installed, and on reducing the travel time while explicitly considering the physical and operational constraints of the system [Bellemans et al., 2006, Hegyi et al., 2005, Kotsialos et al., 2002a, van den Berg et al., 2003]. However, despite the advantages of MPC over other methods, the application of this control scheme in real large-scale systems (such as traffic networks) is rendered impractical due to the computational burden of its centralized nature. In order to make the real-life implementation of MPC in large-scale systems possible, distributed model predictive control (DMPC) approaches have been proposed [Camponogara et al., 2002]. DMPC is a control scheme in which the system is divided into a number of subsystems. Each subsystem is able to share information with other subsystems in order to determine its local control actions [Negenborn et al., 2008, Talukdar et al., 2005, Wang and Cameron, 2007]. The main goal of the DMPC approach is to achieve some degree of coordination among subsystems that are solving local MPC problems with locally relevant variables, costs, and constraints, without solving the centralized MPC problem [Jia and Krogh, 2001, Necoara et al., 2008].

In this chapter, the application of a new bargaining-game-theory-based DMPC for the management of congestion in motorways and urban traffic networks is presented. Game theory is a branch of applied mathematics used in a wide range of disciplines (see [Von Neumann et al., 1947] for a more detailed overview of game theory). Game theory attempts to capture behaviors in strategic situations, or games where the outcome of a player is not only a function of his own choices but also depends on the choices of others [Myerson, 1991]. Some DMPC schemes based on game theory concepts have been reported in the literature. In [Du et al., 2001, Giovanini and Balderud, 2006, Li et al., 2005, Trodden et al., 2009] DMPC schemes based on Nash optimality were proposed. In such approaches, the DMPC problem was formulated as a non-cooperative game, and the convergence of the solution to a Nash equilibrium point of the resulting non-cooperative game was demonstrated. In Rantzer [2006, 2008, 2009] the DMPC problem was related to game theory using a cooperative game framework, as proposed in [Von Neumann et al., 1947]. In these approaches, the Lagrange multipliers of the dual decomposition were conceived as price mechanisms in a market serving to achieve mutual agreements among subsystems, and dynamic price mechanisms were used for decomposing and distributing the optimization problem associated with the original MPC problem. More specifically, the minimization problem was converted into a min-max problem, and again the convergence of the solution to a Nash equilibrium was demonstrated. In [Maestre et al., 2011a,b,c, Muñoz de la Peña et al., 2009] some other DMPC approaches based on cooperative game theory were presented. In these approaches, each subsystem computes local control actions and suggests control actions to the remaining subsystems. The final control decisions are taken by each subsystem based on the local information and the suggested control actions from the other subsystems.

The congestion management described in the current chapter uses the theory of bargaining games as a mathematical framework. In previous bargaining games based approaches [Venkat et al., 2006a,b,c], the authors demonstrated that (in some cases) the convergence of the DMPC solution to a Nash equilibrium point could produce undesired results because it could give an undesirable closed-loop behavior in the controlled system. Moreover, in DMPC the controllers are able to communicate with each other. In this chapter, the communication capabilities of the controllers in a DMPC scheme will be exploited for improving the decision-making of each controller. Such improvements pertains to the knowledge each local controller has about the preferences of the remaining controllers. In this way, local control actions can be chosen in such a way that synergy among controllers arises as a consequence of their cooperative behavior. Note that this is not an additional objective of the proposed control scheme, but only an additional feature which is related to the formulation of DMPC as a bargaining game.

In order to present the proposed congestion management system, in Section 2 non-linear DMPC is formulated as a bargaining game. Then, in Sections 3 and 4 the specific application of game theory to congestion management in motorways and urban traffic networks is shown. Finally, in Section 5 some closing remarks are discussed.

2 Non-Linear Distributed Model Predictive Control: Bargaining Game Approach

Distributed model predictive control (DMPC) is a variant of decentralized control where some information is exchanged among subsystems in order to determine the local control actions [Negenborn et al., 2008, Talukdar et al., 2005, Wang and Cameron, 2007]. Compared with totally decentralized control schemes, DMPC architectures yield better closed-loop behavior due to the communication, cooperation, and perhaps negotiation between subsystems. However, these elements also increase the computational and communication burden [Camponogara et al., 2002, Negenborn et al., 2008]. Nevertheless, DMPC is becoming important because it is effective in supporting the implementation of complex control systems with hard requirements involving fault tolerance and flexibility, it has high control capabilities and allows the implementation of optimal controllers in real-life large-scale systems through system decomposition, reducing the computational burden associated with the solution of one large centralized optimization problem [Pimentel and Salazar, 2002, Yang et al., 2003].

Figure 1 shows a DMPC control scheme. In this figure Process 1 and Process 2 have local MPC controllers. Since these processes interact with each other, sharing information between controllers is required in order to allow them to compute their own control actions. Otherwise, the system may lose performance and/or stability. So, at each time step local controllers must decide on the control actions to be locally applied, transmit them to the other controllers, and negotiate with the other controllers on which control actions will be applied. In the following sections this procedure is mathematically described and discussed using control theory and game theory as mathematical frameworks.

2.1 Problem Statement

Consider the discrete-time non-linear system given by:

$$x(k+1) = f_{\mathrm{dx}}(x(k), u(k)) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ denote the state and input vectors of the dynamic system at time step k, with $f_{\mathrm{dx}}(\cdot)$ a non-linear function describing the time evolution of the dynamical system to be controlled. The general idea of non-linear model predictive control (NMPC) is to determine the sequence of control actions for the system by solving an optimization problem considering the predicted trajectories given by the non-linear discrete-time model of Eq. (1).

Commonly, a quadratic cost function (that may be interpreted as the total energy of the system) is used to measure the performance of the system:

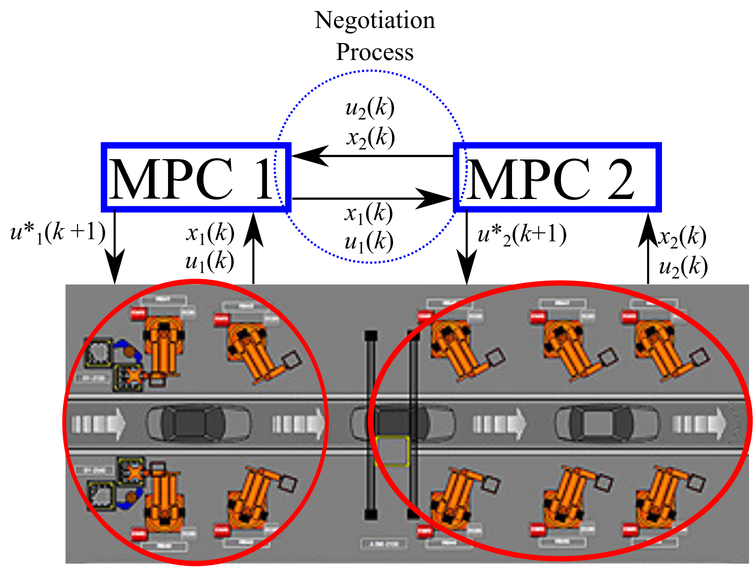


Fig. 1 Schematic diagram of a typical DMPC scheme. Here each process has a local MPC controller with the ability to share information with the other MPC controllers with the purpose of deciding on which control action to apply.

$$L(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k)) = \sum_{h=k}^{k+N_\mathrm{p}-1} [x^T(h+1|k)Qx(h+1|k)] + \sum_{h=k}^{k+N_\mathrm{p}-1} \left[u^T(h)Ru(h)\right] \tag{2}$$

where the superscript T denotes the transpose operation, $x(h|k)$ denotes the predicted value of x at time step h given the conditions at time step k, $u(h)$ denotes the control input u at time step h, $\widetilde{\mathbf{x}}(k) = [x^T(k+1|k),\ldots,x^T(k+N_\mathrm{p}|k)]^T$, $\widetilde{\mathbf{u}}(k) = [u^T(k),\ldots,u^T(k+N_\mathrm{c}),\ldots,u^T(k+N_\mathrm{p}-1)]^T$, where $x(k|k) = x(k)$, and $u(h) = u(k+N_\mathrm{c}-1)$, for $h = k+N_\mathrm{c},\ldots,k+N_\mathrm{p}-1$; Q and R are diagonal matrices with positive diagonal elements, and N_c, N_p are the control and prediction horizons respectively, with $N_\mathrm{c} \le N_\mathrm{p}$. Recall that $\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k)$ are the projections of the state and input vectors along the prediction horizon N_p. Hence, $L(\cdot)$ is a function of $\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k)$ instead of being a function of $x(k),u(k)$.

Let $\mathbb{X} \subset \mathbb{R}^n$ and $\mathbb{U} \subset \mathbb{R}^m$ denote the feasible sets for the states and inputs of the system, i.e., $x(k) \in \mathbb{X}$, $u(k) \in \mathbb{U}$ (these sets are determined by the physical and operational constraints of the system). Then, the NMPC problem can be formulated as the non-linear optimization problem:

$$
\begin{aligned}
&\min_{\widetilde{\mathbf{u}}(k)} L(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))\\
&\textbf{s.t.:}\\
&x(h+1) = f_{\mathrm{dx}}(x(h), u(h))\\
&x(h+1) \in \mathbb{X}; \qquad u(h) \in \mathbb{U};
\end{aligned}
\tag{3}
$$

This optimization problem corresponds to the centralized formulation of the NMPC problem. Although widely studied, the solution of Eq. (3) is hard to compute in real-time for large-scale systems such as traffic networks. This fact motivates the use of distributed predictive control schemes.

For instance, following the approaches in Kotsialos et al. [2002b, 1999], Papageorgiou et al. [1990] for motorways and the approaches presented in Lin et al. [2011, 2012] and the references therein for urban traffic, both traffic systems can be modeled as Eq. (1). Since they are composed of several interacting elements (links in the case of the motorways and intersections in the case of the urban traffic networks), the whole network can be decomposed into those fundamental elements and local predictive control schemes can be used for an optimal local control of each element. A motivation for such a decomposition is that traffic networks are large-scale systems, therefore a centralized optimal solution is not viable due to the lack of flexibility and vulnerability of this control structure (See Table 1 for a comparison between centralized and distributed structures).

Table 1 Comparison between centralized and distributed MPC

	Centralized	Distributed
Objective	Single objective	Both local and global system objectives
Prediction model	Broad system prediction model	Several prediction models (one per local controller)
Communications	All system information should be transmitted to a central unit	Local information is transmitted between local controllers
Processing	Centralized computation of the control actions to be applied to the system under control	Local controllers compute the local control actions based on the available information

For implementing DMPC schemes the whole system must be decomposed into several subsystems. For each subsystem a local MPC is designed, and a negotiation strategy is provided to each controller in order to determine the local control actions to be applied. Assume that the whole system can be decomposed into M subsystems

$$
x_r(k+1) = f_{\mathrm{dx}r}(x(k), u_r(k), u_{-r}(k)), \quad \text{for} \quad r = 1,...,M \tag{4}
$$

where $x_r(k) \in \mathbb{R}^{n_r}$ and $u_r(k) \in \mathbb{R}^{m_r}$ are the local states and inputs, and $u_{-r}(k) = [u_1^T(k), \ldots, u_{r-1}^T(k), u_{r+1}^T(k), \ldots, u_M^T(k)]^T$. Furthermore, assume that the sets $\mathbb{X}_r \subset \mathbb{R}^{n_r}$ and $\mathbb{U}_r \subset \mathbb{R}^{m_r}$ define the local feasible sets for $x_r(k)$ and $u_r(k)$ respectively, where $\mathbb{X} = \Pi_r^M \mathbb{X}_r$ and $\mathbb{U} = \Pi_r^M \mathbb{U}_r$, Π denoting the Cartesian product. From the system decomposition of (4) the cost function $L(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$ can be expressed as

[Venkat et al., 2006b,c]

$$L(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k)) = \sum_{r=1}^{M}\left(\sum_{h=k}^{k+N_\mathrm{p}-1} x_r^T(h+1|k)Q_r x_r(h+1|k) + \sum_{h=k}^{k+N_\mathrm{c}-1} u_r^T(h)R_r u_r(h)\right) \tag{5}$$

Let $\phi_r(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k))$ denote the local cost function, and for the sake of simplicity, $\phi_r(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k))$ is defined as the term inside the brackets in Eq. (5). Therefore, the centralized optimization problem (3) can be equivalently solved through the solution of (6), with $r = 1,\ldots,M$.

$$\begin{aligned}
&\min_{\widetilde{\mathbf{u}}(k)} \sum_{r=1}^{M} \phi_r(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k))\\
&\textbf{s.t.:}\\
&x_r(h+1) = f_{\mathrm{dx}r}(x(h),u_r(h),u_{-r}(h))\\
&x_r(h) \in \mathbb{X}_r; \quad u_r(h) \in \mathbb{U}_r; \quad y_r(h) \in \mathbb{Y}_r
\end{aligned} \tag{6}$$

The optimization problem Eq. (6) defines the non-linear DMPC (NDMPC) formulation. In this formulation, each controller determines its local control actions $\widetilde{\mathbf{u}}_r(k)$ according to its local cost function $\phi_r(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k))$. Note that from Eq. (6) a set of M local optimization problems is derived, all coupled via the cost function and the constraints. In this sense, Eq. (6) defines a situation in which the success of each controller depends upon the decisions of the remaining controllers. This situation defines a **game** referred to in this chapter as the NDMPC game.

Game-theory-based NDMPC has been previously studied by several authors. In these approaches the DMPC was analyzed as a non-cooperative game [Du et al., 2001, Giovanini and Balderud, 2006, Li et al., 2005, Trodden et al., 2009], where local decisions were computed as the solution to the local optimization problem (7). In those cases, the authors demonstrated the existence of at least one Nash equilibrium point and the convergence of the distributed solution to this point.

$$\begin{aligned}
&\min_{\widetilde{\mathbf{u}}_r(k)} \phi_r(\widetilde{\mathbf{x}}(k),u_r(h),u_{-r}(h))\\
&\textbf{s.t.:}\\
&x_r(h+1) = f_{\mathrm{dx}r}(x(h),u_r(h),u_{-r}(h))\\
&x_r(h) \in \mathbb{X}_r; \quad u_r(h) \in \mathbb{U}_r
\end{aligned} \tag{7}$$

Although there exist several strategic situations where achieving a Nash equilibrium point is desired, this is not the case in DMPC. For instance, in [Venkat et al., 2006a,b,c] the authors presented some examples where DMPC approaches with assured convergence to a Nash equilibrium point exhibited an unexpected closed-loop behavior, thereby limiting the applicability of those schemes. Alternatively, [Rantzer, 2006, 2008, 2009] transformed the optimization problem (7) into a min-max optimization problem. Accordingly, the solution to (7) does not require information exchange and also converges to a Nash equilibrium point. However, depending on the dimensions of the system the solution to the min-max problem might not be feasible in real-time. Based on these facts, bearing in mind the work done in [Venkat et al., 2006a,b,c] on feasible cooperation MPC (where convergence of the distributed scheme to the centralized solution was demonstrated), and given that local controllers are able to communicate with each other, a cooperative game framework is used in this chapter for analyzing the situation arising from Eq. (6).

2.2 The Distributed Model Predictive Control Game

As it was stated in Section 2.1, in the NDMPC formulation the success of each local controller is based upon the choices of the remaining controllers. According to [Myerson, 1991], such situations are the object of study of game theory. In the NDMPC case, the game is determined by the physical laws used to model the system to be controlled, by the models locally used to predict the system's behavior, and by the physical and operational constraints of the whole system. Since NDMPC is a discrete-time control strategy, it is played at each time step k, i.e., at each time step an optimal control action is obtained (over the decision space) based on local performance indices. Note that from Eq. (6) at each time step k each local controller has a decision space $\mathbb{U}_r$ for selecting the sequence of control actions $\widetilde{\mathbf{u}}_r(k)$, and this selection obeys the minimization of the local cost function $\phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$ (moves, strategies, and choices in the NDMPC game). So, based on [Nash, 1953, Von Neumann et al., 1947] the NDMPC circumstance has all the elements required for being analyzed within the game theory framework. In order to make this concept clear, Table 2 shows a didactic comparison between Game Theory and DMPC.

Table 2 Comparison between game theory and DMPC

	Game Theory	DMPC
Game	Set of rules used to describe the circumstances.	Local and global system model rules, as well as physical and operational constraints.
Play	Every particular instance at which the game is played.	Each time step k.
Move	The occasion choosing of an alternative under the conditions of the game.	Each time step k (as shown at the end of Section 2).
Strategy	Preference and/or rule followed by each player to select an alternative.	Minimization of the local system-wide-control cost function (as shown at the end of Section 2)
Choice	The selected alternative in a move according to the strategy.	Local control action to be applied to the system, driven by the minimization of the local system-wide-control cost function.

Mathematically, a game G can be defined in its strategic form as a tuple $G = (\mathscr{N}, \{\Omega_r\}_{r\in\mathscr{N}}, \{\phi_r\}_{r\in\mathscr{N}})$ where $\mathscr{N} = \{1,\ldots,M\}$ is the set of players, Ω_r is the decision space (set of feasible decisions) of the r-th player; and $\phi_r\colon \Omega_1 \times \Omega_M \to \mathbb{R}$ is the profit function of the r-th player (i.e., we must maximize instead of minimize as in MPC). Often, ϕ_r quantifies the preferences of player r (and determines its strategy), and gives to each player some degree of rationality [Akira, 2005]. Let $\mathscr{N}$ be the set of local controllers, $\Omega_r = \mathbb{X}_r \cap \mathbb{U}_r$ be the decision space of controller r, and $\{\phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))\}_{i\in\mathscr{N}}$ be the set of profit functions. Then the NDMPC game in its strategic form is a tuple $G_{\text{NDMPC}} = (\mathscr{N}, \{\Omega_r\}_{r\in\mathscr{N}}, \{\phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))\}_{r\in\mathscr{N}})$. In the light of (6), the game G_{NDMPC} involves a group of controllers who have the opportunity to collaborate for a mutual benefit: improving both local and whole system performance. So, G_{NDMPC} is a bargaining game according to the definition provided by Nash in [Nash, 1950b,a, 1953]. Furthermore, the game G_{NDMPC} has a group of individuals involved in the bargaining, a mutual benefit which is the objective of the bargaining, and a utopia point defined by the set of choices where all the individuals involved in the bargaining achieve at the same time their maximum benefit. Thus, the only missing element to define the game G_{NDMPC} as a bargaining game is the disagreement point.

According to Nash [1950b,a, 1953] the disagreement point is the benefit perceived by a player when an agreement is not possible. Such benefit is associated with an alternative plan carried out by the player in this situation, which is determined by the information locally available. Moreover, the disagreement point should give to the players a strong incentive to increase their demands as much as possible without losing compatibility. Following these statements the disagreement point $\eta_r(k) \in \mathbb{R}$ for each player (local controller) in the NDMPC game should be defined such that it reflects the expected cost associated with the non-cooperative behavior. That is, the expected value of the local cost function if the local controller "decides" not to cooperate. Associated with this cost, there is a local control action to be applied that acts as an alternative plan carried out by the controller in a non-cooperative situation. In addition, the disagreement point must be updated following a rule that provides incentives for changing the decision of the controllers that decide not to cooperate, and for enhancing the performance of the controllers that decide to cooperate.

For controller r, assume $\eta_r(k)$ as the expected maximum loss of performance. Then, at time step k: $\eta_r(k) - \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$ denotes the utility of such subsystem. Associated with the utility perceived by controller r there is a plan or sequence of local control actions $\widetilde{\mathbf{u}}_r(k)$. Thereby, as in Nash [1950b,a, 1953], controller r seeks a feasible local control sequence that maximizes its own utility. That is, controller r looks for a control sequence $\widetilde{\mathbf{u}}_r(k)$ such that $\phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)$ is minimum and $\eta_r(k) \geq \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$. If that control sequence exists, the plan of action is locally to apply the first element of the control sequence $\widetilde{\mathbf{u}}_r(k)$, to use a shifted control sequence as the initial condition for making the decision again in the next time step, and to reduce the disagreement according to the expression $\eta_r(k+1) = \eta_r(k) - \alpha(\eta_r(k) - \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)))$, with $\alpha \in \mathbb{R}$, $0 < \alpha < 1$. If that control action does not exists, the plan of action is to keep applying the current local control action, to use a shifted control sequence from the initial condition as a condition for performing the next decision making stage, and to make the value of the disagreement point equal to the value of keeping the current control action, viz., $\eta_r(k) = \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$.

Given the updating conditions of the disagreement point, decreasing its value (which implies the controller r "decides" to cooperate) provides strong incentives for increasing their demand from the cooperative behavior; but, making its value equal to the current value of the cost function (which implies the controller r "decides" not to cooperate) provides incentives to controller r for changing its decision not to cooperate. Indeed, if the expected maximum loss of performance grows the decision space is augmented, the probability of finding a control sequence such that $\phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$ is minimum, and $\eta_r(k) \geq \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$ is increased. Mathematically, the disagreement point is formulated as follows [Valencia, 2012]:

$$\eta_r(k+1) = \begin{cases} \eta_r(k) - \alpha(\eta_r(k) - \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))) & \text{if } \eta_r(k) \geq \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)) \\ \eta_r(k) + (\phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)) - \eta_r(k)) & \text{if } \eta_r(k) < \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)) \end{cases} \tag{8}$$

Despite of the similarities between the bargaining games defined by Nash in [Nash, 1950b,a, 1953] and the game G_{NDMPC} (see Table 3), there are some differences that should be accounted for in order to define a bargaining solution to the NDMPC game (in NDMPC the solution pertains to the control actions to be locally applied by each controller). The main difference is that since it is expected that the system will operate over a long time period, the NDMPC game is a sequence of infinite bargaining games which are played at each time step, in a variable-decision environment influencing the behavior of the local controllers and their decision-making stage. Thus, the original game theory is extended in order to have a mathematical framework for analyzing NDMPC bargaining games. In this way the concept of discrete-time dynamic bargaining game for DMPC (identified as GT-NDMPC in this chapter) is introduced in [Valencia, 2012].

Table 3 Comparison between bargaining games and DMPC

	Bargaining Game Theory	DMPC
Players	A group of individuals involved in the bargaining.	The set of local controllers that are able to communicate among them, and bargain.
Decision Space	The set of all choices available to the individuals involved in the bargaining.	The set of available control actions, determined by the physical and operational constraints.
Disagreement point	Minimum level of satisfaction expected by the individuals from the bargaining.	Maximum expected loss of performance by each local controller from the bargaining.
Utopia point	The set of choices where all the individuals involved in the bargaining achieve at the same time their maximum benefit.	The set of control actions that minimize all the local system-wide-control cost functions at the same time.

A discrete-time dynamic bargaining game refers to a situation where at each time step a bargaining game is solved depending on the dynamic evolution of the decision environment. In this bargaining game the dynamic evolution of the decision environment is determined by the state $x(k) \in \mathbb{R}^n$ and input $u(k) \in \mathbb{R}^m$. Mathematically, a discrete-time dynamic bargaining game is defined as follows:

Definition 1. Discrete-time Dynamic Bargaining Game:
A discrete-time dynamic bargaining game for the set of players N is a sequence of games $\{(\Theta(k),\eta(k))\}_{k=0}^{\infty}$, where:

1. $\Theta(k)$ is a nonempty closed subset of $\mathbb{R}^N$ containing the feasible values for the profit function of each player, at $k=1,2,3,\ldots$.
2. $\eta(k)$ is the disagreement point, $\eta(k)\in\mathrm{int}(\Theta(k))$.
3. $\zeta_r(\Theta(k)):=\min\{\phi_r(k)\colon(\phi_r(k))_{r\in N}\in\Theta(k)\}$ exists for every $r\in N$ at each time step k.
4. There exist functions $f_r\in\mathbb{R}^{n_r}, g\in\mathbb{R}^z, h_r\in\mathbb{R},\ r=1,\ldots,N$, determining the dynamic evolution of the decision environment and the disagreement point of player r, and the dynamic evolution of the feasible set, such that:
$$\begin{aligned} x_r(k+1) &= f_r(x(k),u(k)) \\ \eta_r(k+1) &= h_r(x(k),u(k),\eta(k)) \\ \Theta(k+1) &= g(x(k),u(k),\Theta(k)) \end{aligned} \tag{9}$$
with $x_r(k)\in\mathbb{X}_r$, $\mathbb{X}_r\subset\mathbb{X}$, z the dimensions of the feasible set of values for the profit function, and $u(k)$ the vector of actions taken by the players affecting the decision environment. Here, the function $g(x(k),u(k),\Theta(k))$ is defined by the set of time dependent constraints on $x(k)$ and $u(k)$, and the facts that can reduce the size of the decision space.
5. There exists a tuple $(\phi_1(x(k),u(k)),\ldots,\phi_M(x(k),u(k)))\in\Theta(k)$ with $\phi_r(x(k),u(k))$ the profit function of the r-th player.

Let $\Xi_r(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k))$ be the set resulting from the intersection of Ω_r and the equality constraint given by the local prediction model (4). Then, the set $\Theta(k)$ in the GT-NDMPC game is defined as $\Theta(k)\colon=\{(\phi_1(x(k),u(k)),\ldots,\phi_M(x(k),u(k)))\in\mathbb{R}^M\mid(x(k),u(k))\in\Xi_r(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k))\}$. Here $g(x(k),u(k),\Theta(k))$ is equal to the whole system model (1). Then, the game G_{NDMPC} is a discrete-time dynamic bargaining game. Although in the definition of $\{(\Theta(k),\eta(k))\}_{k=0}^{\infty}$ it is desired that $\eta(k)\in\mathrm{int}(\Theta(k))$, from the definition of $\eta_r(k)$ such a condition cannot be guaranteed. In that case, the value of the disagreement point might lie on the boundary of the feasible set. If this happens and there exists a feasible search direction to minimize the local cost function, then a control action satisfying the constraint $\eta_r(k)\geq\phi_r(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k))$ is achieved. Otherwise, there is no change in the local control actions. This allows each local controller to decide whether or not to cooperate with the remaining subsystems. Note that in these statements there exists an underlying utility concept.

For GT-NDMPC game the utility of each local controller is given by the difference between the disagreement point and the local cost function, i.e., $\eta_r(k)-\phi_r(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k))$. From [Nash, 1953, Harsanyi, 1963, Peters, 1992, Akira, 2005] the solution of a bargaining game is given by the maximization of the Nash products, namely, the product of the utility functions $\Pi_{r=1}^{M}\eta_r(k)-\phi_r(\widetilde{\mathbf{x}}(k),\widetilde{\mathbf{u}}(k))$ in the NDMPC case. Based on the axiomatic characterization proposed in [Valencia, 2012] the outcome of the game G_{NDMPC} is given by the solution of the optimization problem (the $\log(\cdot)$ function arises from the transformation of the Nash products).

$$\begin{aligned}
&\max_{\widetilde{\mathbf{u}}(k)} \sum_{i=r}^{M} w_r \log(\eta_r(k) - \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))) \\
&\textbf{s.t.:} \\
&x_r(h+1) = f_{\mathrm{dx}r}(x(h), u_r(h), u_{-r}(h)) \\
&\eta_r(k) \geq \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)) \\
&x_r(h) \in \mathbb{X}_r; \quad u_r(h) \in \mathbb{U}_r
\end{aligned} \tag{10}$$

Then, the maximization problem of Eq. (10) can be solved in a distributed way by locally solving the system-wide control problem of Eq. (11).

$$\begin{aligned}
&\max_{\widetilde{\mathbf{u}}_r(k)} \sum_{i=r}^{M} w_r \log(\eta_r(k) - \phi_r(\widetilde{\mathbf{u}}_i(k), \widetilde{\mathbf{u}}_{-i}(k))) \\
&\textbf{s.t.:} \\
&x_r(h+1) = f_{\mathrm{dx}r}(x(h), u_r(h), u_{-r}(h)) \\
&\eta_r(k) \geq \phi_r(\widetilde{\mathbf{u}}_i(k), \widetilde{\mathbf{u}}_{-i}(k)) \\
&x_r(h) \in \mathbb{X}_r; \quad u_r(h) \in \mathbb{U}_r
\end{aligned} \tag{11}$$

considering $\widetilde{\mathbf{u}}_{-r}(k)$ to be fixed and optimizing only in the direction of $\widetilde{\mathbf{u}}_r(k)$. For implementing the distributed solution of the GT-NDMPC game, a negotiation model based on the model proposed in [Nash, 1953] for two-player games is used [Valencia, 2012]. In this negotiation model each local controller is:

- fully informed on the structure of the game;
- fully informed on the utility function of the remaining subsystems;
- assumed intelligent and rational, i.e., each controller has a set of preferences, treats, and rational expectations of its future environment.

Additionally, it is assumed that the communication architecture allows each subsystem to communicate with the remaining subsystems in order to transmit their disagreement points and their local measurements of the states and inputs. Such a model adapted for solving the GT-NDMPC game has the steps shown in algorithm 1.

The initial condition for solving Eq. (11) at time step $k+1$ is given by the shifted control input, and $\widetilde{\mathbf{u}}_i(k)$ is a feasible control action used as initial condition for the optimization procedure of subsystem i at time step k (shifted control input from previous time instant). As in the case of the negotiation model proposed in [Nash, 1953], the negotiation model for solving the GT-NDMPC game in a distributed way represents a two-moves game where the decisions are taken in steps 3 and 4.

It is worth noting that the proposed negotiation model allows for the avoidance of iterative procedures. This is the main difference of the proposed control scheme with respect to the approaches based on Lagrange multipliers, or those schemes based on game theory reported in the literature. Moreover, it provides each subsystem enough elements for deciding on whether or not to cooperate, depending of the benefit perceived to result form the cooperative behavior.

Algorithm 1 Negotiation model for GT-NDMPC games

for $k = 1, \ldots$ **do**
  Each subsystem sends the values of $\mathbf{x}_i(k),\ \eta_i(k)$ to the remaining subsystems
  for subsystem $i = 1, \ldots, N$ **do**
    Solve the local optimization problem of Eq. (11)
    if Eq. (11) is feasible **then**
      Select $u_i(k)$ as control action
      Update its disagreement point with $\eta(k+1) = \eta_i(k) - \alpha(\eta_i(k) - \phi_i(\widetilde{\mathbf{u}}(k)))$
    else
      Select the first control action of $\widetilde{\mathbf{u}}_i(k)$
      Update its disagreement point with $\eta(k+1) = \phi_i(\widetilde{\mathbf{u}}(k))$
    end if
  end for
  All subsystems send updated control actions and disagreement points to the others.
end for

The closed-loop stability of a system controlled via the proposed scheme can be derived by combining the feasibility proof in Valencia et al. [2011] and the definition of a disagreement point (the disagreement point provides an upper boundary for local and whole system cost functions). From Algorithm 1, the stability of the proposed GT-NDMPC method depends on the decision of each subsystem on whether or not to cooperate. In order to demonstrate the stability of the closed-loop system, in Valencia [2012] two cases were considered: All subsystems always cooperate, or some subsystems do not cooperate at first but a few time steps ahead they all start to cooperate. Following the same procedure proposed in Valencia [2012] for these cases, closed-loop stability conditions can be derived.

In Sections 3 and 4 the GT-NDMPC game formulation presented in the current section is applied to congestion management on motorways and in urban traffic.

3 Bargaining-Game-Based Coordination for Congestion Management on Motorways

3.1 Motorway traffic model

Let us start by introducing some concepts and notations related to the traffic model used in this section, viz. the METANET model described in Kotsialos et al. [2002b, 1999], Papageorgiou et al. [1990]. In this model, the motorway network is represented as a directed graph in which the links represent homogeneous motorway stretches. Each stretch has uniform characteristics, e.g., no on-/off-ramps, no major changes in the geometry, and no metering lines. A node is placed at the locations where a major change in the road characteristics occurs, as well as at junctions and at the on-/off-ramps. A link is further divided into segments of equal distance. Each segment is characterized by its length (L_m), number of lanes (λ_m), vehicle density

($\rho_{(m,i)}(k)$), mean speed ($v_{(m,i)}(k)$), and output flow ($q_{(m,i)}(k)$), with m denoting the link number, (m,i) denoting the segment i of the link m, and k the time step. For each segment, the dynamic evolution of density of vehicles, mean speed, and length of the queue at the on-ramps is determined. Let $R_{(m,i)}(k)$, $C_{(m,i)}(k)$, and $A_{(m,i)}(k)$ denote the relaxation, convection, and anticipation terms, defined as in Kotsialos et al. [2002b, 1999], Papageorgiou et al. [1990]. Moreover, let the subscript o denote the origin nodes (nodes allowing the access of traffic from an external road; mainstream origin or on-ramp). For instance, d_o denotes the demand at the origin o. This traffic accessing a link by on-ramp o often is limited or controlled by a traffic light (or ramp-metering), where $r_o(k)$ denotes the ramp-metering rate, used to regulate the vehicles accessing the motorway.

The METANET model was used here because this model provides an adequate description of the traffic flow on a motorway with reduced complexity, which is desirable for control purposes. For instance, in Kotsialos et al. [2002b, 1999], Papageorgiou et al. [1990] and the references therein, there are several control strategies where the METANET model was used for representing the traffic flow dynamics. Also in Kejun et al. [2008], Groot et al. [2011], Baskar et al. [2009], Hegyi et al. [2002], Lu et al. [2010] the METANET model was used for representing the dynamic behavior of the traffic flow.

Let $q_o(k)$ be the flow of vehicles incoming from the origin o to the link to which origin o is connected $(1,i)$. The value of $q_o(k)$ is given by:

$$q_o(k) = \min\left[d_o + \frac{w_o(k)}{T_s}, C_o r_o(k), C_o\left(\frac{\rho_{\max,i} - \rho_{(1,i)}(k)}{\rho_{\max,i} - \rho_{\mathrm{cr},i}}\right)\right] \tag{12}$$

where C_o is the capacity of origin o under free flow conditions, $\rho_{\max,i}$ is the maximum density of a segment, w_o denotes the queue of vehicles on the origin node o, T_s is the sample time. Discussions on the meaning of the parameters in (12) and their selection can be found in, e.g., [Kotsialos et al., 2002b, 1999, Papageorgiou et al., 1990].

The dynamic evolution of density, speed, and queues in a traffic segment of a motorway is given by:

$$\rho_{(m,i)}(k+1) = \rho_{(m,i)}(k) + \frac{T_\mathrm{s}}{L_m\lambda_m}(q_{\mathrm{in},(m,i)}(k) - q_{\mathrm{out},(m,i)}(k)) \tag{13}$$

$$\begin{aligned} v_{(m,i)}(k+1) = v_{(m,i)}(k) + R_{(m,i)}(k) + C_{(m,i)}(k) \\ + A_{(m,i)}(k) - \frac{\delta T_\mathrm{s} q_o(k) v_{(m,i)}(k)}{L_m\lambda_m(\rho_{(m,i)}(k) - \mu)} \end{aligned} \tag{14}$$

$$w_o(k+1) = w_o(k) + T_\mathrm{s}(d_o(k) - q_o(k)) \tag{15}$$

with

$$q_{\text{in},(m,i)}(k) = q_o(k) + q_{(m-1,i_{last})}(k) \tag{16}$$

$$q_{\text{out},(m,i)}(k) = \lambda_m \rho_{(m,i)}(k) v_{(m,1)}(k) \tag{17}$$

where $q_{(m-1,i_{last})}(k)$ is the flow from the last segment of the link $m-1$, and the term $\frac{\delta T_s q_o(k) v_{(m,i)}(k)}{L_m \lambda_m (\rho_{(m,i)}(k) - \mu)}$ defines the reduction in the speed in the link (m,i) due to the incoming flow from the origin o.

3.2 Bargaining-game approach to congestion management on motorways

The idea of congestion management on motorways is to provide a control strategy for regulating the number of vehicles entering the traffic network. In this sense, expected travel time is used as the cost function for the NMPC. The travel time is a performance index that relates the amount of vehicles on a motorway at any one time with the changes in the timing of the traffic lights. Let $x(k) = [\rho^T(k), v^T(k), w^T(k)]^T$, and $u(k) = r(k)$, where $\rho(k)$, $v(k)$, $w(k)$, $r(k)$ are the vectors containing the densities, mean speeds, queues, and ramp-metering rates of all links, segments and origins of the motorway respectively. Thus, the performance index of the users of the motorway and the access roads over a prediction horizon N_p is given by:

$$\begin{aligned} L(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)) = T_\text{s} \sum_{h=k}^{k+N_\text{p}-1} \sum_{m \in \mathfrak{M}} \Bigg(& \sum_{i \in \psi_m} \rho_{(m,i)}(h) L_m \lambda_m \\ & + \alpha \sum_{o \in \mathfrak{O}} w_o(h) + \alpha_\text{r} (\Delta r_o(h))^2 \Bigg) \end{aligned} \tag{18}$$

where $\mathfrak{M}$ is the set of links, ψ_m denotes the set of segments of link m, $\mathfrak{O}$ denotes the set of origins, $\Delta r_o(k) = r_o(k) - r_o(k-1)$, and $\alpha, \alpha_\text{r} > 0$ are tuning parameters associated with the time spent by the users in the queues at the origins and with the smoothness of the changes of the control actions. Since the traffic on a motorway is very sensitive to changes in the ramp-metering rates in Eq. (18) the norm of those changes over the prediction horizon is penalized instead of the value itself.

Since motorways are large-scale systems, implementation of centralized NMPC is not advisable [Frejo and Camacho, 2012]. Assume that the whole system can be decomposed into M subsystems r such that the local models have the form (4) for all r.

The decomposition could be made based on the inputs, or merging different segments [Ferrara et al., 2012]. Let $\mathfrak{M}_r$, Ψ_r, and $\mathfrak{O}_r$ denote the set of links, the set of segments, and the set of origins belonging to the subsystem r. Then, NDMPC for congestion management on a motorway is given by:

$$
\begin{aligned}
&\min_{\widetilde{\mathbf{u}}(k)} \sum_{r=1}^{M} \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)) \\
&\textbf{s.t.:} \\
&x_r(h+1) = f_{\mathrm{dx}r}(x(h), u_r(h), \widetilde{\mathbf{u}}_{-r}(k)) \\
&\rho_{\min,(m,i)} \leq \rho_{(m,i)}(k) \leq \rho_{\max,(m,i)} \\
&w_{\min,o} \leq w_o(k) \leq w_{\max,o} \\
&v_{\min,(m,i)} \leq v_{(m,i)}(k) \leq v_{\max,(m,i)} \\
&r_{\min,o} \leq r_o(k) \leq r_{\max,o} \\
&m \in \mathfrak{M}_r,\ i \in \Psi_r,\ o \in \mathfrak{O}_r
\end{aligned}
\tag{19}
$$

with $f_{\mathrm{dx}r}(x(h), u_r(h), \widetilde{\mathbf{u}}_{-r}(k))$ being the local prediction model. Furthermore, the system decomposition $\mathbb{X}_r$ and $\mathbb{U}_r$ are defined by the sets $\mathfrak{M}_r$, Ψ_r, and $\mathfrak{O}_r$ which determine the links and segments belonging to the subsystem r.

Then, the NMPC problem for travel time reduction can be equivalently formulated as in Eq. (6), with the corresponding definition for the variables and sets. Since each subsystem model requires the information from the remaining subsystems for making the prediction, the values of the local travel times $\phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$ are coupled to each other. Thus, a situation arises belonging to the set of games G_{NDMPC}, where $\mathscr{N}$ is the set of local controllers trying to minimize their local cost function, over a feasible set $\Omega_r = \mathbb{X} \times \mathbb{U}$.

In addition, since the travel time of the users of the motorway can be expressed as $L(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)) = \sum_{r=1}^{M} \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$, and since the local controllers are able to communicate with each other, the game G_{NDMPC} is a discrete-time bargaining game $\{(\Theta(k), \eta(k))\}_{k=0}^{\infty}$. The outcome of the game G_{NDMPC} associated with the distributed congestion management scheme described in this section is obtained by the solution to the local optimization problems previously described in Eq. (11) considering for its implementation the algorithm 1, proposed in Section 1.

3.3 Simulation and results

Consider the motorway shown in Figure 2. It consists of a motorway with ten segments and nine on-ramps modeled as origins and it allows the entry of new vehicles to the motorway regulated by the traffic signals $r_i(k),\ i = 1, \ldots, 9$. A period of 12 hours is simulated. The Matlab function *fmincon* is used for solving each local optimization problem. The solver used an interior point method. In order to test the performance of the proposed congestion management scheme a time-varying demand profile is used. Thus, the curve of demand shown in Figure 3 is simulated at each on-ramp. The maximum number of entering cars per input is 600; and an initial queue of 10 vehicles is considered (see Figure 3). For simulation purposes all links are assumed to have the same characteristics. The parameters for the simulations are taken from [Zegeye et al., 2012]. Note that in Figure 2 the current control action is denoted by $r(k-1)$ while the control action to be locally applied is denoted by $r_i^*(k),\ i=1,\ldots,9.$ In this case the prediction and control horizons are $N_\mathrm{p}=10$ and $N_\mathrm{c}=5$, respectively. For implementing the proposed scheme the whole system is divided into ten subsystems. Each subsystem has a link and an on-ramp (Figure 2 shows the system partition). For comparison purposes a centralized NMPC is also implemented (with $N_\mathrm{p}=10$ and $N_\mathrm{c}=5$). The comparison between the proposed GT-NDMPC and a centralized NMPC is done because the centralized solution is the best possible. Therefore, NMPC provides the best baseline for evaluating the loss of performance of the motorway with the GT-NDMPC scheme.

From the formulation of the GT-NDMPC for congestion management purposes on the motorway of Figure 2, the controllers at each on-ramp must share the current local control actions and the measurements of their local states. Based on this information, all controllers are able to identify the current operating conditions of the remaining controllers, and they are also able to decide which control actions should be locally applied with the purpose of minimizing their effect on the performance of the other local controllers. It is worth pointing out that the information exchange can be done using any full duplex communication channel.

Figures 4(a) and 4(b) show the cycle for each on-ramp light. From these figures it is evident that the centralized and the proposed congestion management schemes generate different sequences of control actions at times with higher demands. Moreover, in the off-peak time intervals, the same constant control actions are used for managing the congestion at the motorway. It is noteworthy that although at peak demand the traffic network presents a congestion scenario (600 vehicles are expecting to get onto the motorway at each on-ramp), blocking actions are not required, i.e., flow on the motorway and on the on-ramps is reduced. This is reflected in the behavior of the speed and density of vehicles.

Figures 5(a) and 5(b) present the evolution along the simulation of the speed at the different links when the control actions are computed by the NMPC and the GT-NDMPC respectively. In these figures, it is clear that as the demand at the on-ramps increases the speed at the links decreases, reaching the lowest value at the link $(1,7)$ when the control actions are computed by the NMPC. It is noteworthy that the speed distribution over the motorway depends on the control scheme used for computing the control actions. In fact, the speed distribution with the proposed GT-NDMPC was [60, 100]; lower than the speed distribution with the NMPC (here speed distribution is understood as the range where all the speed trajectories are moving).

Figures 6(a) and 6(b) show the time evolution of the density of vehicles at each link. From these figures it is possible to infer that the centralized NMPC allows a better use of the traffic infrastructure. Since NMPC performed a larger reduction of speed than the GT-NDMPC, the density of vehicles in the motorway increases. Hence, the expected length of the queues at the on-ramps decreases. It is worth pointing out that despite the demand, the control schemes kept the density below the critical density. Thus the traffic system remained stable along the simulation.

Although GT-NDMPC presents a loss of performance with respect to the centralized NMPC, the loss of performance is not significant. Let the total time spent (TTS)

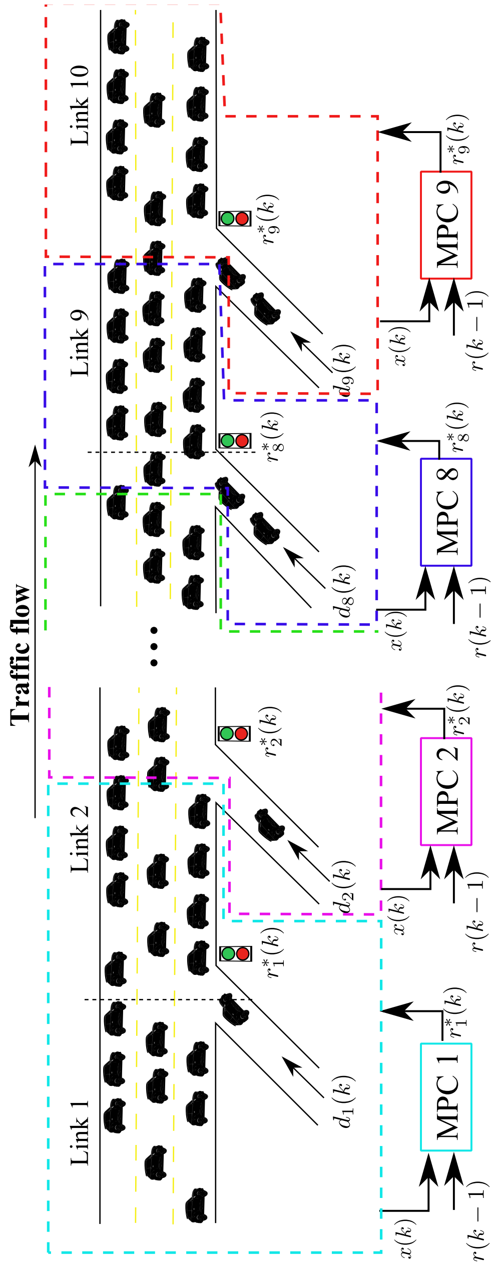


Fig. 2 Motorway used as a testbed for evaluating the performance of the proposed GT-NDMPC approach.

by the vehicles on the motorway and the entrance ramps over the entire simulation

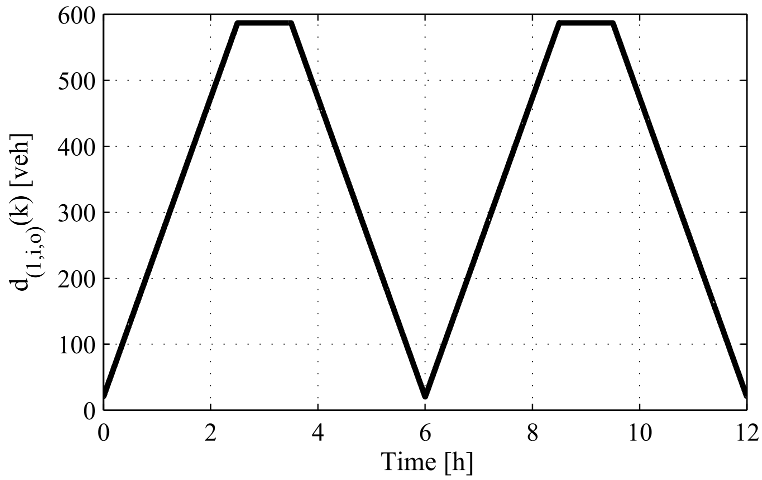


Fig. 3 Simulated demand at each on-ramp in the evaluation of the proposed GT-NDMPC approach in the case study for evaluating the performance of the proposed GT-NDMPC approach.

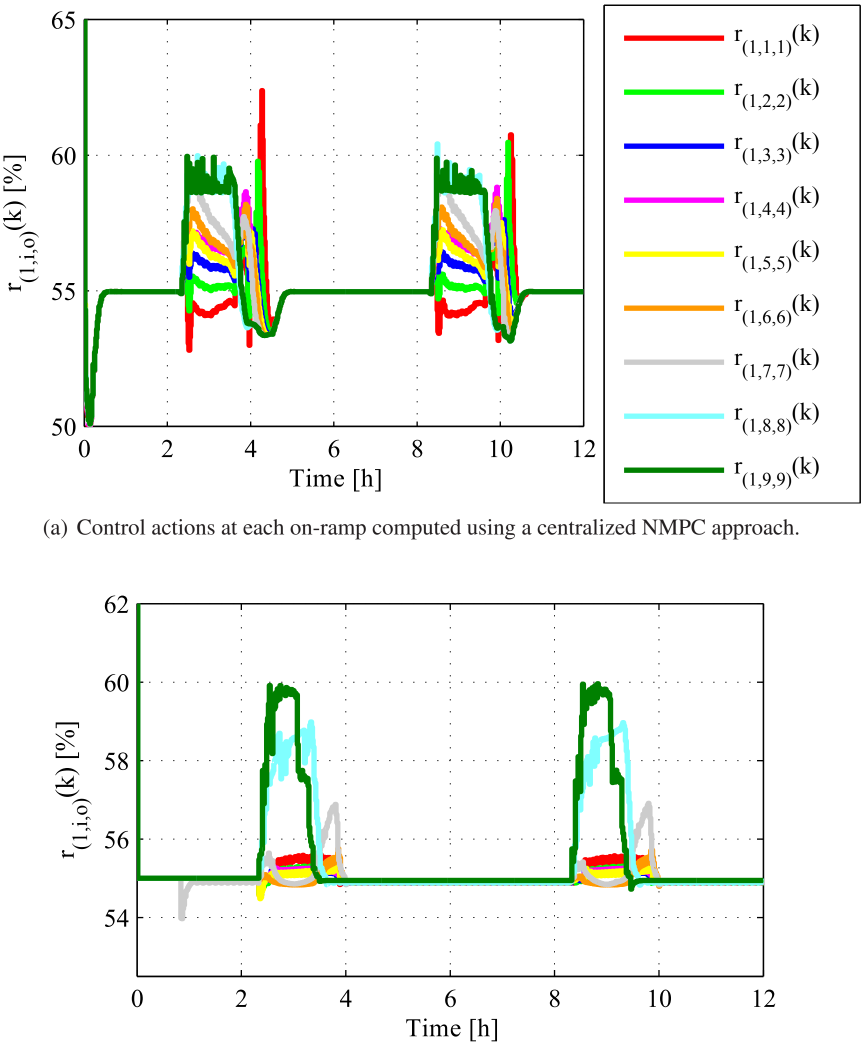
period be defined as:

$$TTS = \sum_{l=1}^{N_{\text{sim}}} \left(\sum_{m\in\mathfrak{M}} \sum_{i\in\psi_m} \rho_{i,m}(l) L_m \lambda_m + \sum_{o\in\mathfrak{O}} w_o(l) \right) T_s \tag{20}$$

where N_{sim} is the number of simulation steps. Another performance index used for evaluating control schemes with application to motorways is the Total Waiting Time (TWT) Treiber and Kesting [2013], this index is computed as:

$$TWT = \sum_{i=1}^{N_{\text{sim}}} \left(\sum_{m\in\psi_0} w_m(l) \right) \tag{21}$$

Table 4 presents the TTS and the TWT for two optimization-based control techniques, namely, the centralized NMPC, the proposed GT-NDMPC, and the ALINEA method reported in Haj-Salem et al. [2001], Papageorgiou et al. [2008] which is a simpler approach for traffic control on motorways. In fact, the control action in the ALINEA method is a sort of integral state-feedback where $r_i(k+1) = r_i(k) + K\left(\rho_{\text{cr},m} - \rho_{(m,i)}(k)\right)$, with K the gain of the controller. Note that, ALINEA is a simpler control law and the optimization-based techniques perform better (in terms of the TTS and the TWT indexes) on the motorway presented in this chapter. In fact, significant improvements are achieved even with the proposed GT-NDMPC, which (due to system partition) has a poorer performance compared to the centralized NMPC. The TTS results shown in Table 4 also confirm the loss of performance using GT-NDMPC; however, a distributed control scheme can be used as an al-

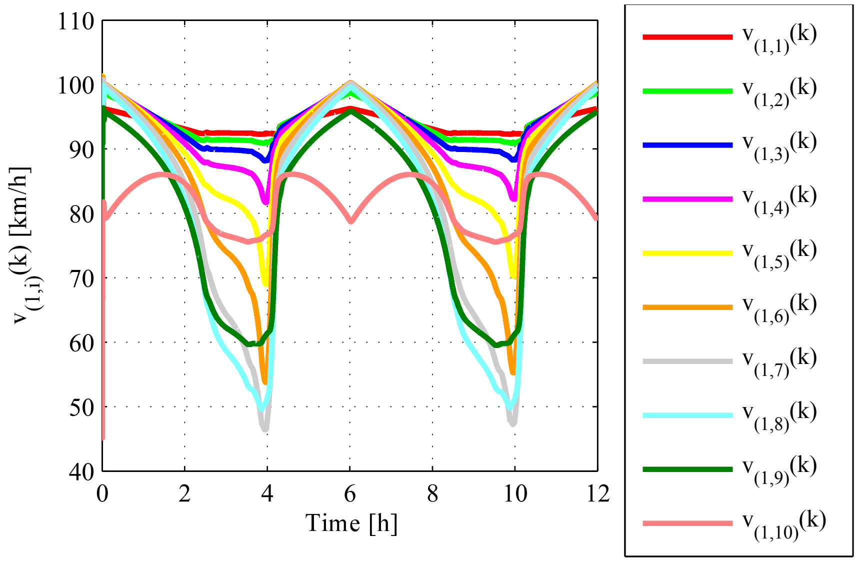


(a) Control actions at each on-ramp computed using a centralized NMPC approach.

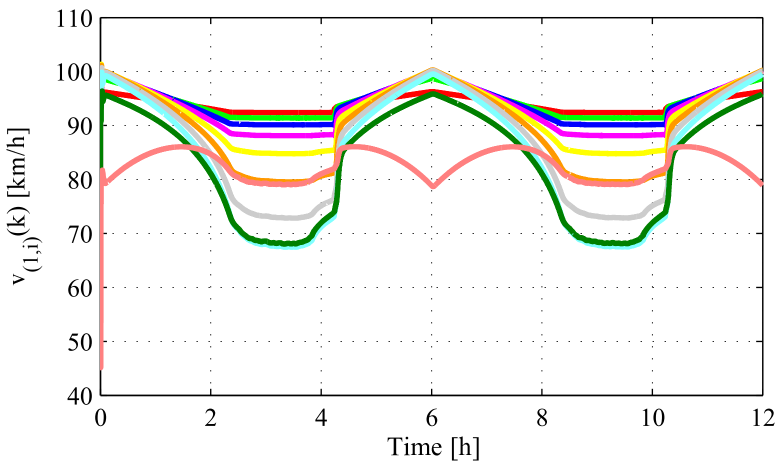
(b) Control actions at each on-ramp computed by the proposed GT-NDMPC approach.

Fig. 4 Comparison of control actions at each on-ramp for the centralized and proposed schemes.

ternative for controlling large-scale traffic networks. Note that although the TWT increases by about 77 % in the GT-NDMPC case with respect to the centralized case, the difference in the TTS is just about 4 %. This means that with GT-NDMPC there are more vehicles waiting in the queues but once they are on the motorway they are efficiently evacuated, resulting in a reduction of their TTS.



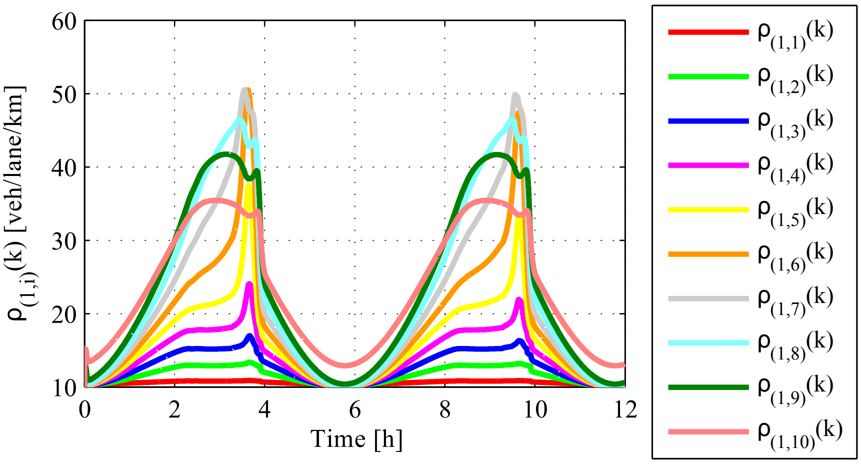
(a) Behavior of the speed with the NMPC approach.



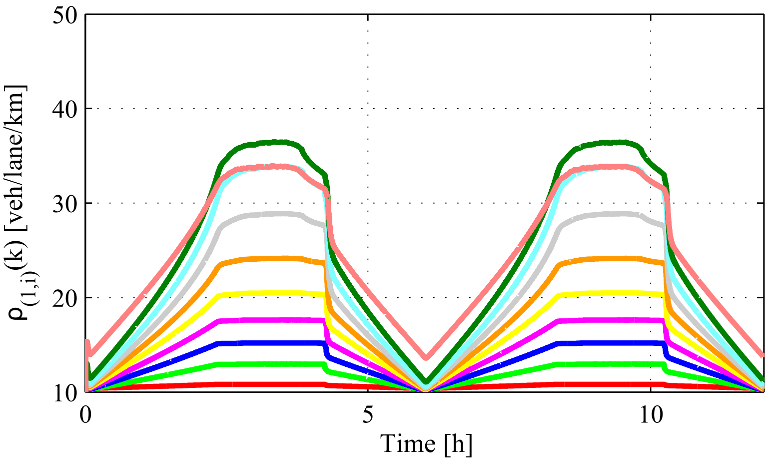
(b) Behavior of the speed with the proposed GT-NDMPC approach.

Fig. 5 Comparison of speed behavior between the proposed and centralized approaches.

Figures 7(a) and 7(b) show the evolution of the queues at each on-ramp. It is evident that the centralized NMPC maintains the queues at almost all on-ramps at the same length while in the case of the motorway controlled by the GT-NDMPC each on-ramp has its own queue length, resulting from the negotiation among controllers. Consequently, a better use of the infrastructure is achieved with the centralized NMPC, which is able to manage the congestion on the motorway. For instance



(a) Time evolution of the density when the control actions are computed by an NMPC approach.



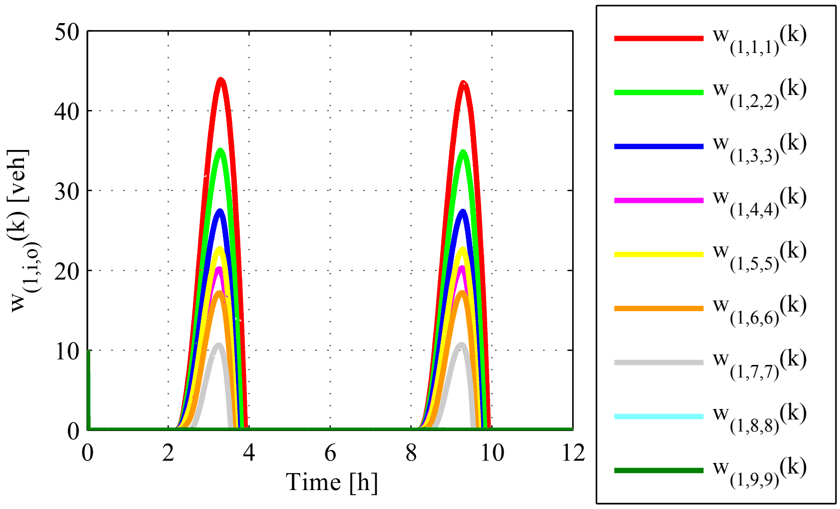
(b) Time evolution of the density when the control actions are computed by the proposed GT-NDMPC approach.

Fig. 6 Comparison of densities between centralized and proposed schemes.

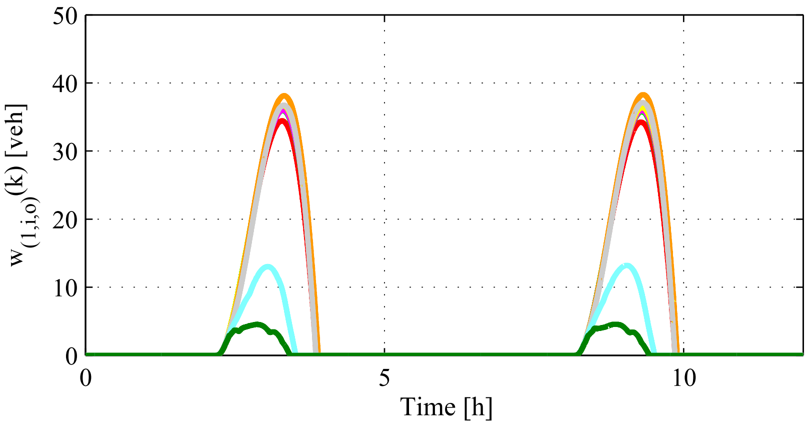
Table 4 TTS and TWT for the ALINEA, NMPC, and GT-NDMPC control schemes.

Controller	TTS	Relative difference (%)	TWT	Relative difference (%)
ALINEA	3998	0	291683	0
NMPC	3677	8.03	131485	54.92
GT-NDMPC	3833	4.127	231830	20.51

the total waiting time index for the centralized MPC is significantly lower than in the case of the GT-NDMPC.



(a) Time evolution of the queues when the control actions are computed by a centralized NMPC approach.



(b) Time evolution of the queues when the control actions are computed by the proposed GT-NDMPC approach.

Fig. 7 Comparison of queues between the centralized and proposed schemes.

Figure 8 presents the computational time associated with both the solution of the centralized NMPC and that of the proposed GT-NDMPC. In Figure 8, the time involved in the solution of each local controller was considered because it is assumed that all local controllers are working at the same time. Thus the computational time of the proposed GT-NDMPC is determined by the slowest local controller. From Figure 8, it is evident that (as expected) the time required by the GT-NDMPC for computing the local control actions is lower than the time required by the centralized NMPC. In fact, the computational time of the centralized NMPC is higher by an order of magnitude. Accordingly, from the point of view of the computational time, the GT-NDMPC scales better than the NMPC. It is worth to point out that the sample time T_s is 60 s. Assume that there is an exponential dependence of the computational time on the number of on-ramps on the motorway. Thus with the centralized NMPC a maximum of 28 on-ramps can be controlled, while with the GT-NDMPC ideally up to 648 on-ramps can be controlled before requiring to increase the sampling time. Previous scaling results were obtained following the rules $t_{\text{NMPC}} = 0.02707 \exp(0.2n)$ and $t_{\text{NDMPC}} = 0.091 \exp(0.01n)$ for centralized NMPC and GT-NDMPC respectively, with n the number of on-ramps.

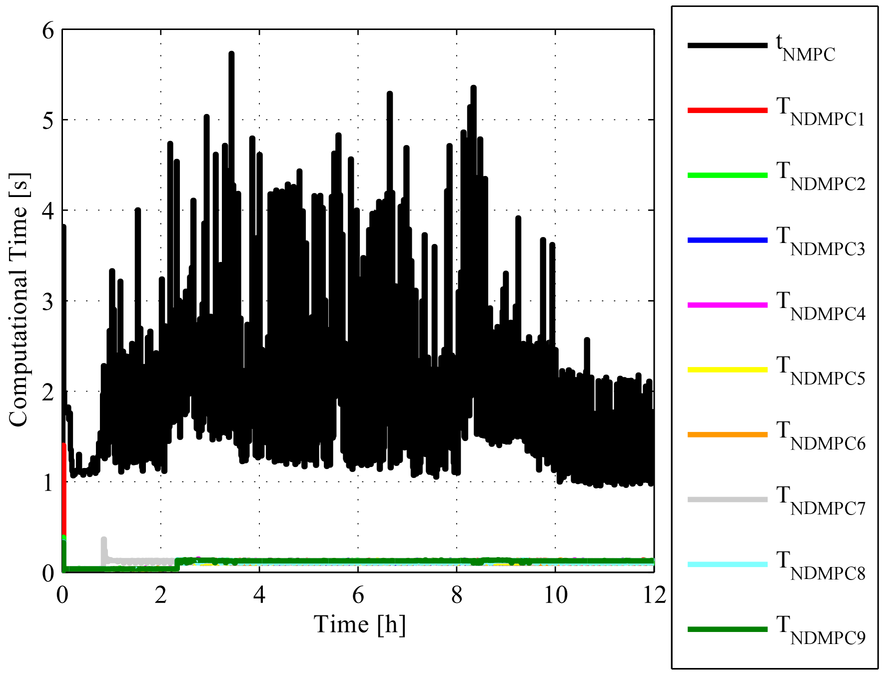


Fig. 8 Comparison of the computational time of both centralized NMPC and GT-NDMPC schemes. Here $t_{\text{NMPC}}(k)$ represents the time taken by the centralized NMPC for computing the control actions at time step k, while $t_{\text{NDMPC}r}(k)$, $r = 1,\dots,9$ represents the time taken by each local controller for computing the local control actions at the same time step.

4 Bargaining-Game-Based Coordination for Urban Congestion Management

4.1 Urban traffic model

As in Section 3, let us begin by introducing some concepts and notations related to the traffic model used here, viz. the Macroscopic Simplified Urban Traffic Model (S model) described in [Lin et al., 2011, 2012]. Similar to the motorway case, in urban traffic models the concepts of links and origins are also used. In this model J denotes the set of nodes or intersections, L denotes the set of links or roads, $I_{(u,d)} \subset J$ denotes the set of input nodes, and $O_{(u,d)} \subset J$ denotes the set of output nodes. Hence, each link is defined by its input and output nodes, i.e., by the pair $(u,d),\ u,d \in J$ marking the starting and ending intersections respectively. The S model has the particularity that each intersection takes the corresponding cycle time as its simulation time interval. Therefore, the simulation time intervals might be different for each intersection. So the input and output flow rates of each link are averaged over the cycle times (the flows leaving or entering links are described with flow rates rather than with numbers of cars [Lin et al., 2012]). For a link $(u,d) \in L$, let c_d be the cycle time with k_d its corresponding time step counter. Figure 9 illustrates the concepts previously introduced.

Let $\alpha^{\text{leave}}_{(u,d,o)}(k_d)$ denote the leaving flow rate of link (u,d) turning to the output link o. Let $g_{(u,d,o)}(k_d)$ be the green time signal duration allowing the vehicles to flow from link (u,d) to output link o. Then, $\alpha^{\text{leave}}_{(u,d,o)}(k_d)$ can be computed as the minimum value out of the capacity of the intersection, the number of cars waiting or arriving to the next intersection, and the available space in the downstream link:

$$\begin{aligned}\alpha^{\text{leave}}_{(u,d,o)}(k_d) = \min\Bigg\{ & \beta_{(u,d,o)}(k_d)\mu_{(u,d)}\frac{g_{(u,d,o)}(k_d)}{c_d}, \frac{g_{(u,d,o)}(k_d)}{c_d} + \alpha^{\text{arrive}}_{(u,d,o)}(k_d), \\ & \frac{\beta_{(u,d,o)}(k_d)}{\sum_{u \in I_{(d,o)}} \beta_{(u,d,o)}(k_d)} \frac{C_{(d,o)} - n_{(d,o)}}{c_d} \Bigg\}\end{aligned} \tag{22}$$

where $\beta_{(u,d,o)}(k_d)$ is the relative fraction of the traffic at link (u,d) turning towards output link o at time step k_d, $\mu_{(u,d)}$ is the saturated flow rate leaving link (u,d), $\alpha^{\text{arrive}}_{(u,d,o)}(k_d)$ is the arriving average flow rate of the substream going towards o, $C_{(d,o)}$ is the storage capacity of the link (d,o) expressed in number of vehicles, and $n_{(d,o)}(k_d)$ is the number of vehicles at link (d,o) at time step k_d. In (22) $\alpha^{\text{arrive}}_{(u,d,o)}(k_d)$ is calculated as the fraction of the input flow rate of link (u,d) the destination of which is the output link o. Let $\alpha^{\text{arrive}}_{(u,d)}(k_d)$ be the average flow rate arriving at the end of the queue at link (u,d) at time step k_d. Thus:

$$\alpha^{\text{arrive}}_{(u,d,o)}(k_d) = \beta_{(u,d,o)}(k_d)\alpha^{\text{arrive}}_{(u,d)}(k_d) \tag{23}$$

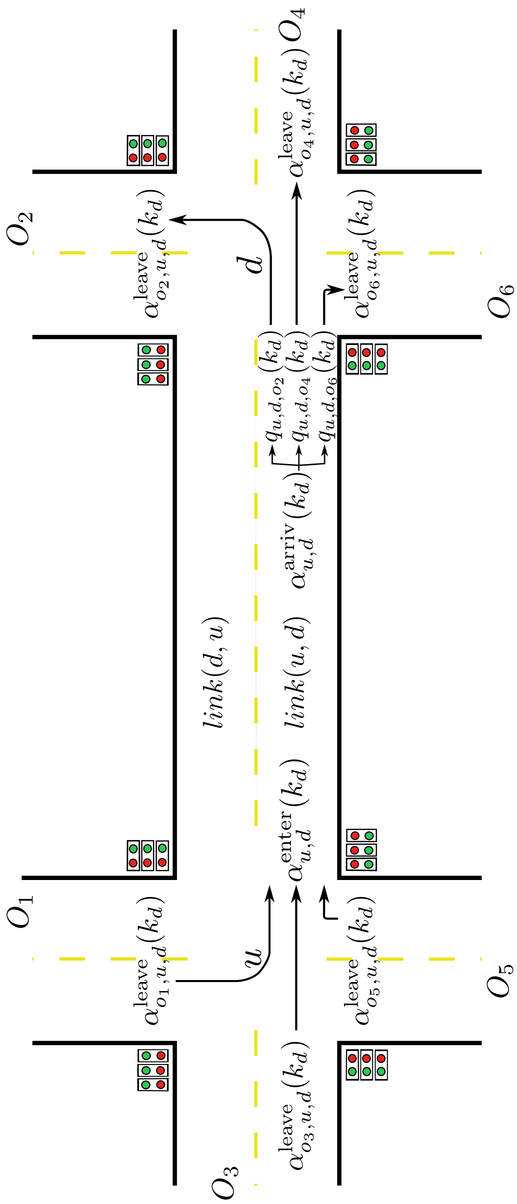


Fig. 9 Two interconnected intersections in an urban traffic network.

with $\alpha^{\mathrm{arrive}}_{(u,d)}(k_d)$ defined as:

$$\alpha^{\mathrm{arrive}}_{(u,d)}(k_d) = \frac{c_d - \gamma(k_d)}{c_d}\alpha^{\mathrm{enter}}_{(u,d)}(k_d - \delta(k_d)) + \frac{\gamma(k_d)}{c_d}\alpha^{\mathrm{enter}}_{(u,d)}(k_d - \delta(k_d) - 1) \quad (24)$$

where $\alpha^{\text{enter}}_{(u,d)}(k_d)$ is the average flow rate entering to the link (u,d) at time step k_d $\gamma(k_d)$, and $\delta(k_d)$ being functions depending on the vehicles that arrived at the queues of the link (u,d) (see Appendix A of [Lin et al., 2012] for details). Note that $\alpha^{\text{enter}}_{(u,d)}(k_d) = \sum_{i \in I_{(i,u,d)}} \alpha^{\text{leave}}_{(i,u,d)}(k_d)$.

In order to derive a model for a traffic network, a balance between entering and leaving vehicles is performed. Taking the definitions for $\alpha^{\text{arrive}}_{(u,d)}(k_d)$ and $\alpha^{\text{leave}}_{(u,d)}(k_d)$ into account, and assuming that the vehicles are in the queue corresponding to their output destination o, the dynamic evolution of the number of vehicles and queues at link (u,d) is given by:

$$n_{(u,d)}(k_d+1) = n_{(u,d)}(k_d) + \left(\alpha^{\text{enter}}_{(u,d)}(k_d) - \alpha^{\text{leave}}_{(u,d)}(k_d)\right) c_d \tag{25}$$

$$q_{(u,d,o)}(k_d+1) = q_{(u,d,o)}(k_d) + \left(\alpha^{\text{arrive}}_{(u,d,o)}(k_d) - \alpha^{\text{leave}}_{(u,d,o)}(k_d)\right) c_d \tag{26}$$

where the number of vehicles waiting in the queue is given by the sum of the vehicles waiting in each individual queue, viz., $q_{(u,o)}(k_d) = \sum_{o \in O_{(u,d)}} q_{(u,d,o)}(k_d)$.

4.2 Bargaining-game approach to congestion management in urban traffic

For an urban traffic network, let $x(k) = [n^T(k), q^T(k)]^T$ be the state vector, where $n(k)$ and $q(k)$ are vectors whose components are the number of vehicles and the queues at each link of the network. Moreover, let $u(k) = g(k)$ be the input vector, with $g(k)$ a vector whose components are the green signal time durations of each traffic light in the network. Furthermore, assume that all intersections in the network have the same cycle time c_d with k its corresponding time step counter. Then, the urban traffic model of (25)-(26) can be written as Eq. (1).

Thus, this urban traffic model can be used as prediction model for implementing NDMPC. Again, the idea of this traffic network is to provide a control strategy for congestion management. Hence, the performance index (travel time) is used as the cost function. From Eqs. (25)-(26) the expected travel time inside a prediction horizon N_{p} is determined by Eq. (27) [Lin et al., 2011, 2012]:

$$L(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)) = c_d \sum_{h=k}^{k+N_{\text{p}}-1} \left(\sum_{(u,d) \in L} n_{(u,d)}(h) \right) \tag{27}$$

As in the case of motorways, urban traffic networks are large-scale systems and therefore solving the optimization problem (3) is not feasible in real-time. Assume that the whole urban traffic network can be decomposed into M subsystems r such that each local model can be expressed as Eq. (4).

Let L_r denote the set of links (u,d) belonging to subsystem r. Let $\mathfrak{P} = \{\mathfrak{p}_1, \mathfrak{p}_2, ..., \mathfrak{p}_T\}$ be the set of intersections in the urban traffic network, where $\mathfrak{p}_{\text{i}},\ i = 1, \ldots, T$ are its elements. Let $\mathfrak{P}_r \subset \mathfrak{P}$ be the set of intersections belonging to subsystem r. Then, from Eq. (27) and the system decomposition, the NMPC for congestion management in an urban traffic network is given by:

$$
\begin{aligned}
&\min_{\widetilde{\mathbf{u}}(k)} \sum_{r=1}^{M} \phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)) \\
&\textbf{s.t.:} \\
&x_r(h+1) = f_{\mathrm{dx}r}(x(h), u_r(h), \widetilde{\mathbf{u}}_{-r}(k)) \\
&0 \le n_{(u,d)}(k) \le C_{(u,d)} \\
&q_{(u,d,o)}(k) \ge 0 \\
&0 \le g_{(u,d,o)}(k) \le c_d \\
&\sum_{(u,d)\in \mathfrak{p}_i} g_{(u,d,o)}(k) = c_d \\
&(u,d) \in L_r,\ \mathfrak{p}_i \in \mathfrak{P}_r
\end{aligned}
\tag{28}
$$

with $f_{\mathrm{dx}r}(x(h), u_r(h), \widetilde{\mathbf{u}}_{-r}(k))$ being the local prediction model. Also, from the system decomposition, the sets $\mathbb{X}_r$ and $\mathbb{U}_r$ are determined by the sets L_r and $\mathfrak{P}_r$ defining the links and intersections belonging to each subsystem. Note that the optimization problem (28) has the same form as the optimization problem (6). Therefore, a calculated circumstance belonging to the discrete-time dynamic bargaining games $\{(\Theta(k), \eta(k))\}_{k=0}^{\infty}$ arises.

In this circumstance or game each local controller has to make a trade-off between its local control objective $\phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$ and the common goal $L(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$. It is worth pointing out that subsystems are able to achieve a mutual benefit because the common goal $L(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$ provides them with the opportunity to collaborate. Moreover, in the resulting game $\{(\Theta(k), \eta(k))\}_{k=0}^{\infty}$ for the traffic network decomposition $\mathscr{N}$ is the set of local controllers, their preferences are determined by the minimization of the local cost $\phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$, and the decision space is given by $\Omega_r = \mathbb{X}_r \times \mathbb{U}_r$. Furthermore, the decision environment evolves according to the model of the traffic network and the local model used for predicting the trajectories of the local states. Let $\Xi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$ be the set resulting from the intersection of Ω_r with the space defined by the local prediction model. Then, in the game associated with the distributed congestion management in urban networks $\Theta(k) :=$ $\{(\phi_1(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k)), \ldots, \phi_M(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))) \in \mathbb{R}^M \mid (\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))_r \in \Xi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))\}$, with $(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))_r$ the tuple defining the value of $\phi_r(\widetilde{\mathbf{x}}(k), \widetilde{\mathbf{u}}(k))$. Finally, a solution to the game G_{NDMPC} resulting from the urban traffic network decomposition can be obtained by solving the same local optimization problems of (11), implemented with the negotiation model proposed in Section 1.

4.3 Simulation and results

For evaluating the performance of the proposed congestion management scheme an urban traffic network with three intersections is proposed. Each intersection has four links with three lanes, where each lane has a length of 452 m, and a capacity of 192 vehicles per lane. For simulation purposes, it is assumed that the vehicles have a length of 7 m and a free flow speed of 50 km/h. Moreover, a cycle time of 50 s and initial queues of 5 vehicles in each link are also considered. As in the case of the motorway, the Matlab function *fmincon* is used for solving each local optimization problem. The solver uses a interior point algorithm. Figure 10 shows the urban network used as a case study. Moreover, for implementing the proposed distributed congestion management scheme the urban traffic network is divided into three subsystems as illustrated in Figure 10. Each subsystem is composed of the four links at interacting in the intersections.

In the network of Figure 10 (and according to the notation used in that figure) the output flow rate for origins o_1 to o_6 is assumed constant and equal to 460 veh/h; the flow of vehicles entering through links $(1,a)$ and $(6,a)$ is assumed constant and equal to 705 veh/h; through links $(2,b)$ and $(7,b)$ 903 veh/h; through links $(3,c)$ and $(8,c)$ 902 veh/h; through link $(5,c)$ 300 veh/h; and through link $(4,a)$ it is assumed to be time-varying with the trapezoidal shape shown in Figure 11(b). Furthermore, in order to reduce the complexity of the optimization problems, two operational modes for the traffic lights are considered. As shown in Figure 11(a), in each operational mode several destinations at each link in an intersection are allowed. As a consequence, one decision variable is required for assigning the green light time to each flow rate at each intersection [Lin, 2011, Lin et al., 2011, 2012] (the sum of the times assigned to each operational mode must be equal to the cycle time).

As in the case of the motorways, in the distributed congestion management scheme for urban traffic the local controllers should exchange their measurements of the local states as well as their current local control actions. This allows each local controller deciding on the control action to be locally applied following the **focusing on others** criteria. That is, each local controller selects the feasible control action that minimizes the effect of the local decisions on the performance of the remaining controllers.

Figure 12(a) shows the evolution of the number of vehicles waiting in the queues of the urban traffic network. In this figure the centralized scheme for congestion management exhibits longer queues than the distributed scheme. However, the duration of the queues with NMPC is not longer than the duration with GT-NDMPC. Recall that centralized NMPC takes all interactions into account in its predictions. Therefore, this control scheme is able to manage the increasing size of the queues more efficiently than the distributed scheme. There is a concept associated with the length of the queues that measures the time spent by the vehicles running with free flow speed from the beginning of a link until reaching the tail of the queue corresponding to its destination o. Figure 12(b) shows the aggregate behavior of the total time of vehicles at free flow speed (delay time) along the network. Note that when

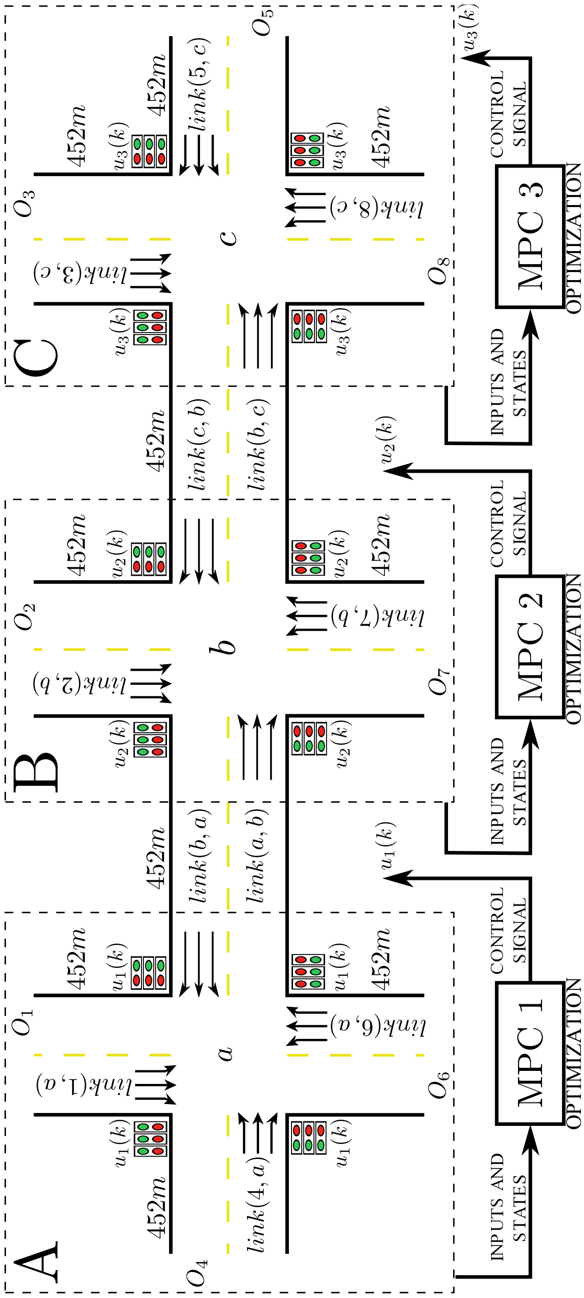
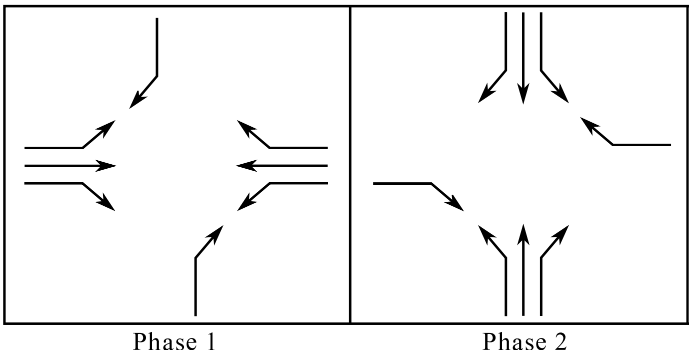
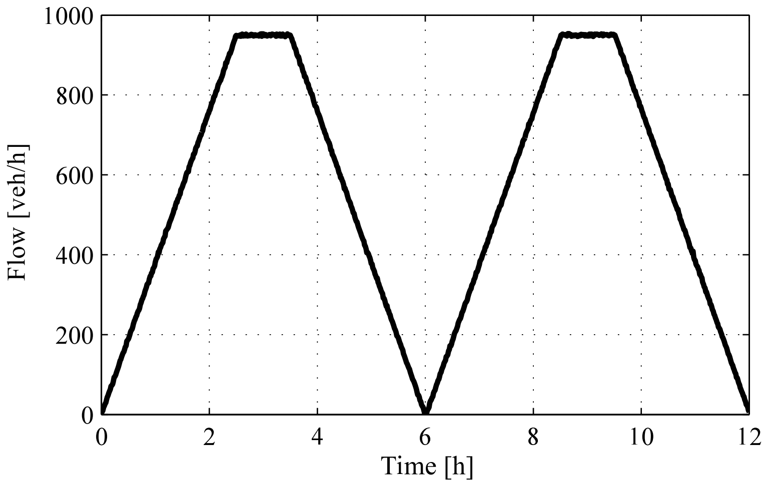


Fig. 10 Urban traffic network used as a testbed for the proposed congestion management scheme.



(a) Operational modes for the traffic lights considered in the simulations.

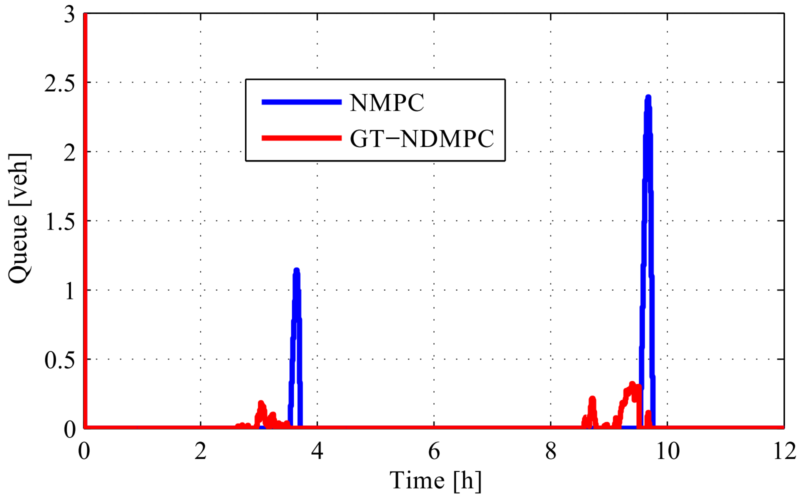


(b) Flow rate of link $(4,a)$ used in the simulations.

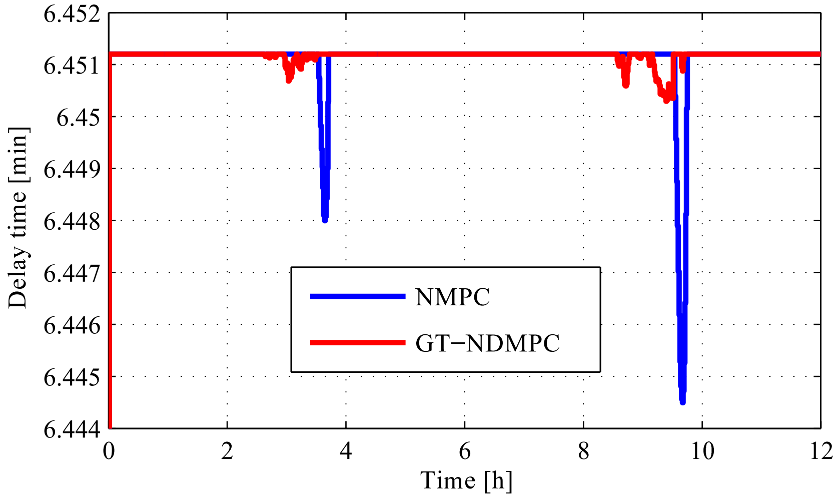
Fig. 11 Parameters of the simulation.

the length of the queues increases, the total time of vehicles traveling in free flow speed decreases, as expected.

For performing congestion management, the green light time must be assigned to each flow rate. Figures 13(a) and 13(b) show the green time for phases 1 and 2 at the intersection a (see Figure 10). Note that the total time of the operative modes is equal to 50 s, which is the cycle time. In addition, from Figures 13(a) and 13(b) it is evident that the GT-NDMPC approach uses more aggressive control actions than the centralized approach, viz. the changes of the control actions of the GT-NDMPC are bigger than the changes of the control actions in the centralized NMPC. This



(a) The total number of vehicles waiting in the queues along the traffic network.

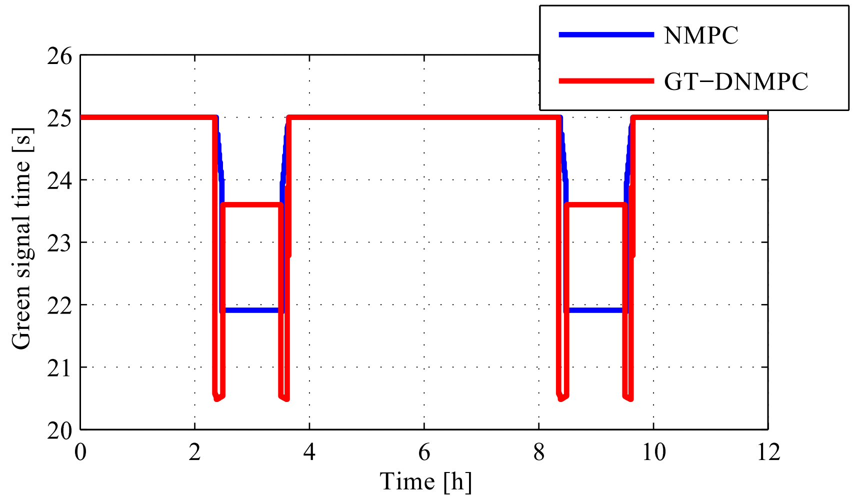


(b) Total time of vehicles at free flow speed in the traffic network with the implemented NMPC schemes.

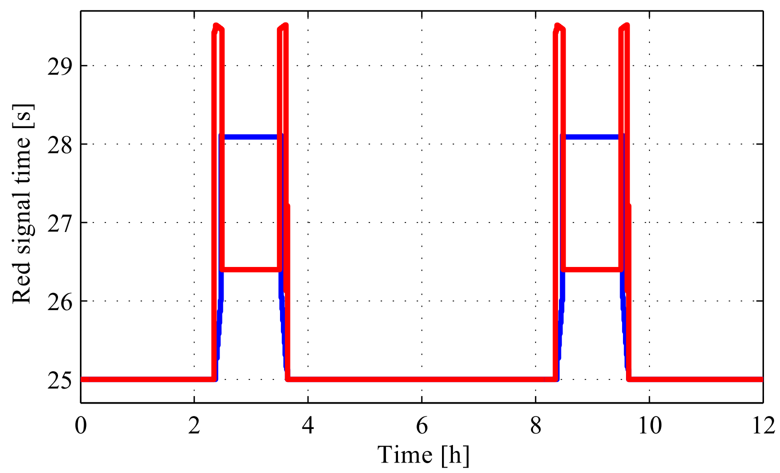
Fig. 12 Comparison of the total number of vehicles in the queue and at free flow speed for both centralized and proposed approaches.

allows the distributed congestion management scheme to have queues that are not longer than the queues with the centralized NMPC approach.

A comparison of the total time spent by the users of the traffic network, the vehicles waiting in the queues, and the total delay time for several congestion management schemes is presented in Table 5. The adaptive SCOOT method was included



(a) Phase 1.



(b) Phase 2.

Fig. 13 Green time signal assigned by the controllers in intersection a.

because it is one of the optimization-based alternatives for traffic control in urban networks. Moreover, the SCOOT method proposed the concept of the green wave in order to reduce the complexity of the resulting optimization problem. In this table it is evident that the performance of centralized and GT-NDMPC schemes implemented for congestion management in urban networks is almost the same, while other schemes severely increased the TVQ, keeping a similar TTS. This validates the possibility of using distributed schemes based on game theory for congestion management in urban traffic networks.

Table 5 Comparison of total time spent (TTS) for a vehicle, and total vehicles in the queues (TVQ) for centralized, distributed, adaptive SCOOT, and fixed time schemes.

Configuration	TTS [veh · h]	TVQ [veh]
Centralized NMPC	3498	7744
GT-NDMPC	3522	9369
State-Feedback	3769	25612
SCOOT	3961	38857
Fixed Time	5142	124125

Table 6 presents a comparison of the computational times in the urban traffic network for the centralized NMPC, the proposed GT-NDMPC, and the adaptive SCOOT method. As shown in Table 6, despite the simplifications involved in the SCOOT method, the proposed GT-NDMPC requires a lower computation time. Furthermore, with the GT-NDMPC the heuristics behind the SCOOT method (which may hinder the application of this method in large urban networks) are avoided.

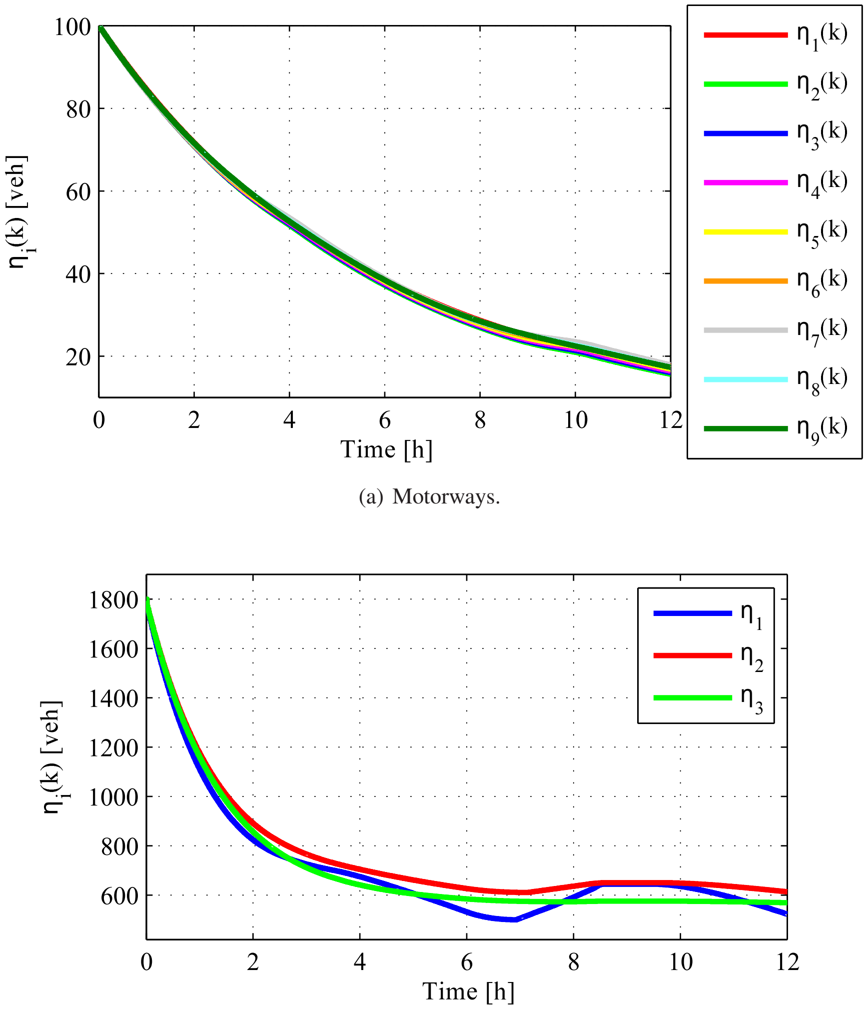
Configuration	Computational time [s]
Centralized NMPC	390.6207
Subsystem 1	60.2311
Subsystem 2	95.2946
Subsystem 3	88.4602
SCOOT	164.6238

Table 6 Comparison between computational times for centralized NMPC, for each subsystem in the proposed GT-NDMPC, and for the adaptive SCOOT method.

4.4 Disagreement point analysis

From a game theory point of view, in both the motorway and urban traffic models it was observed that each local controller behaved according to its own desires and preferences. Figures 14(a) and 14(b) show the evolution of the disagreement point in each case (motorway and urban network respectively).

Note that in Figure 14(a) and 14(b) the evolution of the disagreement point is almost the same for all controllers in the case of the motorway. This is because the similarities of the subsystems in this case (recall that all segment parameters and on-ramp demands are the same). Thus, this game is close to being a symmetric game. However, in the case of the urban traffic network the behavior of the disagreement points is different for each subsystem. This is in accordance with the flow rate specifications for the links (recall that the flow rates at the boundary of the



(a) Motorways.

(b) Urban traffic.

Fig. 14 Evolution of the disagreement point in the congestion management.

network are different). Thus, this game is clearly non-symmetric. Note that the disagreement point trajectories present some oscillations along the simulation, which are more evident in the case of the urban traffic network. Such behavior is due to the decision making each controller performs. When they decide to cooperate the disagreement point decreases, but when they decide not to cooperate the disagreement point increases. The decision regarding cooperation is defined by the perceived benefit from the cooperative behavior. So, if there are no alternatives such that the local performance index is less than the disagreement point, the controller decides not to cooperate.

5 Conclusion

In this chapter, bargaining game theory was used as a mathematical framework for analyzing the game arising from the distributed model predictive control formulation. In this way, the non-linear model predictive control –NMPC– problem was initially presented. Then the system was decomposed; motivated by the fact that implementation of NMPC in real large-scale systems is not advisable. As a consequence of the system decomposition, the NMPC problem became a set of coupled optimization problems. Since each optimization problem was locally solved, a trade-off between local and global system performance was required. Moreover, since the controllers solving the local optimization problems were able to communicate with each other, bargaining among controllers was possible. Hence, the distributed model predictive control –DMPC– formulation resulting from system decomposition can be characterized as a bargaining game (GT-NDMPC).

Once the similarities between bargaining games and GT-NDMPC were established, some extensions to the original theory were performed. Specifically, the discrete-time dynamic bargaining game concept was defined. Such a concept was required because original bargaining game theory does not consider the time evolution of the decision environment, of the decision space, and of the disagreement point. Finally, based on the concepts of a discrete-time dynamic bargaining game and a disagreement point, a solution to the GT-NDMPC game and an algorithm for computing such a solution in a distributed way were proposed.

Despite of game theory often presenting selfish procedures for strategic decision making, in this chapter the bargaining game theory afforded conditions for solving the DMPC problem inside a **focusing on others** frame, without implying that the subsystems have to solve more than one optimization problem at each time step, which would prevent the convergence of the solution on a Nash equilibrium point. This non selfish bargaining game approach is the main difference with the schemes based on game theory previously reported in the literature (see e.g., Trodden et al. [2009], Muñoz de la Peña et al. [2009]). Additionally, a reduction of the computational burden associated with the communications between subsystems is achieved, and avoiding the solution of more than one optimization problem. Furthermore, only local functions that depend on decisions made by the other subsystems were required. This makes the proposed bargaining approach (GT-NDMPC) to the DMPC problem more flexible than almost all the DMPC schemes presented in the literature. This statement has also been validated in Portilla et al. [2012].

The bargaining-game-based formulation for distributed model predictive control –GT-NDMPC– was applied in this chapter for congestion management in motorways and urban traffic networks. To this end, macroscopic models were used to represent the dynamic behavior of the vehicles in the network. Since these models are discrete-time, they were used by the controllers as prediction models. Moreover, an expression for the travel time was derived from these models. Travel time was used as the cost function in both centralized and distributed NMPC approaches. With the models, the cost function, and the constraints defined, the centralized NMPC was formulated. After that, the whole system was decomposed into several subsystems. Also, local models and their corresponding constraints were defined. Furthermore, the bargaining situation associated with the system decomposition was analyzed. At the end, the elements defining the corresponding discrete-time dynamic bargaining games were introduced.

Finally, the proposed scheme for congestion management was tested on a motorway with ten on-ramps, and in an urban traffic network with three intersections. The performance of the GT-NDMPC approach was compared with the performance of a centralized NMPC approach. In conclusion, the distributed congestion management scheme based on game theory presented a performance similar to the one obtained using a centralized scheme.

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References

O. Akira. A noncooperative approach to general n-person cooperative games. Discussion Papers 2005-01, Graduate School of Economics, Hitotsubashi University, 2005.

L.D. Baskar, B. De Schutter, and J. Hellendoorn. Optimal routing for intelligent vehicle highway systems using a macroscopic traffic flow model. In *Intelligent Transportation Systems, 2009. ITSC '09. 12th International IEEE Conference on*, pages 1–6, Oct 2009. doi: 10.1109/ITSC.2009.5309657.

T. Bellemans, B. De Schutter, and B. De Moor. Model predictive control for ramp metering of motorway traffic: A case study. *Control Engineering Practice*, 14(7): 757–767, July 2006. doi: 10.1016/j.conengprac.2005.03.010.

E. Camponogara, D. Jia, B. H. Krogh, and S. Talukdar. Distributed Model Predictive Control. *IEEE Control Systems Magazine*, 22(1):44–52, 2002.

X. Du, Y. Xi, and S. Li. Distributed model predictive control for large-scale systems. *Proceedings of the 2001 American Control Conference, Arlington, USA, 25-27 June*, pages 3142–3143, 2001.

A. Ferrara, A. Nai Oleari, S. Sacone, and S. Siri. Freeway networks as systems of systems: An event-triggered distributed control scheme. In *Proceedings of the 7th IEEE Conference on System of Systems Engineering*, pages 197–202, Genoa, Italy, July 2012.

J.R.D. Frejo and E.F. Camacho. Global versus local MPC algorithms in freeway traffic control with ramp metering and variable speed limits. *IEEE Transactions on Intelligent Transportation Systems*, 13(4):1556–1565, 2012.

L. Giovanini and J. Balderud. Game approach to distributed model predictive control. In *Proceedings of the International Control Conference*, Glasgow, Scotland, United Kingdom, 30th August-1st September 2006.

N. Groot, B. De Schutter, S.K. Zegeye, and H. Hellendoorn. Model-based traffic and emission control using pwa models: A mixed-logical dynamic approach. In *Intelligent Transportation Systems (ITSC), 2011 14th International IEEE Conference on*, pages 2142–2147, Oct 2011. doi: 10.1109/ITSC.2011.6082809.

H. Haj-Salem, P. Poirier, J.-F. Heylliard, and J.-P. Peynaud. Alinea: a local traffic responsive strategy for ramp metering. field results on a6 motorway in paris. In *Intelligent Transportation Systems, 2001. Proceedings. 2001 IEEE*, pages 106–111, 2001. doi: 10.1109/ITSC.2001.948638.

J. C. Harsanyi. A simplified bargaining model for the n-person cooperative game. *International Economic Review*, 4(2):194–220, 1963.

A. Hegyi, B. De Schutter, H. Hellendoorn, and T. Van den Boom. Optimal coordination of ramp metering and variable speed control-an mpc approach. In *American Control Conference, 2002. Proceedings of the 2002*, volume 5, pages 3600–3605 vol.5, 2002. doi: 10.1109/ACC.2002.1024487.

A. Hegyi, B. De Schutter, and H. Hellendoorn. Model predictive control for optimal coordination of ramp metering and variable speed limits. *Transportation Research Part C*, 13(3):185–209, June 2005. doi: 10.1016/j.trc.2004.08.001.

D. Jia and B. H Krogh. Distributed Model Predictive Control. *Proceedings of the 2001 American Control Conference, Arlington, USA, 25-27 June*, pages 2767–2772, 2001.

Long Kejun, Yun Meiping, Zheng Jianlong, and Yang Xiaoguang. Model predictive control for variable speed limit in freeway work zone. In *Control Conference, 2008. CCC 2008. 27th Chinese*, pages 488–493, July 2008. doi: 10.1109/CHICC.2008.4605219.

A. Kotsialos, M. Papageorgiou, and A. Messmer. Optimal coordinated and integrated motorway network traffic control. In *Proceedings of the 14th International Symposium on Transportation and Traffic Theory (ISTTT), Jerusalem, Israel*, pages 621–644, 1999.

A. Kotsialos, M. Papageorgiou, C. Diakaki, Y. Pavlis, and F. Middelham. Traffic flow modeling of large-scale motorway networks using the macroscopic modeling tool metanet. *IEEE Transactions on Intelligent Transportation Systems*, 3(4): 282–292, December 2002a.

A. Kotsialos, M. Papageorgiou, M. Mangeas, and H. Haj-Salem. Coordinated and integrated control of motorway networks via non-linear optimal control. *Transportation Research Part C*, 10:65–84, 2002b.

S. Li, Y. Zhang, and Q. Zhu. Nash-optimization enhanced distributed model predictive control applied to the Shell benchmark problem. *Information Sciences*, 170 (2-4):329–349, 2005.

S. Lin. *Efficient model predictive control for large-scale urban traffic networks*. PhD thesis, Delft University of Technology, Delft, The Netherlands, 2011.

S. Lin, B. De Schutter, Y. Xi, and H. Hellendoorn. Fast model predictive control for urban road networks via MILP. *IEEE Transactions on Intelligent Transportation Systems*, 12(3):846–856, 2011. ISSN 1524-9050. doi: 10.1109/TITS.2011.2114652.

S. Lin, B. De Schutter, Y. Xi, and H. Hellendoorn. Efficient network-wide model-based predictive control for urban traffic networks. *Transportation Research Part C: Emerging Technologies*, 24(0):122–140, 2012. ISSN 0968-090X. doi: http://dx.doi.org/10.1016/j.trc.2012.02.003. URL http://www.sciencedirect.com/science/article/pii/S0968090X12000150.

Xiao-Yun Lu, T.Z. Qiu, P. Varaiya, R. Horowitz, and S.E. Shladover. Combining variable speed limits with ramp metering for freeway traffic control. In *American Control Conference (ACC), 2010*, pages 2266–2271, June 2010.

J. M. Maestre, D. Muñoz de la Peña, and E. F. Camacho. Distributed model predictive control based on a cooperative game. *Optimal Control Applications and Methods*, 32(2):153–176, March–April 2011a.

J. M. Maestre, D. Muñoz de la Peña, A. Jiménez Losada, E. Algaba Durán, and E. F. Camacho. An application of cooperative game theory to distributed control. *Proceedings of the 18th IFAC World Congress, Milan, Italy*, pages 9121–9126, September 2011b.

J.M. Maestre, D. Muñoz de la Peña, E.F. Camacho, and T. Alamo. Distributed model predictive control based on agent negotiation. *Journal of Process Control*, 1(5): 685–697, June 2011c.

J.M. Muñoz de la Peña, D. Maestre, and E.F. Camacho. Distributed MPC based on a cooperative game. *Proceedings of the 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference, Shanghai, China, 15-18 December,*, pages 5390–5395, 2009.

R. B. Myerson. *Game Theory: Analysis of Conflict*. Harvard University Press, 1991. ISBN 978-0-674-34116-6.

J. Nash. The bargaining problem. *Econometrica*, 18(2):155–162, 1950a.

J. Nash. Equilibrium points in N-persons games. *Proceedings of the National Academy of Sciences of United States of America*, 36(1):48–40, 1950b.

J. Nash. Two-person cooperative games. *Econometrica*, 21(1):128–140, 1953.

I. Necoara, D. Doan, and J. A. K. Suykens. Application of the proximal center decomposition method to distributed model predictive control. *Proceedings of the 2008 IEEE Conference on Decision and Control, Cancun, Mexico, 9-11 December*, pages 2900–2905, 2008.

R.R. Negenborn, B. De Schutter, and J. Hellendoorn. Multi-agent model predictive control for transportation networks: Serial versus parallel schemes. *Engineering Applications of Artificial Intelligence*, 21(3):353–366, 2008.

M. Papageorgiou, J.M. Blosseville, and H. Haj-Salemn. Modelling and real-time control of traffic flow on the southern part of boulevard peripherique in Paris: Part ii: coordinated on-ramp metering. *Transportation Research Part A*, 24(5): 361–370, 1990.

M. Papageorgiou, E. Kosmatopoulos, I. Papamichail, and Yibing Wang. A misapplication of the local ramp metering strategy alinea. *Intelligent Transportation Systems, IEEE Transactions on*, 9(2):360–365, June 2008. ISSN 1524-9050. doi: 10.1109/TITS.2008.922975.

Hans J. M. Peters. *Axiomatic Bargaining Game Theory*. Kluwer Academic Publishers, Dordrecht, 1992.

J. Pimentel and M. Salazar. Dependability of distributed control system fault tolerant units. In *Proceedings of IECON*, volume 4, pages 3164–3169, 2002.

C. Portilla, F. Valencia, J.D. López, J. Espinosa, A. Núñez, and B. De Schutter. Non-linear model predictive control based on game theory for traffic control on highways. In *Proceedings of the 4th IFAC Nonlinear Model Predictive Control Conference*, pages 436–441, Noordwijkerhout, The Netherlands, August 2012.

A. Rantzer. Linear quadratic team theory revisited. *Proceedings of the 2006 American Control Conference, Minneapolis, USA, 14-16 June*, 2006.

A. Rantzer. Using game theory for distributed control engineering. In *Proceedings of the 3rd World Congress of the Game Theory Society, Evanston, USA, 13-17 July*, 2008.

A. Rantzer. Dynamic dual decomposition for distributed control. *Proceedings of the American Control Conference 2009, St Louis, USA, 10-12 June*, pages 884–888, 2009.

S. Talukdar, D. Jia, P. Hines, and B. H. Krogh. Distributed model predictive control for the mitigation of cascading failures. *Proceedings of the 44th IEEE Conference on Decision and Control, and the 2005 European Control Conference, Seville, Spain, 12-15 December*, pages 4440–4445, 2005.

Martin Treiber and Arne Kesting. *Traffic flow dynamics*. Sprin, 2013.

P. A. Trodden, D. Nicholson, and A. G. Richards. Distributed model predictive control as a game with coupled contraints. *Proceedings of the European Control Conference, Budapest, Hungary, 23-26 August*, 2009.

F. Valencia. *Game Theory Based Distributed Model Predictive Control: An Approach to Large-Scale Systems Control*. PhD thesis, Facultad de Minas, Universidad Nacional de Colombia, Medellín, 2012.

F. Valencia, J.J. Espinosa, B. De Schutter, and K. Staňková. Feasible-cooperation distributed model predictive control scheme based on game theory. In *Proceedings of the 18th IFAC World Congress*, pages 386–391, Milan, Italy, August–September 2011.

M. van den Berg, A. Hegyi, B. De Schutter, and J. Hellendoorn. A macroscopic traffic flow model for integrated control of freeway and urban traffic networks. In *Proceedings of the 42nd IEEE Conference on Decision and Control, Maui, HI, 09-12 December, 2003*, pages 2774–2779, 2003.

A. N. Venkat, J. B. Rawlings, and S. J. Wright. Implementable distributed model predictive control with guaranteed performance properties. *Proceedings of the 2006 American Control Conference, Minneapolis, USA, 14-16 June*, pages 613–618, 2006a.

A.N. Venkat, I.A. Hiskens, J.B. Rawlings, and S.J. Wright. Distributed MPC strategies for automatic generation control. In *Proceedings of the IFAC Symposium on Power Plants and Power Systems Control, Canada*, pages 383–388, 2006b.

A.N. Venkat, J.B. Rawlings, and S.J. Wright. Stability and optimality of distributed, linear model predictive control. Part I: State feedback. *Texas-Wisconsin Modeling and Control Consortium Technical Report*, 3, 2006c.

J. Von Neumann, O. Morgenstern, H.W. Kuhn, and A. Rubinstein. *Theory of Games and Economic Behavior*. Princeton University Press, Princeton, NJ, 1947.

F.Y. Wang and I.T. Cameron. A multi-form modelling approach to the dynamics and control of drum granulation processes. *Powder Technology*, 179(1-2):2–11, 2007.

S.H. Yang, X. Chen, and J.L. Alty. Design issues and implementation of Internet-based process control systems. *Control Engineering Practice*, 11(6):709–720, 2003.

S.K. Zegeye, B. De Schutter, J. Hellendoorn, E.A. Breunesse, and A. Hegyi. A predictive traffic controller for sustainable mobility using parameterized control policies. *IEEE Transactions on Intelligent Transportation Systems*, 13(3):1420–1429, September 2012. doi: 10.1109/TITS.2012.2197202.