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Effects of water flow on energy consumption and travel times of micro-ferries for energy-efficient transport over water

M. Burger and B. De Schutter

Abstract Controlling the transport *of* water by adjusting water flows in rivers and canals, inevitably will have an effect on the transport *over* water by vessels as well. We will discuss the effect of flowing water on scheduling micro-ferries (small autonomous water-taxis) using the least amount of energy, while aiming at satisfying customer demands with respect to pick-up times. This trade-off will be made by optimizing the assignment of micro-ferries to customers in a specific order, and by searching for the best travel speeds.
The interplay between controlling transport of water and scheduling transport over water will become clear by the explicit relation between the speed of the water (influenced by water management) on travel times and energy consumption, derived in this chapter. It is shown that on average the travel times (and thereby the energy consumption) will increase with increasing magnitudes of the current. Hence, decisions made on water management have a direct effect on the performance of the transport system, and the interests of both parties should be taken into account to obtain a well-functioning water transport system.

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1 Introduction

In this chapter the problem of scheduling pick-ups and deliveries of people with water-taxis is discussed, where –besides the common issue of scheduling within time-windows– we take into account varying speeds of vessels and water flows and their effects on travel times and energy consumption. As such it is a problem involving *transport over water* (of people using water-taxis) when influenced by *transport of water* (via varying speeds of the water flows).

1.1 Micro-ferry scheduling

We consider the transport of people using small, autonomous water-taxis that travel between several stations along a river in a city. These water-taxis will pickup customers on-demand, and are envisioned to be powered electrically to reduce emissions and noise (although fuel tanks or a hybrid system could also be used). There is a fixed number of stations (see Figure 1) where customers can (dis)embark the vessels and where the batteries can be charged. We will refer to this kind of water-taxi as *micro-ferries*.

1.1.1 Work related to the micro-ferry scheduling problem

The problem consists in finding a route for each individual micro-ferry that ensures that each transport request is handled at minimum cost, similar to the (multi-depot) *traveling salesman problem* [3, 17]. Often, variants of the traveling salesman problem –such as the *vehicle routing problem* [16, 21] and the *pick-up and delivery*

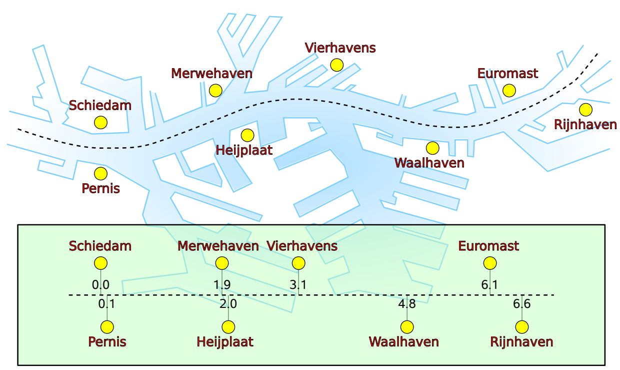

Fig. 1 The micro-ferry scheduling problem for the Rotterdam harbor: find an energy-efficient schedule for transporting people between stations along the river. The distances (in kilometers) of the stations along the river are indicated in the lower plot.

problem [19]– are concerned with minimizing the travel time or distance. However, these problems do not take into account that the vehicles will have a limited driving range, and hence they might need to charge in between jobs.

Recently, some papers have appeared on energy consumption within scheduling problems. An extension to the vehicle routing problem with (constant speeds and) energy consumption (defined as the multiplication of the vehicle load and the travel distance) is presented in [15]. In the *pollution routing problem* [2] a trade-off is made between minimizing travel distances, travel times, transport costs, and greenhouse emissions. The emissions are related to the energy consumption, which is dependent on both the speed and the load of a vehicle. A vehicle routing problem with fuel cost minimization is proposed in [22], where the fuel costs are defined as the product of unit fuel costs, fuel consumption rates, and road lengths.

1.1.2 Previous results on micro-ferry scheduling by the authors

The *micro-ferry scheduling problem* consists in finding routes that minimize the total *energy consumption*, while satisfying the desired pick-up times as much as possible by using *soft time windows* [10]. By considering the *vehicle speeds* as optimization variables, both the energy consumption and travel times will be variable.

Since reducing the energy consumption is our main focus, the vessels should be light-weight (i.e. a small vehicle load) and hence the batteries (or fuel tanks) shall be small. Therefore, recharging of the batteries (or alternatively refueling of the tank) will be needed during operation. These *recharging times* therefore take a non-negligible amount of time with respect to the travel times, and we took them into account in the scheduling problem in [9].

The energy consumption will be a non-linear function of the vehicle speed, but a (computationally faster) linear function can be used by approximating the energy consumption by a piece-wise affine function[1]. This approach was used in the above-mentioned work, but due to the extra decision variables that were needed to formulate the piece-wise affine functions, only very small (in terms of fleet size and number of requests) problems could be solved efficiently. Exploiting the fact that the non-linear energy consumption function is *convex*, we proposed an alternative formulation of the *function approximation using linear constraints* only [8]. This modeling method greatly reduced the computation times.

For calm water the discussed work would be sufficient, but for *flowing water* one cannot use the same formulations any more. Both the energy consumption and travel times are dependent on the speed of the flowing water; disregarding this fact could result in schedules where micro-ferries run out of energy while transporting customers, and the calculated pick-up times would become incorrect. While the first side-effect is obviously worse than the second, both can be seen as a degradation of the service. To ensure correct pick-up times and to avoid empty batteries a reformu-

[1] Actually in [8] we showed that the energy consumption is linear in the speed u and pace w (the reciprocal of speed [13]), and there we approximated the function $u = \frac{1}{w}$ by a piece-wise affine function.

lation of the micro-ferry scheduling problem for flowing water was presented in [7]. Since both the pick-up times and the energy consumption are non-linear functions in both the vehicle speed and the water flow speed, it was decided to consider the vehicle speed as a constant in that work to reduce the complexity.

1.2 Contributions

This chapter will consist of a complete overview of the previous results discussed above, extended with the introduction of *variable speeds for flowing water*. The theoretical results will be provided first, followed by a discussion of the modeling aspects of the micro-ferry scheduling problem.

1.2.1 Theory

The micro-ferry scheduling problem can be seen as an extension of the *multi-depot vehicle routing problem*, where the addition of variable speeds and environmental disturbances results in *convex constraints*. To the best of our knowledge, the authors' work has introduced two topics into the field of operations research, namely

- **reformulation**: modeling of the multi-depot traveling salesmen/vehicle routing problem using the same amount of decision variables as the single-depot variants
- **disturbances**: inclusion of environmental disturbances (e.g. water flows or wind) in scheduling problems

Reformulation of multi-depot traveling salesman problems

The multi-depot traveling salesman problem (and its variants) can be stated using the following mixed-integer linear program with 3-index binary variables [3, 21]:

$$\min \quad \sum_{i\in\mathscr{R}}\sum_{j\in\mathscr{R}}\sum_{k\in\mathscr{M}} \mathsf{c}_{ij}x_{ijk} \tag{1}$$

$$\text{s.t.} \quad \sum_{j\in\mathscr{R}}\sum_{k\in\mathscr{M}} x_{ijk} = 1, \qquad \sum_{h\in\mathscr{R}}\sum_{k\in\mathscr{M}} x_{hik} = 1 \quad \forall i \in \mathscr{R} \tag{2}$$

$$\{subtour\ elimination\ constraints\} \tag{3}$$

where $\mathscr{M} = \{1,\ldots,\mathsf{M}\}$ denotes the set of M depots, $\mathscr{N} = \{\mathsf{M}+1,\ldots,\mathsf{M}+\mathsf{N}\}$ denotes the set of N customers, and $\mathscr{R} = \mathscr{M}\cup\mathscr{N} = \{1,\ldots,\mathsf{R}\}$ is the set of $\mathsf{R} = \mathsf{M}+\mathsf{N}$ locations in the problem.

Theorem 1. *The mixed-integer linear program with 2-index binary variables* x_{ij}

$$\min \quad \sum_{i\in\mathscr{R}}\sum_{j\in\mathscr{R}} \mathsf{c}_{ij}x_{ij} \tag{4}$$

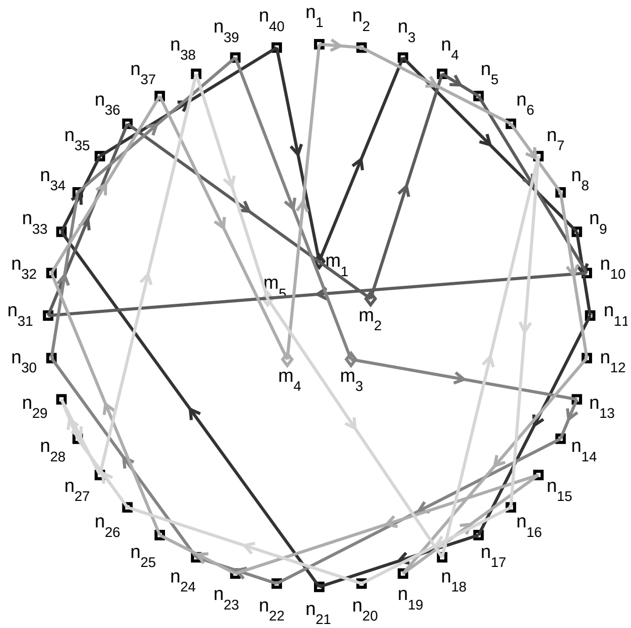

Fig. 2 Assignment of new requests to micro-ferries. Each micro-ferry has a unique color, and the requests are ordered with increasing desired pick-up times.

$$\text{s.t.} \quad \sum_{j\in\mathscr{R}} x_{ij} = 1, \qquad \sum_{h\in\mathscr{R}} x_{hi} = 1 \quad \forall i \in \mathscr{R} \tag{5}$$

$$k_m = m \quad \forall m \in \mathscr{M} \tag{6}$$

$$k_i - k_j + (\mathsf{M}-1)(x_{ij}+x_{ji}) \le (\mathsf{M}-1) \quad \forall i, j \in \mathscr{R} \tag{7}$$

$$\{\mathit{subtour\ elimination\ constraints}\} \tag{8}$$

$$x_{ij} \in \{0,1\}, \quad k_i \in [1,\mathsf{M}] \quad \forall i, j \in \mathscr{R} \tag{9}$$

is equivalent to the mixed-integer linear program (1)–(3).

A proof for this theorem can be found in [6]. In this formulation the continuous variables k_i can be seen as *node currents*, which are the dual to the *node potentials* in the Miller-Tucker-Zemlin subtour elimination constraints [18].

Figure 2 shows an example solution to the above-mentioned formulation for $\mathsf{M} = 5$ depots (shown in the inner circle) and $\mathsf{N} = 40$ customers (shown in the outer circle). By (6) each depot gets a unique index number (represented as a unique color). When a variable $x_{ij} = 1$ it means that the trip from location i to j is part of the selected tour, where the direction of the tour is indicated by the arrows in the figure. Along the path the index number will be assigned to the customers through (7), resulting in exactly M cycles in the set of $\mathsf{R} = \mathsf{M} + \mathsf{N}$ nodes, each originating from another depot.

The mean travel time increases with increasing disturbances

In the case study that is presented later on in this chapter –where the current flows in parallel to the riverbed– the travel times depend on the water flow. The effect of the magnitude of the flow on the travel times can be stated as follows.

Theorem 2. *The travel time for a round-trip (from one location to another and back) will increase with increasing water flow magnitudes, and therefore both the mean travel time and total energy consumption will increase with increasing magnitudes.*

A formal proof will be provided in Section 2.1.4, after introducing the necessary variables and equations. The difference in travel times for going one way or the other is to be expected from experience, but the increase in times for round-trips is less intuitive. Since the micro-ferries will be traveling in both upstream and downstream direction, this means that the total travel time for handling all transport requests will increase with increasing flows, and hence the mean travel times will be higher when the current is stronger. Due to the longer travel times also the energy consumption will increase with the water flow magnitude.

1.2.2 Application

With the extension to variable speeds for flowing water, the variant of the micro-ferry problem as discussed in this chapter provides a complete modeling framework for scheduling vehicles with variable speeds under environmental disturbances. The model takes the following aspects into account:

- **(soft) time windows**: each customer is picked up at the desired time (if possible)
- **varying speeds**: the micro-ferries can travel within a given speed range
- **energy consumption**: the schedules are energy-efficient through minimization of the total energy consumption of the micro-ferry fleet
- **charging**: empty batteries on the water are avoided by keeping track of energy levels and by recharging
- **flowing water**: the effect of currents on travel times and energy consumption is taken into account

2 Micro-Ferry Scheduling Problem for Flowing Water

The research on micro-ferry scheduling originated as a fictitious (but realizable) case study for the city of Rotterdam, the Netherlands. With the creation of new container terminals at Maasvlakte II the current harbor activity near the city center is expected to partially move towards the sea. This leaves space for redesigning the riversides and for creating new housing and offices. To avoid more traffic by car via the already busy roads, alternative transport over the river is a viable option.

Envisioned is a personal transport system with small, autonomous vessels, which we will refer to as micro-ferries. Customers can embark and disembark the ferries at specific locations along the river, and transport requests can be (pre)ordered. To avoid empty batteries while on the river, which is inconvenient on a lake but dangerous when drifting towards a harbor with large container ships and oil tankers, the energy levels are taken into account in the scheduling and a recharge is planned when necessary.

This section starts with a description of the effects of flowing water on travel times and energy consumption, followed by a mathematical formulation of the problem. This formulation is then used to compute a transport schedule in a case study example.

2.1 Effects of flowing water

With respect to the scheduling problem for micro-ferries, the velocity of the micro-ferry has an effect on two distinct properties; both the travel time and the energy consumption of the micro-ferries change when changing the velocity. The *travel time* will depend on the velocity *relative to the land*, whereas the *energy consumption* will depend on the velocity *relative to the water*. For calm water these two velocities will be equal, but for flowing water one can no longer use a single notion of velocity.

To obtain schedules that take into account the effects of water flows within reasonable computation times, some assumptions are made for modeling the problem:

A1: *The water flow is uniform and time-invariant over the scheduling horizon (in the order of a few hours),*
A2: *Side-slip of the micro-ferries can be neglected,*
A3: *The acceleration and deceleration close to the stations can be neglected, as well as the changes of the water flows near the stations,*
A4: *The water flow is slower than the speed of the micro-ferries.*

Assumption A1 states that the water flow will not change over time nor depends on the location on the river, which means that the water flow is constant. Assumption A2 will hold for reasonable speeds and accelerations (i.e. no sharp turns), which is expected to be valid due to safety reasons. Assumption A3 is valid if the traveled distances are long enough to neglect differences in vessel speeds and water flow speeds at the start and end of a traveled path. Finally, assumption A4 ensures controllability of the micro-vessel on the water by always being able to move forwards relative to the water flow.

2.1.1 Velocities and paths

The flow velocity of a river will influence the perception of speed both on a vessel and of a vessel with respect to the shore. When a vessel travels upstream, it will travel faster relative to the water than relative to the land. This effect can conveniently be described by using three different velocity vectors:

$\mathbf{v}_\mathrm{b}$: vessel velocity relative to the water,
$\mathbf{v}_\mathrm{i}$: vessel velocity relative to the land,
$\mathbf{v}_\mathrm{r}$: water velocity relative to the land.

As shown in Figure 3 these three velocities relate to each other as

$$\mathbf{v}_\mathrm{i} = \mathbf{v}_\mathrm{r} + \mathbf{v}_\mathrm{b}. \tag{10}$$

The velocities can be decomposed into the speed components in the x and y direction of the inertial reference frame; for each $* \in \{\mathsf{b}, \mathsf{i}, \mathsf{w}\}$ we have

$$\mathbf{v}_* = \dot{x}_* \mathbf{i} + \dot{y}_* \mathbf{j}, \tag{11}$$

where $\dot{x}_*$ and $\dot{y}_*$ denote the speeds in the x_i and y_i direction respectively, whereas $\mathbf{i}$ and $\mathbf{j}$ denote the unit vector in the x_i and y_i direction of the inertial reference frame[2] respectively. The speed u_* associated with a velocity $\mathbf{v}_*$ can be determined as

$$u_* = |\mathbf{v}_*| = \sqrt{\dot{x}_*^2 + \dot{y}_*^2}. \tag{12}$$

Paths of a micro-ferry are defined as displacements over time in a certain reference frame. Due to assumptions A1 and A3, we can model a river as a straight waterway (i.e. like a canal), and the micro-ferries will travel in straight-line paths from one location to another. Hence, the path of a micro-ferry can be modeled by a vector with the same direction as its associated velocity. We define the paths $\mathbf{p}_\mathrm{i}$, $\mathbf{p}_\mathrm{b}$, and $\mathbf{p}_\mathrm{r}$ associated with the velocities discussed above. A displacement $\mathbf{p}_*$ can be decomposed as

$$\mathbf{p}_* = x_* \mathbf{i} + y_* \mathbf{j}, \tag{13}$$

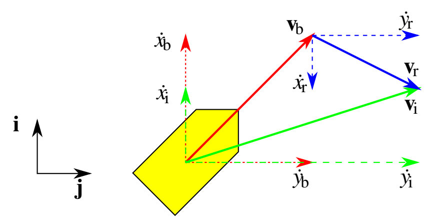

Fig. 3 The velocity $\mathbf{v}_\mathrm{i}$ with respect to the land is the sum of the velocity $\mathbf{v}_\mathrm{b}$ of the micro-ferry plus the velocity $\mathbf{v}_\mathrm{r}$ of the water.

[2] The inertial reference frame is the reference frame that is fixed with respect to the land.

with the associated path length

$$l_* = |\mathbf{p}_*| = \sqrt{x_*^2 + y_*^2}. \tag{14}$$

Based on these definitions of velocities and paths, we can now analyze the effects of flowing water on the micro-ferry scheduling problem.

2.1.2 Effect on travel times

For a vessel that travels with a constant velocity relative to the water, it will take longer to travel from some location a to another location z in upstream direction than from z to a in downstream direction. This section explains how the currents affect the travel times.

Calculation of travel times

The travel time of a micro-ferry equals the distance traveled divided by the travel speed; more specifically it is the time it takes to travel from one station to the next. The path $\mathbf{p}_\text{i}$ from one location to the other will not change with the water flow, but the path $\mathbf{p}_\text{b}$ of the micro-ferry relative to the water will be dependent on the velocity $\mathbf{v}_\text{r}$ of the water relative to the land, and the travel time T. Note that for a micro-ferry traveling with a velocity $\mathbf{v}_\text{i}$ relative to the land, the travel time T and the traveled path $\mathbf{p}_\text{i}$ are related as

$$\mathbf{p}_\text{i} = T\mathbf{v}_\text{i}. \tag{15}$$

Combined with the relation between the different velocities as given in (10), the velocity of the micro-ferry relative to the water can be written as

$$\mathbf{v}_\text{b} = \frac{1}{T}\mathbf{p}_\text{b} = \frac{1}{T}(\mathbf{p}_\text{i} - \mathbf{p}_\text{r}) = \frac{1}{T}\mathbf{p}_\text{i} - \mathbf{v}_\text{r}. \tag{16}$$

Both $\mathbf{p}_\text{i}$ and $\mathbf{v}_\text{r}$ are constants, and the velocity of the micro-ferry through the water therefore only varies with the travel time T. The speed u_b is related to the water flow and the displacement by

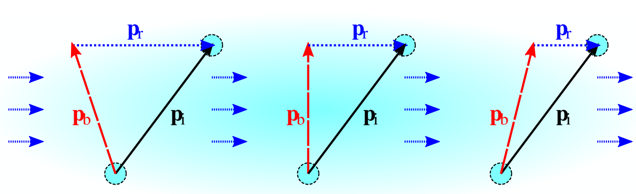

Fig. 4 The same path $\mathbf{p}_\text{i}$ in the inertial frame can be accomplished at different velocities, resulting in different paths $\mathbf{p}_\text{b}$ in the body frame.

$$\begin{aligned} u_{\mathrm{b}}^2 = |\mathbf{v}_{\mathrm{b}}|^2 = \dot{x}_{\mathrm{b}}^2 + \dot{y}_{\mathrm{b}}^2 &= \left(\frac{x_{\mathrm{i}}}{T} - \dot{x}_{\mathrm{r}}\right)^2 + \left(\frac{y_{\mathrm{i}}}{T} - \dot{y}_{\mathrm{r}}\right)^2 \\ &= \left(\dot{x}_{\mathrm{r}}^2 + \dot{y}_{\mathrm{r}}^2\right) - 2\left(x_{\mathrm{i}}\dot{x}_{\mathrm{r}} + y_{\mathrm{i}}\dot{y}_{\mathrm{r}}\right)\frac{1}{T} + \left(x_{\mathrm{i}}^2 + y_{\mathrm{i}}^2\right)\frac{1}{T^2} \\ &= u_{\mathrm{r}}^2 - 2\left(x_{\mathrm{i}}\dot{x}_{\mathrm{r}} + y_{\mathrm{i}}\dot{y}_{\mathrm{r}}\right)\frac{1}{T} + l_{\mathrm{i}}^2\frac{1}{T^2}, \end{aligned} \tag{17}$$

where l_{i} is the traveled distance of the micro-ferry with respect to the land (i.e. the distance between two locations). Since the traveled distance of the micro-ferry with respect to the water equals $l_{\mathrm{b}} = u_{\mathrm{b}}T$ by (12), (14) and $\mathbf{p}_{\mathrm{b}} = \mathbf{v}_{\mathrm{b}}T$, we have

$$l_{\mathrm{b}}^2 = u_{\mathrm{b}}^2T^2 = u_{\mathrm{r}}^2T^2 - 2\left(x_{\mathrm{i}}\dot{x}_{\mathrm{r}} + y_{\mathrm{i}}\dot{y}_{\mathrm{r}}\right)T + l_{\mathrm{i}}^2, \tag{18}$$

which is a quadratic equation of the form $\alpha T^2 + \beta T + \gamma = 0$ with

$$\alpha = u_{\mathrm{r}}^2 - u_{\mathrm{b}}^2, \quad \beta = -2\left(x_{\mathrm{i}}\dot{x}_{\mathrm{r}} + y_{\mathrm{i}}\dot{y}_{\mathrm{r}}\right), \quad \gamma = l_{\mathrm{i}}^2. \tag{19}$$

We have $\alpha < 0$ since –by assumption A4– the speed of the micro-ferry u_{b} will be larger than the speed of the water u_{r} to ensure controllability of the vessel. Furthermore, $\gamma > 0$ since it represents the distance between two stations. Therefore, using the variables defined in (19) the discriminant of (18) satisfies

$$\Delta = \beta^2 - 4\alpha\gamma > \beta^2 > 0. \tag{20}$$

The second inequality shows that there are two distinct real-valued solutions for T, whereas the first inequality shows that

$$-\beta + \sqrt{\Delta} > -\beta + |\beta| \geq 0, \quad -\beta - \sqrt{\Delta} < -\beta - |\beta| \leq 0. \tag{21}$$

Since $\alpha < 0$, positive travel times T can be found by

$$\begin{aligned} T &= \frac{-\beta - \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} = \frac{-\frac{1}{2}\beta - \sqrt{\frac{1}{4}\beta^2 - \alpha\gamma}}{\alpha} \\ &= \frac{\left(x_{\mathrm{i}}\dot{x}_{\mathrm{r}} + y_{\mathrm{i}}\dot{y}_{\mathrm{r}}\right) - \sqrt{\left(x_{\mathrm{i}}\dot{x}_{\mathrm{r}} + y_{\mathrm{i}}\dot{y}_{\mathrm{r}}\right)^2 - (u_{\mathrm{r}}^2 - u_{\mathrm{b}}^2)l_{\mathrm{i}}^2}}{u_{\mathrm{r}}^2 - u_{\mathrm{b}}^2}. \end{aligned} \tag{22}$$

Note that the water flow-related coefficients $\dot{x}_{\mathrm{r}}, \dot{x}_{\mathrm{r}}, u_{\mathrm{r}}$ are constant for all possible trajectories between the stations, whereas the coefficients $x_{\mathrm{i}}, y_{\mathrm{i}}, l_{\mathrm{i}}$ depend on the start and end location a and z for a certain trip. As opposed to the work in [8], here we will treat the speed u_{b} of the micro-ferry in the water as an optimization variable. Therefore, the travel time T_{az} from location a to z will depend on the chosen speed u_{b}, as can be seen in the plot of (22) in Figure 5.

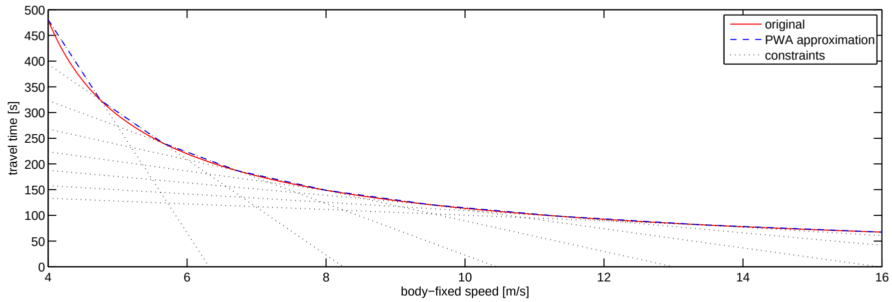

Fig. 5 Travel times at different speeds. The convex function can be approximated using constraints.

Linear approximation for travel times

Equation (22) is non-linear in the micro-ferry speed u_b. Analysis of the function in (22) shows that it is a strictly decreasing, convex function in u_b, for which we can obtain an accurate approximation using a continuous piece-wise affine function

$$\hat{T}_{a,z}(u_\mathsf{b}) = \begin{cases} \mathsf{a}^1_{a,z} u_\mathsf{b} + \mathsf{b}^1_{a,z}, & \mathsf{u}_0 \leq u_\mathsf{b} \leq \mathsf{u}_1 \\ \vdots & \\ \mathsf{a}^\mathsf{P}_{a,z} u_\mathsf{b} + \mathsf{b}^\mathsf{P}_{a,z}, & \mathsf{u}_{\mathsf{P}-1} \leq u_\mathsf{b} \leq \mathsf{u}_\mathsf{P} \end{cases} \tag{23}$$

Note that by increasing the number of segment P one can increase the accuracy of the approximation at the cost of increasing the computational effort. Since the function $T_{a,z}$ is convex in u_b, we will have $\mathsf{a}^i_{a,z} < \mathsf{a}^j_{a,z}$ for $i < j$, and the function can be written as the maximum of a set of lines

$$\hat{T}_{a,z}(u_\mathsf{b}) = \max_{i=1,\dots,\mathsf{P}} \left(\mathsf{a}^i_{a,z} u_\mathsf{b} + \mathsf{b}^i_{a,z} \right). \tag{24}$$

For such a function the value of $\hat{T}_{a,z}(u_\mathsf{b})$ can be found using the linear program [5]

$$\min \quad T \tag{25}$$

$$\text{s.t.} \quad \mathsf{a}^i_{a,z} u_\mathsf{b} + \mathsf{b}^i_{a,z} \leq T \quad \forall i \in \{1, \dots, \mathsf{P}\} \tag{26}$$

where the optimal value T^* will equal the travel time approximation $\hat{T}_{a,z}(u_\mathsf{b})$. An example of the relation (22) between the travel time and the travel speed is shown in Figure 5 as the continuous, red line. The blue, dashed lines show the continuous piece-wise affine approximation $\hat{T}_{a,z}(u_\mathsf{b})$ from (23), and the black, dotted lines show the constraints used to approximate the travel time in (26).

2.1.3 Effect on energy consumption

Due to the flowing water the micro-ferries might need more or less energy to travel from one location to another as compared to still water, depending on whether or not they are traveling against the current. This section explains how the currents affect the energy consumption.

Calculation of energy consumption

The dynamics of a vessel can be modeled using the vectorial representation [14]

$$\mathsf{M}\dot{\nu}+\mathsf{C}\nu+\mathsf{D}\nu+\tau_{\mathrm{e}}=\tau_{\mathrm{c}}, \tag{27}$$

where $\nu=[u, v, r]^{\top}$ is the velocity vector consisting of the surge speed u, the sway speed v, and the rotational speed r, M is a symmetric, positive definite mass matrix, C is a skew-symmetric Coriolis and centripetal forces matrix, D is a symmetric, positive definite damping matrix, τ_{e} is a force vector representing external disturbances (e.g. wind and currents), and τ_{c} is the control vector representing the forces exerted by the actuators. Using the force balance (27) we can write the kinetic energy of a surface vessel as

$$E_{\mathrm{kin}}=\frac{1}{2}\nu^{\top}\mathsf{M}\nu, \tag{28}$$

and the associated power (due to movement) becomes the quadratic function

$$\begin{aligned} P_{\mathrm{kin}} &= \frac{d}{dt}E_{\mathrm{kin}}=\frac{1}{2}\left[\dot{\nu}^{\top}\mathsf{M}\nu+\nu^{\top}\mathsf{M}\dot{\nu}\right]=\nu^{\top}\mathsf{M}\dot{\nu} \\ &= \nu^{\top}[-\mathsf{C}\nu-\mathsf{D}\nu+\tau_{\mathrm{c}}-\tau_{\mathrm{e}}]=[\tau_{\mathrm{c}}-\tau_{\mathrm{e}}]^{\top}\nu-\nu^{\top}\mathsf{D}\nu. \end{aligned} \tag{29}$$

In order to take the energy consumption of the micro-ferries into account, we use a simplified expression for the power based on the along-path speed u only. Besides the quadratic and linear terms of (29) due to the kinetic energy, we also add a constant term to include the energy losses due to a running motor when the micro-ferries are not moving. Therefore, the power of the micro-ferries will be a quadratic function of the speed, written as [10]

$$P=\pi_2 u^2+\pi_1 u+\pi_0. \tag{30}$$

The energy consumption will depend on the speed u_{b} relative to the water, and it is kept constant during a trip but used as a variable in the optimization problem. Then, the energy consumption from a to z becomes

$$E_{a,z}(u_{\mathrm{b}})=P(u_{\mathrm{b}})T_{a,z}(u_{\mathrm{b}})=\left(\pi_2 u_{\mathrm{b}}^2+\pi_1 u_{\mathrm{b}}+\pi_0\right)T_{a,z}(u_{\mathrm{b}}) \tag{31}$$

which is a non-linear equation in the variables u_{b} representing vessel speed; the travel time T_{az} is a non-linear function in the vessel speed u_{b} as given by (22).

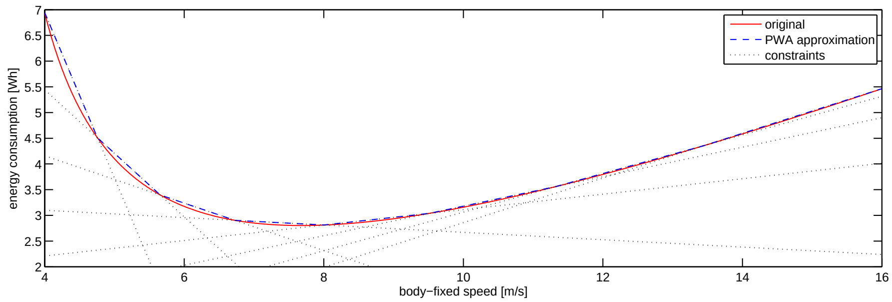

Fig. 6 Energy consumption at different speeds. The energy consumption is a convex function of the speed, and can efficiently be approximated using linear constraints.

Linear approximation of the energy consumption

Equation (31) is non-linear in the micro-ferry speed u_b. Analysis of the function shows that it is a convex function in u_b; hence also this function can be approximated accurately using a continuous piece-wise affine function[3]

$$\hat{E}_{a,z}(u_\mathrm{b}) = \begin{cases} \mathsf{c}^1_{a,z} u_\mathrm{b} + \mathsf{d}^1_{a,z}, & \mathsf{u}_0 \leq u_\mathrm{b} \leq \mathsf{u}_1 \\ \vdots & \\ \mathsf{c}^\mathsf{P}_{a,z} u_\mathrm{b} + \mathsf{d}^\mathsf{P}_{a,z}, & \mathsf{u}_{\mathsf{P}-1} \leq u_\mathrm{b} \leq \mathsf{u}_\mathsf{P} \end{cases} \tag{32}$$

Since the function $E_{a,z}$ is convex in u_b, the value of $\hat{E}_{a,z}(u_\mathrm{b})$ can be found using [5]

$$\min \quad E \tag{33}$$

$$\text{s.t.} \quad \mathsf{c}^i_{a,z} u_\mathrm{b} + \mathsf{d}^i_{a,z} \leq E \quad \forall i \in \{1, \ldots, \mathsf{P}\} \tag{34}$$

where the optimal value E^* will equal the energy consumption approximation $\hat{E}_{a,z}(u_\mathrm{b})$. An example of the relation (31) between the energy consumption and the travel speed is shown in Figure 6 as the continuous, red line. The blue, dashed lines show the continuous piece-wise affine approximation $\hat{E}_{a,z}(u_\mathrm{b})$ from (32), and the black, dotted lines show the constraints used to approximate the travel time in (34).

2.1.4 Proof of Theorem 2

With the equations for the travel times and energy consumption derived above, we can now prove the statement of Theorem 2. We consider a uniform and time-invariant current, which –without loss of generality[4]– flows along the x-axis of the inertial reference frame. Hence, the relative velocity becomes $\mathsf{v}_\mathrm{r} = (\dot{x}_\mathrm{r}, \dot{y}_\mathrm{r}) = (u_\mathrm{r}, 0)$

[3] Actually, neither the number of partitions P nor the speeds $\mathsf{u}_0, \ldots, \mathsf{u}_\mathsf{P}$ need to be the same for the travel time approximation (23) and the energy consumption approximation (32).

[4] The inertial reference frame can always be chosen to be aligned with the water flow.

such that the travel time defined in (22) becomes

$$T = \frac{\sqrt{(x_\mathrm{i}\dot{x}_\mathrm{r})^2 + (u_\mathrm{b}^2 - u_\mathrm{r}^2) l_\mathrm{i}^2} - x_\mathrm{i}\dot{x}_\mathrm{r}}{u_\mathrm{b}^2 - u_\mathrm{r}^2}.$$

The travel time T_{az} from location a to z can be found by considering the displacement $\mathbf{p}_{\mathrm{i},az} = (x_{\mathrm{i},az}, y_{\mathrm{i},az})$ in the inertial reference frame. Then the return trip from z to a is given by $\mathbf{p}_{\mathrm{i},za} = -\mathbf{p}_{\mathrm{i},az} = (-x_{\mathrm{i},az}, -y_{\mathrm{i},az})$, such that by (14) we have $l_{\mathrm{i},az} = l_{\mathrm{i},za}$. When[5] $u_\mathrm{b}^2 - u_\mathrm{r}^2 > 0$ the difference in travel time $\Delta_T = T_{az} - T_{za}$ of going one way or the other between arbitrary locations a and z is

$$\begin{aligned} \Delta_T &= \frac{\sqrt{(x_\mathrm{i}\dot{x}_\mathrm{r})^2 + (u_\mathrm{b}^2 - u_\mathrm{r}^2) l_\mathrm{i}^2} - x_\mathrm{i}\dot{x}_\mathrm{r}}{(u_\mathrm{b}^2 - u_\mathrm{r}^2)} - \frac{\sqrt{(-x_\mathrm{i}\dot{x}_\mathrm{r})^2 + (u_\mathrm{b}^2 - u_\mathrm{r}^2) l_\mathrm{i}^2} + x_\mathrm{i}\dot{x}_\mathrm{r}}{(u_\mathrm{b}^2 - u_\mathrm{r}^2)} \\ &= \frac{-2x_\mathrm{i}\dot{x}_\mathrm{r}}{\sqrt{u_\mathrm{b}^2 - u_\mathrm{r}^2}}. \end{aligned} \tag{35}$$

This shows that the travel times are different if there is a current (that is for $\dot{x}_\mathrm{r} \neq 0$), as could be expected; if we travel against the current from a to z (such that $x_\mathrm{i} > 0$ and $\dot{x}_\mathrm{r} < 0$) then $T_{az} > T_{za}$ and indeed $\Delta_T > 0$.

Perhaps less obvious is the fact that the travel time $\Sigma_T = T_{az} + T_{za}$ for a round trip (from a to z and back to a) has a larger travel time when the current's magnitude $u_\mathrm{r} = |\dot{x}_\mathrm{r}|$ increases:

$$\begin{aligned} \Sigma_T &= \frac{\sqrt{(x_\mathrm{i}\dot{x}_\mathrm{r})^2 + (u_\mathrm{b}^2 - u_\mathrm{r}^2) l_\mathrm{i}^2} - x_\mathrm{i}\dot{x}_\mathrm{r}}{(u_\mathrm{b}^2 - u_\mathrm{r}^2)} + \frac{\sqrt{(-x_\mathrm{i}\dot{x}_\mathrm{r})^2 + (u_\mathrm{b}^2 - u_\mathrm{r}^2) l_\mathrm{i}^2} + x_\mathrm{i}\dot{x}_\mathrm{r}}{(u_\mathrm{b}^2 - u_\mathrm{r}^2)} \\ &= \frac{2\sqrt{(x_\mathrm{i}\dot{x}_\mathrm{r})^2 + \left(u_\mathrm{b}^2 - u_\mathrm{r}^2\right) l_\mathrm{i}^2}}{u_\mathrm{b}^2 - u_\mathrm{r}^2}, \end{aligned} \tag{36}$$

which has a minimum

$$\Sigma_{T,\min} = 2\frac{l_\mathrm{i}}{\sqrt{u_\mathrm{b}^2 - u_\mathrm{r}^2}}, \tag{37}$$

for $\dot{x}_\mathrm{r} = 0$, and Σ_T increases for larger currents. Hence, the larger $|\dot{x}_\mathrm{r}|$, the larger the travel times within the micro-ferry network, and —by (31)— the larger the energy consumption needed to handle the requests.

[5] This holds for $|u_\mathrm{b}| > |u_\mathrm{r}|$, which is a necessary condition to be able to move forwards under all circumstances, as desired under normal operations and stated as assumption A4.

2.2 Problem definition

To formulate the micro-ferry scheduling problem, several optimization variables will be used. First, an overview of these variables is given, followed by a detailed explanation of the relations between them. This will lead to a mixed-integer linear program for finding the transport schedule of the micro-ferries, as shown in the example provided at the end of this section.

2.2.1 Optimization variables

The optimization variables used for the micro-ferry scheduling problem are summarized next, separated by type. The variables are defined per *request*, which consists of the transport of a customer from one location to another within a certain time window, and the possible relocation, charging and waiting that are associated to the specific transport. The set of non-negative scalars is defined as R_+.

Decision variables:

- $x_{ij} \in \{0,1\}$: binary variable representing whether ($x_{ij} = 1$) or not ($x_{ij} = 0$) request j succeeds request i,
- $y_j \in \{0,1\}$: binary variable representing whether ($y_j = 1$) or not ($y_j = 0$) the micro-ferry is recharged after request j,
- $k_j \in [1, \mathsf{M}]$: continuous variable[6] representing the index number of the micro-ferry that handles the request.

Energy variables:

- $e_j \in [0, \mathsf{E}]$: energy level after completion of transport j,
- $f_j \in \mathsf{R}_+$: energy increase (by recharging or refueling) during request j,
- $g_j \in \mathsf{R}_+$: energy consumed during the relocation phase of request j,
- $h_j \in \mathsf{R}_+$: energy consumed during the transportation phase of request j.

Time variables:

- $p_j \in \mathsf{R}_+$: pick-up time for the passengers of request j,
- $q_j \in \mathsf{R}_+$: charging time after handling request j,
- $r_j \in \mathsf{R}_+$: relocation time for request j,
- $s_j \in \mathsf{R}_+$: time window mismatch for request j,
- $t_j \in \mathsf{R}_+$: transportation time for request j.

Speed variables:

- $u_j \in [\underline{\mathsf{u}}, \overline{\mathsf{u}}]$: speed of the micro-ferry during the relocation phase of request j,
- $v_j \in [\underline{\mathsf{v}}, \overline{\mathsf{v}}]$: speed of the micro-ferry during the transportation phase of request j.

[6] Although this variable is continuous, due to the constraints it will always attain an integer value.

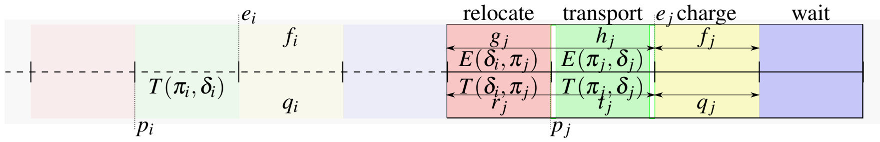

Fig. 7 The four phases of a request consist of relocating, transporting, charging, and waiting. The associated energy (top) and time (bottom) variables show the relations between energy levels e and pick-up times p for successive requests.

2.2.2 Phases of a request

A transportation of passengers associated with request j (preceded by request i) consists of the following phases (see Figure 7):

- a micro-ferry should (optionally) *relocate* from delivery location δ_i towards the pick-up location π_j for request j,
- the micro-ferry will *transport* the customer(s) from pick-up location π_j to the delivery location δ_j,
- the micro-ferry will (optionally) charge after the transportation at location δ_j,
- when charged the micro-ferry will (optionally) wait until it can handle the next request.

Each of these four steps takes time and alters the energy level, which should be accounted for in the scheduling of the pick-up times p_j and the charging actions for increasing the energy levels e_j.

Time variables

For embarking and disembarking the micro-ferries a (combined) duration $\mathsf{t_d}$ can be chosen by the operator, which will be a trade-off between giving the customers enough time to safely enter and exit the micro-ferry, and not wasting time at the station. Furthermore, the time it takes to couple and decouple the ferry to the power source when charging is represented by $\mathsf{t_c}$, and the rate at which the batteries are charged is given by $\mathsf{r_c}$.

The duration for the relocation of the micro-ferry from the delivery station of request i towards the pick-up location of station j is given by the travel time $T(\delta_i, \pi_j)$ calculated using (22), where δ_i and π_j denote the index number of the delivery station of request i and the index number of the pick-up station of request j respectively. The duration for the transportation of the customers from pick-up station π_j to delivery station δ_j is given by the travel time $T(\pi_j, \delta_j)$ in (22), where π_j and δ_j denote the index number of the pick-up and delivery station of request j respectively.

Let $\hat{T}(a,z)$ denote the continuous piece-wise affine approximation travel time $T(a,z)$ from location a to z, given by (23). Using the parameters $\mathsf{a}^p_{a,z}$ and $\mathsf{b}^p_{a,z}$ (which can be determined a priori) the *relocation time* r_j of request j from location $a = \delta_i$ to location $z = \pi_j$ can be determined by the linear program

$$\min \quad r_j \tag{38}$$
$$\text{s.t.} \quad \mathsf{a}^p_{\delta_i,\pi_j} u_j + \mathsf{b}^p_{\delta_i,\pi_j} \leq r_j \quad \forall p \in \{1,\ldots,\mathsf{P}\} \tag{39}$$

where $u_\mathrm{b} = u_j$ is the travel speed during the relocation. Similarly, the *transportation time* t_j of request j from location $a = \pi_j$ to location $z = \delta_j$ can be determined using

$$\min \quad t_j \tag{40}$$
$$\text{s.t.} \quad \mathsf{a}^p_{\pi_j,\delta_j} v_j + \mathsf{b}^p_{\pi_j,\delta_j} \leq t_j \quad \forall p \in \{1,\ldots,\mathsf{P}\} \tag{41}$$

where $u_\mathrm{b} = v_j$ is the travel speed during the transportation. Note that these constraints are defined on the entire domain of u_j and v_j; due to convexity of the original function the optimal value r_j^* / t_j^* will always lie on the line segment associated to the speed domain in the piece-wise affine approximation associated with the optimal speed u_j^* / v_j^* (see Figure 5).

Energy variables

For the energy consumption during the relocation phase and the transportation phase we will use the continuous piece-wise affine approximation (32). The *relocation energy* g_j can then be found by solving the linear program

$$\min \quad g_j \tag{42}$$
$$\text{s.t.} \quad \mathsf{c}^p_{\delta_i,\pi_j} u_j + \mathsf{d}^p_{\delta_i,\pi_j} \leq g_j \quad \forall p \in \{1,\ldots,\mathsf{P}\} \tag{43}$$

where $u_\mathrm{b} = u_j$ is the travel speed during the relocation. Similarly, the *transportation energy* h_j of request j from location $a = \pi_j$ to location $z = \delta_j$ can be determined by the linear program

$$\min \quad h_j \tag{44}$$
$$\text{s.t.} \quad \mathsf{c}^p_{\pi_j,\delta_j} v_j + \mathsf{d}^p_{\pi_j,\delta_j} \leq h_j \quad \forall p \in \{1,\ldots,\mathsf{P}\} \tag{45}$$

where $u_\mathrm{b} = v_j$ is the travel speed during the transportation. Note that these constraints are defined on the entire domain of u_j and v_j; due to convexity of the original function the optimal value g_j^* / h_j^* will always lie on the line segment associated to the speed domain in the piece-wise affine approximation associated with the optimal speed u_j^* / v_j^* (see Figure 6).

2.2.3 Micro-ferries and requests

Consider a fleet of M micro-ferries that could be either traveling or waiting at a station. These ferries will already have a pick-up time $p_{\mathrm{o},j}$, an energy level $e_{\mathrm{o},j}$, and a micro-ferry index number $k_{\mathrm{o},j}$. Besides the M current requests (which might be handled when a micro-ferry is docked at a station) there will be N new requests to schedule, resulting in a total of $\mathsf{R} = \mathsf{M} + \mathsf{N}$ requests. The sets

$$\mathscr{M} = \{1, \dots, \mathsf{M}\}, \quad \mathscr{N} = \{\mathsf{M}+1, \dots, \mathsf{R}\}, \quad \mathscr{R} = \mathscr{M} \cup \mathscr{N} \tag{46}$$

denote the set of current requests, new requests, and all requests respectively. These sets can be seen as the set of depots $\mathscr{M}$, the set of customers $\mathscr{N}$, and the total set of locations $\mathscr{R}$, as used in vehicle routing problems.

2.2.4 Mixed-integer linear programming formulation

The micro-ferry scheduling problem for flowing water and variable speeds can be solved using the following mixed-integer linear program.

$$\min \quad \sum_{i \in \mathscr{R}} (g_i + \alpha h_i + \gamma s_i - \eta e_i + \rho r_i + \theta t_i) \tag{47}$$

$$\text{s.t.} \quad \sum_{i \in \mathscr{R}} x_{ij} = 1; \quad \sum_{i \in \mathscr{R}} x_{ji} = 1 \qquad \forall\, j \in \mathscr{R} \tag{48}$$

$$k_i - k_j \leq (M-1)(1 - x_{ij} - x_{ji}) \qquad \forall\, i, j \in \mathscr{R} \tag{49}$$

$$\mathsf{a}^k_{\delta_i,\pi_j} u_j + \mathsf{b}^k_{\delta_i,\pi_j} \leq r_j + \mathsf{T}(1 - x_{ij}) \qquad \forall\, i \in \mathscr{R}, j \in \mathscr{N}, k \in \mathscr{P} \tag{50}$$

$$\mathsf{a}^k_{\pi_j,\delta_j} v_j + \mathsf{b}^k_{\pi_j,\delta_j} \leq t_j + \mathsf{T}(1 - x_{ij}) \qquad \forall\, i \in \mathscr{R}, j \in \mathscr{N}, k \in \mathscr{P} \tag{51}$$

$$p_i + t_i + q_i + r_j + \mathsf{t_d} \leq p_j + \mathsf{T}(1 - x_{ij}) \qquad \forall\, i \in \mathscr{R}, j \in \mathscr{N} \tag{52}$$

$$\mathsf{p}_{\mathrm{a},j} - p_j \leq s_j; \quad p_j - \mathsf{p}_{\mathrm{b},j} \leq s_j \qquad \forall\, j \in \mathscr{R} \tag{53}$$

$$\mathsf{t_c} y_j \leq q_j \qquad \forall\, j \in \mathscr{R} \tag{54}$$

$$f_j = \mathsf{r_c}(q_j - \mathsf{t_c} y_j) \qquad \forall\, j \in \mathscr{R} \tag{55}$$

$$f_j \leq \mathsf{E} y_j \qquad \forall\, j \in \mathscr{R} \tag{56}$$

$$e_j + f_j \leq \mathsf{E} \qquad \forall\, j \in \mathscr{R} \tag{57}$$

$$\mathsf{c}^k_{\delta_i,\pi_j} u_j + \mathsf{d}^k_{\pi_j,\delta_j} \leq g_j + \mathsf{E}(1 - x_{ij}) \qquad \forall\, i \in \mathscr{R}, j \in \mathscr{N}, k \in \mathscr{P} \tag{58}$$

$$\mathsf{c}^k_{\pi_j,\delta_j} v_j + \mathsf{d}^k_{\pi_j,\delta_j} \leq h_j + \mathsf{E}(1 - x_{ij}) \qquad \forall\, i \in \mathscr{R}, j \in \mathscr{N}, k \in \mathscr{P} \tag{59}$$

$$\left| e_i - h_i + f_i - g_j - e_j \right| \leq \mathsf{E}(1 - x_{ij}) \qquad \forall\, i \in \mathscr{R}, j \in \mathscr{N} \tag{60}$$

$$p_j = \mathsf{p}_{\mathrm{o},j}; \quad e_j = \mathsf{e}_{\mathrm{o},j}; \quad k_j = \mathsf{k}_{\mathrm{o},j} \qquad \forall\, j \in \mathscr{M} \tag{61}$$

$$x_{ij} \in \{0,1\}, \quad y_j \in \{0,1\} \qquad \forall\, i, j \in \mathscr{R} \tag{62}$$

where E is the upper bound on the energy levels e_j, and T should be chosen larger than the latest expected pick-up time (conform to the big-M method [20]).

The objective function (47) consists of the total energy consumption during relocation (first term) and transportation (second term), the total time window misfit (third term), it puts a penalty on low energy levels (fourth term), and ensures correct estimation of the travel times (fifth term). A trade-off between using less energy, assigning less pick-up times outside the desired time windows, and keeping the batteries charged can be made by changing the weights $\alpha > 0$, $\gamma > 0$, and $\eta > 0$. The weights $\rho > 0$ and $\theta > 0$ can be used to reduce travel times, but when chosen small the energy consumption is minimized while ensuring correct travel times.

Equalities (48) are the assignment constraints ensuring that every request is handled once and only once, and (49) assigns the micro-ferry index numbers to the requests. Relocation times and transportation times are determined using (50) and (51) respectively; variables r_j and t_j are minimized indirectly through (52). When request j proceeds i, inequality (52) ensures that the pick-up time for request j is later than the pick-up time for request i, plus the transportation time, charging time, and relocation time (see Figure 7). The time window mismatch s_j will be zero if the pick-up time p_j is scheduled within the desired time window $[\mathsf{p}_{\mathrm{a},j}, \mathsf{p}_{\mathrm{b},j}]$ through inequalities (53); otherwise, it will be equal to the time outside the time window.

With (54) we ensure that when the micro-ferry will charge, the charging time is at least equal to the (dis)connection time t_{c}. Furthermore, (55) couples the charged energy to the charging time, (56) ensures that the energy level will not increase when the micro-ferry will not charge, and (57) avoids overcharging of the batteries.

The energy consumed during the relocation phase and transportation phase are set using inequalities (58) and (59) respectively; the variables g_j and h_j are minimized directly through (47). Unlike the pick-up times (where time can pass while waiting), the energy level of request j is exactly equal to the energy level of request i minus the transportation and relocation energy consumption plus the charged energy, when request i precedes j (see Figure 7). This conditional equality is enforced using the inequality constraints (60).

Finally, constraints (61) will set the initial conditions for the pick-up times, energy levels, and index numbers of the micro-ferries, and (62) are the integrality constraints for the binary variables used in this formulation.

2.2.5 Case study example

As a case study we use the Rotterdam harbor example as shown in Figure 1, with $\mathsf{M} = 5$ micro-ferries and $\mathsf{N} = 40$ new requests. The water flow has a speed of 3 $[\mathrm{m/s}]$, and the micro-ferries are allowed to travel at speed between 4 and 16 $[\mathrm{m/s}]$.

The energy levels are given in percentages, with $\mathsf{E} = 100[\%]$ indicating a fully charged battery. The coefficients in expression (30) for the power are chosen as $\mathsf{p}_0 = 0.1, \mathsf{p}_1 = -0.02, \mathsf{p}_2 = 0.002$, resulting in a minimum power consumption of 0.05 $[\%/\mathrm{s}]$ at the optimal speed of 5 $[\mathrm{m/s}]$, resulting in a radius of 10 $[\mathrm{km}]$. The time for (dis)embarking the micro-ferries is set to $\mathsf{t}_{\mathrm{d}} = 60$ $[\mathrm{s}]$, and charging can be done

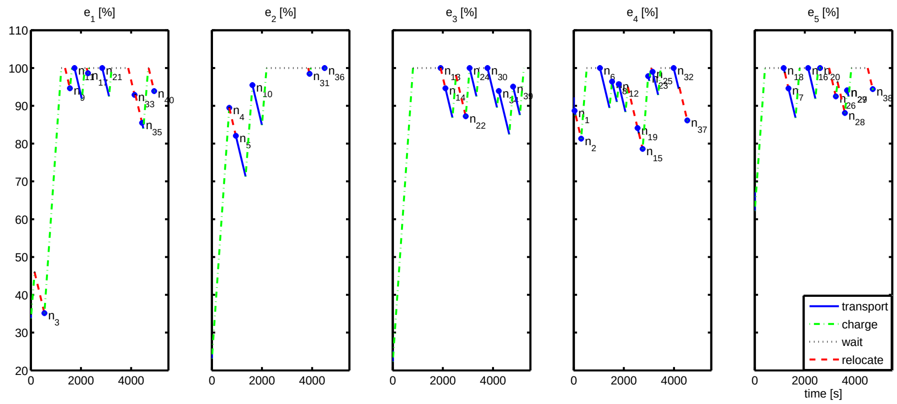

Fig. 8 Energy levels over time. Micro-ferries will charge when possible (dash-dotted green lines) to avoid empty batteries.

with a fixed setup time $t_c = 60$ [s] at a rate of $r_c = 0.1$ [%/s]. At the optimal speed it will take 22 minutes to travel the largest distance of 6.6 [km] (from the station at Schiedam to the station at Rijnhaven).

An example schedule resulting from a randomly generated test case is shown in Figures 2 and 8. Pick-up times are determined such that on average there will be 5 minutes in between consecutive requests per micro-ferry, and pick-up and delivery locations are chosen randomly (but not equal to each other). All micro-ferries start at a (randomly chosen) station with an energy level between 0 and 100 %.

Figure 2 shows the assignment of the $N = 40$ new requests to the $M = 5$ micro-ferries. Starting from the micro-ferry node, the arrows indicate the order in which the requests will be handled. The requests are sorted based on their desired pick-up time p_a, and all time windows $[p_{a,j}, p_{b,j}]$ are 60 [s] long. As can be seen the order of the pick-ups is not always consistent with the desired pick-up times, indicating that it was more efficient (in terms of the objective function (47)) to change the order.

In Figure 8 the energy levels are shown over time. The different phases of a request are indicated using different line types: red-dashed lines are relocations, continuous blue lines are transportations, dash-dotted green lines indicates charging, and the black-dotted are associated with waiting. The blue stars indicate the pick-up times of the requests. As can be seen the micro-ferries will charge when possible.

3 Linking Transport of Water and Transport over Water

Transport over water

The micro-ferry scheduling problem discussed in this chapter is an example of transport over water. Based on transport requests of customers along a river, the proposed optimization problem (47)–(62) provides an energy-efficient schedule for transporting customers over the flowing water. The formulation ensures that the micro-ferries will not run out of energy while traveling on the water; the energy level is not allowed to become too low, and charging of the micro-ferries is taken into account in the schedule. Taking into account the velocity of the current is crucial to ensure punctuality and correct predictions of energy levels.

Transport of water

As discussed in this chapter –and formalized in Theorem 2– the influence of flowing water on the travel times and energy consumption is important for transport over water. Stronger currents result in larger mean travel times, and overall the energy consumption will increase. Since the transport of water will influence the strength of the currents in the rivers and canals, this effect should be considered when managing the water network.

Contribution to a unified framework

The micro-ferry scheduling problem can be seen as a transportation problem over water, which is influenced by the transport of water via the strength of the currents in the water network. For the short time-horizon of this problem (a few hours at most) the current can be seen as a constant, and due to the narrow time windows for the pick-up times there is little flexibility in handling the requests during more suitable water flow conditions. Nonetheless, the analysis of this problem shows that there is a direct relation between the strength of the current and the energy consumption.

This work could be extended to transport over water on a larger scale (e.g. between a harbor and the hinter land), such that people or freight will be transported within a large water network with distances of several hundred kilometers, and the time-scale will be in days. In this case the expected water flows –due to transport of water– can be taken into account in the scheduling of the transport of goods. With freight the time windows are usually much larger, thereby creating more flexibility in planning the barges at times that the energy consumption would be low (i.e. when the current is relatively weak).

On the other hand, when given a freight transportation schedule, one can determine how the water network should be managed such that the objectives for the transport of water are met while also reducing the energy consumption for the barges. Eventually, this might lead to a combined optimization problem for both transport of and over water.

Global performance measurement

The performance measures for the transport over water as presented here are the total energy consumption of the vessels, and the time window misfit for picking up the customers. Both values should be as small as possible. When combining this problem with transport over water, performance measures such as minimum deviations from target water levels (combined with constraints on minimum and maximum levels) and energy consumption (e.g. for pumping water) can be taken into account in one large optimization problem.

4 Open Topics

The mixed-integer linear program (47)–(62) can be solved with standard hardware and software for small instances, but when considering many requests at once the computation times become too large to be useful in practice. To reduce computation times one could solve the problem over a limited time horizon, and recompute the optimal schedule periodically, hence using a rolling horizon approach. Furthermore decomposition methods [12] can be used to exploit the structure of the problem.

5 Conclusions and Future Research

5.1 Conclusions

In this chapter we have discussed a modeling framework for transport over water. Autonomous micro-ferries are used to transport customers over the water between different stations. A trade-off is made between energy consumption and picking up the customers on time, and charging of the micro-ferries is scheduled to avoid empty batteries. The speeds of the micro-ferries are also taken as optimization variables to increase the flexibility in scheduling the transport requests.

The micro-ferry scheduling problem can be seen as a variant of the multi-depot vehicle routing problem, and a mixed-integer linear program with 2-index decision variables has been presented to find appropriate schedules. This scheduling problem contains soft time windows, variable speeds, energy levels, and takes into account the effect of water flows. The effect of water flow speeds on both travel times and energy consumption is derived, and we conclude that the mean travel times and total energy consumption will increase with increasing magnitudes of the water flow.

5.2 Future research

The micro-ferry scheduling problem consists in finding a transport schedule with times in the order of minutes, and with travel distances within a city. On this time-scale the water flow is expected to be almost constant. For future research one could consider long-distance transport over water (e.g. from the harbor to the hinterland) where both distances and time scales will be larger. In this case the water flow can no longer be considered a constant, and the influence of the (planned) transport of water becomes even more important.

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References

1. L.H. Appelgren. Integer programming methods for a vessel scheduling problem. *Transportation Science*, 5(1):64–78, 1971.
2. T. Bektaş and G. Laporte. The pollution-routing problem. *Transportation Research Part B: Methodological*, 45(8):1232–1250, 2011.
3. T. Bektas. The multiple traveling salesman problem: an overview of formulations and solution procedures. *Omega*, 34(3):209–219, 2006.
4. A. Bemporad and M. Morari. Control of systems integrating logic, dynamics, and constraints. *Automatica*, 35(3):407–427, 1999.
5. S.P. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, United Kingdom, 8th edition, 2010.
6. M. Burger. Exact and compact formulation of the fixed-destination travelling salesman problem by cycle imposement through node currents. In *Proc. of the 2013 International Conference on Operations Research*, Rotterdam, the Netherlands, September 2013. Springer.
7. M. Burger and B. De Schutter. Energy-efficient transportation over flowing water. In *Proc. of the 10th IEEE International Conference on Networking, Sensing and Control*, pages 226–231, Evry, France, April 2013. IEEE.
8. M. Burger, B. De Schutter, and H. Hellendoorn. An improved method for solving micro-ferry scheduling problems. In *Proc. of the 1st European Symposium on Quantitative Methods in Transportation Systems*, pages 164–171, Lausanne, Switzerland, September 2012.
9. M. Burger, B. De Schutter, and H. Hellendoorn. Micro-ferry scheduling problem with charging and embarking times. In *Proc. of the 13th IFAC Symposium on Control in Transportation Systems*, Sofia, Bulgaria, September 2012.
10. M. Burger, B. De Schutter, and H. Hellendoorn. Micro-ferry scheduling problem with time windows. In *Proc. of the 2012 American Control Conference*, pages 3998–4003, Montréal, Canada, June 2012. IEEE.
11. M. Christiansen, K. Fagerholt, and D. Ronen. Ship routing and scheduling: Status and perspectives. *Transportation Science*, 38(1):1–18, 2004.
12. A.J. Conejo, E. Castillo, R. Mínguez, and R. García-Bertrand. *Decomposition Techniques in Mathematical Programming*. Springer, Berlin, Germany, 1st edition, 2006.
13. C.F. Daganzo. *Fundamentals of Transportation and Traffic Operations*. Pergamon Press, United Kingdom, 3rd edition, 1997.
14. T.I. Fossen. *Marine Control Systems - Guidance, Navigation and Control of Ships, Rigs and Underwater Vehicles*. Marine Cybernetics, 1st edition, 2002.

15. İ. Kara, B.Y. Kara, and M.K. Yetis. Energy minimizing vehicle routing problem. In *Combinatorial Optimization and Applications*, volume 4616 of *Lecture Notes in Computer Science*, pages 62–71. Springer Berlin / Heidelberg, 2007.
16. R.V. Kulkarni and P.R. Bhave. Integer programming formulations of vehicle routing problems. *European Journal of Operational Research*, 20(1):58–67, 1985.
17. G. Laporte. The traveling salesman problem: an overview of exact and approximate algorithms. *European Journal of Operational Research*, 59(2):231–247, 1992.
18. C.E. Miller, A.W. Tucker, and R.A. Zemlin. Integer programming formulation of traveling salesman problems. *Journal of the ACM*, 7(4):326–329, 1960.
19. M.W.P. Savelsbergh and M. Sol. The general pickup and delivery problem. *Transportation Science*, 29(1):17–29, February 1995.
20. H.A. Taha. *Operations Research: An Introduction*. Macmillan Publishing Company, New York, 4th edition, 1987.
21. P. Toth and D. Vigo, editors. *The Vehicle Routing Problem*. Monographs on Discrete Mathematics and Applications. SIAM, 2002.
22. Y. Xiao, Q. Zhao, I. Kaku, and Y. Xu. Development of a fuel consumption optimization model for the capacitated vehicle routing problem. *Computers & Operations Research*, 39(7):1419–1431, 2012.