

Technical report 14-033

# **Hierarchical control of irrigation canals in the presence of disturbances: Framework and comparison\***

A. Sadowska, B. De Schutter, and P.-J. van Overloop

*If you want to cite this report, please use the following reference instead:*

A. Sadowska, B. De Schutter, and P.-J. van Overloop, “Hierarchical control of irrigation canals in the presence of disturbances: Framework and comparison,” *Proceedings of the 2014 European Control Conference*, Strasbourg, France, pp. 1349–1354, June 2014.

Delft Center for Systems and Control  
Delft University of Technology  
Mekelweg 2, 2628 CD Delft  
The Netherlands  
phone: +31-15-278.51.19 (secretary)  
fax: +31-15-278.66.79  
URL: <http://www.dcsc.tudelft.nl>

---

\*This report can also be downloaded via [http://pub.deschutter.info/abs/14\\_033.html](http://pub.deschutter.info/abs/14_033.html)

# Hierarchical control of irrigation canals in the presence of disturbances: framework and comparison

Anna Sadowska, Bart De Schutter and Peter-Jules van Overloop

**Abstract**—We study a control problem of delivering water to farmers through an irrigation canal and introduce a hierarchical controller with a Coordinator that by employing Model Predictive Control principles coordinates local canal reaches by modifying setpoints only when it is needed. Once the setpoints are set, the Coordinator does not interfere with the functioning of the local sites and the canal is fully controlled by local PI controllers located at each gate. Therefore the communication between the centralized controller and the local sites is kept minimal, which is motivated by the communication restrictions that are present in the field of irrigation. We consider three predictive control designs, namely a nominal controller, and two robust designs: a constraint tightening controller adapted to fit our application and a min-max controller. We present a numerical example to compare the performance obtained by the three controllers. It is found that for the given case study with a small disturbance realization, the nominal controller performs better than the robust controllers, the behaviors of which prove overly conservative.

## I. INTRODUCTION

Several control strategies have been introduced in the literature to date for the purpose of controlling irrigation canals, see e.g. [1], [6], [9], [13], [20], [22]. Despite the relatively high technological advance in the field of water resources engineering in theory, in many places the operation of the canal still resorts to a human operator manually adjusting the control structures based on his or her own judgment. However, since the operator changes the settings based on local observations only, the overall performance of this method may be compromised. A possible remedy to this problem is introducing automation to the field of canal control with the help of various feedforward or feedback controllers [17]. However, oftentimes the simplest controllers are preferable due to their advantageous quality of functioning to cost ratio. In fact, possibly the most widely used controllers are PI controllers applied to control gates in all reaches along the canal [10], [21]. Their popularity amongst the practitioners can be attributed to the fact that they are very simple, do not require an internal model and, if properly tuned, function satisfactorily.

An important practical feature limiting acceptable control designs is the fact that in the field of irrigation communication needs to be paid for and maintained by the farmers and is thus considered expensive. This is due to the harsh environment that the communication links are located in:

A. Sadowska and B. De Schutter are with the Delft Center for Systems and Control, Delft University of Technology, The Netherlands; P.-J. van Overloop is with the Water Resources Management section, Delft University of Technology, The Netherlands (e-mail: {A.D.Sadowska, B.DeSchutter, P.J.A.T.M.vanOverloop}@tudelft.nl).

the radio communication that is employed can be prone to disturbances and as such should ideally not be used in the regular control loop but rather infrequently. Therefore, we propose to use the local PI controllers at individual gates to control water levels in the canal – as sole controllers in the normal operating conditions, thus eliminating any communication in such circumstances. However, to improve the performance of the local PI controllers in order to facilitate speedy water deliveries, we introduce a hierarchical control structure. Accordingly, as any of the farmers requests a sudden delivery, a higher-layer centralized controller – the Coordinator – is invoked to coordinate the local controllers by modifying their setpoints, and thus to make the water available to the farmers faster than when only local PI controllers would be employed. When only PI controllers are used to control the canal as it is often done in the current implementations in the field, there may be a considerable delay between the moment water released from the head gate reaches the offtake point where it is requested. To improve this situation, we propose that as one of the farmers makes a delivery request, the Coordinator activates and uses Model Predictive Control [11] to compute its control actions: the setpoint modification profiles and the head gate flow profile. Yet, it communicates to the local sites only when changes are needed. Moreover, with the proposed set-up, we make sure that even if the communication lines fail for some time, the control remains acceptable due to the presence of PI controllers that can operate autonomously.

Since the Coordinator only acts in response to delivery requests, it is event-driven as opposed to time-driven. We assume that setpoints may change twice per delivery per canal reach, and use the Time Instant Optimization MPC [19], which enables the optimization of the setpoint switching moments.

To date, a hierarchical control problem in which a higher-layer controller provides the lower layer with modified setpoints was studied in e.g. [15] for a power network and in e.g. [23] for a water system. Moreover, various MPC approaches can be pursued to deal with the disturbances in the model equations, see e.g. [2]–[5], [12], [14], [18]. In this paper we consider the dynamic model of the canal to comprise unknown disturbances, such as leakage, model mismatches in offtake gate and head gate etc. We first introduce the hierarchical predictive controller and afterwards apply three set-ups to tackle the disturbance terms in the model. In the first instance, we study the nominal controller. Then, we examine a robust controller in which hard state constraints are tightened recursively over the prediction horizon taking

into account the upper bounds of the disturbances as motivated by the results in [7], [16]. This method is similar to the third method applied - the min-max MPC, in the sense that they both consider maximum bounds on the disturbances and hence guarantee that the resulting input is feasible for the system regardless of the magnitude of the actual disturbance as long as it does not exceed the given upper bounds. We compare the performance yielded by the three controllers.

The paper is outlined as follows. In Section II we show the dynamic model of the canal and discuss principles of TIO-MPC. In Section III we present the hierarchical control design and show how the nominal and robust controllers are applied, and analyze the differences. We then present a case study in Section IV and conclude the paper in Section V.

## II. PRELIMINARIES

### A. Model of an irrigation canal

The flow of water in a canal consisting of  $N$  reaches can be modeled via discretizing and linearizing the nonlinear partial differential equations, the so-called Saint Venant's equations [8], [13], which for reach  $i \in \{1, \dots, N\}$  reads

$$\begin{aligned} h_i(k+1) &= h_i(k) + \frac{T_m}{c_i}(u_{i-1}(k - k_{di}) - u_i(k) \\ &\quad + d_i(k) + g_i(k)) + w_i(k), \\ u_i(k) &= u_i(k-1) + K_{Pi}(e_i(k) - e_i(k-1)) \\ &\quad + K_{Ii}e_i(k), \quad (\text{PI controller}) \\ u_0(k) &= Q_S(k), \\ e_i(k) &= y_i(k) - h_i^{\text{ref}}(k), \\ y_i(k) &= h_i(k) + v_i(k), \end{aligned} \quad (1)$$

in which  $h_i$  denotes water level at the downstream end of reach  $i$ ,  $T_m$  denotes the sampling period,  $c_i$  is the surface area, and  $k_{di}$  is a time delay (in sampling steps) representing the time required for an inflow from upstream gate  $i-1$  to influence the water level at the downstream end of reach  $i$ . For  $i=1$ , the inflow is the flow from the head gate  $Q_S$ . Moreover,  $d_i$  denotes a water offtake from the canal due to a request made by the user,  $g_i$  is a known disturbance in the reach  $i$  due to e.g. rainfall, and  $w_i$  is an unknown disturbance. Furthermore,  $e_i$  denotes the deviation between  $h_i$  and the given setpoint  $h_i^{\text{ref}}$ , and  $v_i$  is a measurement uncertainty.

### B. Time Instant Optimization MPC (TIO-MPC)

Time instant optimization [19] is an approach to MPC that may be used for on/off control structures, in which for the whole length of the prediction horizon it is first selected how many times the structure's state can change. Then, given the chosen number of changes of the structure's state, one optimizes the real-valued time instants when these changes should occur. This results in a more computationally efficient optimization problem.

## III. HIERARCHICAL EVENT-DRIVEN CONTROL DESIGN

Now we present the design of the hierarchical event-driven controller, see Figure 1, to control the irrigation canals to aid water deliveries to the users through the canal, in which a

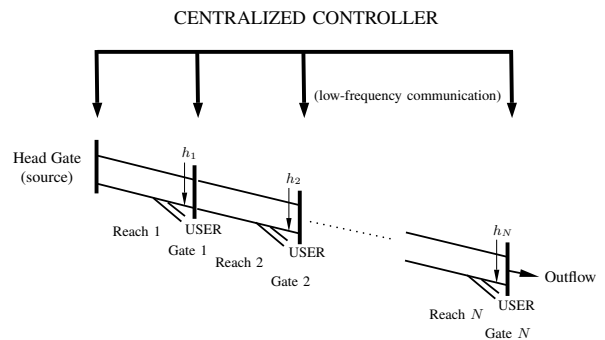


Fig. 1. Hierarchical controller proposed in the paper.

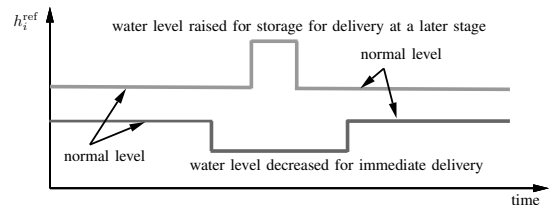


Fig. 2. Examples of admissible setpoint profiles.

delivery denotes an offtake in a pool of a given duration and magnitude. The Coordinator coordinates the local PI controllers by adjusting their setpoints, which consequently speeds up the delivery process, thereby making water available to users faster than with local PI controllers only. To aid presentation of our concept in a simplified way, we assume that no overlap of the requests of individual users is allowed in the sense that a new delivery can only be requested when water levels in all reaches have returned back to the steady state after a previous delivery. However, the method can be extended to multiple requests when adequate modifications are made. The timing of the Coordinator works as follows. It is considered that as a new delivery request is conveyed to the Coordinator, the time and step counter variables are reset to 0 and then are incremented until the Coordinator is re-activated for another delivery etc. We use  $A_c = T_c/T_m \in \mathbb{N}$ , where  $T_c$  is the length of the control cycle of the Coordinator, which is a multiple of the sampling time of the model  $T_m$ .

It is assumed that the model of the irrigation canal is given by (1) and that for all  $k$  we have

$$w_i(k) \in \mathcal{W}, \quad v_i(k) \in \mathcal{V}, \quad (2)$$

where  $\mathcal{W}, \mathcal{V} \subset \mathbb{R}$  are known convex and compact sets containing the origin.

The Coordinator provides the local controllers with a block-shaped setpoint profile: it finds a modified value of the setpoint and the time instants when this modified value should be switched on and back off to return to the normal operating value of the setpoint. Examples of possible setpoint profiles are given in Figure 2. Importantly, the Coordinator only needs to communicate once to each local site with the information on how a setpoint profile should be changed. This is an essential feature because it implies that there is no need for frequent communication with the local sites, which is a practical requisite of the system.

The control inputs to be determined by the Coordinator for each delivery request are  $\mathcal{U}^T =$

$(\tilde{Q}_{S,\text{demand}}^T, (H^{\text{ref, delivery}})^T, (T^{\text{on}})^T, (T^{\text{off}})^T)$ , where  $\tilde{Q}_{S,\text{demand}}$  denotes the optimum profile of the extra flow from the head gate to the first canal reach:  $\tilde{Q}_{S,\text{demand}} = (Q_{S,\text{demand}}(0), \dots, Q_{S,\text{demand}}(N_p - 1))^T$ . Knowing the base flow in the canal  $Q_{S,\text{base}}$ , from  $Q_{S,\text{demand}}(j)$ ,  $j = 0, \dots, N_p - 1$ , we can determine the overall flow from the head gate to be used in (1) as  $Q_S(jA_c + i) = Q_{S,\text{base}} + Q_{S,\text{demand}}(j)$ ,  $i = 0, \dots, A_c - 1$  starting from  $k = 0$  i.e. the moment the Coordinator is triggered until the end of the prediction horizon  $N_p$ .

Further control inputs in are  $H^{\text{ref, delivery}} = (h_1^{\text{ref, delivery}}, \dots, h_N^{\text{ref, delivery}})^T$ ,  $T^{\text{on}} = (t_1^{\text{on}}, \dots, t_N^{\text{on}})^T$ , and  $T^{\text{off}} = (t_1^{\text{off}}, \dots, t_N^{\text{off}})^T$ , where  $h_i^{\text{ref, delivery}} \in \mathbb{R}$ , and where in the spirit of TIO-MPC,  $t_i^{\text{on}} \in \mathbb{R}$  and  $t_i^{\text{off}} \in \mathbb{R}$  are the switching time instants such that

$$h_i^{\text{ref}}(k) = \begin{cases} h_i^{\text{ref, normal}} & \text{if } k \leq k_i^{\text{on}} \text{ or } k \geq k_i^{\text{off}}, \\ h_i^{\text{ref, delivery}} & \text{otherwise,} \end{cases} \quad (3)$$

with  $k_i^{\text{on}} = \lceil \frac{t_i^{\text{on}}}{T_m} \rceil$  and  $k_i^{\text{off}} = \lceil \frac{t_i^{\text{off}}}{T_m} \rceil$ , where  $\lceil x \rceil$  denotes the value of  $x \in \mathbb{R}$  rounded to the nearest integer assuming a round-half-up rule. Moreover,  $h_i^{\text{ref, normal}}$  is the normal operating level of the setpoint in canal reach  $i$ .

For the Coordinator to fulfill its task, the following cost function is proposed

$$J = \alpha \sum_{j=1}^{A_c N_p} (u_N(j-1) - Q_{S,\text{base}})^2 \quad (4)$$

$$+ \sum_{i=1}^N \sum_{j=1}^{A_c N_p} \left[ \gamma_1 \left( \max(h_i(j) - h_i^{\text{max, des}}, 0) \right)^2 \right. \quad (5)$$

$$\left. + \gamma_2 \left( \min(h_i(j) + h_i^{\text{min, des}}, 0) \right)^2 \right] \quad (6)$$

$$+ \sum_{i=1}^N \sum_{j=1}^{A_c N_p} \beta (h_i(j-1) - h_i^{\text{ref}}(j-1))^2 \quad (7)$$

$$+ \sum_{i=1}^N \mu \left( h_i^{\text{ref, normal}} - h_i^{\text{ref}}(N_p A_c - 1) \right)^2, \quad (8)$$

in which  $\alpha$ ,  $\gamma_1$ ,  $\gamma_2$ , and  $\beta$  are positive weighting coefficients, and  $u_N$  denotes the flow through gate  $N$ . Therefore, the Coordinator will modify the head gate and the setpoints, thus enabling speedy deliveries, in a way to minimize the disruptive effect of the Coordinator's actions on the canal when a sudden delivery request occurs.

The hard constraints are as follows:

$$h_i^{\text{min}} \leq h_i(\ell) \leq h_i^{\text{max}}, \quad (\ell = 1, \dots, N_p A_c), \quad (9)$$

$$h_i^{\text{min}} \leq h_i^{\text{ref, delivery}} \leq h_i^{\text{max}}, \quad (10)$$

$$t_i^{\text{off}} \geq t_i^{\text{on}} + T_m, \quad (11)$$

$$t_i^{\text{on}} \geq 0 \quad (12)$$

$$Q_{S,\text{demand}}(n) \geq 0, \quad (n = 0, \dots, N_p - 1), \quad (13)$$

$$0 \leq Q_S(m) \leq Q_{\text{capacity}}, \quad (m = 0, \dots, N_p A_c - 1), \quad (14)$$

for all  $i \in \{1, \dots, N\}$ . Furthermore, (3) is also treated as a hard constraint defining the possible shape of the setpoint profiles.

We study three control designs. The nominal controller

disregards the effect of possible disturbances when evaluating the cost function. In contrast, the second approach uses approximate values of the state and an approximate model when calculating state predictions. At the same time, the controller tightens the hard constraints applied to the state estimates to ensure that the control actions found by the Coordinator guarantee that the actual state remains within the feasible bounds. The third controller is based on an application of the standard min-max MPC.

#### A. Nominal controller

In the case of the nominal controller, the noise terms  $w_i(k)$  and  $v_i(k)$ ,  $i = 1, \dots, N$  are neglected in (1). Hence, the model of the canal (1) is used with  $w_i(k) = 0$  and  $v_i(k) = 0$ ,  $i = 1, \dots, N$  for all  $k$ .

The functioning of the nominal controller can be detailed as follows. The Coordinator is triggered when a new delivery needs to be accounted for. It then uses the noise-free model to find the optimal control action  $\mathcal{U}$  subject to constraints (3) and (9)–(14). Then, the optimal setpoint profiles defined by the triple  $(t_i^{\text{on}}, t_i^{\text{off}}, h_i^{\text{ref, delivery}})$ ,  $i = 1, \dots, N$ , are sent to the local sites and the optimal profile of head gate flow  $Q_S$  is set up according to  $\tilde{Q}_{S,\text{demand}}$ .

#### B. Constraint tightening controller

In this approach, the main idea, based on the results presented in [7], [16], is to tighten the constraint set (9) to accommodate for the unknown effects of the disturbances  $w_i(k)$  and  $v_i(k)$  and thus to achieve robust constraint satisfaction. We denote  $\tilde{p}_{i,j}(k) = (p_i(k-j), \dots, p_i(k))^T$  for a variable  $p_i(k)$  and  $\mathbb{1}_\alpha = (1, \dots, 1)^T \in \mathbb{R}^{\alpha \times 1}$  for a positive constant  $\alpha$ .

For simplicity, at time step  $k$  we take the estimate  $\hat{h}_i(k|k)$  to be equal to the measured output

$$\hat{h}_i(k|k) = y_i(k), \quad (15)$$

where we use the notation  $(k_1|k_2)$  to highlight that it is the estimate for step  $k_1$  using the information available at step  $k_2$ . In general  $\hat{h}_i(k|k)$  is not equal to the actual value of the state variable  $h_i(k)$  and so if we take  $\hat{h}_i(k|k) = h_i(k) + \epsilon_i(k|k)$ , in which  $\epsilon_i(k|k)$  is the estimation error, this error clearly is

$$\epsilon_i(k|k) = v_i(k). \quad (16)$$

Considering the known bounds (2), we can specify that

$$\epsilon_i(k|k) \in \mathcal{E}_i(k|k) = \mathcal{V}. \quad (17)$$

To obtain further state estimates  $h_i(k+j|k)$ ,  $i = 1, \dots, N$ ,  $j = 1, \dots, A_c N_p$ , we use the model of the canal (1) using  $\hat{h}_i(k|k)$  as the initial condition, and discount the effect of unknown disturbances, i.e. we set  $w_i(k+j|k) = 0$  and  $v_i(k+j|k) = 0$ ,  $j = 0, \dots, A_c N_p - 1$ . Moreover, for each step we check how much such an estimate may differ from the actual state. In doing so, it is assumed that known values from the past (i.e. past measured output and delayed inflow, which de facto depends on past measured output) are taken as is since they have already occurred and as such do not introduce any

uncertainty to the model. We introduce

$$e_i^*(k) = \begin{cases} y_i(k) - h_i^{\text{ref}}(k) & \text{if } k \leq k_{\text{current}}, \\ h_i(k) + v_i(k) - h_i^{\text{ref}}(k) & \text{if } k > k_{\text{current}}. \end{cases} \quad (18)$$

where  $k_{\text{current}}$  denotes the current model sample step at which the formula is calculated. Using the above variables and the recursive formula of a PI controller as in (1), we have

$$u_i^*(k+j) = \begin{cases} u_i(k) & \text{if } j = 0, \\ u_i(k) + K_{\text{PI}i}e_i^*(k+j) + \xi(j)K_{\text{I}i}\mathbb{1}_{j-1}^T\tilde{e}_{i,j-2}^*(k+1) \\ \quad - K_{\text{P}i}e_i^*(k) & \text{if } j \geq 1, \end{cases} \quad (19)$$

and similarly

$$u_{i-1}^*(k-k_{\text{di}}+j) = \begin{cases} u_{i-1}(k-k_{\text{di}}+j) & \text{if } k_{\text{di}} \geq j, \\ u_{i-1}(k) + K_{\text{PI}(i-1)}e_{i-1}^*(k-k_{\text{di}}+j) \\ \quad + \xi(j-k_{\text{di}})K_{\text{I}(i-1)}\mathbb{1}_{j-k_{\text{di}}-1}^T\tilde{e}_{i-1,j-k_{\text{di}}-2}^*(k+1) \\ \quad - K_{\text{P}(i-1)}e_{i-1}^*(k) & \text{if } k_{\text{di}} < j, \end{cases} \quad (20)$$

where  $K_{\text{PI}i} = K_{\text{P}i} + K_{\text{I}i}$  and  $\xi(j) = \min(1, j-1)$ . We start from evaluating the state predictions and approximate predictions for step  $k+1$  and obtain (cf. (1))

$$h_i(k+1) = h_i(k) + z_i(u_{i-1}(k-k_{\text{di}}) - u_i(k) + d_i(k) + g_i(k) + w_i(k)), \quad (21)$$

and

$$\hat{h}_i(k+1|k) = \hat{h}_i(k|k) + z_i(u_{i-1}(k-k_{\text{di}}) - u_i(k) + d_i(k) + g_i(k)), \quad (22)$$

where  $z_i = \frac{T_m}{c_i}$  and all terms on the right-hand sides except  $w_i(k)$  are measured or known. Consequently, the estimation error  $\epsilon_i(k+1|k) = \hat{h}_i(k+1|k) - h_i(k+1)$  is

$$\epsilon_i(k+1|k) = \epsilon_i(k|k) - w_i(k) \quad (23)$$

and thus

$$\epsilon_i(k+1|k) \in \mathcal{E}_i(k+1|k) = \mathcal{E}_i(k|k) \oplus (-1)\mathcal{W}, \quad (24)$$

where  $\oplus$  denotes the Minkowski summation ( $A \oplus B = \{a+b \mid a \in A, b \in B\}$ ). In general, using formula (1) recursively, we have for  $j = 1, \dots, A_c N_p - 1$

$$h_i(k+j+1) = h_i(k) + z_i(u_{i-1}^*(k-k_{\text{di}}+j) - u_i^*(k+j) + d_i(k+j) + g_i(k+j) + w_i(k+j)) \quad (25)$$

and

$$\hat{h}_i(k+j+1|k) = \hat{h}_i(k+j|k) + z_i(\check{u}_{i-1}^*(k-k_{\text{di}}+j) - \check{u}_i^*(k+j) + d_i(k+j) + g_i(k+j)), \quad (26)$$

where we use

$$\check{u}_i^*(k+j) = u_i(k) + K_{\text{PI}i}\hat{e}_i(k+j|k) + \xi(j)K_{\text{I}i}\mathbb{1}_{j-1}^T\tilde{e}_{i,j-2}(k+1|k) - K_{\text{P}i}e_i(k) \quad (27)$$

and

$$\check{u}_{i-1}^*(k-k_{\text{di}}+j) = \begin{cases} u_{i-1}(k-k_{\text{di}}+j) & \text{if } k_{\text{di}} \geq j, \\ u_{i-1}(k) + K_{\text{PI}(i-1)}\hat{e}_{i-1}(k-k_{\text{di}}+j|k) \\ \quad + \xi(j-k_{\text{di}})K_{\text{I}(i-1)}\mathbb{1}_{j-k_{\text{di}}-1}^T\tilde{e}_{i-1,j-k_{\text{di}}-2}(k+1|k) \\ \quad - K_{\text{P}(i-1)}e_{i-1}(k) & \text{otherwise,} \end{cases} \quad (28)$$

and  $\hat{e}_i(k_1|k_2) = \hat{h}_i(k_1|k_2) - h_i^{\text{ref}}(k_1)$ . Basically, the formulas

for  $\check{u}_i^*(k+j)$  and  $\check{u}_{i-1}^*(k-k_{\text{di}}+j)$  are the noise-free counterparts of  $u_i^*(k+j)$  and  $u_{i-1}^*(k-k_{\text{di}}+j)$ , respectively. The estimation error for each step  $(k+j+1|k)$  in the future,  $j = 1, \dots, A_c N_p - 1$ , can be determined by subtracting (25) from (26):

$$\epsilon_i(k+j+1|k) = \epsilon_i(k+j|k) + z_i(\check{u}_{i-1}^*(k-k_{\text{di}}+j) - u_{i-1}^*(k-k_{\text{di}}+j) - \check{u}_i^*(k+j) + u_i^*(k+j)), \quad (29)$$

from which we find that

$$\epsilon_i(k+j+1|k) \in \mathcal{E}_i(k+j+1|k) =: \mathcal{E}_i(k+j|k) \oplus \bar{\mathcal{E}}_i(k+j+1|k), \quad (30)$$

in which  $\bar{\mathcal{E}}_i(k+j+1|k)$  is defined as

$$\bar{\mathcal{E}}_i(k+j+1|k) = z_i(K_{\text{PI}i}(\mathcal{E}_i(k+j|k) \oplus \mathcal{V}) \oplus \xi(j)K_{\text{I}i} \bigoplus_{\ell=j-1}^1 (\mathcal{E}_i(k+\ell|k) \oplus \mathcal{V})) \oplus \mathcal{W} \oplus (-z_i) \begin{cases} 0 & \text{if } k_{\text{di}} \geq j, \\ K_{\text{PI}(i-1)}(\mathcal{E}_{i-1}(k-k_{\text{di}}+j|k) \oplus \mathcal{V}) \oplus \xi(j-k_{\text{di}})K_{\text{I}(i-1)} \bigoplus_{\ell=\ell_0}^1 (\mathcal{E}_{i-1}(k+\ell|k) \oplus \mathcal{V}) & \text{otherwise,} \end{cases} \quad (31)$$

where  $\ell_0 = -k_{\text{di}} + j - 1$  and  $\bigoplus_{j=a}^b A_j = A_a \oplus \dots \oplus A_b$ .

The way to proceed from here is to determine the optimal control actions of the Coordinator  $\mathcal{U}$  using the approximate model  $\hat{h}_i(k+j|k)$ ,  $i = 1, \dots, N$  to evaluate the cost function (4)–(8) and to optimize it subject to constraints (3) and (9)–(14). However, to compensate for the difference between the actual state predictions  $h_i(k+j)$  and the approximate state predictions  $\hat{h}_i(k+j|k)$ ,  $i = 1, \dots, N$ ,  $j = 1, \dots, A_c N_p$  and de facto ensure that the actual state remains within the feasible bounds (9), we recursively tighten (9) using the derived bounds of the estimation errors  $\epsilon_i(k+j+1|k)$ . Therefore, let us rewrite constraint (9) as  $h_i(\ell) \in \mathcal{H}_i = [h_i^{\min}, h_i^{\max}]$ . In view of the above discussion, the modified constraints for every  $i \in \{1, \dots, N\}$  are

$$\hat{h}_i(k+j+1|k) \in \hat{\mathcal{H}}_i(k+j+1|k) \quad (32)$$

$$\text{with } \hat{\mathcal{H}}_i(k|k) = \mathcal{H}_i \ominus \mathcal{E}_i(k|k), \quad (33)$$

$$\hat{\mathcal{H}}_i(k+j+1|k) = \hat{\mathcal{H}}_i(k+j|k) \ominus \bar{\mathcal{E}}_i(k+j+1|k), \quad (34)$$

where  $\ominus$  denotes the Minkowski difference ( $A \ominus B = \{x \mid x+b \in A, \forall b \in B\}$ ). This process ensures feasibility of the solution.

### C. Min-max MPC

The standard min-max MPC approach [18] aims at finding a control action  $\mathcal{U}$  for the worst possible realization of the disturbances over the whole prediction horizon given the bounds (2). Then, the optimization problem becomes:

$$\min_{\mathcal{U}} \max_{\substack{W_i(k) \\ V_i(k) \\ i=1, \dots, N}} J \quad \text{in (4)–(8)} \quad (35)$$

$$\text{subject to (3), (9)–(14) and (2),} \quad (36)$$

where the sequences  $W_i(k)$  and  $V_i(k)$  for  $i = 1, \dots, N$  are defined as  $W_i(k) = (w_i(k), \dots, w_i(k + A_c N_p - 1))^T$  and  $V_i(k) = (v_i(k), \dots, v_i(k + A_c N_p - 1))^T$ .

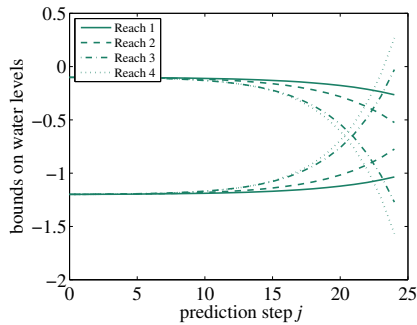


Fig. 3. Constraint profile as obtained by the constraint tightening controller.

#### D. Discussion

The robust controllers, i.e. the constraint tightening controller and the min-max MPC guarantee feasibility of the solution provided that it exists. The nominal controller does not verify whether or not the solution satisfies the state constraints. However, for a very long prediction horizon or overly stretched definitions of sets  $\mathcal{W}$  and  $\mathcal{V}$ , we may end up using the constraint tightening controller with an empty constraint set  $\hat{\mathcal{H}}_i(k+j+1|k)$  for some  $j$ , resulting in no feasible solution, whereas in reality we may still be well within the constraints.

At the same time the solution provided by the constraint tightening and min-max predictive controllers might turn out to be rather conservative, thus compromising the performance of the controller. In addition, the min-max MPC approach is known to be computationally very challenging and is executed online. Similarly, computations of the tightened constraint sets may be demanding. Yet, as this is only done once offline, the constraint tightening approach has an edge over the standard min-max MPC in that respect.

#### IV. CASE STUDY

Now we present a small numerical example to illustrate how the different controllers work and to compare their performance. For brevity, we do not explicitly write units in the text but SI units are assumed throughout this section.

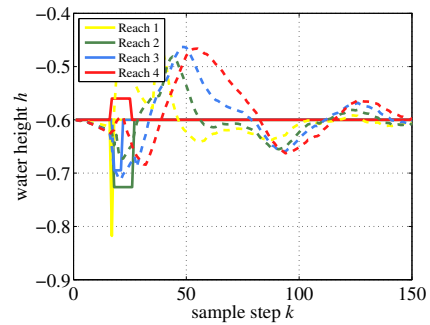
We use a virtual canal consisting of  $N = 4$  canal reaches. The parameters used in simulations are as follows:  $T_m = 180$ ,  $T_c = 3T_m$ ,  $A_c = 3$ ,  $K_{P_i} = 0.85$ ,  $K_{I_i} = 0.02$ ,  $N_p = 8$  (equivalent to 24 sample steps of the model),  $\alpha = 21$ ,  $\beta = 4$ ,  $\gamma_1 = \gamma_2 = 1$ ,  $\mu = 6$ ,  $c_1 = 740$ ,  $c_2 = 843$ ,  $c_3 = 730$ ,  $c_4 = 867$ ,  $k_{d1} = k_{d3} = k_{d4} = 1$ ,  $k_{d2} = 2$ ,  $\forall i: h_i^{\text{ref,normal}} = -0.6$ ,  $h_i^{\text{min}} = -1.2$ ,  $h_i^{\text{max}} = -0.1$ , and  $Q_{\text{capacity}} = 2$ . Moreover, the disturbances are modeled as random numbers satisfying  $-0.005 \leq w_i(k) \leq 0.005$  and  $-0.005 \leq v_i(k) \leq 0.005$ .

The delivery request used in the simulations occurs in Reach 3 and has the magnitude of 0.1. It takes place from  $k = 18$  to  $k = 27$ .

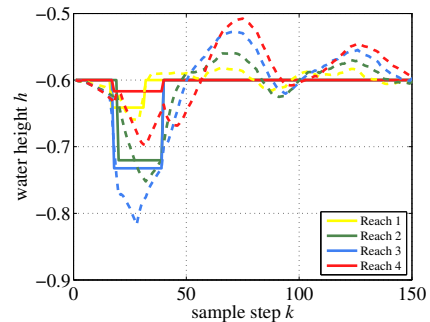
To compare the performance of the controllers, we consider the a posteriori performance index  $J_{\text{post}}$  defined as

$$J_{\text{post}} = \alpha \sum_{k=1}^{T_f/T_m} (u_N(k) - Q_{S,\text{base}})^2 + \sum_{i=1}^N \sum_{k=1}^{T_f/T_m} \beta (e_i(k))^2, \quad (37)$$

where  $T_f = 50T_c$  is the duration of the simulation.



(a)



(b)

Fig. 4. Water levels (dashed line) and setpoints (solid line) for all reaches for (a) the nominal controller and (b) min-max controller.

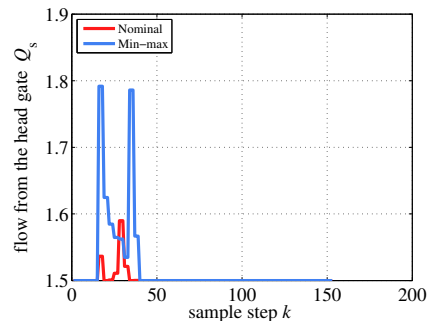


Fig. 5. Flow from the head gate.

We start by discussing the constraint tightening controller. As it turns out, because of the particular nature of the tightening process, the method proves unsuitable for the given  $N_p$ . The reasons for that is the open-loop implementation and the cascaded nature of an irrigation canal, which imply that the constraint tightening yields too much accumulation of uncertainty for further reaches and thus the constraint set is rendered empty after 21 prediction steps, see Figure 3. Another factor contributing to the constraint set ending up being empty, is the fact that with an event-driven controller even for a small case study like this one, the prediction horizon needs to be rather long since the optimization problem is not solved in the receding horizon manner as it is customary in MPC, but only once per delivery.

The other two methods, the nominal and min-max controllers, do not suffer from such a problem. We present the evolution of the water levels and the corresponding setpoints in all reaches in Figure 4 and in Figure 5 we depict the inflow from the head gate as found by the Coordinator in the case

TABLE I  
PERFORMANCE COMPARISON

$N_p$	nominal	constraint tightening	min-max
8	8.88	–	14.80
6	9.33	10.41	17.90

of the nominal and min-max controllers. The values of the performance indicator  $J_{\text{post}}$  are given in Table I. It is seen that the min-max controller performs worse than the nominal controller, which can be explained by the fact that the min-max controller is inclined to choose more conservative and hence more performance-compromising control actions than the nominal controller, which assumes no disturbances. However, the nominal controller does not necessarily provide a feasible solution in the presence of disturbances.

For the sake of completeness, we also consider a prediction horizon of  $N_p = 6$  (i.e. 18 sample steps), which is short enough to prevent the feasible sets  $\mathcal{H}_i$  from becoming empty. The corresponding solutions are given in Table I. Again, the nominal controller outperforms the other two, but the constraint tightening robust controller yields a similar value of  $J_{\text{post}}$ . Yet, with larger disturbances, the performance of the nominal controller is likely to deteriorate and in particular a control action returned by the solver as feasible may disobey the constraints in reality. At the same time, the two discussed robust designs may not be able to find a feasible solution altogether because of their conservative approach. This indeed advocates the analysis of more flexible control methods that would result in a satisfactory performance with little computation effort.

## V. CONCLUSIONS

We have introduced a hierarchical event-driven controller that enables speedy water deliveries to users through an irrigation canal. The hierarchical controller consists of local upstream PI controllers in the lower control layer, and a higher-layer controller coordinating the local controllers and acting in response to delivery requests only, therefore necessitating only minimal communication. This is an important feature as in the field of irrigation communication is considered expensive.

We have examined and compared three different set-ups to deal with the uncertainties: the nominal controller, the constraint tightening controller and the min-max controller, all based on MPC. It has been shown that in the particular set-up considered in the paper, the nominal controller outperforms the two other strategies but without guaranteeing feasibility, while the two robust controllers resulted in more cautious control actions. As it is expected that less conservatism can be achieved, without compromising on the feasibility guarantee, if the specific nature of the disturbances is analyzed in more depth, in the future we want to examine other methods to deal with model uncertainties, e.g. examining explicitly the particular stochastic nature of the disturbances [3]–[5].

## VI. ACKNOWLEDGMENTS

Research supported by the European Union Seventh Framework Programme [FP7/2007-2013] under grant agreement no. 257462 HYCON2 Network of Excellence.

## REFERENCES

- [1] G. Bastin, A. Bayen, C. D'Apice, X. Litrico, and B. Piccoli, "Open problems and research perspectives for irrigation channels," *Networks and Heterogeneous Media*, vol. 4, no. 2, pp. i–v, 2009.
- [2] A. Bemporad and M. Morari, "Robust Model Predictive Control: A Survey," in *Robustness in Identification and Control*. Springer Berlin Heidelberg, 1999, vol. 245, pp. 207–226.
- [3] D. Bernardini and A. Bemporad, "Stabilizing model predictive control of stochastic constrained linear systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 6, pp. 1468–1480, 2012.
- [4] G. Calafiore and L. Fagiano, "Robust model predictive control via scenario optimization," *IEEE Transactions on Automatic Control*, vol. 58, no. 1, pp. 219–224, 2013.
- [5] M. Cannon, B. Kouvaritakis, S. Rakovic, and Q. Cheng, "Stochastic tubes in model predictive control with probabilistic constraints," *IEEE Transactions on Automatic Control*, vol. 56, no. 1, pp. 194–200, 2011.
- [6] M. Cantoni, E. Weyer, Y. Li, S. K. Ooi, I. Mareels, and M. Ryan, "Control of large-scale irrigation networks," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 75–91, 2007.
- [7] L. Chisci, J. Rossiter, and G. Zappa, "Systems with persistent disturbances: predictive control with restricted constraints," *Automatica*, vol. 37, no. 7, pp. 1019–1028, 2001.
- [8] V. T. Chow, *Open-Channel Hydraulics*, ser. McGraw-Hill Civil Engineering. London: McGraw-Hill, 1959.
- [9] Y. Li and B. De Schutter, "Stability and performance analysis of an irrigation channel with distributed control," *Control Engineering Practice*, vol. 19, no. 10, pp. 1147–1156, Oct. 2011.
- [10] X. Litrico, V. Fromion, J.-P. Baume, and M. Rijo, "Modelling and PI controller design for an irrigation canal," in *Proceedings of the 2003 European Control Conference, Cambridge, UK, 2003*.
- [11] J. M. Maciejowski, *Predictive Control with Constraints*. Essex, England: Prentice Hall, 2002.
- [12] L. Magni and R. Scattolini, "Robustness and robust design of MPC for nonlinear discrete-time systems," in *Assessment and Future Directions of Nonlinear Model Predictive Control*. Springer Berlin Heidelberg, 2007, pp. 239–254.
- [13] P.-O. Malaterre and J.-P. Baume, "Modeling and regulation of irrigation canals: existing applications and ongoing researches," in *Proceedings of the 1998 IEEE International Conference on Systems, Man, and Cybernetics, San Diego, CA, vol. 4, 1998*, pp. 3850–3855.
- [14] D. Mayne, J. Rawlings, C. Rao, and P. Sokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, vol. 36, no. 6, pp. 789–814, 2000.
- [15] R. R. Negenborn, S. Leirens, B. De Schutter, and J. Hellendoorn, "Supervisory nonlinear MPC for emergency voltage control using pattern search," *Control Engineering Practice*, vol. 17, no. 7, pp. 841–848, July 2009.
- [16] A. Richards and J. How, "Robust model predictive control with imperfect information," in *Proceedings of the 2005 American Control Conference, Portland, OR, 2005*, pp. 268–273.
- [17] J. Schuurmans, "Control of water levels in open-channels," Ph.D. dissertation, Delft University of Technology, The Netherlands, 1997.
- [18] P. Sokaert and D. Mayne, "Min-max feedback model predictive control for constrained linear systems," *IEEE Transactions on Automatic Control*, vol. 43, no. 8, pp. 1136–1142, 1998.
- [19] H. van Ekeren, R. R. Negenborn, P. J. van Overloop, and B. De Schutter, "Hybrid model predictive control using time-instant optimization for the Rhine-Meuse delta," in *Proceedings of the 2011 IEEE International Conference on Networking, Sensing and Control, Delft, The Netherlands, 2011*, pp. 216–221.
- [20] P.-J. van Overloop, A. Clemmens, R. Strand, and R. Wagemaker, "Real-time implementation of model predictive control on MSIDD's WM canal," *Journal of Irrigation and Drainage Engineering-ASCE*, vol. 136, no. 11, pp. 747–756, 2010.
- [21] P.-J. van Overloop, J. Schuurmans, R. Brouwer, and C. Burt, "Multiple-model optimization of proportional integral controllers on canals," *Journal of Irrigation and Drainage Engineering-ASCE*, vol. 131, no. 2, pp. 190–196, 2005.
- [22] E. Weyer, "Control of irrigation channels," *IEEE Transactions on Control Systems Technology*, vol. 16, no. 4, pp. 664–675, 2008.
- [23] A. Zafra-Cabeza, J. M. Maestre, M. A. Ridao, E. F. Camacho, and L. Sanchez, "Hierarchical distributed model predictive control for risk mitigation: An irrigation canal case study," in *Proceedings of the 2011 American Control Conference, San Francisco, CA, 2011*, pp. 3172–3177.