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Tractable Robust Predictive Control Approaches for Freeway Networks

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Bart De Schutter, Anna Sadowska, Hans Hellendoorn, Eduardo F. Camacho

Abstract—Robust control aims to maintain predefined performance specifications for a wide range of uncertainties. In this paper, we consider the robust control problem for freeway networks, including the uncertainties explicitly in the control design step. We use min-max scheme for handling the uncertainties occurring in freeway networks. In order to reduce the computational complexity of min-max scheme, we propose scenario-based min-max Model Predictive Control (MPC) and scenario-based Receding-Horizon Parametrized Control (RHPC) in this paper, which solve the complete robust problem approximately. In addition, a new objective function is proposed to ensure the satisfaction of queue length constraints. A case study is implemented to assess the effectiveness of the proposed approaches. The results show that nominal MPC and nominal RHPC may result in a better performance than scenario-based min-max MPC and scenario-based min-max RHPC. However, nominal MPC and nominal RHPC cannot ensure the satisfaction of the queue length constraint. By applying scenario-based min-max MPC and scenario-based min-max RHPC, the queue length constraint is satisfied conservatively at the cost of an increase in the performance index.

I. INTRODUCTION

With the increasing load of freeway networks, traffic congestion becomes a critical problem, leading to waste of time, higher risk of accidents, environmental problems, and so on. Many methods have been proposed in traffic management for reducing congestion, and on-line model-based traffic control is one of the most popular approaches in the literature [1–3]. Nonlinear Model Predictive Control (MPC) [4] is a model-based control approach that has been successfully tested in simulations of traffic systems. However, in real-life traffic networks there are various types of uncertainties or disturbances, such as demand uncertainties, model uncertainties, missing samples, sensor errors, and delays. Including these uncertainties when determining control strategies offers a significant potential for obtaining a better control performance. Hence, it is important to develop robust MPC approaches for traffic networks that maintain performance specifications for a given range of uncertainties.

In general, there are two ways to ensure robustness in nonlinear MPC: one way is based on Lyapunov functions

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and the other on exploiting the optimality of MPC controllers. Regarding the methods based on Lyapunov functions, the basic idea is to guarantee that the cost function is a strictly decreasing Lyapunov function considering also the effect of uncertainties [5, 6]. Another way to ensure robustness is based on the optimality of MPC controllers, and it consists in explicitly considering the uncertainties in the MPC design step. The inclusion of the effects of uncertainties in the design of MPC controllers is a complex problem because it is essential to deal not only with the optimality of performance, but also with constraints, for the whole set of possible uncertainties. Various robust control schemes that include uncertainties in the MPC design have been proposed in the literature. One of the most used is the min-max approach, which considers the effect of uncertainties in the worst case scenario [7, 8]. This way of accounting for uncertainties is intuitive; however, the solutions obtained are conservative due to the worst case assumption.

Robust control approaches for traffic have mainly been developed for urban networks, which are quite different from freeway networks. In [9], a robust optimal traffic signal control approach is proposed where the future demand is assumed to be uncertain. A robust dynamic system optimal model with an embedded cell transmission model is formulated, and numerical analysis is performed on a test network to illustrate the benefits of accounting for uncertainty and robustness. In [10], an efficient min-max MPC approach for urban networks is proposed to obtain green time combinations that minimize the objective function that corresponds to the worst-case scenarios. This approach is able to explicitly handle norm-bounded traffic modeling uncertainty, and has been proved to be an appropriate choice for traffic control in uncertain urban networks. In [11], the near-Bayes near-Minimax strategy is proposed for robust traffic signal control for an urban network where the uncertainty in the origin-destination (OD) demands is considered. A good compromise solution between two different solutions is obtained: one is the Bayes case in which a probability density is assumed on the possible OD matrices, and the second is the Minimax solution, which minimizes the worst possible costs. This results in a conservative approach with a risk-averse performance.

Some control approaches for freeway networks that take robustness into account are also available in literature. In [12], fuzzy intervals are adopted to model traffic measurements, since prediction methods do not provide all

the information about the traffic dynamics in every scenario of a real freeway network. In [13], an optimal control approach for freeway networks is given by solving a set of recursive coupled Riccati difference equations. However, robustness is considered by using a min-max scheme, aiming to reduce the effect of disturbances, and to act robustly with respect to parameter misidentification.

In this paper, we develop two robust control approaches for freeway networks: scenario-based min-max Model Predictive Control (MPC) and scenario-based min-max Receding-horizon Parametrized Control (RHPC). The min-max optimization problem is solved by searching within a limited number of disturbance scenarios for the maximum objective function value. In addition, a new way to include queue length constraints and other constraints on traffic states is proposed to ensure the satisfaction of this constraint.

This paper is organized as follows. In Section II, we describe the basic concepts of MPC and robust MPC. In Section III, we present the traffic flow model METANET that is used for MPC in this paper. In Section IV we analyze the disturbances appearing in freeway networks, and in Section V we design two different robust MPC approaches for freeway networks. A case study is implemented to assess the efficiency of the proposed approaches in the Section VI.

II. ROBUST MPC BACKGROUND

A. Model Predictive Control

Model Predictive Control (MPC) [4] can be used for on-line traffic management, considering its capability to deal with nonlinear systems, multi-criteria optimization, and constraints. Assume that the model of traffic networks is a discrete-time nonlinear system in the following form:

$$x(k+1) = f(x(k), u(k), D(k)) \quad (1)$$

where k is the discrete time step counter, corresponding to the time instant $t = kT$ with T the simulation time step, x is the system state (e.g. flow, density, and speed), u is the control input (e.g. ramp metering rate and variable speed limit), and D is the non-controllable input (e.g. demand).

There are two main elements in MPC: dynamic model prediction and a receding horizon scheme. The essence of dynamic model prediction is that the future performance of the controlled system is included in the objective function $J(\tilde{x}(k), \tilde{u}(k), \tilde{D}(k))$ of the closed-loop control problem. The variables in the objective function are respectively the prediction $\tilde{x}(k)$ of the states over the prediction period with the length of N_p , the control inputs $\tilde{u}(k)$ over the control period with the length of N_c , and the non-controllable input $\tilde{D}(k)$ over the prediction period:

$$\tilde{x}(k) = [x^T(k+1), \dots, x^T(k+N_p)]^T \quad (2)$$

$$\tilde{u}(k) = [u^T(k), \dots, u^T(k+N_c-1)]^T \quad (3)$$

$$\tilde{D}(k) = [D^T(k), \dots, D^T(k+N_p-1)]^T \quad (4)$$

where $\tilde{x}(k)$ is the future state over the prediction period N_p , the control input $u(k+l)$ equals to $u(k+N_c-1)$ for $l = k+N_c, \dots, k+N_p$, and the non-controllable input $\tilde{D}(k)$ is assumed to be known.

The MPC problem at time step k can be formulated as a nonlinear optimization problem:

$$\begin{aligned} & \min_{\tilde{u}(k), \tilde{x}(k)} J(\tilde{x}(k), \tilde{u}(k), \tilde{D}(k)) \\ & \text{subject to} \\ & x(k+l+1) = f(x(k+l), u(k+l), D(k+l)), \\ & \quad \quad \quad l = 0, 1, \dots, N_p - 1, \\ & x(k) = x_k, \\ & u(k+l) = u(k+N_c-1), \quad l = N_c, \dots, N_p - 1, \\ & x(k+l) \in \mathbb{X}, \quad l = 1, 2, \dots, N_p, \\ & u(k+l) \in \mathbb{U}, \quad l = 0, 1, \dots, N_c - 1 \end{aligned} \quad (5)$$

where x_k is the state at time step k , \mathbb{X} is the set containing all the feasible state values, \mathbb{U} contains all the feasible control inputs values, and both \mathbb{X} and \mathbb{U} are determined by the physical and operational conditions of the controlled system. Solving this nonlinear optimization problem gives an optimal control sequence. The receding horizon scheme adapted in MPC means that only the first element $u(k)$ of optimal input sequence $\tilde{u}(k)$ is applied to the controlled system. Then the prediction horizon is moved to the next control step, and the control inputs are optimized again.

B. Robust Model Predictive Control

The non-controllable inputs are usually assumed to be equal to known nominal values when predicting the future evolution of the system states. In case the real values of the non-controllable inputs and the nominal values are significantly different, the performance of MPC will be reduced.

Consider now the inclusion of uncertainties $\omega(k)$ in the dynamic evolution of the discrete-time nonlinear system, as follows:

$$x(k+1) = f_\omega(x(k), u(k), D(k), \omega(k)) \quad (6)$$

with $x(k)$, $u(k)$, and $D(k)$ as in (1), and where $\omega(k)$ is the vector representing uncertainties. The predicted trajectories given by the nonlinear model will depend on the realization $\tilde{\omega}(k)$ of the uncertainties over the prediction period with

$$\tilde{\omega}(k) = [\omega^T(k), \dots, \omega^T(k+N_p-1)]^T \quad (7)$$

As the numerical solution of the robust MPC problem is computationally very intensive, and the realization of the uncertainties $\tilde{\omega}(k)$ is not previously known, different approaches can be used to approximate the original robust MPC problem. One of these approaches is the min-max approach, in which the optimization of the cost is based on considering the possible worst-case scenario. Therefore, the min-max formulation [7] is

$$\min_{\bar{u}(k), \bar{x}(k)} \max_{\bar{\omega}(k) \in \mathbb{W}} \{J_{\omega}(\bar{x}(k), \bar{u}(k), \bar{D}(k), \bar{\omega}(k))\} \quad (8)$$

subject to

$$x(k+l+1) = f_{\omega}(x(k+l), u(k+l), D(k+l), \omega(k+l)), \quad l = 0, 1, \dots, N_p - 1, \quad (9)$$

$$x(k) = x_k, \quad (10)$$

$$u(k+l) = u(k+N_c-1), \quad l = N_c, \dots, N_p - 1, \quad (11)$$

$$x(k+l) \in \mathbb{X}, \quad l = 1, 2, \dots, N_p, \quad (12)$$

$$u(k+l) \in \mathbb{U}, \quad l = 0, 1, \dots, N_c - 1 \quad (13)$$

where \mathbb{W} represents the set of all the possible realizations of uncertainties over the prediction horizon.

III. TRAFFIC FLOW MODEL: METANET

In this paper, we choose METANET as a prediction model for traffic flow, note however that other models can be used as well. The METANET model [14] is a second-order macroscopic traffic flow model, in which links are used for representing freeway stretches, and nodes are used for representing on-ramps, off-ramps, and changes in geometry. Each link (denoted by m) is divided into several segments (denoted by i) of equal length L_m (e.g. 1 km), and each segment is characterized by basic traffic states: traffic outflow of vehicles $q_{m,i}(k)$ (veh/h), segmental density of vehicles $\rho_{m,i}(k)$ (veh/km/h), and space-mean speed of vehicles $v_{m,i}(k)$ (km/h).

The evolution of the basic traffic states is described as

$$q_{m,i}(k) = \rho_{m,i}(k)v_{m,i}(k)\lambda_m \quad (14)$$

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_m\lambda_m}(q_{m,i-1}(k) - q_{m,i}(k)) \quad (15)$$

$$v_{m,i}(k+1) = v_{m,i}(k) + \frac{T}{\tau}(V(\rho_{m,i}(k)) - v_{m,i}(k)) + \frac{T}{L_m}v_{m,i}(k)(v_{m,i-1}(k) - v_{m,i}(k)) - \frac{T\eta}{L_m\tau} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa} \quad (16)$$

$$V(\rho_{m,i}(k)) = v_{\text{free},m} \exp \left[-\frac{1}{a_m} \left(\frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}} \right)^{a_m} \right] \quad (17)$$

where λ_m is the number of lanes in link m , τ , η , κ , and a_m are model parameters, $V(\rho_{m,i}(k))$ is the desired speed at density $\rho_{m,i}(k)$, $v_{\text{free},m}$ is the free flow speed, $\rho_{\text{crit},m}$ is the critical density.

In addition to the original model, a way to include variable speed limit is as follows [1]:

$$V(\rho_{m,i}(k)) = \min \left(v_{\text{free},m} \exp \left[-\frac{1}{a_m} \left(\frac{\rho_{m,i}(k)}{\rho_{\text{crit},m}} \right)^{a_m} \right], (1 + \alpha)v_{\text{control},m,i}(k) \right) \quad (18)$$

in which $1 + \alpha$ is a non-compliance factor, and $v_{\text{control},m,i}(k)$ is the speed limit of segment i of link m .

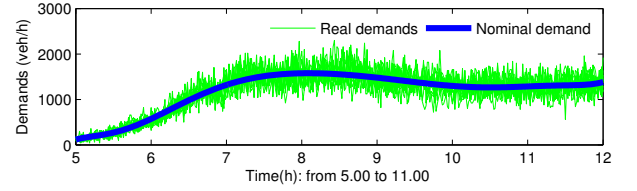


Fig. 1: Real demands and nominal demand

The outflow q_o of an on-ramp origin o is determined by

$$q_o(k) = \min \left[d_o(k) + \frac{w_o(k)}{T}, C_o r_o(k), C_o \left(\frac{\rho_{\text{max},m} - \rho_{m,1}(k)}{\rho_{\text{max},m} - \rho_{\text{crit},m}} \right) \right] \quad (19)$$

in which C_o is the capacity flow of on-ramp origin o , $\rho_{\text{max},m}$ is the maximum density of the link m that is connected to the on-ramp, and $r_o \in [0, 1]$ is the on-ramp metering rate.

Both the queue length at a mainstream origin and the queue length at an on-ramp origin are described by a simple queue model:

$$w_o(k+1) = w_o(k) + T(d_o(k) - q_o(k)) \quad (20)$$

where $w_o(k)$ is the queue length, and $d_o(k)$ is the origin demand.

IV. DISTURBANCE ANALYSIS FOR FREEWAY NETWORKS

In MPC for freeway networks, it is important to appropriately include non-controllable inputs (\bar{D}). These inputs include the origin demands, the on-ramp demands, and for METANET also the upstream speed for the first segment and the downstream density for the last segment. These inputs are used for predicting the future traffic states in MPC for freeway networks. Here we only focus on the uncertainty in the traffic demand.

In conventional MPC for freeway networks, the nominal demand is used for predicting the future evolution of traffic states. The estimation of the nominal demand can be done in several ways. One easy way is to use the average of historical demand profiles as the nominal demand for the future. It is also possible to scale or shift the nominal demand according to on-line measurements in order to get a better estimation. Here we consider a setting where the average of historical demand profiles is used as the nominal demand for the future.

Fig. 1 shows the real mainstream demands of the Dutch A13 freeway near Rijswijk on several Fridays in 2013 and 2014 and the nominal demand that is generated from these real demands. All the real demands are plotted with the same color, and the nominal demand is plotted with a different color. We can see that most of the demand values fall within a certain confidence band around the nominal demand profile.

V. ROBUST MPC FOR FREEWAY NETWORK

In MPC control for freeway networks, one often uses the Total Time Spent (TTS) as performance index. Here we use TTS as the performance index, and METANET model is used as the prediction model. We choose Ramp Metering

(RM) and Variable Speed Limits (VSL) as control measures, because they are easy to implement, and they can result in a large decrease in TTS. The objective function without considering the robustness is

$$\begin{aligned}
J(k_c) = & \xi_{\text{TTS}} \frac{\text{TTS}(k_c)}{\text{TTS}_{\text{nom}}} \\
& + \xi_{\text{ramp}} \sum_{j=k_c}^{k_c+N_c-1} \sum_{o \in O_{\text{ramp}}} (r_o^{\text{ctrl}}(j) - r_o^{\text{ctrl}}(j-1))^2 \\
& + \xi_{\text{speed}} \sum_{j=k_c}^{k_c+N_c-1} \sum_{(m,i) \in I_{\text{speed}}} \left(\frac{v_{m,i}^{\text{ctrl}}(j) - v_{m,i}^{\text{ctrl}}(j-1)}{v_{\text{free},m}} \right)^2 \quad (21)
\end{aligned}$$

where k_c is the control time step counter, corresponding to time instant $t = k_c T_c$ with T_c the control time step, TTS_{nom} is the value of the TTS for some nominal control profile, ξ_{TTS} is the positive weight for the normalized TTS, $r_o^{\text{ctrl}}(k)$ is the ramp metering rate at origin o at control step k_c with $r_o(k) = r_o^{\text{ctrl}}(k_c)$ for $k = Mk_c + 1, \dots, (M+1)k_c$, $v_{m,i}^{\text{ctrl}}(k)$ is the variable speed limit in segment i of link m at control step k_c with $v_{\text{control},m,i}(k) = v_{m,i}^{\text{ctrl}}(k_c)$ for $k = Mk_c + 1, \dots, (M+1)k_c$, where $M = T_c/T$ is assumed to be an integer, the second term and the third term in (21) penalize the variations of the control inputs with ξ_{ramp} and ξ_{speed} as corresponding nonnegative weights, O_{ramp} represents all metered origins, and I_{speed} represents all the segments with speed limits.

The TTS can be defined as follows:

$$\text{TTS}(k_c) = T \sum_{j=k_c M}^{(k_c+N_p)M-1} \left(\sum_{(m,i) \in I_{\text{all}}} \rho_{m,i}(j) L_m \lambda_m + \sum_{o \in O_{\text{all}}} w_o(j) \right) \quad (22)$$

where I_{all} is the set of indices of all pairs of segments and links, and O_{all} is the set of indices of all origins. The traffic state variables, which are necessary for computing the objective function (21), are predicted using a nominal demand profile (or an on-line updated version of this profile) in nominal MPC. However, if the demands are constantly very different from the predicted values, the performance of the controller is expected to be reduced. The idea is to design an MPC controller to compute a control input that will improve the behavior of the controlled system by taking the uncertainties into account. In order to limit the computational burden, we consider a scenario-based min-max scheme. Based on this scheme, we propose two tractable robust approaches for freeway networks.

A. Scenario-Based Min-Max MPC

We can apply the min-max MPC for freeway networks. Here we give a way to realize the min-max approach¹:

$$\begin{aligned}
J(\tilde{x}(k_c), \tilde{u}(k_c), \tilde{D}(k_c)) = \\
\max_{\tilde{\omega}(k_c) \in \tilde{\Omega}} \{ J_{\omega}(\tilde{x}(k_c), \tilde{u}(k_c), \tilde{D}(k_c), \tilde{\omega}(k_c)) \} \quad (23)
\end{aligned}$$

¹Note that in Section II we assume $T_c = T$. Now we consider the general case with $T_c \neq T$.

where the set $\tilde{\Omega} = \{\tilde{\omega}_1, \dots, \tilde{\omega}_H\} \subset \mathbb{W}$ comprises H possible scenarios that will be considered for the control design. This set can be constructed by building a library of possible disturbance profiles.

In freeway networks, the queue lengths at on-ramps are often constrained. Queue length and equity of ramp metering schemes based on the queue length management and maximum queue constraint are discussed in [15]. In this paper, we adopt a queue length penalty which is a kind of soft constraint like in [16, 17] to ensure that queue lengths exceeding the permissible maximum queue lengths are penalized and queue length constraints are satisfied as much as possible under uncertainties. The objective function of the closed-loop control problem is defined as

$$\begin{aligned}
J(\tilde{x}(k_c), \tilde{u}(k_c), \tilde{D}(k_c)) = \max_{\tilde{\omega}(k_c) \in \tilde{\Omega}} \left\{ J_{\omega}(\tilde{x}(k_c), \tilde{u}(k_c), \tilde{D}(k_c), \tilde{\omega}(k_c)) \right. \\
\left. + \gamma \sum_{o \in O_{\text{ramp}}} \max \left(\frac{\max_{j=k_c M}^{(k_c+N_p)M-1} w_o(j)}{w_{\text{max},o}} - 1, 0 \right) \right\} \quad (24)
\end{aligned}$$

where $w_{\text{max},o}$ is the permitted maximum queue length at on-ramp o , and the weight γ is a large positive number to ensure that the queue length constraint is satisfied for all $\tilde{\omega}(k_c)$. By including the queue length penalty, we can ensure that J is optimized only when the constraint is not violated. The inner max operator of the queue length penalty is used to impose a soft constraint on the queue length and thus to only penalize queue lengths exceeding the maximum permissible lengths. Note that in this newly proposed scheme, state constraint (12) is moved into the objective function (24).

B. Scenario-Based Min-max Receding-Horizon Parametrized Control (RHPC)

The RHPC approach is developed by Zegeye et al. [18] based on the receding-horizon control scheme and parametrized control law, in which the parameters are optimized instead of full input sequence $\tilde{u}(k_c)$. The variable speed limit and the ramp metering rate are defined as follows:

$$\begin{aligned}
v_{m,i}^{\text{ctrl}}(k_c + 1) = & \theta_{m,0}(k_c) v_{\text{free},m} + \theta_{m,1}(k_c) \frac{v_{m,i+1}(k_c) - v_{m,i}(k_c)}{v_{m,i+1}(k_c) + \kappa_v} \\
& + \theta_{m,2}(k_c) \frac{\rho_{m,i+1}(k_c) - \rho_{m,i}(k_c)}{\rho_{m,i+1}(k_c) + \kappa_\rho} \quad (25)
\end{aligned}$$

$$r_o^{\text{ctrl}}(k_c + 1) = r_o^{\text{ctrl}}(k_c) + \theta_{m,3}(k_c) \frac{\rho_{\text{crit},m} - \rho_{m,1}(k_c)}{\rho_{\text{crit},m}} \quad (26)$$

in which κ_v and κ_ρ small positive values to prevent the divisors to be 0, and $\theta_{m,0}$, $\theta_{m,1}$, $\theta_{m,2}$, and $\theta_{m,3}$ are the parameters to be optimized.

In order to limit variable speed limit and ramp metering rate within respective lower bound and upper bound, we propose to apply the following scheme:

$$v_{m,i}^{\text{ctrl}}(k_c + 1) = \max(\min(v_{m,i}^{\text{ctrl}}(k_c + 1), v_{\text{max}}), v_{\text{min}}) \quad (27)$$

$$r_o^{\text{ctrl}}(k_c + 1) = \max(\min(r_o^{\text{ctrl}}(k_c + 1), r_{\text{max}}), r_{\text{min}}) \quad (28)$$

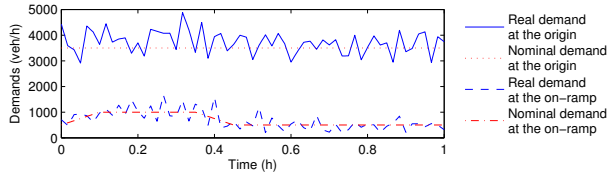


Fig. 2: Real demands and nominal demands used for case study

where v_{\max} the maximum speed, v_{\min} the minimum speed, r_{\max} is the maximum ramp metering rate, and r_{\min} is the minimum ramp metering rate.

VI. SIMULATION EXPERIMENT

A. Analyzed Network

A benchmark network reported in [1] is chosen as case study. This network consists of two links, one origin, one on-ramp, and one destination. The lengths of link 1 and link 2 are respectively 4 km and 1 km. Both links consists of two lanes, and are divided into homogeneous segments with the length of 1 km. The origin connects to the main road, the single-lane on-ramp is located in between link 1 and link 2, and the destination has unrestricted outflow. Variable speed limits are applied in the third and fourth segments of link 1, and the on-ramp is metered.

The model parameters are taken from [1]: $\tau=18$ s, $\kappa=40$ veh/km/lane, $\eta=60$ km²/h, $\rho_{\max}=180$ veh/km/lane, $a_1=a_2=1.867$, $v_{\text{free},1}=v_{\text{free},2}=102$ km/h, $\rho_{\text{crit},1}=\rho_{\text{crit},2}=102$ veh/km/lane, and $\alpha=0.1$.

The queue length at O_2 is assumed to be limited to 100 veh to avoid spill-back to a surface street intersection. As for the control parameters, we select $\xi_{\text{TTS}}=1$, $\xi_{\text{ramp}}=\xi_{\text{speed}}=0.001$, $T=10$ s, $T_c=60$ s, $N_p=7$, $N_c=5$, and $\gamma=100$.

The nominal demand profiles and the real demand profiles are shown in Fig. 2. The real demand is generated by adding disturbances to the nominal demand. The disturbances are generated from the real data of A13 freeway network in the Netherlands.

B. Results

Six different control approaches are implemented: MPC using real demands for predicting, nominal MPC, scenario-based min-max MPC, RHPC using real demands for predicting, nominal RHPC, and scenario-based min-max RHPC. The control problem here is a nonlinear optimization problem, and we solve it with sequential quadratic programming. In scenario-based min-max MPC and scenario-based min-max RHPC, it is assumed that there are 10 random noise scenarios subjecting to the lower bound and upper bound of all probable disturbances. The corresponding results are shown in Table 1.

According to Table 1, when real demands are used for predicting in MPC and RHPC, TTS can be optimized with queue length constraint satisfied. Nominal MPC and nominal RHPC result in a similar TTS with MPC and RHPC using real demands. However, the queue length

Table 1 Simulation results

Scenario	TTS (veh-h)	TTS _{improve}	Maximum queue length (veh)
No control	594.3		0
MPC + Real demands	554.6	6.7%	100
Nominal MPC	557.5	6.2%	113
Min-max MPC	574.3	3.4%	97
RHPC + Real demands	562.7	5.3%	100
Nominal RHPC	567.5	1.4%	33

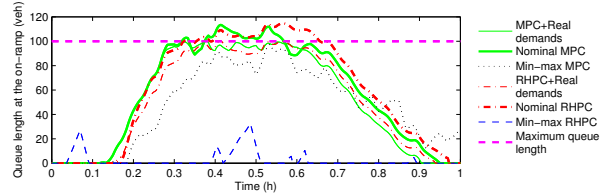


Fig. 3: Queue length at the on-ramp

constraint cannot be satisfied in nominal MPC and nominal RHPC. For scenario-based min-max MPC and scenario-based min-max RHPC, the queue length at the on-ramp always stays below 100 veh. Note however that the TTS is larger in comparison with nominal MPC and nominal RHPC. This is caused by the fact that the real demands do not always lead to the worst case, and the scenario-based min-max approach is conservative. The queue length curves for different control scenarios are shown in Fig. 3.

VII. CONCLUSIONS

In this paper, we have developed scenario-based min-max MPC and scenario-based min-max RHPC for freeway networks. Thereby, we have mainly considered the disturbance in the traffic demands. In these approaches, we have optimized the worst-case scenario over a limited number of disturbance scenarios. Moreover, an objective function is proposed to ensure that queue length constraints are satisfied as much as possible. A case study is implemented to assess the effectiveness of the proposed approaches. The results show that for the given settings nominal MPC and nominal RHPC result in a smaller TTS than scenario-based min-max MPC and scenario-based min-max RHPC. For the given case study, nominal MPC and nominal RHPC cannot ensure that the queue length constraint at the on-ramp is always satisfied; however, this constraint are satisfied by scenario-based min-max MPC and scenario-based min-max RHPC.

In the future, we will focus on probabilities of disturbance scenarios and statistical among them; other disturbances such as turning rates at junctions will be considered. We will also explore other more suitable parametrized control laws, simulate more scenarios and layouts, compare with queue override scheme, and develop new robust approaches, including robust approaches that are better in avoiding congestion than nominal approaches.

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REFERENCES

- [1] A. Hegyi, B. De Schutter, and H. Hellendoorn. Model predictive control for optimal coordination of ramp metering and variable speed limits. *Transportation Research Part C*, 13(3):185–209, 2005.
- [2] J. Yan and R. R. Bitmead. Incorporating state estimation into model predictive control and its application to network traffic control. *Automatica*, 41(4):595–604, 2005.
- [3] L. B. de Oliveira and E. Camponogara. Multi-agent model predictive control of signaling split in urban traffic networks. *Transportation Research Part C: Emerging Technologies*, 18(1):120–139, 2010.
- [4] E. F. Camacho and C. Bordons. *Model Predictive Control in the Process Industry*. Springer-Verlag, Berlin, Germany, 1995.
- [5] P. O. M. Scokaert, J. B. Rawlings, and E. S. Meadows. Discrete-time stability with perturbations: Application to model predictive control. *Automatica*, 33(3):463–470, 1997.
- [6] G. De Nicolao, L. Magni, and R. Scattolini. Stabilizing receding-horizon control of non-linear time-varying systems. *IEEE Transactions on Automatic Control*, 43(8):1030–1036, 1998.
- [7] P. J. Campo and M. Morari. Robust model predictive control. In *Proceedings of the American Control Conference*, pages 1021–1026, Minneapolis, USA, June 1987.
- [8] H. Michalska and D. Q. Mayne. Robust receding horizon control of constrained nonlinear systems. *IEEE Transactions on Automatic Control*, 38(11):1623–1633, 1993.
- [9] S. V. Ukkusuri, G. Ramaduraiand, and G. Patil. A robust transportation signal control problem accounting for traffic dynamics. *Computers & Operations Research*, 37(5):869–879, 2010.
- [10] T. Tettamanti, T. Luspay, and B. Kulcsár. Robust control for urban road traffic networks. *IEEE Transactions on Intelligent Transportation Systems*, 15(1):385–398, 2014.
- [11] L. K. Jones, R. Deshpande, N. H. Gartner, C. Stamatiadis, and F. Zou. Robust controls for traffic networks: The near-Bayes near-Minimax strategy. *Transportation Research Part C: Emerging Technologies*, 27:205–218, 2013.
- [12] A. Núñez and B. De Schutter. Distributed fuzzy confidence interval for traffic measurements. In *Proceedings of the 51th Conference on Decision and Control*, pages 6995–7000, Maui, Hawaii, USA, 2012.
- [13] R. X. Zhong, A. Sumalee, T. L. Pan, and W. H. K. Lam. Optimal and robust strategies for freeway traffic management under demand and supply uncertainties: An overview and general theory. *Transportmetrica A: Transport Science (ahead-of-print)*, pages 1–29, 2014.
- [14] A. Messmer and M. Papageorgiou. METANET: A macroscopic simulation program for motorway networks. *Traffic Engineering and Control*, 31(8-9):466–470, 1990.
- [15] A. Kotsialos and M. Papageorgiou. Efficiency and equity properties of freeway network-wide ramp metering with amoc. *Transportation Research Part C: Emerging Technologies*, 12(6):401–420, 2004.
- [16] A. Rosenfeld, R. Hummel, and S. Zucker. Scene labeling by relaxation operations. *IEEE Transactions on Systems, Man, and Cybernetics*, 6(6):173–184, 1976.
- [17] A. Kotsialos, M. Papageorgiou, M. Mangeas, and H. Haj-Salem. Coordinated and integrated control of motorway networks via non-linear optimal control. *Transportation Research Part C: Emerging Technologies*, 10(1):65–84, 2002.
- [18] S. K. Zegeye, B. De Schutter, J. Hellendoorn, and E. A. Breunese. A predictive traffic controller for sustainable mobility using parameterized control policies. *IEEE Transactions on Intelligent Transportation Systems*, 13(3):1420–1429, 2012.