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Technical report 15-009

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A. Sadowska, P.-J. van Overloop, C. Burt, and B. De Schutter

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A. Sadowska, P.-J. van Overloop, C. Burt, and B. De Schutter, "Hierarchical operation of water level controllers: Formal analysis and application on a large scale irrigation canal," *Water Resources Management*, vol. 28, no. 14, pp. 4999–5019, Nov. 2014.

Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: https://www.dcsc.tudelft.nl

Hierarchical operation of water level controllers: formal analysis and application on a large scale irrigation canal

A. Sadowska · P.-J. van Overloop ·
C. Burt · B. De Schutter

Abstract We introduce a hierarchical controller, the purpose of which is to speed up the water delivery process as compared to the standard method applied currently in the field. The lower layer of the hierarchical control consists of local proportional integral filter controllers (PIF controllers) for upstream control at each gate; specifically they are proportional integral controllers with a low-pass filter. In contrast, the higher layer is composed of a centralized model-based predictive controller, which acts by controlling the head gate and by coordinating the local PIF controllers by modifying their setpoints when needed. The centralized controller is event-driven and is invoked only when there is a need for it (a water delivery request) and as such it contributes scarcely to the communication burden. The scheme is robust to temporary communication losses as the local PIF controllers are fully able to control the canal in their normal independent automatic upstream control mode until the communication links are restored. We discuss the application of the hierarchical controller to a precise numerical model of the Central California Irrigation District Main Canal. This shows the improved performance of the new hierarchical controller over the standard control method.

Keywords Model Predictive Control · hierarchical control · irrigation canals

A. Sadowska · B. De Schutter
Delft Center for Systems and Control, Delft University of Technology, Delft, The Netherlands, E-mail: {A.D.Sadowska,B.DeSchutter}@tudelft.nl

P.-J. van Overloop
Water Resources Management, Delft University of Technology, Delft, The Netherlands, E-mail: P.J.A.T.M.vanOverloop@tudelft.nl

C. Burt
Irrigation Training and Research Center (ITRC), California Polytechnic State University, San Luis Obispo, California, E-mail: CBurt@calpoly.edu

1 Introduction

1.1 Setting the scene

The Central California Irrigation District (CCID) is situated approximately 225 km south of Sacramento, California with services covering a farming land area of approximately 580 km^2. This comprises over 1600 fields (CCID, 2013; Burt et al, 2005), which places CCID among the largest irrigation districts in the region. The CCID Main Canal consists of two parts: the upper part, which is discussed in this paper, and the lower part are connected with one another with a reservoir. The reservoir provides a buffer for flow rate errors/changes; yet, there is a need to move flow rate changes from the head gate to the reservoir more rapidly. This will allow for much more flexible deliveries along the complete length of the Main Canal.

While the CCID Main Canal and Outside Canal are automated and modernized (Burt et al, 2005; Richardson, 2008), both in CCID and in many other irrigation districts, communication links are still not considered to be reliable enough to be employed in a continuous communication loop, with equipment breakdowns associated with radios that can be attributed to the harsh outdoor environment they are located in. Given these restrictions, possibly the most widely used controller in canal control is a local upstream PIF (Proportional Integral Filter) controller (Åström and Hägglund, 1995) applied to control gates in all pools (Litrico et al, 2003; Van Overloop et al, 2005; Litrico et al, 2007; Ooi and Weyer, 2008), as it is a decentralized controller requiring only local information about the current water level. The popularity of PIF controllers is due to their simplicity, model-independence, and satisfactory functioning when tuned adequately. Indeed, utilizing local PIF controllers can serve for stable water level control at delivery points along the canal and hence to deliver water through the canal to farmers as required; however, local PIF controllers cannot quickly move changes in flow from the canal inlet to downstream points and the time delay to execute a flow change may be significant (e.g. a few hours), which is rather undesirable. With the current water delivery method the lag times are inevitable as water needs to be delivered directly from the head gate, indicating possibly a considerable travel distance for long canals. While the geometric properties of a canal cannot be modified, to overcome the problem of time lags we propose a hierarchical controller consisting of two layers: the lower layer is constituted by the local upstream PIF controllers and the higher one by a centralized controller developed using the principles of Model Predictive Control (MPC)(Maciejowski, 2002; Camacho and Bordons, 1999), and working by controlling the head gate and by modifying the setpoints of the PIF controllers for the delivery. The controller is named the *Coordinator* as it coordinates the PIF controllers. By the actions of the Coordinator, it is no longer necessary to wait a long time before an announced offtake can actually start. Importantly, as the higher layer only activates in response to a delivery request, there is no need for a continuous communication between the Coordinator and the local sites.

1.2 Previous work on control of irrigation canals

Due to the event-driven nature, the high-level controller is compliant with the practical restrictions on the feasible amount of communication. Nevertheless, many control methods proposed in the literature to control an irrigation canal rely on frequent communication. For instance, (Negenborn et al, 2009) introduces a distributed MPC-based scheme, where each gate finds a local control action, using information about its own pool, (i.e. the pool just upstream of a gate) and pools immediately upstream and downstream (the neighbors). To find a globally optimal solution, the local controllers communicate to the neighbors multiple times at each control step to negotiate the control action to be ultimately applied. Also (Álvarez et al, 2013) studies distributed controllers but employs cooperative game theory (Maestre et al, 2011) to find local controls. Understandably, methods such as the ones of (Negenborn et al, 2009; Álvarez et al, 2013) yield adequate performance, if sufficient communication is guaranteed. Yet, with unreliable communication links, the control performance may be significantly compromised.

Distributed controllers were also discussed in (Cantoni et al, 2007; Li and Cantoni, 2008; Li and De Schutter, 2010; Li and De Schutter, 2012), focusing on distant downstream control, where a controller at each gate controls the water level at the downstream end of a subsequent pool, as opposed to upstream control, in which a water level immediately upstream of the gate is controlled. It was argued in (Malaterre and Baume, 1999) that downstream control facilitates water deliveries more effectively than upstream control. However, downstream control that uses a target at the downstream end of a pool inherently relies on continuous communication as the controlled variables (e.g. the water levels) are distant from the control variables (e.g. the gate positions). Hence, as communication links may be prone to damages, downstream canal control may prove unreliable.

In contrast to the aforementioned decentralized or distributed schemes, centralized algorithms as studied in (Xu et al, 2012; van Overloop et al, 2005, 2010; Silva et al, 2007) consider the canal as one entity with control actions for the gates provided by a central controller looking at the whole system, not at individual subsystems separately. This gives a very good performance, particularly in simulations, but at the cost of a higher computational power required as the size of the control problem is larger than in the decentralized or distributed case, in which only a partition of the overall problem is considered at a time (see also (Weyer, 2008) for a discussion on differences between centralized and decentralized control strategies for irrigation canals). The common factor in (Xu et al, 2012; van Overloop et al, 2005, 2010; Silva et al, 2007) is that, as a new set of control actions needs to be found in every control step, the volume of communication between the control center and the local sites may exceed the practicable amount, which may result in the control signal not being conveyed to the gates, and consequently in the canal not being managed properly.

Another perspective on the problem of employing MPC for water systems is taken in (Xu and Schwanenberg, 2012), which offers an analysis of sequential and simultaneous MPC realizations. They differ in that while in sequential MPC, the controlled variable is calculated using the system dynamical equations after the optimization routine, in simultaneous MPC the controlled variable stems directly from solving the optimization problem with system dynamics as equality constraints. Furthermore, a different study concerning MPC for water systems is given in (Lemos et al, 2009). The authors of that paper consider adaptive and non-adaptive controllers based on MPC for controlling an irrigation canal and compare their performance on a pilot canal.

A comment about communication effort beyond the practicable limits, similar to the one in the articles reported earlier, can be raised regarding (Zafra-Cabeza et al, 2011). That paper introduces a hierarchical MPC-based control approach: the lower layer consists of distributed controllers and the higher one of a centralized controller. The higher control layer in (Zafra-Cabeza et al, 2011) acts by modifying the setpoints of the local controllers. However, as the setpoint can be changed in (Zafra-Cabeza et al, 2011) in general in every control step, the communication links need to be reliable to allow a continuous and dependable communication, which may not always be guaranteed. This may raise some questions on practical applicability of such results despite their theoretical soundness.

1.3 Contributions and outline of the paper

In this paper, we contribute to the field of irrigation canal control in a twofold manner. First, we present a hierarchical controller to expedite the water delivery process. Second, we apply the hierarchical controller to an accurate computational model of CCID Main Canal. The purpose of such a numerical study is to examine how the controller functions with the canal being simulated using a precise model as opposed to when only a simplified model is applied to simulate the canal. This allows drawing conclusions as to what kind of behavior is expected on the real system in an experimental analysis, which will be the next stage of our investigation.

The current paper extends the preliminary results in (Sadowska et al, 2013b) by considering a more practical setting. In that spirit, the canal dynamics are significantly more precise here than in (Sadowska et al, 2013b) and the controller is formulated in a way that makes it directly applicable to a real canal, e.g. the control input for the head gate is its required position, not flows, mimicking the situation present in real settings. In addition, the present study reports on the application of the controller using a numerical model of a real canal - the CCID Main Canal, whereas in (Sadowska et al, 2013b) a far more simplistic model was examined. Note that different aspects of the hierarchical controller discussed in this paper are also analyzed in (Sadowska et al, 2013a), which concentrates on differences between the controller's operation in the event-driven and the time-driven manner.

The outline of this paper is as follows. In Section 2 we introduce the concept of MPC. In Section 3 we describe the CCID Main Canal, provide its mathematical model, and discuss how it is simulated in the case study. In Section 4 we introduce the hierarchical controller. Subsequently, in Section 5 we present the case study results. To summarize, we give our concluding remarks in Section 6.

2 Preliminaries

2.1 Model Predictive Control

In this section we summarize the concept of Model Predictive Control (MPC) (Maciejowski, 2002; Camacho and Bordons, 1999) for discrete-time systems. MPC is an established technique that is widely used in many fields, e.g. process engineering and power systems. It is a powerful tool due to, amongst others, its ability to take care of state and control input constraints and to deal with multivariable systems. MPC is an optimal control method that supplies the system with a control action suitable for the control objective under consideration, taking into account a current situation $x(k|k)$ (a feedback component) and predictions of the future situation[1] $x(k+1|k), \dots, x(k+N_\mathrm{p}|k)$ for the following N_p steps using the internal model of the process (a feedforward component), where N_p is the length of the prediction horizon. The objective of MPC is to find a suitable sequence of control actions[2] $u^*(k|k), \dots, u^*(k+N_\mathrm{p}-1|k)$ minimizing the cost function. Once the sequence of optimal controls is found, the first control action $u^*(k|k)$ is applied to the plant and the process is repeated at the next time step $k+1$ looking again N_p steps into the future and using new information available.

2.2 Time Instant Optimization Model Predictive Control

Time instant optimization is a special case of the classical MPC, and it was first introduced for traffic control (De Schutter and De Moor, 1998). For a water system, it can be a useful approach to deal with on/off hydraulic structures (Van Ekeren et al, 2011)[3]. Note that when using the classical MPC for on/off control structures, a decision needs to be made at each sampling step about the optimal control sequence for the prediction horizon, i.e. a chain of N_p elements of *on* or *off* inputs needs to be found. This is a combinatorial problem resulting in a mixed-integer programming problem with N_p binary

[1] Here, $x(k+j|k)$ denotes the state prediction for time step $k+j$ obtained at time step k.
[2] Here, $u^*(k+j|k)$ denotes the optimal control found at step k to be applied at step $k+j$.
[3] (Cristea et al, 2011) uses a similar scheme to deal with hybrid components in the realm of MPC for the application to a wastewater treatment plant.

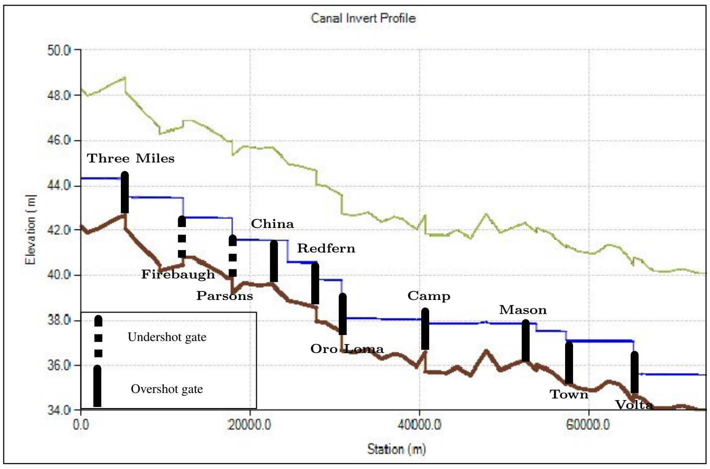


Fig. 1 Longitudinal profile of the CCID with the control structures shown (Picture adopted from (Burt and Piao, 2003)).

variables for each control structure present in the system, which may prove intractable (Garey and Johnson, 1979). In contrast, using time instant optimization MPC one first needs to decide how many switches of the control structure should occur during the prediction horizon and consequently write down the optimization problem with the on/off switching time instants as the direct real-valued control variables. Therefore, by recasting the problem into a programming problem with real variables only, it may be solved more computationally efficiently.

We have now introduced the required control techniques used in the paper. In the following section we present the description of the benchmark system considered in this paper: the CCID Main Canal.

3 Central California Irrigation District - Main Canal

3.1 General description

The upper part of the CCID Main Canal is a trapezoidal channel consisting of ten pools and ten control gates, see Figure 1. The gates starting from the upstream end of the canal are named: Three Mile, Firebaugh, Parsons, China, Redfern, Oro Loma, Camp, Mason, Town, and Volta. The numerical values characterizing the canal are given in Table 4 in the appendix.

The water flow in CCID Main Canal is by gravity only, with no pumping power involved. The gates in the CCID Main Canal are mainly overshot gates with the exception of the second and third gates, which are undershot (radial) gates.

3.2 Mathematical modeling of the canal

In this section we describe mathematical models of the canal as studied in the paper. We consider two models: the *process model*, which is used to accurately capture how the actual system (the canal) behaves and thus to simulate it precisely, and the *prediction model*, which is a simplification of the process model used to design the controller. This model is used because high computational requirements of the process model do not allow for utilizing it for a real-time controller design.

3.2.1 Process model

Canal dynamics are described with nonlinear partial differential equations, the so-called Saint Venant's equations (Chow, 1959; Van Overloop, 2006; Malaterre and Baume, 1998), relating water flow Q, time t, water level h, and spatial distance x as follows:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_{\text{lat}}, \qquad \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q}{A}\right)^2 + gA\frac{\partial h}{\partial x} + \frac{gQ|Q|}{C^2RA} = 0, \tag{1}$$

where A is the cross-section area, q_{lat} is the lateral unitary net inflow, g is the gravitational acceleration, R is the hydraulic radius, and C is the Chézy constant (Chow, 1959; Van Overloop, 2006; Malaterre and Baume, 1998; Malaterre et al, 1998).

To obtain water levels h and flows Q from (1) for the whole canal, each pool is divided along its longitudinal axis into small pieces for which (1) is numerically integrated for each piece individually. This yields water levels and flow profiles in each segment, and consequently a precise model is obtained. However, in order to do that, boundary conditions such as inflow to and outflow from each pool are required. These result from the flows through consecutive gates, which are managed through PIF controllers. Below we explain the operating principles of such controllers.

Assume the canal has n control structures. Denote by $h_{i,\text{up}}(k)$ (respectively $h_{i,\text{down}}(k)$) the water level immediately upstream (respectively downstream) of gate i at sampling step k. The first phase of the operation of the PIF controller is a low-pass filter (Oppenheim et al, 1996) described by

$$h_{i,\text{up}}^{\text{filtered}}(k) = K_{\text{F},i} h_{i,\text{up}}^{\text{filtered}}(k-1) + (1 - K_{\text{F},i}) h_{i,\text{up}}(k), \tag{2}$$

where $K_{\text{F},i} \in [0, 1)$ is the filter gain. We then define the tracking error $e_i(k) = h_{i,\text{up}}^{\text{filtered}}(k) - h_{i,\text{up}}^{\text{ref}}(k)$ for gate i, and denote by q_i the flow through gate i, which is de facto the outflow from pool i and equals

$$q_i(k) = \max\left(q_i(k-1) + K_{\text{P},i}\left(e_i(k) - e_i(k-1)\right) + K_{\text{I},i} e_i(k), 0\right). \tag{3}$$

In the formula, the maximum function represents the fact that the flow is only gravity-powered and hence it cannot be negative, as it would indicate an upstream flow. The constants $K_{\text{P},i} > 0$ and $K_{\text{I},i} > 0$ are the gains of the proportional and integral components of the controller. Formula (3) denotes the desired flow through gate i. In reality, a different value can result, if for instance the physical limits of a control structure are reached (e.g. the gate's opening is maximal). In particular, the actual flow u_i relates to a certain gate position φ_i by

$$u_i(k) = c_i w_i \mu_i \varphi_i(k) \sqrt{2g(h_{i,\mathrm{up}}(k) - h_i^{\mathrm{crest}} - 1/2\varphi_i(k))}, \tag{4}$$

$$u_i(k) = c_i w_i \mu_i \sqrt{2g(h_{i,\mathrm{up}}(k) - h_{i,\mathrm{down}}(k))}, \tag{5}$$

$$u_i(k) = \frac{2}{3} c_i w_i \mu_i \sqrt{2/3g}\, (h_{i,\mathrm{up}}(k) - \varphi_i(k))^{\frac{3}{2}}\,, \tag{6}$$

for a free-flowing undershot gate, a submerged undershot gate, and an overshot gate[4], respectively, where c_i denotes a calibration coefficient, w_i is the gate's width, μ_i is the contraction coefficient, and h_i^{crest} is the crest level, see the appendices for details.

To determine the realizable flow $u_i(k)$ from $q_i(k)$ in (3), it is first checked whether $h_{i,\mathrm{down}}(k) \leq h_{i,\mathrm{up}}(k)$. Otherwise, we set $u_i(k) = 0$ regardless of $q_i(k)$. Second, as $u_i(k)$ accounts for gate physical restrictions (e.g. the maximum opening/width: $0 \leq \varphi_i(k) \leq \bar{\varphi}_i$, and the maximum change rate: $|\varphi_i(k) - \varphi_i(k-1)| \leq \Delta_{\varphi,i}$), the actual settings $\varphi_i(k)$ of the gates are:

$$\varphi_i(k) = \begin{cases} \hat{\varphi}_i(k) & \text{if } \hat{\varphi}_i(k) \geq -\Delta_{\varphi,i} + \varphi_i(k-1) \\ & \text{and } \hat{\varphi}_i(k) \leq \Delta_{\varphi,i} + \varphi_i(k-1), \\ \Delta_{\varphi,i} + \varphi_i(k-1) & \text{if } \hat{\varphi}_i(k) > \Delta_{\varphi,i} + \varphi_i(k-1), \\ -\Delta_{\varphi,i} + \varphi_i(k-1) & \text{if } \hat{\varphi}_i(k) < -\Delta_{\varphi,i} + \varphi_i(k-1), \end{cases} \tag{7}$$

where $\hat{\varphi}_i(k) = \min\left(\max\left(\varphi_i(k), 0\right), \bar{\varphi}_i\right)$. Then, u_i follows from (4)-(6) and (7).

3.2.2 Prediction model

In this section we present the prediction model of the canal, developed from (1) by means of a more coarse discretization yielding a linear model (Schuurmans, 1997; Schuurmans et al, 1999; Malaterre, 2007; Van Overloop et al, 2005):

$$h_{i,\mathrm{up}}(k+1) = h_{i,\mathrm{up}}(k) + \frac{T_\mathrm{m}}{A_i}(u_{i-1}^{\mathrm{prediction}}(k - k_{\mathrm{d}i}) - u_i^{\mathrm{prediction}}(k) + d_i(k)), \tag{8}$$

where $k_{\mathrm{d}i}$ is a time delay (in sampling steps) before an upstream inflow affects $h_{i,\mathrm{up}}(k)$, T_m is the model sampling time (equal for all pools), A_i is the average surface area of pool i, d_i is the net inflow to pool i due to e.g. an offtake ($d_i < 0$) or rainfall ($d_i > 0$), and $u_i^{\mathrm{prediction}}$ denotes the flow through gate i, with $u_0^{\mathrm{prediction}} = Q_\mathrm{S}$ denoting the inflow from the head gate. As the prediction model does not include information about local gates constraints, the flow through the local gates in the prediction model is assumed to be unconstrained; hence $u_i^{\mathrm{prediction}}(k)$ equals $q_i(k)$ with $K_{\mathrm{F},i} = 0$.

[4] The free flow occurs when the downstream water level is less than the available gap between the crest height and the gate opening; otherwise the flow is considered submerged.

Table 1 Parameters of the prediction model (8).

	1	2	3	4	5	6	7	8	9	10
k_{di}	13	26	21	25	13	11	44	57	15	36
A_i	154842	132722	121181	154842	73346	58066	163951	139358	67979	154842

Table 2 VAF for the responses of the process model and the prediction model.

1	2	3	4	5	6	7	8	9	10
94%	88%	91%	88%	97%	97%	93%	87%	96%	86%

3.2.3 Validation of the prediction model

In this section we validate pool by pool the prediction model given in the preceding section, with values of the parameters k_{di} (in sampling steps) and A_i (in m^2) for $T_m = 1$ minute given in Table 1, against the process model described in Section 3.2.1.

The validation process consists of applying a step increase of 1.5 m^3/s in the upstream inflow $u_{i-1}^{\text{prediction}}$ to each pool with the outflow $u_i^{\text{prediction}}$ from the pool at the downstream end kept constant. Then, water levels $h_{i,\text{up}}$ are measured, compared, and illustrated in Figure 2. Apart from a visual test, we also examine the fitness of the prediction model in comparison to the process model by inspecting the variance-accounted-for (VAF) values calculated according to the formula

$$\text{VAF}(m, n) = \left(1 - \frac{\text{var}(m - n)}{\text{var}(m)}\right) \cdot 100\%, \tag{9}$$

for the responses to the aforementioned stimuli of the process model (signal n) and the prediction model (signal m). The corresponding values are given in Table 2. The observed values in the validation experiment allow to conclude that the prediction model resembles the process model close enough to be utilized to derive the controller. This is done in the subsequent section.

4 Delivery accelerating hierarchical controller design

In this section we introduce the hierarchical centralized controller to coordinate the local PIF controllers and thus to accelerate the water delivery process, see Figure 3. Recall that the hierarchical controller is composed of two layers: the local PIF controllers are in the lower layer and the centralized predictive controller is in the higher layer. Accordingly, the lower layer is based on the equipment already present in the field, and so is the higher layer, which in addition is invoked on an event-driven basis with events associated with delivery requests. It is assumed that a single delivery (flow change) request is described by its flow per second and time instant when the delivery should start. In this paper, to facilitate a clear presentation of our concept, we assume that no overlapping of the requests of individual users is allowed. So a new request

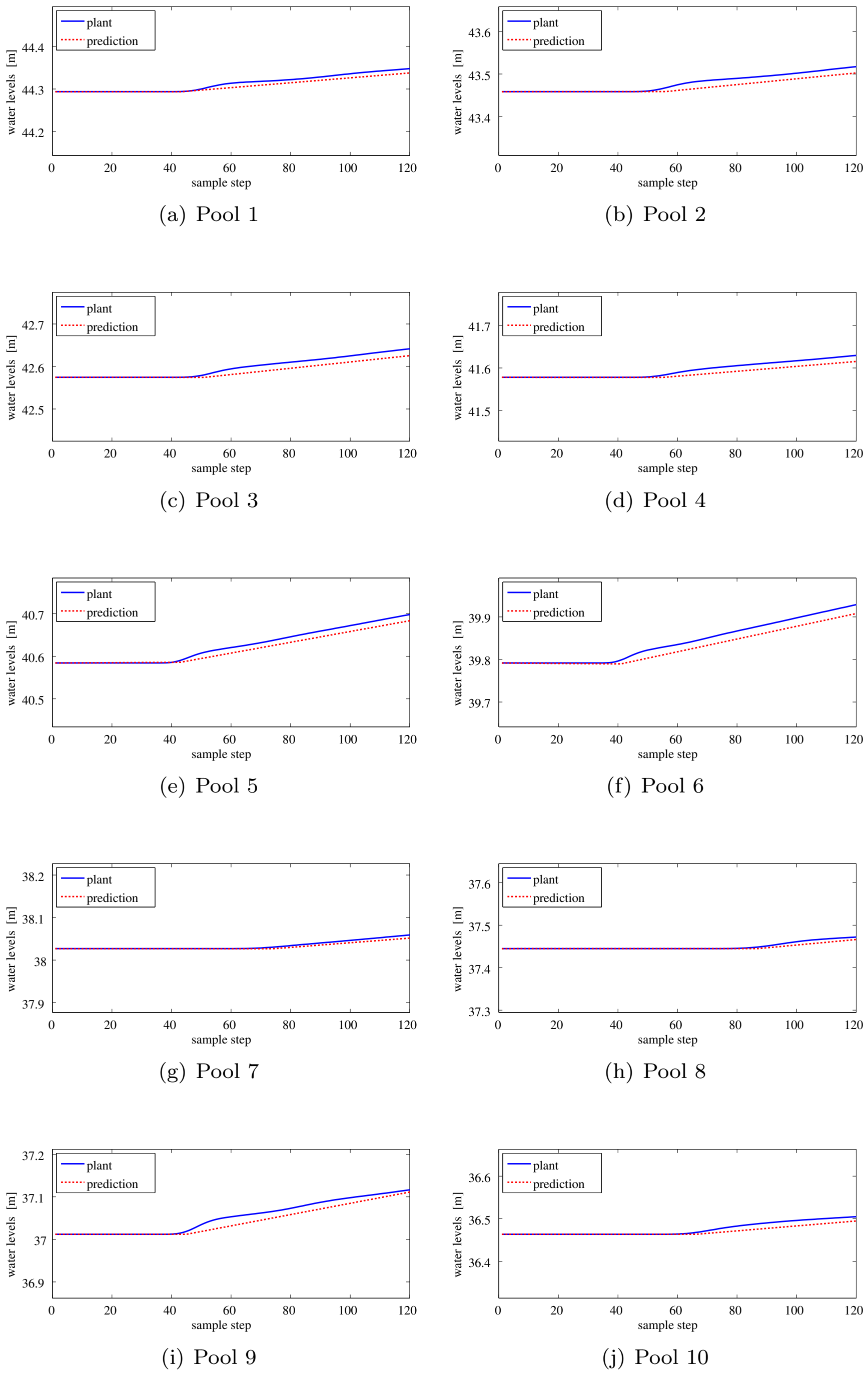


Fig. 2 Comparison between the process model and the prediction model.

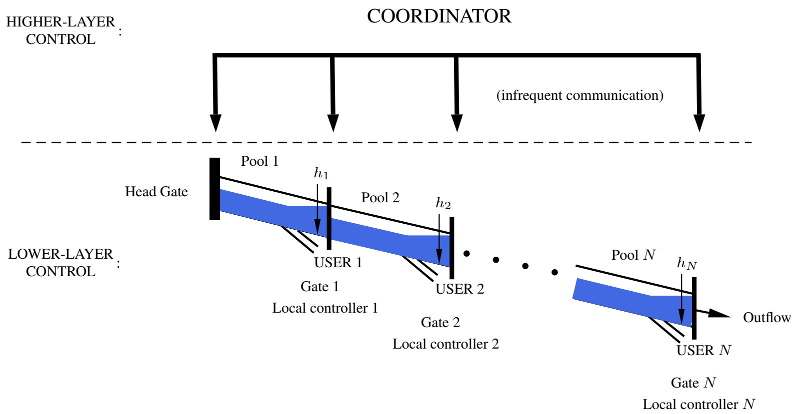


Fig. 3 The structure of the hierarchical controller proposed in the paper.

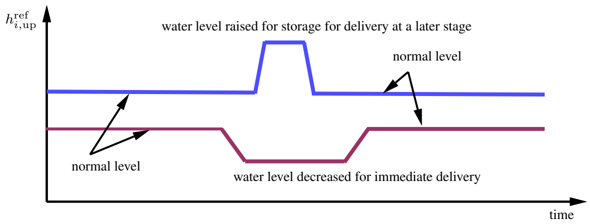


Fig. 4 Admissible setpoint profiles provided by the Coordinator.

can only be made after steady state has been restored after a preceding request. The method, however, is not limited in this way and can be extended to multiple overlapping requests by establishing how multiple requests are dealt with and how they activate the higher-layer control.

The Coordinator coordinates the water deliveries to the users by controlling the water flow through the head gate as well as by manipulating the reference levels in individual canal pools at appropriate times. The setpoint changes provided by the Coordinator are ramp-shaped, see Figure 4, and they are characterized by a maximum modified value of the setpoint, setpoint change rate per sampling step, and the time instants when the setpoint should start changing and be back at the normal operating value of the setpoint. This implies that the Coordinator only needs to communicate once to each local site to provide information about the modification of the setpoint profile. This is an essential feature given the communication limitations present in the system. With the event-driven design as presented in the paper, there is no need for frequent communication with the local sites. What is more, the local sites are capable of autonomously controlling the canal in case of communication breakdowns, which makes the strategy robust to operate.

When a new delivery request is conveyed to the Coordinator, the time t and step counter k variables are reset to 0 and then are incremented until the Coordinator is re-activated for another delivery. Define T_c as the length of the control cycle of the Coordinator, which is assumed to be an integer multiple of the sampling time of the model T_m. So $A_\mathrm{c} = T_\mathrm{c}/T_\mathrm{m} \in \mathbb{N}$.

The Coordinator controls the head gate by establishing its movements $M_{\text{head gate}} \in \mathbb{R}$ for a certain duration to accommodate the delivery. This is done with the help of a variable $\widetilde{M}^{\text{control}}_{\text{head gate}}$, which is a profile of the head gate setting for the whole duration of the prediction horizon, i.e. from the current moment until N_p control steps of the Coordinator ahead:

$$\widetilde{M}^{\text{control}}_{\text{head gate}} = (M^{\text{control}}_{\text{head gate}}(0), \ldots, M^{\text{control}}_{\text{head gate}}(N_\mathrm{p} - 1))^T, \tag{10}$$

and, consequently, the head gate setting is found according to the formula

$$M_{\text{head gate}}(jA_\mathrm{c} + \ell) = M^{\text{control}}_{\text{head gate}}(j), \text{ for } \begin{array}{l} \ell = 0, \ldots, A_\mathrm{c} - 1, \\ j = 0, \ldots, N_\mathrm{p} - 1, \end{array} \tag{11}$$

and $M_{\text{head gate}}(k) = M^{\text{steady state}}_{\text{head gate}} = M^{\text{control}}_{\text{head gate}}(N_\mathrm{p} - 1)$ for $k > N_\mathrm{p}A_\mathrm{c} - 1$, where we exercised the fact that upon activation of the Coordinator, the step counter k is reset to 0. Notice that since we assign $M^{\text{steady state}}_{\text{head gate}} = M^{\text{control}}_{\text{head gate}}(N_\mathrm{p} - 1)$ after the transient in the system associated with a new delivery, i.e. for $k > N_\mathrm{p}A_\mathrm{c} - 1$, steady-state settings of the head gate are changed and are set to be $M^{\text{control}}_{\text{head gate}}(N_\mathrm{p} - 1)$.

The Coordinator also modifies the setpoints of the local PIF controllers by determining ramped-block-shaped setpoint changes. To define each block, four quantities are used: the time instant $t_i^{\text{on}} \in \mathbb{R}$ when the setpoint should be altered from its predefined level, the modified value $h_i^{\text{ref, delivery}} \in \mathbb{R}$ of the setpoint, the change rate $r_i \in \mathbb{R}$ of a setpoint, and the time instant $t_i^{\text{off}} \in \mathbb{R}$ when the setpoint should be back at its predefined level. Consequently, we construct vectors

$$\begin{array}{ll} H^{\text{ref, delivery}} & = [h^{\text{ref, delivery}}_{1,\text{up}}, \ldots, h^{\text{ref, delivery}}_{N,\text{up}}]^T, \\ R & = [r_1, \ldots, r_N]^T, \\ T^{\text{on}} & = [t_1^{\text{on}}, \ldots, t_N^{\text{on}}]^T, \\ T^{\text{off}} & = [t_1^{\text{off}}, \ldots, t_N^{\text{off}}]^T. \end{array} \tag{12}$$

Then, we define how the setpoint profiles should change for a delivery as

$$h^{\text{ref}}_{i,\text{up}}(k) = \begin{cases} \kappa_i^{\text{on}}(k) & \text{if } k_i^{\text{on}} \le k \le k_i^{\text{on}\prime}, \\ h_i^{\text{ref, delivery}} & \text{if } k_i^{\text{on}\prime} < k < k_i^{\text{off}\prime}, \\ \kappa_i^{\text{off}}(k) & \text{if } k_i^{\text{off}\prime} \le k \le k_i^{\text{off}}, \\ h_i^{\text{ref, normal}} & \text{otherwise}, \end{cases} \tag{13}$$

in which k_i^{on} and k_i^{off} are discrete-time equivalents of the continuous variables t_i^{on} and t_i^{off}:

$$k_i^{\text{on}} = \left[\frac{t_i^{\text{on}}}{T_\mathrm{m}}\right] \qquad \text{and} \qquad k_i^{\text{off}} = \left[\frac{t_i^{\text{off}}}{T_\mathrm{m}}\right], \tag{14}$$

and

$$k_i^{\mathrm{on}\prime} = k_i^{\mathrm{on}} + \left[\frac{|\Delta h_i^{\mathrm{ref}}|}{r_i}\right] \qquad \text{and} \qquad k_i^{\mathrm{off}\prime} = k_i^{\mathrm{off}} - \left[\frac{|\Delta h_i^{\mathrm{ref}}|}{r_i}\right], \tag{15}$$

where $[x]$ denotes the value of x rounded to the nearest integer assuming a round-half-up rule, and $\Delta h_i^{\mathrm{ref}} = h_i^{\mathrm{ref,\,delivery}} - h_i^{\mathrm{ref,\,normal}}$. Moreover, $h_i^{\mathrm{ref,\,normal}}$ is the normal operating level of the setpoint in canal pool i, and the functions $\kappa_i^{\mathrm{on}}(k)$ and $\kappa_i^{\mathrm{off}}(k)$ are defined as

$$\kappa_i^{\mathrm{on}}(k) = \begin{cases} \min(h_{i,\mathrm{up}}^{\mathrm{ref}}(k-1) + r_i, h_i^{\mathrm{ref,\,delivery}}), & \text{if } \Delta h_i^{\mathrm{ref}} > 0, \\ \max(h_{i,\mathrm{up}}^{\mathrm{ref}}(k-1) - r_i, h_i^{\mathrm{ref,\,delivery}}), & \text{otherwise,} \end{cases} \tag{16}$$

$$\kappa_i^{\mathrm{off}}(k) = \begin{cases} \max(h_{i,\mathrm{up}}^{\mathrm{ref}}(k-1) - r_i, h_i^{\mathrm{ref,\,normal}}), & \text{if } \Delta h_i^{\mathrm{ref}} > 0, \\ \min(h_{i,\mathrm{up}}^{\mathrm{ref}}(k-1) + r_i, h_i^{\mathrm{ref,\,normal}}), & \text{otherwise.} \end{cases} \tag{17}$$

Following the above discussion, the Coordinator finds the control action $\mathcal{U} = \left(\widetilde{M}_{\mathrm{head\,gate}}^{\mathrm{control}}, H^{\mathrm{ref,\,delivery}}, R, T^{\mathrm{on}}, T^{\mathrm{off}}\right)$ to minimize the cost function

$$J = \alpha \sum_{j=1}^{A_\mathrm{c} N_\mathrm{p}} \left(u_N(j-1) - Q_{\mathrm{S,\,base}}\right)^2 \tag{18}$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{A_\mathrm{c} N_\mathrm{p}} \left[\gamma_1 \left(\max(h_{i,\mathrm{up}}(j) - h_i^{\mathrm{max,des}}, 0)\right)^2\right. \tag{19}$$

$$\left. + \gamma_2 \left(\min(h_{i,\mathrm{up}}(j) + h_i^{\mathrm{min,des}}, 0)\right)^2\right] \tag{20}$$

$$+ \beta \sum_{i=1}^{N} \sum_{j=1}^{A_\mathrm{c} N_\mathrm{p}} \left(h_{i,\mathrm{up}}(j) - h_{i,\mathrm{up}}^{\mathrm{ref}}(j)\right)^2 \tag{21}$$

$$+ \mu \sum_{i=1}^{N} \left(t_i^{\mathrm{off}} - t_i^{\mathrm{on}}\right)^2 + \zeta \sum_{i=1}^{N} \left(\Delta h_i^{\mathrm{ref}}\right)^2, \tag{22}$$

in which α, β, γ_1, γ_2, μ and ζ are positive weighting coefficients. Moreover, $u_N(k)$ is used to denote the flow through gate N at time step k.

The first term in the cost function J (18) vanishes when the outflow from the last gate is exactly equal to the given base flow $Q_{\mathrm{S,\,base}}$ and grows as that outflow diverges from the base flow. This condition is added due to a possible existence of further downstream users beyond the stretch of the canal with N pools under consideration. The second term (19) and the third term (20) in the objective function penalize control actions resulting in the water levels departing from the desired operational range $[h_i^{\mathrm{min,des}}, h_i^{\mathrm{max,des}}]$, $i = 1, \ldots, N$. This penalty is incorporated into the objective function to stimulate good performance of the system, in which water levels do not fluctuate excessively. Furthermore, the fourth term (21) adds a penalty on the setpoint tracking errors. While the local controllers are in charge of maintaining the water levels at their respective setpoints, adding (21) to the cost function aids the local controllers by inducing the higher-layer controller to apply control actions for which the setpoint tracking errors are smaller. Finally, the last two terms (22) are used to induce the Coordinator to switch the setpoints back to their normal level as soon as possible. This is meant to prompt the controller to swiftly bring the system to the normal operating conditions with the normal setpoint levels after changing the setpoints for a delivery.

The Coordinator must also comply with the following hard constraints:

$$h_i^{\min} \leq h_{i,\mathrm{up}}(j) \leq h_i^{\max}, \qquad j = 1, \ldots, N_\mathrm{p}A_\mathrm{c}, \tag{23}$$

$$h_i^{\min} \leq h_{i,\mathrm{up}}^{\mathrm{ref}}(j) \leq h_i^{\max}, \qquad j = 0, \ldots, N_\mathrm{p}A_\mathrm{c} - 1, \tag{24}$$

$$t_i^{\mathrm{off}} \geq t_i^{\mathrm{on}} + T_\mathrm{m}, \tag{25}$$

$$t_i^{\mathrm{on}} \geq 0 \tag{26}$$

$$0 \leq M_{\text{head gate}}(j) \leq M_{\text{head gate}}^{\max} \qquad j = 0, \ldots, N_\mathrm{p}A_\mathrm{c} - 1, \tag{27}$$

$$M_{\text{head gate}}(j) - M_{\text{head gate}}(j-1) \leq \Delta M_{\text{head gate}}^{\max} \qquad j = 0, \ldots, N_\mathrm{p}A_\mathrm{c} - 1, \tag{28}$$

$$M_{\text{head gate}}(0) \leq \Delta M_{\text{head gate}}^{\max} + M_{\text{head gate}}^{\text{steady state}}, \tag{29}$$

$$M_{\text{head gate}}(N_\mathrm{p}A_\mathrm{c} - 1) \leq \Delta M_{\text{head gate}}^{\max} + M_{\text{head gate}}^{\text{steady state}}, \tag{30}$$

$$t_i^{\mathrm{off}} \leq T_\mathrm{c}N_\mathrm{c}, \tag{31}$$

$$M_{\text{head gate}}^{\text{control}}(j) = M_{\text{head gate}}^{\text{control}}(N_\mathrm{c}), \qquad j = N_\mathrm{c} + 1, \ldots, N_\mathrm{p} \tag{32}$$

$$M_{\text{head gate}}(j) = M_{\text{head gate}}^{\text{control}}(N_\mathrm{p}A_\mathrm{c} - 1), \qquad j > N_\mathrm{p}A_\mathrm{c} - 1 \tag{33}$$

for all $i \in \{1, \ldots, N\}$, in which $N_\mathrm{c} \leq N_\mathrm{p}$ is the duration of the control horizon. Constraints (23) and (24) correspond to the physical constraints of the depth of the canal. Thus, the water levels and the setpoints should always be no less than the bottom of the canal and no more than the canal banks. We use $h_i^{\min} \leq h_i^{\min,\mathrm{des}} < h_i^{\max,\mathrm{des}} \leq h_i^{\max}$.

Further constraints (25) and (26) limit the possible choice of the switching time instants in that the first switch t_i^{on} should only occur after the moment the Coordinator has been activated and the second moment t_i^{off} needs to occur at least one sampling step after the first one. In addition, the movements of the head gate ordered by the Coordinator need to satisfy the head gate minimum and maximum position (constraint (27)) and the maximum change rate (constraints (28)–(30)). Lastly, constraints (31) and (32) impose that all changes are made during the control horizon, and constraint (33) enforces that new steady-state settings of the head gate are given by the last component of vector $\widetilde{M}_{\text{head gate}}^{\text{control}}$, i.e. element $M_{\text{head gate}}^{\text{control}}(N_\mathrm{p}A_\mathrm{c} - 1)$.

The Coordinator only works in response to requested offtakes and in normal operating conditions the PIF controllers are deemed to be sufficient for adequate canal control performance. The triggering condition that activates the Coordinator can be described as follows. The Coordinator optimizes (18)–(22) subject to constraints (13) and (23)–(28) once per delivery, when it learns about a new delivery[5]. Then, the Coordinator finds suitable values for the con-

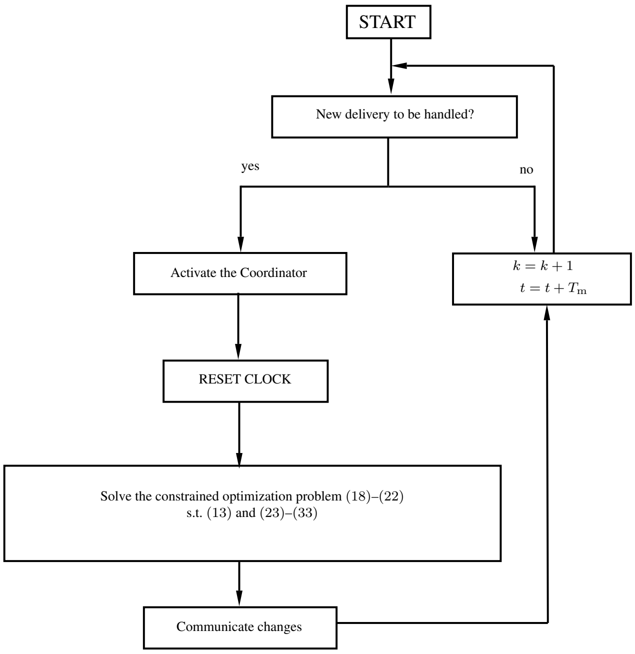


Fig. 5 The functioning of the system with the Coordinator.

trol inputs $\mathcal{U} = \left(\widetilde{M}^{\mathrm{control}}_{\mathrm{head\,gate}}, H^{\mathrm{ref,\,delivery}}, R, T^{\mathrm{on}}, T^{\mathrm{off}}\right)$. After finding the suitable control action $\mathcal{U}$ for a given delivery, the Coordinator is switched off until the steady state in the canal is re-established, at which time the Coordinator can be reactivated as soon as a new delivery comes along.[6] The functioning of the system governed by the Coordinator can be illustrated by the algorithm shown in Figure 5.

The constrained optimization problem defined in (18)–(22) subject to (13) and (23)–(33) is a nonconvex and nonsmooth nonlinear programming problem, where the nonsmoothness stems from the time instants being rounded off. To solve such an optimization problem, several methods can be used, e.g. multi-start genetic (Mitchell, 1996), simulated annealing (Kirkpatrick et al, 1983), or pattern search algorithms (Hooke and Jeeves, 1961; Torczon, 1997).

[5] For this strategy to be successful, the prediction horizon N_{p} needs to be long enough to capture the canal's dynamics.

[6] To enable reactivations of the Coordinator before the steady state in the canal has been restored, the proposed algorithm needs to be adapted so as to specify how the changed setpoints and changed head gate settings can be further modified in subsequent activations of the Coordinator. Specifically, formulae (13), (16), (17) for the setpoints changes and formula (11) for the head gate settings need to be modified to fit the multiple activation framework. This, however, is out of the scope of the current paper.

5 Case study results

In this section we present the results obtained through applying the hierarchical control algorithm discussed in the paper to the precise numerical model of the CCID Main Canal. The performance of the hierarchical controller is analyzed in comparison to the standard method, which, as mentioned earlier, relies on releasing the extra water required from the head gate at the time when the delivery request becomes known and letting the PIF controllers transport the water through the successive pools to the offtake point, see Figures 6–8.

To compare the standard method and the method proposed in the paper, we use an a posteriori cost function

$$J_{\text{post}} = \alpha \sum_{j=k_{\text{known}}}^{N_{\text{F}}} \left(u_N\left(j-1\right) - Q_{\text{S, base}}\right)^2 + \beta \sum_{i=1}^{N} \sum_{j=k_{\text{known}}}^{N_{\text{F}}} e_i^2\left(j\right), \tag{34}$$

where $k_{\text{known}} = 180$ is the sample step when the delivery request becomes known and $N_{\text{F}} = 780$ is the duration of the simulation. The weighting parameters in J_{post} above and in (18)–(22) are $\alpha = 5$, $\beta = 10$, $\gamma = 1$, $\mu = 1$, $\zeta = 1$. The delivery request used in the case study corresponds to a representative situation in the field: the request is for a sustained offtake in pool 10 of magnitude 2.5 m^3/s starting immediately at the moment when it becomes known, i.e. at step $k = k_{\text{known}}$. We use $A_{\text{c}} = 15$, $N_{\text{c}} = 16$, and $N_{\text{p}} = 36$.

To solve the optimization problem defined in Section 4 we use a pattern search algorithm implemented in the Global Optimization Toolbox in MATLAB R2013a. Most of the options are taken to be the default ones but we set both the polling strategy and the search strategy to be the mesh adaptive direct search with the Positive basis 2N pattern, and we use the initial mesh size of 10.

The simulation results of the case study are given in Figures 6–8 for the standard method and in Figures 9–11 for the hierarchical controller proposed. In Figure 9 we depict the head gate settings throughout the simulation obtained with the hierarchical controller. It can be observed that we start from a steady state and at the moment when the offtake is announced and starts, the head gate settings diverge from the steady-state level to provide water for the offtake. Then, after some transient time, the head gate settings reach a new level, accounting for the extra flow needed for the continuing offtake on top of

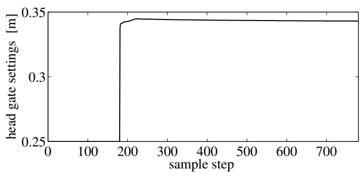
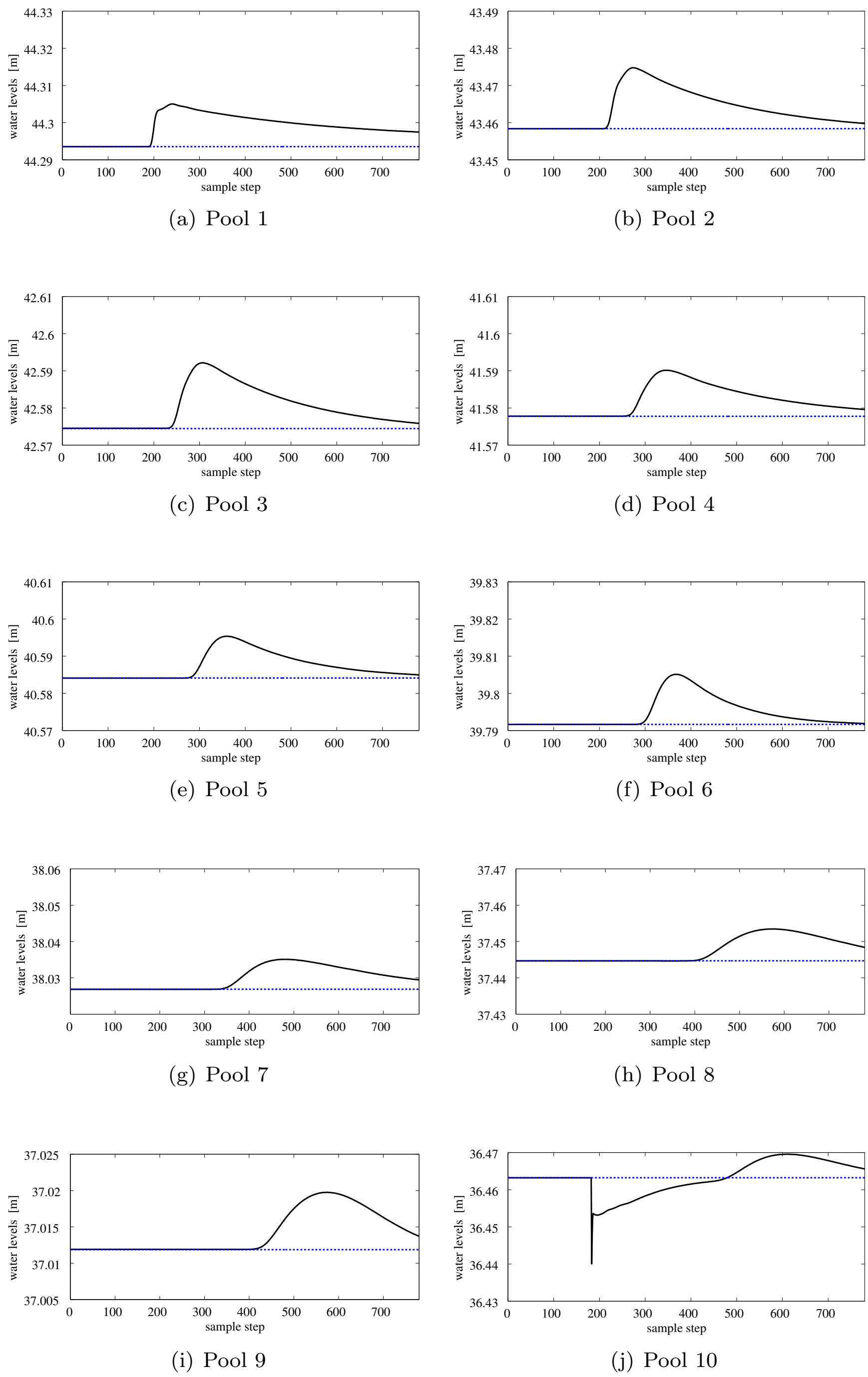


Fig. 6 Setting of the head gate obtained in the standard method.



(a) Pool 1 (b) Pool 2

(c) Pool 3 (d) Pool 4

(e) Pool 5 (f) Pool 6

(g) Pool 7 (h) Pool 8

(i) Pool 9 (j) Pool 10

Fig. 7 Water levels (solid line) and setpoints (dashed line) obtained in the standard method.

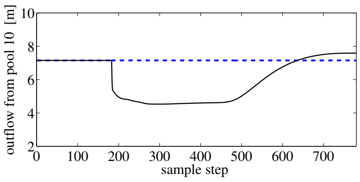


Fig. 8 Outflow from pool 10 obtained in the standard method.

the base flow. In contrast, in the standard method the head gate is changed from one steady state directly to another steady state, see Figure 6.

The setpoints and the resulting water levels in all pools are shown in Figures 7 and 10 for the standard method and the new hierarchical controller, respectively. Understandably, when the standard method is applied, the setpoints remain unchanged, whereas we observe that when the hierarchical controller is used, the setpoints in the canal pools do change. In particular, the setpoints are mostly lowered to use the water that becomes available by lowering the setpoints in the pools for immediate use in pool 10 for the delivery. This is to compensate for the delay time that is needed for the offtake water to be transported all the way from the head gate and indeed illustrates the essence of functioning of the hierarchical controller proposed in the paper. Notice that there are a few increased setpoints in some of the pools. Keeping in mind that the hierarchical controller needs to find its actions balancing multiple objectives (cf. (18)–(22)), they are not striking or unexpected. On the contrary, because of the multi-objective cost function (18)–(22) that the controller needs to minimize through its control actions, by appropriate scheduling of the setpoint decrease in some pools and increase in others, the controller is able to meet the objectives as closely as possible.

To analyze how closely the required objectives are met with the hierarchical controller applied, we show in Figure 11 the outflow from the canal (that is from the tenth pool). Recall that one of the objectives of the controller is to diminish deviations in this flow u_{10} with respect to the base flow of $Q_{\mathrm{S,\,base}} = 7.14\ \mathrm{m}^3/\mathrm{s}$. We see in Figure 11 that while there is a short period in which the flow u_{10} diverges from the desired value $Q_{\mathrm{S,\,base}}$, overall the performance is satisfactory (cf. Figure 8 for the standard method). In fact, the performance of the hierarchical controller proposed in the paper clearly sur-

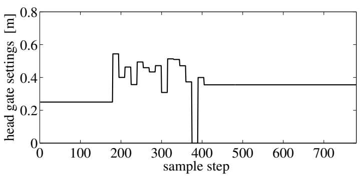
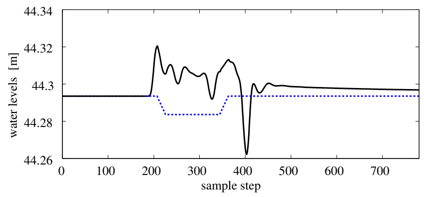
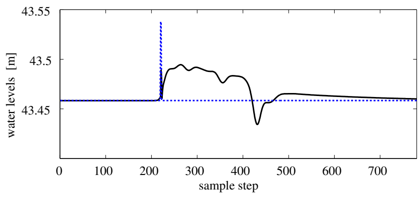


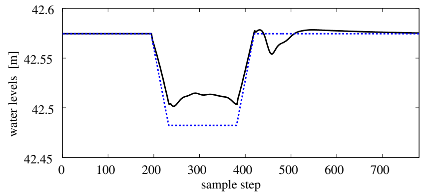
Fig. 9 Setting of the head gate obtained with the hierarchical controller.



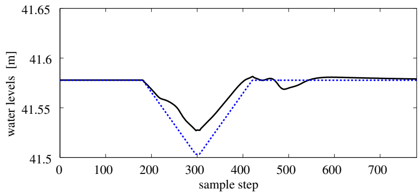
(a) Pool 1



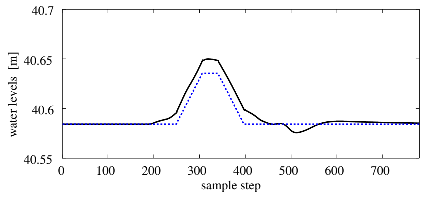
(b) Pool 2



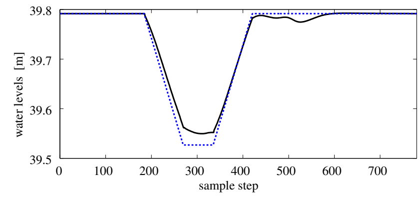
(c) Pool 3



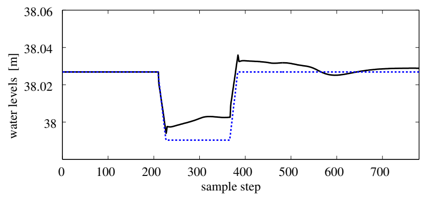
(d) Pool 4



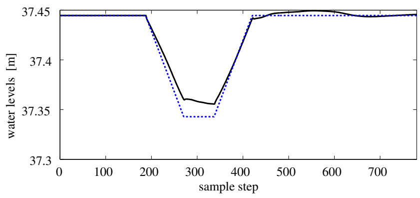
(e) Pool 5



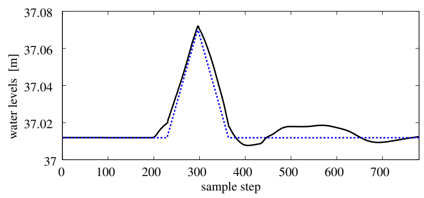
(f) Pool 6



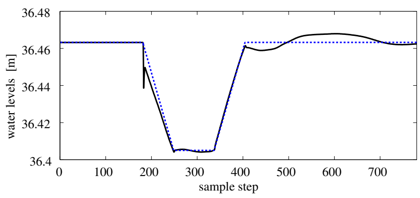
(g) Pool 7



(h) Pool 8



(i) Pool 9



(j) Pool 10

Fig. 10 Water levels (solid line) and setpoints (dashed line) obtained with the hierarchical controller.

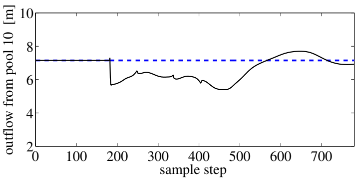


Fig. 11 Outflow from pool 10 obtained with the hierarchical controller.

Table 3 Comparison between the values of J_{post} obtained with the standard method and with the hierarchical controller.

	standard method	proposed method
J_{post}	10800.1	2539.3

passes the performance that is achieved for the standard approach as demonstrated in Table 3, where we give values of the a posteriori cost function J_{post} for the standard method and for the new method proposed in the paper. With $J_{\text{post}} = 2539.3$ for the new hierarchical approach and over four times more, i.e. $J_{\text{post}} = 10800.1$ for the standard method, the hierarchical controller evidently manifests its superiority over the standard method.

Nonetheless, there are ways to further improve the results. In particular, in our work we assume the prediction model to be a linear model (8), apart from the head gate flow, which is calculated from the head gate settings through a nonlinear relation. As illustrated in Section 3.2.3, the model (8) captures the basic dynamics of the system, which justifies the utilization of the model for the controller design purpose. However, if a more accurate, nonlinear model was used, the controller's action could explicitly account for the various phenomena that model (8) does not represent. In that respect, however, one needs to consider the issues of tractability of the controller: if the prediction model becomes more involved, the controller will take longer to come up with control actions. This shows a trade-off between the accuracy of the model used for predictions and the time that the controller needs to provide a presumably better solution. Nevertheless, the results communicated in this paper demonstrate that even the simple model (8) already gives a significant advantage over the standard method.

6 Conclusions

In this paper, we have proposed a hierarchical controller for speeding up the water delivery process in an irrigation canal. The hierarchical controller consists of two layers: the lower layer is based on the upstream PIF controllers at each local gate as already existing in practical applications in the field, and the higher layer is a centralized controller designed using the principles of MPC. We have formulated the controller, which achieves its goals by modifying the settings of the head gate, and by temporarily altering the setpoints of the local controllers. Importantly, the controller modifies the setpoints in such a way that it only requires to communicate changes to the local sites once per delivery request, thus is not relying on continuous communication and so the proposed controller adds up little to the communication effort required. We have presented the results of the application of the controller to the accurate numerical model of CCID Main Canal and compared the performance of the controller against the standard method. This study indicates that the new hierarchical controller introduced in the paper outperforms the standard method.

Further work on the topic could include an experimental study to verify the proposed scheme on a real-system application. In addition, as the controller is designed in the current paper in such a way that the setpoints are modified with ramped-block-characteristics, the practical implementation of the controller could be further simplified if stepwise setpoint changes are used. Given the high gains of the PI controllers in CCID Main Canal, see Table 4, that would necessitate temporary modifications of the PI gains to avoid excessive oscillations and hence to allow for the water to be released from pools steadily when setpoints are altered as it is achieved in the present study for ramped-block-changes. Moreover, in the spirit of reducing the wear and tear of the equipment, the control of the head gate could be set up using time instant optimization too. More specifically, one could assume that the head gate settings could only change twice to lessen its wear and tear: once to let the extra water into the canal, and once more to establish new steady-state settings. Then, the time instants of when the changes should occur could be optimized, resulting in a reduced number of modifications to the head gate settings and thus in prolonging its lifespan.

7 Acknowledgments

Research supported by the European Union Seventh Framework Programme [FP7/2007-2013] under grant agreement no. 257462 HYCON2 Network of Excellence and by the California State University Agricultural Research Initiative.

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A Quantitative characterization of the Central California Irrigation District Main Canal

Table 4 Data of the Central California Irrigation District Main Canal.

i	h_i^{crest}	$K_{\text{P},i}$	$K_{\text{I},i}$	$K_{\text{F},i}$	c_i	w_i	μ_i
1	42.7	186	0.5	0.74	1	10.67	0.63
2	40.5	157	0.6	0.74	1	7.32	-
3	39.9	143	0.6	0.74	1	7.32	-
4	38.9	182	0.7	0.74	1	3.96	0.63
5	38.6	190	1.0	0.51	1	5.79	0.63
6	37.5	152	1.1	0.46	1	4.88	0.63
7	36.6	208	0.9	0.74	1	7.62	0.63
8	38.0	168	1	0.74	1	4.57	0.63
9	35.8	165	1.8	0.50	1	3.66	0.63
10	37.0	182	1.8	0.74	1	2.74	0.63

B Calculation of the contraction coefficient μ_i for undershot gates

For undershot gates, μ_i depends on the water level $h_{i,\mathrm{up}}$ upstream of a gate and is obtained according to the following rule-based procedure (ITRC, 2001):

$$\mu_i = \begin{cases} 0.745 + 2.55(\frac{\varphi_i(k)}{\Delta h_i} - 0.9) & \text{if } \frac{\varphi_i(k)}{\Delta h_i} \geq 0.9, \\ 0.5 + 0.268\frac{\varphi_i(k)}{\Delta h_i} & \text{if } 0.55 \leq \frac{\varphi_i(k)}{\Delta h_i} < 0.9, \\ 0.65 & \text{otherwise,} \end{cases}$$

where $\Delta h_i = h_{i,\mathrm{up}} - h_i^{\mathrm{crest}}$.