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* This report can also be downloaded via https://pub.bartdeschutter.org/abs/15_021.html

Integration of Resource Allocation Coordination and Branch-and-Bound

Renshi Luo[1], Romain Bourdais[2], Ton J.J. van den Boom[1] and Bart De Schutter[1]

Abstract— **In general, integer programming problems are computationally very hard to solve, which makes solving an integer programming problem with a large number of decision variables in a centralized way intractable. In this paper, we propose a novel integer optimization method for strategic planning by integrating the resource allocation coordination method into a branch-and-bound paradigm. Thanks to the distributed computation of the resource allocation coordination method and distributed evaluation of nodes in the branch-and-bound paradigm, our method is capable of solving an integer programming problem in a distributed way. Moreover, since in the branch-and-bound paradigm the size of solution space decreases monotonically as the iteration proceeds, it is guaranteed that the globally optimal solution to an integer programming problem is obtained by using our method. Finally, we apply our method to the optimal charging control problem of electric vehicles under constrained grid conditions in a simulation study.**

I. INTRODUCTION

Integer programming problems are computationally very hard to solve, which makes solving an integer programming problem with a large number of decision variables in a centralized way intractable. So far, the inevitability of solving (mixed) integer programming problems has been hindering the development of intelligent control of large-scale hybrid systems. Therefore, it is important to seek a method to solve the integer programming problem in a distributed way.

In strategic planning, resource allocation is a plan for using available resources to achieve goals for the future. Actually, primal decomposition is naturally applicable to resource sharing scenarios, where the allocation of resources can be represented by auxiliary variables and these variables are optimized via using a master problem to coordinate the resource allocation to the subproblems [1]. In fact, a resource allocation coordination method, which is a distributed optimization method based on the primal decomposition of the overall problem, has already been developed for continuous optimization problems with global capacity constraints in [2].

However, if the resource allocation coordination method is used directly in distributed optimization for an integer programming problem, there could arise problems, such as oscillatory behavior of the decision variables, which will then not converge anymore.

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Actually, in [3], a mechanism integrating a distributed optimization method based on dual decomposition of the overall problem into the branch-and-bound [4] paradigm has been proposed to deal with the oscillatory behavior of the integer decision variables. However, that mechanism cannot guarantee that the constraints are always satisfied during iterations since the constraints are relaxed as penalties to the original objective function in dual decomposition. In the contrast, satisfaction of the constraints during the iterations is always guaranteed by using the resource allocation coordination method since it is based on primal decomposition of the overall problem.

A general framework of embedded optimization based on the branch-and-bound paradigm has been presented in [5] and it has been pointed out that optimization methods developed by using that framework can be used for implementing predictive control of hybrid systems on embedded systems.

In this paper, we integrate the resource allocation coordination method into the paradigm of generic branch-and-bound, and then develop a new optimization method for a class of integer programming problems with capacity constraints. As an illustration example, we apply the proposed optimization method to the optimal charging control problem of electric vehicles under constrained grid conditions.

As the increasing number of electric vehicles (EVs) will inevitably cause an additional load to the electrical power distribution grids [6], [7], the current capacity of the distribution grid will not be sufficient. The most direct way to solve this problem is to increase the capacity of the distribution grid. However, this will require huge investment costs. Alternatively, a smart charging control strategy that balances the charging demands of EVs can help to solve the problem in a more sustainable way and therefore is highly preferred by the distribution grid operators. So far, intelligent control of electric vehicles charging has been addressed by using distributed integer linear optimization method [8], sequential quadratic optimization [9], [10], dynamic programming [11], and heuristic methods [12].

Since a whole fleet of vehicles has to share the limited amount of charging power provided by the grid, we use our method which is based on the resource allocation coordination method to solve the optimal charging control problem of a fleet of electric vehicles.

II. RESOURCE ALLOCATION COORDINATION METHOD

In the resource allocation coordination method, the shares of resources that are allocated to subproblems are represented by auxiliary variables and then optimized by a master problem to achieve the coordination of resource allocation [1].

A. General separable problem

Let us consider the following general optimization problem

$$
\begin{aligned}
\underset{\boldsymbol{x}}{\text{minimize}} \quad & \sum_{n=1}^{N} f_n(x_n) \\
\text{subject to} \quad & x_n \in X_n,\ n = 1,\dots,N \\
& \sum_{n=1}^{N} g_n(x_n) \le r
\end{aligned} \tag{1}
$$

where all x_n are scalar variables. The problem can be interpreted as the optimization of N independent agents sharing a common resource r. More specifically, the variable x_n denotes the decision of agent n and the function value $g_n(x_n)$ denotes the amount of resources required by agent n if it makes decision x_n.

B. Primal decomposition

By introducing auxiliary variables θ_n, which denote the amount of resources allocated to each agent n, we can rewrite (1) as

$$
\begin{aligned}
\underset{\boldsymbol{x}}{\text{minimize}} \quad & \sum_{n=1}^{N} f_n(x_n) \\
\text{subject to} \quad & x_n \in X_n,\ n = 1,\dots,N \\
& g_n(x_n) \le \theta_n,\ n = 1,\dots,N \\
& \sum_{n=1}^{N} \theta_n = r
\end{aligned} \tag{2}
$$

Let us define

$$
p_n(\theta_n) = \underset{g_n(x_n)\le\theta_n, x_n\in X_n}{\text{minimize}} \quad f_n(x_n) \tag{3}
$$

Then, problem (2) can be written as

$$
\begin{aligned}
\underset{\boldsymbol{\theta}}{\text{minimize}} \quad & \sum_{n=1}^{N} p_n(\theta_n) \\
\text{subject to} \quad & \sum_{n=1}^{N} \theta_n = r
\end{aligned} \tag{4}
$$

This problem is called the master problem. We will discuss how to solve it in the next subsection.

C. Optimize the resource allocation

Actually, if all x_n are continuous decision variables, the problem (4) can be solved efficiently by using subgradient methods, which are simple iterative methods for solving convex optimization problems [13]. Suppose y is the decision variable of a convex problem, classical subgradient methods search for the solution to the problem by using the iteration

$$
y^{(z+1)} = \Pi\Big(y^{(z)} - \alpha^{(z)} h\big(y^{(z)}\big)\Big)
$$

where $h\big(y^{(z)}\big)$ denotes a subgradient of the objective function of the problem at $y^{(z)}$, $\alpha^{(z)}$ denotes the stepsize and $\Pi(\cdot)$ denotes the projection onto the constrained solution space.

It can be seen that a subgradient of $p_n(\theta_n)$ at θ_n is equal to $-\lambda_n$, where λ_n is the Lagrange multiplier corresponding to the constraint $g_n(x_n) \le \theta_j$ in the definition of $p_n(\theta_n)$ [14, Chapter 6.4.2]. In particular, the projected subgradient method in [2] is given by

$$
\theta_n^{(z+1)} = \theta_n^{(z)} + \xi^{(z)} \left(\lambda_n^{(z)} - \frac{1}{N} \sum_{j=1}^{N} \lambda_j^{(z)} \right) \tag{5}
$$

where z is the iteration counter and $\xi^{(z)}$ is a square-summable but not summable stepsize which satisfies

$$
\xi^{(z)} > 0,\ \sum_{z=1}^{+\infty} \xi^{(z)} = +\infty,\quad \sum_{z=1}^{+\infty} (\xi^{(z)})^2 < +\infty \tag{6}
$$

Note that (5) uses $-\lambda_n^{(z)}$ as the subgradient of $p_n(\cdot)$ at $\theta_n^{(z)}$ and guarantees the constraint $\sum_{n=1}^{N} \theta_n^{(z)} = r$ for is satisfied for all iterations.

In fact, the Lagrange multiplier corresponding to the constraint $g_n(x_n) \le \theta_n$ in the definition of $p_n(\theta_n)$ can be generally computed by

$$
\lambda_n = \begin{cases} -\frac{f_n'(x_n^*)}{g_n'(x_n^*)}, & \text{if} -\frac{f_n'(x_n^*)}{g_n'(x_n^*)} > 0 \\ 0, & \text{otherwise} \end{cases} \tag{7}
$$

where

$$
x_n^* = \underset{g_n(x_n)\le\theta_n, x_n\in X_n}{\arg\min} \quad f_n(x_n)
$$

In summary, the explicit resource allocation coordination algorithm is:

Resource allocation coordination algorithm

---

i) Initialize $\theta_n^{(1)}$ for all n and set $z = 1$.
ii) At each iteration z, each agent n solves
$$
x_n^{*,(z)} = \underset{g_n(x_n)\le\theta_n^{(z)}, x_n\in X_n}{\arg\min} \quad f_n(x_n)
$$
and obtains $\lambda_n^{(z)}$ by using equation (7).
iii) Obtain $\theta_n^{(z+1)}$ by using (5).
iv) Stop if $|\theta_n^{(z+1)} - \theta_n^{(z)}| \le \varepsilon$ for all n are satisfied or the maximum number of iterations is reached; otherwise set $z \leftarrow z+1$ and go back to ii).

III. INTEGRATE RESOURCE ALLOCATION COORDINATION METHOD INTO BRANCH-AND-BOUND

Actually, the resource allocation coordination method can guarantee that the global optimum is attained if the overall problem (1) is strictly convex. However, even given $f_n(\cdot)$ and $g_n(\cdot)$ are strictly convex functions, if x_n is integer decision variable or subject to discrete values i.e. X_n is a set of integers or discrete values, the overall problem is not convex. In fact, in that case the integer or discrete decision variables will

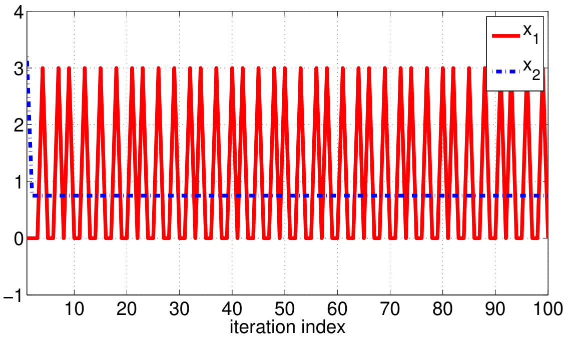

Fig. 1. Discrete decision variables exhibit oscillatory behaviors

exhibit oscillatory behaviors and the global optimum will not be attained by using the resource allocation coordination method. To be more specific, a simple numerical example describing the problem of directly using the resource allocation coordination on an integer optimization problem is given here:

$$\begin{aligned} \underset{x_1,x_2}{\text{minimize}} \quad & (x_1-2)^2+(x_2-2)^2 \\ \text{subject to} \quad & x_1 \in \{-3, 0, 3, 3.5, 3.7\} \\ & x_2 \in \{-2.7, 0.75, 3.2, 3.6, 3.8\} \\ & x_1 + x_2 \leq 4 \end{aligned}$$

and Figure 1 shows the behaviors of the decision variables.

A. Generic branch-and-bound

Branch-and-bound is an algorithm design paradigm for combinatorial optimization problems that involve integer or discrete variables. Actually, disregarding the computation time, an algorithm designed according to the branch-and-bound paradigm can always find the global optimum of a combinatorial optimization problem. Therefore, we propose to integrate the resource allocation coordination method into the branch-and-bound paradigm.

B. Using resource allocation coordination method as a bounding technique

Although there is no guarantee for all cases that the global optimum is attained, the resource allocation coordination method has a nice economic interpretation and has already been proposed as a heuristic [2]. Therefore, we use the resource allocation coordination method as a heuristic to compute an upper bound of the minimum of value of the overall problem within a given subset of the overall solution space. Meanwhile, a lower bound of the minimum of the problem within the same subset of the overall solution space is obtained by solving the relaxed problem where integer variables are relaxed to real variables.

C. Branching based on the outcome of resource allocation coordination method

If x_j for all j are integer decision variables, their values may oscillate between two different integers during the iterations when calling the resource allocation coordination method on a subspace of the overall solution space. Actually, we can make use of the oscillation of the decision variables to help with the further branching of the solution space.

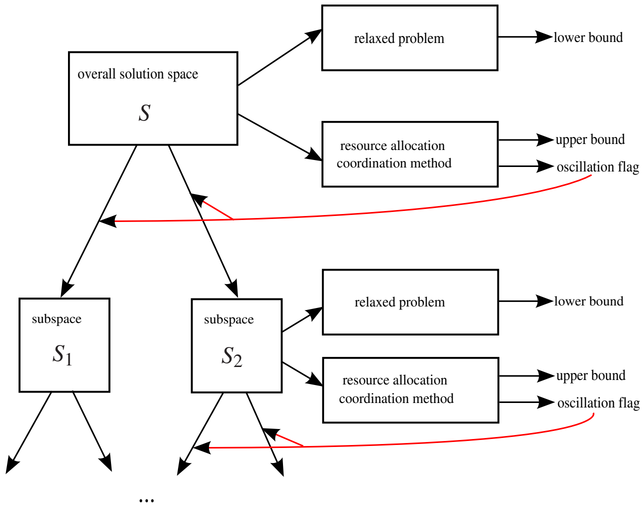

Fig. 2. The overall method

We propose the following way to branch the solution space when the oscillations of decision variables are diagnosed:

- assume $S = X_1 \times X_2 \times \ldots \times X_N$ and call resource allocation coordination method on S.
- when oscillations of decision variables are diagnosed, choose a decision variable x_n which oscillates between $\alpha_n \in X_n$ and $\beta_n \in X_n$.
- perform branching

$$S_1 = X_1 \times \ldots \times \left(X_n \setminus \{x_n | x_n \leq \alpha_n\}\right) \times \ldots \times X_N$$
$$S_2 = X_1 \times \ldots \times \left(X_n \setminus \{x_n | x_n > \alpha_n\}\right) \times \ldots \times X_N$$

Note that the oscillation of decision variable x_n is characterized by

$$x_n^{*,(z+1)} \neq x_n^{*,(z)}, \qquad \text{sgn}\left(\Delta\theta_n^{(z+1)}\right) \neq \text{sgn}\left(\Delta\theta_n^{(z)}\right) \tag{8}$$

with $\Delta\theta_n^{(z+1)} = \theta_n^{(z+1)} - \theta_n^{(z)}$ and $\Delta\theta_n^{(z)} = \theta_n^{(z)} - \theta_n^{(z-1)}$. Therefore, we diagnose the oscillation of decision variables by detecting the condition (8) for each n.

Even if none of the decision variables oscillates, in general, it may happen that the global optimum is not yet reached. Therefore, if no oscillation of any decision variable is diagnosed during the calling of resource allocation coordination method, we propose to arbitrarily choose a decision variable x_n and a $\gamma_n \in X_n$ and then perform branching

$$S_1 = X_1 \times \ldots \times \left(X_n \setminus \{x_n | x_n \leq \gamma_n\}\right) \times \ldots \times X_N$$
$$S_2 = X_1 \times \ldots \times \left(X_n \setminus \{x_n | x_n > \gamma_n\}\right) \times \ldots \times X_N$$

D. The overall method

Finally, the flowchart of the integration of resource allocation coordination method into the branch-and-bound paradigm is given in Figure 2. More specifically, the overall method consists of two levels:

- the inner procedure of resource allocation coordination at each node in the tree
- the outer procedure of branch-and-bound

Since branch-and-bound is used as the outer procedure, the stopping criterion of the overall method is the same as that of the general branch-and-bound.

The global optimal solution to the integer programming problem is always obtained by using the proposed method. This is guaranteed by the fact that in the branch-and-bound paradigm, the size of solution space decreases monotonically until the global optimal solution is found.

Compared with the classical integer optimization method based on the branch-and-bound paradigm but with a predefined branching sequence, our method has advantages in both *bounding* and *branching*. More specifically, since the resource allocation coordination method is a distributed method, it can find an upper bound of the minimum of the problem within a shorter time span. Moreover, since the resource allocation coordination method is also a heuristic method [2], branching based on the heuristic results is in general better than that based on a predefined sequence if no prior knowledge of the problem is given.

IV. CHARGING CONTROL OF ELECTRIC VEHICLES

By assuming that the profile of the electricity price and the arrival and the departure times of all EVs at a charging station given, we focus on the optimal charging control of a fleet of EVs at a charging station. We aim to achieve that all the EVs charge up to the required level within the required time and the total cost on charging the EVs is minimized.

A. Definitions

Let k be the discrete-time counter and T be the simulation interval whose typical value is 15 minutes. Let N_{v} be total number of EVs. Let $T_{i,\mathrm{arrival}}$ be the arrival time of EV i at the charging station and $T_{i,\mathrm{departure}}$ be the departure time of EV i from the charging station. Without loss of generality, we assume $T_{i,\mathrm{arrival}}$ and $T_{i,\mathrm{departure}}$ are integer multiples[1] of T. Let $s_{i,k}$ be the state of charge of EV i at time kT and s_i^{d} be required state of charge of EV i when it departs from the station. Let C_i be the capacity of the onboard battery of EV i and $d_{i,\mathrm{tol}}$ be the tolerance of difference between the state of charge of EV i at its departure time and s_i^{d}. Finally, let $p_{i,k}$ be the consumed power by EV i at time kT and $u_{i,k}$ be the binary decision variable indicating whether EV i is charging at time kT or not.

B. Model of the charging of an individual electric vehicle

The charging power of an electric vehicle at time step $k-1$ is given by

$$p_{i,k-1} = \begin{cases} F\left(s_{i,k-1}\right), & \text{if } u_{i,k-1} = 1 \\ 0, & \text{if } u_{i,k-1} = 0 \end{cases} \tag{9}$$

[1]If vehicles arrive earlier or depart later than a sampling time instant, they will not be charging in the partial time slot of the simulation interval within which they arrive or depart.

where the nonlinear function $F(\cdot)$ describes how the consumed power of EV i depends on its state of charge.

Therefore, the dynamics of the state of charge of an electric vehicle is given by

$$p_{i,k-1} = F\left(s_{i,k-1}\right) \cdot u_{i,k-1} \tag{10}$$

$$s_{i,k} = s_{i,k-1} + \frac{p_{i,k-1} \cdot T}{C_i} \tag{11}$$

In general, there are two charging options available for EV chargers, namely *Constant Current - Constant Voltage* (CCCV) option and *Constant Power - Constant Voltage* (CPCV) option [15]. With the CPCV option, the vehicle is first charged with constant power until the critical state of charge is reached. After that, it is charged with constant voltage until it is fully charged.

C. Coupling constraints

At any time, the total power consumption of all the EVs should not be more than the maximum power that can be provided by the grid. Therefore, the coupling constraints on all the EVs imposed by the capacity of the grid are given by

$$\sum_{i=1}^{N_{\mathrm{v}}} p_{i,k} \leq P^{\max}, \quad k = 1, \ldots, k_{\mathrm{d}} \tag{12}$$

where $P^{\max}$ denotes the steady maximum power limit[2] provided by the grid.

D. Charging cost

Given the profile of price of electricity, the total cost on charging all the EVs is given by:

$$J = \sum_{i=1}^{N_{\mathrm{v}}} \sum_{k=k_{i,\mathrm{arrival}}}^{k_{i,\mathrm{departure}}-1} p_{i,k} \cdot T \cdot c_k \tag{13}$$

where c_k denotes the price of electricity at time kT and

$$k_{i,\mathrm{arrival}} = \frac{T_{i,\mathrm{arrival}}}{T}, \quad k_{i,\mathrm{departure}} = \frac{T_{i,\mathrm{departure}}}{T}$$

E. Problem formulation

According to [15], if the CPCV option is used, an electric vehicle can be charged with constant power up to more than 90% of the capacity of its onboard battery. After that, the charging power will decrease dramatically along with the improvement of state of charge of the vehicle. If we define $s_{i,\mathrm{critical}}$ as the critical state of charge of vehicle i right after which the charging power starts to decrease, then after vehicle i is charged up to $s_{i,\mathrm{critical}}$, it will still take much time for vehicle i to be fully charged (100%) since the charging power decreases dramatically. Therefore, we assume that

$$s_i^d \leq s_{i,\mathrm{critical}} \tag{14}$$

holds for all i. Then, we have $p_{i,k} = p_{i,\mathrm{constant}}$ for all i and all k.

[2]The maximum power limit might also be time-dependent.

By making assumption (14), the model of the charging of an electric vehicle can be simplified to

$$s_{i,k_{i,\text{departure}}} = s_{i,k_{i,\text{arrival}}} + \sum_{k=k_{i,\text{arrival}}}^{k_{i,\text{departure}}-1} \frac{p_{i,\text{constant}} \cdot u_{i,k} \cdot T}{C_i}$$

and the charging requirement constraint $s_i^d - s_{i,k_{i,\text{departure}}} \leq d_{i,\text{tol}}$ for each vehicle i can be written as

$$\frac{s_i^d - s_{i,k_{i,\text{arrival}}} - d_{i,\text{tol}}}{p_{i,\text{constant}} \cdot T} \cdot C_i \leq \sum_{k=k_{i,\text{arrival}}}^{k_{i,\text{departure}}-1} u_{i,k}$$

Note that the decision variables are all binary values. Then, the constraint can be further rewritten as

$$\sum_{k=k_{i,\text{arrival}}}^{k_{i,\text{departure}}-1} u_{i,k} \geq m_i \tag{15}$$

where m_i is an integer constant determined by

$$m_i = \text{ceiling}\left(\frac{s_i^d - s_{i,k_{i,\text{arrival}}} - d_{i,\text{tol}}}{p_{i,\text{constant}} \cdot T} \cdot C_i \right)$$

Finally, the constraint (15) is added as penalty term to the objective function with sufficiently large weight β_i for $i = 1, \ldots, N_\text{v}$. Then, the optimal charging control problem of electric vehicles can be formulated as

$$\begin{aligned} \underset{\boldsymbol{U}}{\text{minimize}} \sum_{i=1}^{N_\text{v}} \Bigg(& \frac{1}{J_{i,\text{cost,typical}}} \sum_{k=k_{i,\text{arrival}}}^{k_{i,\text{departure}}-1} p_{i,\text{constant}} \cdot T \cdot c_k \cdot u_{i,k} \\ & + \frac{\beta_i}{k_{i,\text{departure}} - k_{i,\text{arrival}}} \left| m_i - \sum_{k=k_{i,\text{arrival}}}^{k_{i,\text{departure}}-1} u_{i,k} \right| \Bigg) \end{aligned} \tag{16}$$

$$\text{subject to} \quad \sum_{i=1}^{N_\text{v}} p_{i,\text{constant}} \cdot u_{i,k} \leq P^{\max}, \; k = 1, \ldots, k_\text{d}$$

where $\boldsymbol{U} = [U_1^\text{T}, U_2^\text{T}, \ldots, U_{N_\text{v}}^\text{T}]^\text{T}$ is a compact decision variable symbol with $U_i = [u_{i,k_{i,\text{arrival}}}, u_{i,k_{i,\text{arrival}}+1}, \ldots, u_{i,k_{i,\text{departure}}-1}]^\text{T}$ and $J_{i,\text{cost,typical}}$ is the typical value for the charging cost of vehicle i, which is used to normalize the real charging cost and is given by

$$J_{i,\text{cost,typical}} = \left(k_{i,\text{departure}} - k_{i,\text{arrival}} \right) \cdot T \cdot \bar{c} \cdot p_{i,\text{constant}}$$

with $\bar{c}$ denoting the average price of electricity. It is clear that the optimal charging control problem (16) is a specific case of the general problem (1). Therefore, the resource allocation coordination method is naturally applicable to problem (16).

In the simulation study, we will apply our optimization method based on the integration of resource allocation coordination method into branch-and-bound paradigm to solve problem (16).

V. NUMERICAL SIMULATION STUDY

We consider a case where 6 electric vehicles need to be charged. The information of all the vehicles is summarized in Table I. Note that the number of binary decision variables in this case is $\sum_{i=1}^{6}(k_{i,\text{departure}} - k_{i,\text{arrival}}) = 24$. The other

TABLE I

FLEET INFORMATION

i	k_arrival	$k_\text{departure}$	s_initial	s^d	C (kWh)	p_constant (kW)
1	3	6	0.6	0.8	9	3.5
2	1	4	0.35	0.45	7.1	2.5
3	2	5	0.4	0.6	8	3
4	5	10	0.6	0.9	8.5	2.7
5	4	8	0.5	0.7	7.5	3.2
6	3	9	0.3	0.5	7.8	3.1

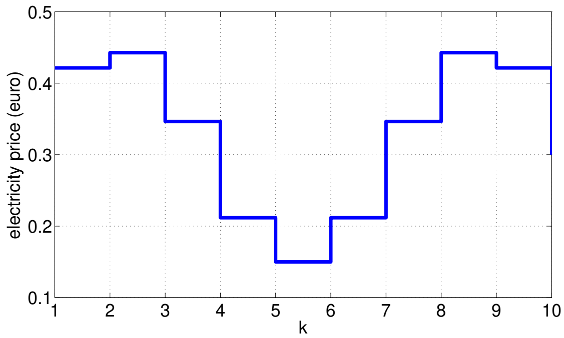

Fig. 3. Profile of electricity price

parameters used in the simulation are $T = 15$ min, $P^{\max} = 9$ kW, $d_{i,\text{tol}} = 0.02$ and $\beta_i = 200$ for $i = 1, \ldots, 6$; the profile of electricity price is shown in Figure 3.

In the simulation, we solve problem (16) by using three different optimization methods. The first two methods use *glcSolve* and *minlpBB* of the Tomlab/MINLP optimization toolbox for Matlab, respectively. More specifically, *glcsolve* is a solver for global optimization problems with both non-linear and integer constraints by implementing an extended version of DIRECT algorithm [16]. The solver *glcsolve* is run for a predefined number of function evaluations and considers the best function value found as the optimal one. It is possible to restart *glcsolve* with a warm-start, where all parameters are set to the final status from the previous run. The solver *minlpBB* is for large, sparse or dense mixed-integer linear, quadratic and nonlinear programming problems by implementing a branch-and-bound algorithm searching a tree whose nodes correspond to continuous non-linearly constrained optimization problems that are solved using *filterSQP* [17]. The third one is the method proposed in this paper.

The simulations are performed using Matlab 2013b on a desktop computer with an Intel® Core™ i5-2400 CPU with 3.10 GHz and 4 GB RAM. The simulation results of using the three optimization methods are summarized in Table II. It is clear that our method outperforms the first two methods in both solution quality and computation time. In addition, by using the distributed nature of the resource allocation coordination method and the distributed evaluation of nodes in the branch-and-bound paradigm, our method can be further accelerated if executed on several processors in parallel. More specifically, at each node of the tree in the branch-and-bound paradigm, the resource allocation coordination

TABLE II

COMPARISON OF THE THREE OPTIMIZATION METHODS

Optimization method	J_{opt}	computation time (s)
glcSolve (30 warm start runs)	0.2086	2056.3
minlpBB	0.2502	1620.2
our method	0.2047	647.3

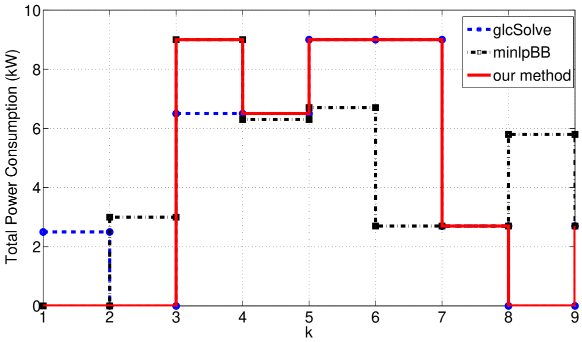

Fig. 4. Total power consumption of the EVs

method can be implemented in a distributed way by assigning one processor for each subproblem. Besides, since the nodes on different branches of the tree in the branch-and-bound paradigm are independent from each other, the evaluation of those nodes can be assigned to different processors.

Finally, the total power consumption of all time steps and the final state of charge of all vehicles corresponding to the solution found by the three methods are given in the Figure 4 and Table III, respectively.

It is clearly seen from Table III that the vehicles are all charged up to the required levels under the charging control using the three optimization methods. However, Figure 4 together with Figure 3 clearly shows that the solution found by our method is best since under the charging control using our method, the fleet of vehicles only charge at the times when the electricity price is the low.

VI. CONCLUSION

In this paper, a novel integer optimization method has been proposed by integrating the resource allocation coordination method into the branch-and-bound paradigm. Thanks to the distributed computation of the resource allocation coordination method and distributed evaluation of nodes in the branch-and-bound paradigm, the proposed method is capable of solving an integer programming problem in a distributed way. Moreover, the optimality of the proposed method is proved by the fact that in the branch-and-bound paradigm the size of solution space decreases monotonically until the global optimum is attained. Finally, the proposed method has been applied to the optimal charging control of electric vehicles under constrained grid conditions in a numerical simulation study. It is shown in the simulation results that the proposed method outperforms the commercial MINLP solvers in both solution quality and computation time.

In our future work, we will first further improve the efficiency of the proposed method, e.g., by directly projecting the update of resource allocation onto the feasible discrete space, and then apply the resulting integer optimization method to distributed model predictive control of large-scale hybrid systems and in particular, of power system and transportation system.

TABLE III

FINAL STATE OF CHARGE OF EVS AFTER CHARGING

Method	EV 1	EV 2	EV 3	EV 4	EV 5	EV 6
glcSolve	0.7944	0.438	0.5875	0.9176	0.7133	0.4987
minlpBB	0.7944	0.438	0.5875	0.9176	0.7133	0.4987
our method	0.7944	0.438	0.5875	0.9176	0.7133	0.4987

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