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Model Predictive Control for Maintenance Operations Planning of Railway Infrastructures

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Abstract. *This paper develops a new decision making method for optimal planning of railway maintenance operations using hybrid Model Predictive Control (MPC). A linear dynamic model is used to describe the evolution of the health condition of a segment of the railway track. The hybrid characteristics arise from the three possible control actions: performing no maintenance, performing corrective maintenance, or doing a replacement. A detailed procedure for transforming the linear system with switched input, and recasting the transformed problem into a standard mixed integer quadratic programming problem is presented. The merits of the proposed MPC approach for designing railway track maintenance plans are demonstrated using a case study with numerical simulations. The results highlight the potential of MPC to improve condition-based maintenance procedures for railway infrastructure.*

Keywords: Health Condition Monitoring and Maintenance, Model Predictive Control (MPC), Track Maintenance, Railway Engineering

1 Introduction

A railway track infrastructure system is composed of a set of different assets. All of those assets are distributed and interconnected over the railway network, continuously working together to keep the railway service reliable, safe, and fast. Each asset has a different need for maintenance, at different times and according to its degradation process, which is influenced by geographic position, tonnage of the track, health condition of the rolling stock, among many other factors. Thus, to sustainably manage railway assets, a step forward from the current policy of "find and repair" towards a more integrated methodology containing condition-based monitoring and predictive maintenance is required to improve the entire whole system performance. In the Netherlands, over forty percent of the maintenance costs is related to track maintenance [1]. Due to this fact, a condition-based maintenance decision support system can facilitate the infrastructure manager to decide where and which type of maintenance should be performed. Moreover, if a prediction capability is incorporated in the decision making, we can expect maintenance actions that will anticipate problems and will take corrective measures before a failure become costly or unsafe for the users. This study proposes a model based predictive maintenance strategy using condition-based monitoring. We in particular show how this strategy can be applied to maintenance planning for ballast degradation and for treating squats.

The role of ballast is to provide support to the tracks with hard stones aiming to distribute loads over the sleepers while the train is passing. It also allows rain and snow to drain, thermal expansion, and weight variance; and it inhibits the growth of weed and vegetation. In the deterioration process of ballast, some stones may be displaced due to the vibrations and some others will get a white rounded shape losing their properties (see Figure 1a). When the ballast is in a bad condition, it will be reflected in the rail geometry. In order to keep the performance level at a satisfactory condition, tamping (packing the track ballast under the railway track) and ballast replacement are the principal maintenance actions to consider. In the literature, different studies have been proposed on how to predict changes in the track geometry condition. In some studies the ballast deterioration is modeled deterministically based on the average growth of track irregularities [2,3,4]. Other studies have linked stochastic degradation process with the effects of possible maintenance and renewal options [5,6].

Squats are a type of Rolling Contact Fatigues (RCFs) that initiate on the rail surface and evolve into a network of cracks beneath the surface of the track that when not treated on time can evolve into rail breakage. Treating RCFs to avoid a reduced life cycle of the track is very expensive. Early stage squats can be efficiently treated with grinding, while the only solution for late stage squats is track replacement (See Figure 1b). There are different methods to detect squats, like non-automatic inspection using human inspectors, photo/video records, and non-destructive testing such as ultrasonic and eddy current test [7,8]. For automatic detection of squats in an early stage, axle box acceleration (ABA) systems can be efficiently employed [9]. Predictive and robust models for squats evolution have been proposed in [10]. In [11], it is suggested to consider clusters of squats to facilitate grinding maintenance operations; however, the maintenance actions were obtained under static scenarios. In this paper, the main contribution is to propose a suitable model that incorporates the dynamics in the decision process of railway track maintenance operations, together with a rolling horizon methodology that can deal with discrete integer control actions.

The major benefit of applying Model Predictive Control (MPC) to maintenance operations planning is that the resulting strategy is flexible. By updating the degradation model using health condition monitoring methodologies regularly, the maintenance plans can be adapted dynamically. Especially when severe problems are predicted within the prediction horizon, MPC will suggest more frequent or more effective maintenance operations, resulting in a more efficient plan. This is a big step from current practice in railway maintenance, where cyclic preventive maintenance prevails, which, as a myopic strategy, is unable to predict the evolution of the degradation process, and treats severe problems only when they occur. Another merit of the proposed predictive methodology is that the objective function explicitly captures the trade-off between maintenance costs and the health condition of the track. This is crucial for railway infrastructure managers, who require transparent tools that facilitate the decision making process. Moreover, other factors concerning the management of railway infrastructures, like closure time due to maintenance, can also be conveniently added to the MPC optimization problem. In addition, limits on the admissible degradation level can be included effortlessly as constraints in MPC, which is especially useful for the maintenance of safety-critical components.

This paper is organized as follows: first a brief introduction to MPC is presented in Section 2; then Key Performance Indicators (KPIs) and maintenance options for railway infrastructures are explained in Section 3; the proposed MPC approach is explained in detail in Section 4 and illustrated by the case study in Section 5; finally a short summary and remarks on future work is provided in Section 6.

2 Model Predictive Control (MPC)

Model Predictive Control (MPC) is an advanced design methodology for control systems, which has gained wide popularity in the process industry since the last decade. MPC was pioneered simultaneously by Richalet et al. [12,13] and Cutler and Ramaker [14] in the late 1970s. The main reasons for the success of MPC in the process industry are:

- Easy handling of multi-input-multi-output (MIMO) processes, non-minimum phases processes, processes with large time delay, and unstable processes;
- Easy tuning of parameters (in principle only three parameters need to be tuned);
- Natural embedding of constraints in a systematic way;
- Easy handling of structural changes by regularly updating the process model.

Five elements are essential for MPC:

1. A process model
2. A cost criterion

(a) Loose ballast indicated by the presence of white dust

(b) Severe squats (type C)

Fig. 1: Two different defects in railway infrastructures

3. Constraints
4. Optimization algorithms
5. Receding horizon principle.

For maintenance operations planning in railway network, the process model can be the degradation model or the performance indicator of an asset, the cost criterion can be a trade-off between track condition and maintenance cost, and the constraints can be an upper bound on the maximum degradation level and budget. Depending on the process model, which might contain continuous and discrete variables, the cost criterion, and the imposed constraints, the computation of a sequence of future control actions at each sample step will result in different optimization problems, which must be solved by some mathematical programming algorithm. For railway maintenance, since the degradation of the track condition is a continuous process and maintenance operations are intrinsically discrete, the planning of maintenance operations should in general result in a mixed integer programming problem. This optimization problem must be solved at each sample step, providing a sequence of optimal discrete control actions. The length of the sequence is called prediction horizon when using a receding horizon approach. Instead of using the whole sequence of predicted control actions, only the first entry is applied, and a new sequence is computed at the next sample step with updated information, e.g. new measurements of the track condition.

Despite its success in the process industry, the applications of MPC in railway maintenance operations are scarce, although the application of mathematical models and optimization is not uncommon in maintenance [15,16]. The process model associated with most operations planning problems usually contains both continuous and discrete dynamics. In [17] an MPC scheme is applied to plan risk mitigation actions together with other control variables, using a mixed integer quadratic formulation. For complex decision making problems, a hierarchical or distributed approach is often applied to render the problem tractable. See [18] for an application of hierarchical distributed MPC to risk management of a network of irrigation canals, and [19] for applications of hybrid MPC to interventions in behavioral health and inventory management in supply chains. As a model-based decision making approach, MPC can be viewed as an extension of condition-based maintenance, as it does not only take into consideration the current track condition, but also predicts the track condition, using updated track measurements, as well as knowledge on the process, e.g. degradation models.

3 Railway Maintenance

A brief introduction on railway maintenance is presented, explaining how the track condition is measured in practice, as well as typical operations on track maintenance.

3.1 Key Performance Indicators (KPIs) for maintenance planning

To ensure the proper functioning of railway tracks, both temporal and spatial characteristics need to be considered in the maintenance decisions. For this purpose, Key Performance Indicators (KPIs) are developed to capture the dynamics of track deterioration and the evolution of defects. These KPIs usually consider a broad set of measurements from different sources and define the health condition of the track as a single number. When normalized, 0 would mean a healthy track, while 1 a track with very bad condition. This health number changes over time, and when reaching some threshold corrective maintenance is performed.

In the case of ballast, the track bed positioning and alignment changes in terms of speed and number of passing trains have to be considered in the design of KPI. The following measurements are usually considered [20,21]: (1) cyclic top, which is a measure of resonant frequencies, (2) rail spacing compared to the standard gauge, (3) a measure of cant variation over 3 m and 5 m, (4) lateral geometry (alignment) - rail alignment, averaged over the left and right rail, expressed as short and long wave standard deviations.

In [10] some KPIs are proposed for maintenance operations related to squats using axle box acceleration measurements. The number of squats of type A (light squats in early stage), number of squats of type B or C (severe squats), number of potential risk points, and density of squats are the KPIs proposed. Field observations also revealed that different squats have different rates of growth, thus three different evolution scenarios were proposed: slow growth, average growth, and fast growth. Due to the big number of KPIs, temporal dependence and scenarios, one global KPI using a fuzzy inference system was proposed to facilitate maintenance decisions. This global KPI for squats represents the health condition of a cluster of defects and it is represented by a score between zero (representing a healthy state of the track) and two (indicating an unhealthy condition of the track).

3.2 Maintenance options

This paper evaluates three possible maintenance options: (1) to do nothing, (2) corrective maintenance: tamping for ballast or grinding for squats, (3) replacement: full ballast replacement or track replacement. Next, a short description of the maintenance actions is given.

In the case of tamping, the track geometry is adjusted when the track alignment is outside the accepted tolerances. Tamping machines are able to pack the ballast under the sleepers in order to correct the alignment of the rails using measurements of the track geometry, estimating the needed adjustments, lifting/inserting the track and at the same time vibrating tamping arms [22]. Tamping can improve the track condition. However, in the worst case, vibrating arms into the ballast could lead some break-up of the stones which can accelerate the ballast degradation. When the ballast has reached the end of its useful life, then ballast renewal must be carried out.

In the case of grinding, the Dutch railways use a cyclic grinding strategy that is not based on the condition of the track. This mean that sections that are healthy are ground, and also sections that need replacement are ground in spite of its inefficiency. Squats in early stage can be removed by grinding, because they have not developed the network of crack beneath yet. Therefore, early detection of RCF rail defects is cost-effective when a grinding campaign is scheduled appropriately in time. Furthermore, grinding is not efficient for cracks deeper than 5-7 mm. Grinding severe squats can delay rail replacement but may accelerate the squat evolution, because the cracks are not yet removed.

When the conditions of either ballast or track has worsened, and tamping or grinding is ineffective, then replacement will be the only option. With predictive models that incorporate the degradation process of the ballast or the track, it is possible to estimate the life cycle of the track and to inform the infrastructure manager about the tentative time for replacing. This is crucial information because replacement campaigns are extremely costly and the track needs to be unavailable for a long period. In the case of ballast replacement, keeping a similar total volume of stones before and after the replacement is essential. Long segments with deteriorated ballast will require a higher packing level of the damaged ballast [23]. In the case of rails, they are typically replaced according to predefined estimated life cycle, tonnage, wear limit, fatigue, and weather conditions. Determining the optimal rail replacement interval is a critical issue for rail industry, due to its high cost and consequences over the entire performance of the infrastructure.

Figure 2 shows a generic KPI and the typical effect of corrective maintenance or replacement. While a corrective maintenance (grinding or tamping) will improve the performance, a general drop of the performance is usually observed. In the case of replacement, while more expensive than corrective maintenance, it usually leads into a healthy status for a longer period of time. In the next section, a hybrid MPC methodology is proposed to capture the principal dynamics of railway maintenance operation.

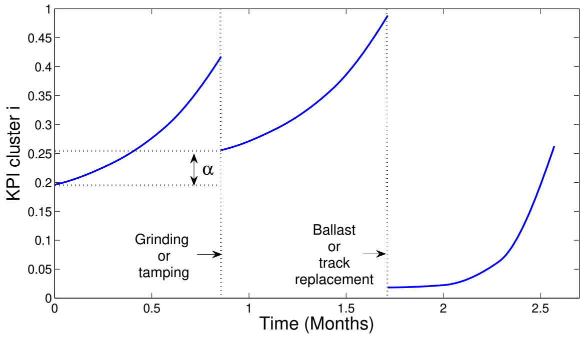


Fig. 2: Generic KPI evolution over time.

4 MPC for Maintenance Operation Planning

A maintenance model describing the degradation dynamic of the track condition is presented, with three maintenance operations as inputs. This linear model with discrete inputs is then transformed into a standard Mixed Logic Dynamic (MLD) system, and an MPC scheme is designed for this MLD system.

4.1 Maintenance Model Description

We consider the following discrete-time state space model for the degradation dynamic (unhealthy condition) of a certain asset (e.g. track with squats, or ballast) in a segment of track:

$$\begin{aligned} x_1(k+1) &= a_1 x_1(k) + f_1(x(k),\, u(k)) \\ x_2(k+1) &= x_2(k) + f_2(x(k),\, u(k)) \end{aligned} \tag{1}$$

with constraint

$$m \leq x_i(k) \leq M \quad \forall i \in \{1,\, 2\} \tag{2}$$

The state vector is $x(k) = \left[x_1(k)\ x_2(k)\right]^{\mathrm{T}}$. In particular, x_1 represents the level of degradation of a part of the track regarding a certain type of fault, e.g. the average length of the squats in a cluster. The parameter a_1 is the degradation rate of the track, which is assumed to be larger than 1 so that the track condition deteriorates exponentially as observed in late stage of degradation. The state x_2 records the level of degradation upon the last corrective maintenance operation. This is necessary because corrective operation will be unlikely to bring the track back to a condition healthier than the previous corrective operation. The states are bounded in the interval $[m,\, M]$ by the constraint (2), with m and M representing the lowest and highest admissible degradation level, respectively.

The input $u(k) \in \{1, 2, 3\}$ is a discrete input representing the three major maintenance operations: do nothing, corrective maintenance, and replacement, respectively. The interpretation of the input u is given in Table 1.

Table 1: Interpretation of maintenance operations and the corresponding value of input

Input value	**Maintenance operation**	**Effect on the track condition**
1	do nothing	no effect
2	corrective maintenance	bring the track condition to a healthier level, but usually not as healthy as the condition upon the last corrective maintenance
3	replacement	bring the track condition to a state with no degradation

The effects of the maintenance operations are captured by the discontinuous functions f_1 and f_2, which take the following representation:

$$f_1(x(k),\, u(k)) = \begin{cases} 0 & \text{if } u(k) = 1 \\ -a_1 x_1(k) + x_2(k) & \text{if } u(k) = 2 \\ -a_1 x_1(k) + m & \text{if } u(k) = 3 \end{cases} \tag{3}$$

and

$$f_2(x(k),\, u(k)) = \begin{cases} 0 & \text{if } u(k) = 1 \\ \alpha & \text{if } u(k) = 2 \\ -x_2(k) + m & \text{if } u(k) = 3 \end{cases} \tag{4}$$

with α, which usually takes a small positive value, indicating the offset of the effect of one corrective maintenance from the last corrective maintenance, i.e. a corrective maintenance can only bring the condition to a level which is α worse than that just after the previous corrective maintenance operation.

4.2 Transformation into an Mixed Logic Dynamic (MLD) Systems

The system (1) can be viewed as a hybrid system with linear plant model (degradation process) and switching control. The difficulty of designing a controller for such systems lies in the different conditions triggered by different values of the control input (equations (3)–(4)). By associating the three conditions triggered by the three control actions with two binary variables $\delta_1(k)$ and $\delta_2(k)$, the system (1) can be transformed into a Mixed Logic Dynamic (MLD) system. The translation from the control input $u(k)$ to the binary variables $\delta_1(k)$ and $\delta_2(k)$ is shown in Table 2.

Table 2: Translation from control input to binary variables

$u(k)$	$\delta_1(k)$	$\delta_2(k)$
1	0	0
2	0	1
3	1	0

We eliminate the fourth option ($\delta_1 = \delta_2 = 1$) by adding the constraint:

$$\delta_1(k) + \delta_2(k) \leq 1 \tag{5}$$

After translating the control input $u(k)$ into binary variables $\delta_1(k)$ and $\delta_2(k)$, system (1) can be rewritten as

$$\begin{aligned} x_1(k+1) &= a_1 x_1(k) + \delta_2(k)(-a_1 x_1(k) + x_2(k)) + \delta_1(k)(-a_1 x_1(k) + m) \\ &= a_1 x_1(k) - a_1 \delta_1(k) x_1(k) - a_1 \delta_2(k) x_1(k) + \delta_2(k) x_2(k) + \delta_1(k) m \\ x_2(k+1) &= x_2(k) + \alpha \delta_2(k) + \delta_1(k)(-x_2(k) + m) \\ &= x_2(k) - \delta_1(k) x_2(k) + m\delta_1(k) + \alpha \delta_2(k) \end{aligned} \tag{6}$$

The system (6) is non-linear in the state $x(k)$ and in the two binary variables $\delta_1(k)$ and $\delta_2(k)$. The non-linear system can be transformed into the following linear system:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} m & 0 \\ m & \alpha \end{bmatrix} \begin{bmatrix} \delta_1(k) \\ \delta_2(k) \end{bmatrix} + \begin{bmatrix} -a_1 & -a_1 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \\ z_3(k) \\ z_4(k) \end{bmatrix} \tag{7}$$

by introducing four auxiliary variables

$$\begin{aligned} z_1(k) &= \delta_1(k) x_1(k) & \qquad z_2(k) &= \delta_2(k) x_1(k) \\ z_3(k) &= \delta_2(k) x_2(k) & \qquad z_4(k) &= \delta_1(k) x_2(k) \end{aligned}$$

The equation for each auxiliary variable

$$z_p(k) = \delta_i(k) x_j(k) \quad p \in \{1,\, 2,\, 3,\, 4\},\ i,\ j \in \{1,\, 2\}$$

is equivalent to the following four inequality constraints [24,25]:

$$\begin{cases} z_p(k) \le M\delta_i(k) \\ z_p(k) \ge m\delta_i(k) \\ z_p(k) \le x_j(k) - m(1 - \delta_i(k)) \\ z_p(k) \ge x_j(k) - M(1 - \delta_i(k)) \end{cases} \tag{8}$$

which results in sixteen inequality constraints in total.

Finally, the mixed logical dynamic (MLD) system [24] described by the linear dynamics (7) and linear constraints (2), (5), (8) can be formulated in the following compact form [1]:

$$x(k+1) = Ax(k) + B_1 \delta(k) + B_2 z(k) \tag{9}$$

$$E_1 x(k) + E_2 \delta(k) + E_3 z(k) \le g \tag{10}$$

with $\delta(k) = \begin{bmatrix} \delta_1(k) \ \delta_2(k) \end{bmatrix}^{\mathrm{T}}$ and $z(k) = \begin{bmatrix} z_1(k) \ z_2(k) \ z_3(k) \ z_4(k) \end{bmatrix}^{\mathrm{T}}$

4.3 The MLD-MPC Problem and its solution via MIQP

Consider the MLD system (9)–(10). Denote by $\hat{x}(k+j|k)$ the estimated state at sample step k with information available at sample step $k+j$. Likewise, define $\hat{z}(k+j|k)$ as the estimate of the auxiliary variable at sample step $k+j$. Let N_{p} be the prediction horizon[2], and define

$$\tilde{x}(k) = \left[\hat{x}^{\mathrm{T}}(k+1|k) \ ... \ \hat{x}^{\mathrm{T}}(k+N_{\mathrm{p}}|k)\right]^{\mathrm{T}}$$

---

[1] Since the introduced binary variable δ is a full replacement of the original discrete control input u, which no longer appears in the resulting MLD system, we treat δ as control input for the MLD-MPC problem

[2] Here we set the control horizon equal to the prediction horizon

as the estimates of the future state at sample step k. The estimates of input and auxiliary variables ($\tilde{\delta}(k)$ and $\tilde{z}(k)$) can be defined in a similar way. Grouping the input and auxiliary variables together we further define

$$\tilde{V}(k) = \left[\tilde{\delta}^{\mathrm{T}}(k)\ \tilde{z}^{\mathrm{T}}(k)\right]^{\mathrm{T}}$$

After successive substitution of (9), the state prediction equation can then be written in the following compact form

$$\tilde{x}(k) = M_1\tilde{V}(k) + M_2x(k) \tag{11}$$

The goal of maintenance operations planning is to minimize degradation with the lowest possible maintenance cost, which can be formulated by the following objective function

$$J(k) = J_{\mathrm{Perf}}(k) + \lambda J_{\mathrm{Cost}}(k) \tag{12}$$

with $J_{\mathrm{Perf}}(k)$ and $J_{\mathrm{Cost}}(k)$ representing the performance and the cost of maintenance operations, respectively, while the weighting parameter $\lambda > 0$ captures the trade-off of the two conflicting objectives.

More specifically, if a weighted 2-norm is considered for performance, i.e. $J_{\mathrm{Perf}}(k) = \|\tilde{x}(k)\|_P^2$, with P positive definite weighting matrix, a mixed integer quadratic programming (MIQP) problem will be obtained. Alternatively, a 1-norm or an $\infty-$norm can also be applied, with $J_{\mathrm{Perf}}(k) = \|P\tilde{x}(k)\|_1$ or $J_{\mathrm{Perf}}(k) = \|P\tilde{x}(k)\|_\infty$, and P a matrix with non-negative entries. If a 1-norm or an $\infty-$norm is applied, the optimization problem can be recast as a mixed integer linear programming (MILP) problem, which is generally easier to solve than an MIQP problem.

The cost of maintenance operations $J_{\mathrm{Cost}}(k)$ has the following linear representation:

$$J_{\mathrm{Cost}}(k) = R\tilde{\delta}(k) = Q\tilde{V}(k)$$

with R a matrix with non-negative entries assigning weights to different maintenance operations, i.e. the cost of corrective maintenance and replacement. Since no weights are assigned to the auxiliary variables, we have $Q = \left[R\ 0\right]$.

For illustration purposes, we consider now a weighted 2-norm for J_{Perf}. Given the weighting matrices P and Q, the objective function can be rewritten as:

$$\begin{aligned} J(k) &= \tilde{x}(k)^{\mathrm{T}}P\tilde{x}(k) + \lambda Q\tilde{V}(k) && (13)\\ &= (M_1\tilde{V}(k) + M_2x(k))^{\mathrm{T}}P(M_1\tilde{V}(k) + M_2x(k)) + \lambda Q\tilde{V}(k) && (14)\\ &= \tilde{V}^{\mathrm{T}}(k)S_1\tilde{V}(k) + \left(S_2 + x^{\mathrm{T}}(k)S_3\right)\tilde{V}(k) + S_4 && (15) \end{aligned}$$

Note that S_4 is a constant and can be removed from the objective function without changing the solution of the optimization problem. Finally we can formulate the MLD-MPC problem as a standard MIQP problem with decision variable $\tilde{V}(k)$:

$$\min_{\tilde{V}(k)} \tilde{V}^{\mathrm{T}}(k)S_1\tilde{V}(k) + (S_2 + x^{\mathrm{T}}(k)S_3)\tilde{V}(k) \tag{16}$$

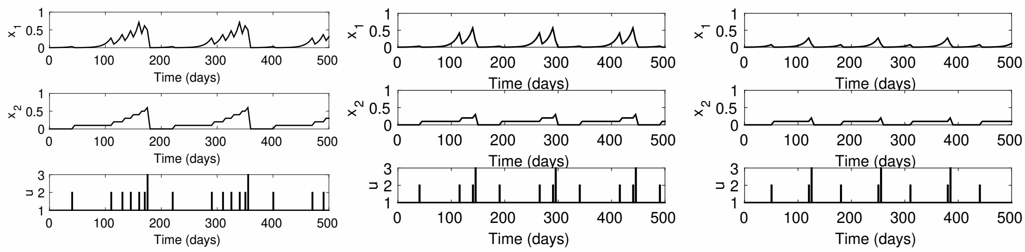
$$\text{s.t. } F_1\tilde{V}(k) \leq F_2 + F_3x(k) \tag{17}$$

MIQP problems are identified as NP-hard [26], indicating that in practice the solution time might grow exponentially with the problem size. Among the major algorithms for solving MIQP and MILP problems, branch-and-bound methods [27] are generally regarded as the most efficient ones.

5 Case Study

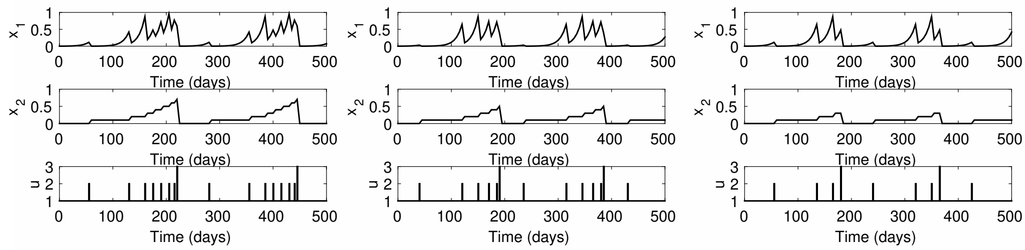
Now we consider a simple case study to illustrate the proposed approach. An MPC controller for the MLD system (9)–(10) with quadratic objective function (16) is implemented in Matlab, and the MIQP optimization problem at each sample step is solved using the Gurobi Optimizer 5.6.3, which can solve MIQP and MILP problems. The parameters for the degradation dynamics, as well as the performance criteria (in particular, the parameters for the objective function) are given in the appendix. We present the simulation results with three different prediction horizons ($N_\mathrm{p} = 5$, 10, 15), and three different weights ($\lambda = 0.1$, 1, 10).

A simple cyclic preventive maintenance approach with the same objective function is also implemented and optimized in Matlab for a comparison. The two decision variables for the cyclic approach are the period for corrective maintenance (T_c) and the period for replacement (T_r). The latter is usually fixed as a multiple of the former, i.e. $T_\mathrm{r} = nT_\mathrm{c}$, thus we determine the multiple n instead of T_r. The planning horizon is the whole simulation time, and an iteration of all possible combinations of T_c and n is applied to generate the best possible cyclic maintenance plan for a given cost criterion.



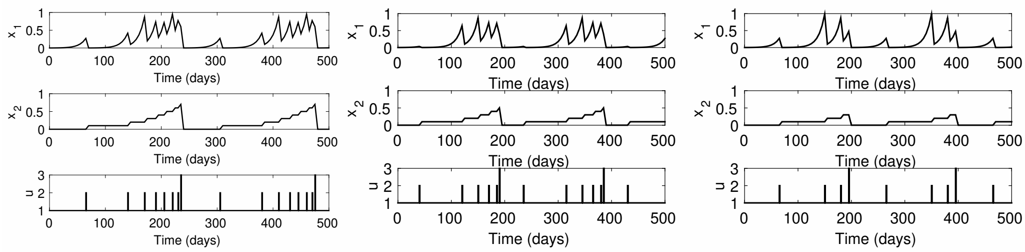
(a) $N_\mathrm{p} = 5$, $\lambda = 0.1$ (b) $N_\mathrm{p} = 10$, $\lambda = 0.1$ (c) $N_\mathrm{p} = 15$, $\lambda = 0.1$

Fig. 3: Predicted state x and control input u for different prediction horizons and a small λ



(a) $N_\mathrm{p} = 5$, $\lambda = 1$ (b) $N_\mathrm{p} = 10$, $\lambda = 1$ (c) $N_\mathrm{p} = 15$, $\lambda = 1$

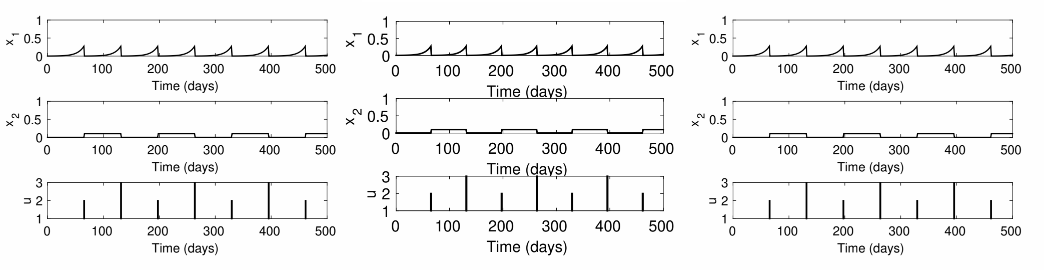
Fig. 4: Predicted state x and control input u for different prediction horizons and a medium λ



(a) $N_\mathrm{p} = 5$, $\lambda = 10$ (b) $N_\mathrm{p} = 10$, $\lambda = 10$ (c) $N_\mathrm{p} = 15$, $\lambda = 10$

Fig. 5: Predicted state x and control input u for different prediction horizons and a big λ

The simulation results from the MPC approach are given in Figure 3–5 and the results from the cyclic approach are given in Figure 6. The flexibility of MPC is clearly demonstrated by the resulting maintenance plans, which all suggest more frequent corrective maintenance as the track condition deteriorates, despite the differences in weighting parameters and the prediction horizon.



(a) $\lambda = 0.1$ (b) $\lambda = 1$ (c) $\lambda = 10$

Fig. 6: Maintenance plans and the corresponding effect on the track degradation level with different λ

Figure 3–5 also indicate that a longer prediction horizon results in a more cautious maintenance plan, i.e. earlier replacement and lower overall degradation level. The weight λ in the objective function represents the trade-off between the track condition and the maintenance cost. This is also shown in Figure 3–5, where it can be noticed that for a larger λ, the first replacement is suggested at a later time step, for a given N_{p}. On the contrary, the cyclic approach suggests the same optimal maintenance plan for the three different λ (corrective maintenance every 65 days and replacement every 130 days).

The values of the closed-loop objective function for both approaches are given in Table 3. It can be seen that with a long enough prediction horizon, the maintenance plan generated by MPC always out performs the plan generated by the cyclic approach.

Table 3: Evaluation of closed-loop objective function for MPC (J_{MPC}) and cyclic approach (J_{CYC}). For a given λ, the lowest objective function value is marked in bold.

λ	$J_{\mathbf{MPC}}$			$J_{\mathbf{CYC}}$
	$N_{\mathbf{p}} = 5$	$N_{\mathbf{p}} = 10$	$N_{\mathbf{p}} = 15$	
0.1	31.85	22.08	**11.88**	12.65
1	125.63	107.80	**90.23**	97.25
10	797.53	726.80	**702.25**	943.25

6 Conclusions

In this contribution a hybrid Model Predictive Control (MPC) approach with discrete input has been developed to support decision making in railway maintenance. We have provided a detailed procedure to illustrate how to transform the model-based optimization problem into a standard mixed integer quadratic programming problem. Numerical simulations have been performed for a case study with different parameter settings, and the results indeed demonstrate the potential of MPC as an optimization-based approach to aid decision making in maintenance of railway infrastructures.

Future work includes developing a more extensive process model with parameters obtained from track measurements, and extending the current approach from maintenance operations planning for a single segment of track to joint decision making for multiple segments.

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Appendix

The values and interpretation of the parameters for the degradation dynamics (1) are given in Table 4. The initial condition is $x(0) = [m\, m]^{\mathrm{T}}$, and the sampling time is 5 days.

Table 4: Parmaters for degradation dynamics

Parameter	**Value**	**Interpretation**
a_1	1.09	degradation rate
α	0.1	offset for corrective maintenance
m	0.001	minimum degradation level
M	1	maximum allowed degradation level

The matrix P is a square matrix consisting of diagonal replications of the diagonal matrices $P_{\mathrm{Sub}} = \mathrm{diag}(1,\, 0)$:

$$P = \mathrm{diag}\left(\underbrace{P_{\mathrm{Sub}}, \ldots, P_{\mathrm{Sub}}}_{N_{\mathrm{p}} \text{ times}}\right)$$

The matrix Q is a block-row matrix containing N_{p} horizontal replications of the diagonal matrix $Q_{\mathrm{Sub}} = \mathrm{diag}(r,\, 1,\, 0,\, 0,\, 0,\, 0)$:

$$Q = \left(\underbrace{Q_{\mathrm{Sub}}, \ldots, Q_{\mathrm{Sub}}}_{N_{\mathrm{p}} \text{ times}}\right)$$

where the parameter r is the ratio of the cost of one replacement over the cost of one corrective maintenance. For the case study we assume replacement is 30 times more costly than corrective maintenance, thus $r = 30$.