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An Iterative Procedure for Estimating the Generalized Average Speed Using Microscopic Point Measurements

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***Abstract*—Various traffic applications including model-based analysis and control of traffic need the average speed. The average speed is used as a measure of performance, and as an input for traffic models. It is also used to obtain the density from a known flow or vice versa. In this paper, a new iterative procedure is presented that uses point measurements from inductive loop detectors for estimating the time-space-mean speed (TSMS), which is an equivalent for the generalized average speed introduced by Edie (1965). An important subject that is missing in the literature considering estimation of the average speed is how to handle the vehicles that remain on a given road section, i.e., a part of the road that is extended between two consecutive loop detectors, for more than one sampling cycle. The problem occurs for the vehicles that are still in the road section at the end of the cycle. These vehicles are detected by the upstream loop detector once they enter the road section. However, in future cycles the data of these vehicles will not be considered by the loop detector. The iterative approach of the new procedure makes it possible to adequately take into account the vehicles that will stay on the same road section for several sampling cycles when estimating the TSMS. To evaluate the new procedure, the NGSIM data, which provides detailed information of a collection of vehicle trajectories on the I-880 highway in the San Francisco Bay Area is utilized. The simulation results illustrate the excellent performance of the iterative procedure for estimating the TSMS compared with previous approaches.**

***Index Terms*—time-space-mean speed; microscopic point measurements; iterative procedure**

I. Introduction

A. General Framework

In traffic theory, the average speed of the vehicles together with the flow and the density are known as the *fundamental variables* of traffic [1,2]. A basic relationship has been established between these three variables where the average speed of the vehicles is the ratio of the flow and the density [1,3,4]. Therefore, knowing the estimated value of each pair of these variables, the third variable can also be estimated. Estimation of any of the two fundamental variables would be based on traffic measurements. Such traffic measurements are commonly obtained using inductive loop detectors.

Inductive loop detectors (single and double) were introduced in the early 1960s [5]. They became common on traffic roads and nowadays they are the most widely used detection tool for point measurements [5,6]. The main advantage of a double-loop detector over a single-loop detector is that it provides information that can be used to compute the individual vehicle speeds and individual headways, i.e., the time between consecutive vehicle observations at a fixed location [7,8].

According to the literature, see e.g. [1,4,9], the appropriate average speed that should be used as the ratio of the flow and the density is the generalized average speed introduced by Edie in 1963 [4], which we call the *time-space-mean* speed (TSMS) here. However, the currently used loop detectors usually report the captured data in the form of an aggregated value, i.e., the arithmetic mean of the observed speeds known as the *time-mean* speed (TMS). These two averages (i.e., the TMS and the TSMS) are in general not equal. Therefore, we cannot apply the output of the loop detectors directly in the traffic applications that require the average speed.

The extensive application of inductive loop detectors in traffic networks worldwide (in particular double-loop detectors), and the high expenses of replacing them with modern detecting equipment, motivated the authors to find an approach which utilizes the same point measurements as the ones the current loop detectors perform, and produces more accurate estimates of the fundamental traffic variables.

B. An Overview of Previous Work

Estimation of the average speed has been considered by many authors (see [3,10-11]). In [12] an algorithm is presented for estimation of the average speed using data from a **single**-loop detector. Two new estimation methods are introduced; the first one is the *root finding* method, which yields an unbiased estimator for $\bar{v}$ when there are idealized noiseless measurements, and the second one is the *filtering* method, which addresses the reliability of the measurements.

Based on the classical definitions for the fundamental variables (see [1]), Rakha and Zhang [11] consider the SMS as the desired average speed. They develop a formula that relates the SMS and the TMS, where this formula also requires the standard deviation of the temporal distribution of the speeds to be known. The old-fashioned loop detectors, however, only report the value of the TMS. To solve this problem, Soriguera and Robusté [10] propose to assume a normal distribution for the speed of vehicles on a particular lane of the road. A confidence interval is also formulated in [10] for the estimated value of the SMS, which delimits the error for a desired confidence level.

In [3] a relation between the TMS and the SMS is developed, where the standard deviation of the spatial distribution of the speeds is involved. Han et al. [13] present a combined theoretical-empirical approach for eliminating the standard deviation from the formula proposed by Wardrop [3]. However, a new term appears in the resulting formula, namely, the mean of the squared speeds.

In [14], Jamshidnejad and De Schutter have introduced an upper and a lower bound for the TSMS, where a convex combination of these bounds is proven to produce excellent results compared with the formulas proposed by Wardrop [3] (and its modified form given in [13]) and by Rakha and Zhang [11]. While the formulas by Wardrop and by Rakha and Zhang find an estimate of the SMS rather than the TSMS (where in general these two are not the same since the TSMS is a spatiotemporal variable and the SMS is a spatial variable), the formula proposed by Jamshidnejad and De Schutter produces an estimate of the TSMS.

One important aspect that is missing in the discussed literature is how to deal with the vehicles that remain on a given road section for more than one sampling cycle. These vehicles will be detected by the loop detector at the upstream of the road section once they enter it. However, if at the end of the sampling cycle they are still on the same road section and have not reached the loop detector at the downstream end of the road section, then they will not be detected by either of these loop detectors.

C. Contribution of the Paper

The iterative procedure proposed in this paper considers the vehicles that will stay on the same sampling road section during a number of sampling cycles. As a result, the produced values by the iterative procedure are more accurate than the values produced by the formulas in [3] and [11].

The rest of the paper is organized as follows; in Section II we give definitions for some basic concepts that will be used frequently in this paper, such as the sampling window, the sampling cycle, etc, and then we introduce the sampling windows in the time-space plane. In Section III, an iterative procedure is introduced that estimates the fundamental traffic variables for the sampling windows, using computations that produce the initial conditions for future sampling windows. In Section IV the simulations are performed using the NGSIM real-life data (for I-880 highway in the San Francisco Bay Area) and the results are discussed. Section V presents conclusions, and also introduces some topics for future work.

II. Processing the Sampling Windows Within a Grid

Consider a road of length L_{road}, with n_{loops} inductive loop detectors installed at equal distances L from one another and having sampling cycles T, i.e., equal observation time periods. Note however that the assumption of same distances and sampling times could easily be relaxed as it causes no significant difference in the procedure that we will present, and is applied here just to make the notations less complicated. The traffic conditions on the road will be investigated for a total time period of length T_{road} using data from the loop detectors.

The most comprehensive way to represent traffic data is to plot the vehicle trajectories in the time-space plane [15]. Processing of the data captured from the road can also be illustrated in the time-space plane (see Figure 1), where the processing time is shown on the horizontal axis, and the processed length on the vertical axis (we assume that the vehicles are moving in direction of the space axis). This way a rectangular frame with length L_{road} and width T_{road} will be formed (see the main frame in Figure 1). The positions of the inductive loop detectors are represented by x_j, for $j = 1,\ 2,\ \dots,\ n_{\text{loops}}$.

Suppose that we want to find the fundamental traffic variables on a piece of the road between two successive loop detectors (which we will call a *sampling road section*), for which we have gathered data through one or more sampling cycles.

Figure 1 shows the time-space rectangular windows of length L and width T within the time-space plane. These windows altogether form the rectangular frame of length L_{road} and width n_{loops} discussed above. We will call each time-space rectangular window a *sampling window*. The lower edges of these sampling windows are located at the lines $x = x_j$ for $j = 1,\ 2,\ \dots,\ n_{\text{loops}}$, i.e., the positions of the inductive loop detectors.

Assume that the starting point of the procedure is point "o", for which $x = x_1$ and $t = t_1$. Now we define n_{loops} and n_{cycles} as the number of rows and columns in the grid respectively:

$$n_{\text{loops}} := \frac{L_{\text{road}}}{L} \tag{1}$$

$$n_{\text{cycles}} := \frac{T_{\text{road}}}{T} \tag{2}$$

To indicate each sampling window within the grid we can use indices i (for the time axis) and j (for the space axis), with $i \in \{1,\ 2,\ \dots,\ n_{\text{cycles}}\}$ and $j \in \{1,\ 2,\ \dots,\ n_{\text{loops}}\}$. For a sampling window (i, j), its right, left, top, and bottom edges are denoted by respectively $\mathrm{E}(i, j)|$, $|\mathrm{E}\ (i, j)$, $\overline{\mathrm{E}}\,(i, j)$, and $\underline{\mathrm{E}}\,(i, j)$ (see Figure 1). Moreover, the number of vehicles detected by the loop detector according to the sampling window (i, j), i.e., the number of trajectories that cross $\underline{\mathrm{E}}\,(i, j)$, is denoted by $n(i, j)$.

We will check if any of the trajectories corresponding to vehicles in the window (i, j) will intersect $|\mathrm{E}(i, j)$, i.e., should they be considered as initial conditions to process (i, j). This is because for the sampling window (i, j), only the vehicles that pass through the loop detector at $x = x_j$ will be detected. In this paper by *initial conditions* for the sampling window (i, j) we mean the information that should be available at $t = t_i$ in addition to the loop detector data. The initial conditions include the speed and location at $t = t_i$ of the vehicles that are on the same sampling road section from previous cycle(s).

The main question here is *whether we need to check all*

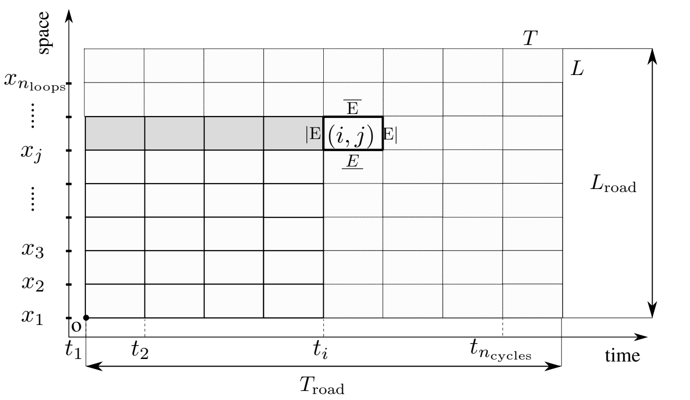


Fig. 1: A grid composed of sampling windows corresponding to the sampling road sections on a road of length L_{road} being processed during a total time of T_{road} using loop detector data (loop detectors are located at equal distances L with the same sampling times T)

previous windows (ℓ, m) *with* $\ell = 1, 2, \ldots, i-1$ *and* $\ell = 1, 2, \ldots, j$ *to process the sampling window* (i, j)*, or whether it is possible to reduce the effort*. The answer is that considering the rectangles in the same row as (i, j) (i.e., their lower edges are located along the line $x = x_j$) will be sufficient, because all trajectories that enter (i, j) through the sampling windows that are not in the same row with (i, j) should definitely cross the line $x = x_j$, and hence would be observed by the loop detector at x_j.

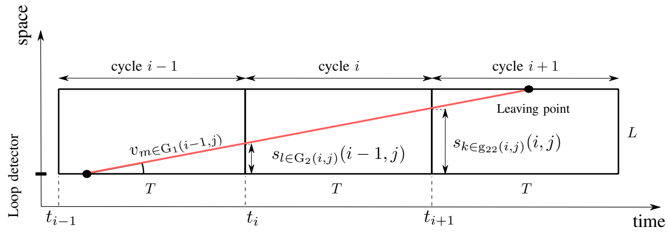
In summary, we just need to know the history of the same sampling road section, independent of the other sampling road sections, in order to calculate the fundamental traffic variables in that section. To find the initial conditions for each sampling window, we will introduce an iterative procedure in the following section.

III. Iterative Procedure for Calculation of the Fundamental Variables Using the Initial Conditions

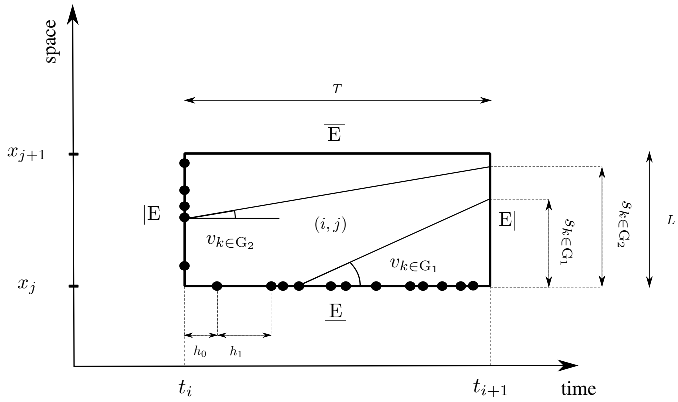
Figure 2(a) illustrates three sampling windows that represent the same sampling road section during successive sampling cycles, for which the upstream and downstream loop detectors are positioned at $x = x_j$ and $x = x_{j+1}$. We assume that speeds of the vehicles remain constant from the time they are detected by the loop detector at $x = x_j$ until they pass through the next loop detector at $x = x_{j+1}$; this assumption was also made by [3] and by [11] in the development of their formulas.

Figure 2(b) demonstrates the vehicles entering the sampling window (i, j), where two groups of vehicles could be distinguished; the first includes vehicles, the trajectories of which pass through the lower edge, $\underline{\mathrm{E}}(i, j)$, of the sampling window (i, j), and hence are detected by the loop detector at $x = x_j$, and the second group includes those vehicles for which the trajectories intersect the left edge, $|\mathrm{E}(i, j)$, of the sampling window (i, j) and therefore are not observed by the loop detector at $x = x_j$. We use G_1 to denote the vehicles in group 1, and G_2 for group 2.

The distance traveled by the vehicle ℓ within the sampling window (i, j) at the end of the sampling cycle i is denoted



(a) Successive sampling cycles



(b) The sampling window (i, j)

Fig. 2: Sampling windows that stand in the same row corresponding to the same sampling road section during successive sampling cycles presenting the trajectory of a vehicle from group one corresponding to the sampling window (i, j), and a sampling window demonstrating two vehicles from group one and two

by $d_\ell(i, j)$, and correspondingly its position w.r.t. $x = x_j$ at $t = t_{i+1}$ is denoted by $s_\ell(i, j)$. From Figure 2(a), for a vehicle $\ell \in \mathrm{G}_1$ we can say that:

- if $s_\ell(i, j) \geq L$, then the vehicle does not enter the sampling window $(i+1, j)$ at all; vehicles from G_1 that satisfy this condition will be put into subgroup one of group one, where this subgroup is denoted by $\mathrm{G}_{1,1}$
- if $s_k(i, j) < L$, then the vehicle enters the sampling window $(i+1, j)$ through its left edge, $|\mathrm{E}(i+1, j)$; vehicles that satisfy this condition will be put into subgroup two of group one, where this subgroup is denoted by $\mathrm{G}_{1,2}$

Similarly, for a vehicle $\ell \in \mathrm{G}_2$ within the sampling window (i, j) we have:

- if $s_\ell(i, j) \geq L$, the vehicle does not enter the sampling window $(i+1, j)$ at all; such vehicles from group two will be put into subgroup one of group two, i.e., $\mathrm{G}_{2,1}$
- if $s_k(i, j) < L$, the vehicle enters the sampling window $(i+1, j)$ through its left edge, $|\mathrm{E}(i, j)$; such vehicles will be put into subgroup two of group two, i.e., $\mathrm{G}_{2,2}$

Therefore, $\mathrm{G}_{1,1}(i, j)$ and $\mathrm{G}_{2,1}(i, j)$ will form $\mathrm{G}_1(i, j+1)$ and will not play any role in the sampling window $(i+1, j)$, while $\mathrm{G}_{1,2}(i, j)$ and $\mathrm{G}_{2,2}(i, j)$ will form $\mathrm{G}_2(i+1, j)$.

From Figure 2(b) it is easily seen that the traveled distances by the *first* group of vehicles in one sampling cycle and also their position at $t = t_{i+1}$ can be calculated as follows:

$$d_\ell(i, j) = v_\ell(i, j) \left(T - \sum_{r=0}^{\ell-1} h_r(i, j) \right), \quad \text{for } \ell \in \mathrm{G}_1(i, j) \quad (3)$$

$$s_\ell(i,j) = d_\ell(i,j), \qquad \text{for } \ell \in \mathrm{G}_1(i,j) \quad (4)$$

where $h_r(i,j)$ denotes the time headway between the vehicles $r-1$ and r corresponding to the sampling window (i,j). Furthermore, the travel times for the vehicles within group $\mathrm{G}_1(i,j)$ in the sampling cycle i are as follows:

$$t_\ell(i,j) = T - \sum_{r=0}^{\ell-1} h_r(i,j) \;, \qquad \text{for } \ell \in \mathrm{G}_1(i,j) \quad (5)$$

Additionally, the distances traveled by the vehicles in group $\mathrm{G}_2(i,j)$ in one sampling cycle are:

$$d_\ell(i,j) = v_\ell(i,j)T, \qquad \text{for } \ell \in \mathrm{G}_2(i,j) \quad (6)$$

and hence their positions are:

$$s_\ell(i,j) = d_\ell(i,j) + s_\ell(i-1,j), \qquad \text{for } \ell \in \mathrm{G}_2(i,j) \quad (7)$$

Note that for $i = 1$, $s_\ell(0,j)$ should be given ahead as the initial condition, and could not be calculated using (7). The travel times according to the i^{th} sampling cycle for the vehicles in group two are:

$$t_\ell(i,j) = T, \qquad \text{for } \ell \in \mathrm{G}_2(i,j) \quad (8)$$

The new procedure that we give in this paper iteratively uses (4), (5), (6), (7), and (8) in order to calculate the fundamental traffic variables, i.e., the average speed, the flow, and the density using the *generalized / time-space* definitions given by [4] for these concepts, i.e.,

$$\rho(i,j) = \frac{1}{\mathrm{A}(i,j)} \sum_{\ell \in \mathrm{G}_1(i,j) \cup \mathrm{G}_2(i,j)} t_\ell(i,j), \quad (9)$$

$$q(i,j) = \frac{1}{\mathrm{A}(i,j)} \sum_{\ell \in \mathrm{G}_1(i,j) \cup \mathrm{G}_2(i,j)} s_\ell(i,j), \quad (10)$$

$$\mathrm{TSMS}(i,j) = \frac{q(i.j)}{\rho(i,j)} = \frac{\displaystyle\sum_{\ell \in \mathrm{G}_1(i,j) \cup \mathrm{G}_2(i,j)} s_\ell(i,j)}{\displaystyle\sum_{\ell \in \mathrm{G}_1(i,j) \cup \mathrm{G}_2(i,j)} t_\ell(i,j)} \quad (11)$$

where $\rho(i,j)$ and $q(i,j)$ are the time-space density and the time-space flow corresponding to the sampling window (i,j), $\mathrm{A}(i,j)$ is the area of the window, i.e., $\mathrm{A}(i,j) = TL$, and $t_\ell(i,j)$ is the total time that the ℓ^{th} vehicle spends within the sampling window (i,j). Here is how the iterative procedure performs.

The procedure requires as its input the microscopic data that are available by **double**-loop detectors, i.e., the individual speeds and headways of the vehicles for each sampling cycle. Till now we have discussed an arbitrary window (i,j) with $i \in \{1,2,\ldots,n_{\text{cycles}}\}$ and $j \in \{1,2,\ldots,n_{\text{loops}}\}$. Here we consider how to apply (3), (4), (5), (6), (7), and (8) in practice. For traffic applications, in particular where calculations should be done online, we will consider the procedure such that once a group of vehicles enters the detection zone of a loop detector, all calculations are performed by the procedure both for the current and the future cycles.

Consequently, the procedure starts when the first group of vehicles ($i = 1$) is observed by the loop detector at $x = x_j$. For the given cycle, the vehicles will form the group $\mathrm{G}_1(1,j)$; using (4) it is determined whether each vehicle will belong to subgroup $\mathrm{G}_{1,1}(1,j)$ or $\mathrm{G}_{1,2}(1,j)$.

Afterwards, the procedure sets $i = 2$ and considers $\mathrm{G}_{1,2}(1,j) \subseteq \mathrm{G}_2(2,j)$. The vehicles that have joined $\mathrm{G}_{1,1}(1,j)$ will not be considered for the rest of the procedure.

Then the calculations will be continued to see whether each vehicle joins $\mathrm{G}_{2,1}(2,j)$ or $\mathrm{G}_{2,2}(2,j)$. The procedure then sets $i = 3$ and considers $\mathrm{G}_{2,2}(2,j) \subseteq \mathrm{G}_2(3,j)$. Then it performs the same calculations for $\mathrm{G}_{2,2}(2,j)$, as it has done for $\mathrm{G}_{1,2}(1,j)$. The iteration will continue in this way.

In summary, for every i, $i = 1,\ 2,\ \ldots,\ n_{\text{cycles}}$, the procedure performs the calculations corresponding to the last *observed* group of vehicles. As the **first** step (*calculations for the current cycle*), $\mathrm{G}_1(i,j)$ is considered to be formed by the last observed vehicles. Then (4) is used to determine whether each vehicle belongs to $\mathrm{G}_{1,1}(i,j)$ or $\mathrm{G}_{1,2}(i,j)$. Then $\mathrm{G}_{1,1}(i,j)$ is omitted and $\mathrm{G}_{1,2}(i,j)$ will enter the second step of the procedure.

For the **second** step (*calculations for the future cycles*), the procedure performs (7) on $\mathrm{G}_{1,2}(i,j)$ to detect whether each vehicle belongs to $\mathrm{G}_{2,1}(i+1,j)$ or $\mathrm{G}_{2,2}(i+1,j)$, where $\mathrm{G}_{2,1}(i+1,j)$ will be eliminated and $\mathrm{G}_{2,2}(i+1,j)$ will enter the third step.

In the **third** step, (7) is applied to $\mathrm{G}_{2,2}(i+1,j)$ like the second step. Then $\mathrm{G}_{2,1}(i+2,j)$ will be omitted from the procedure, and $\mathrm{G}_{2,2}(i+2,j)$ will be sent to the fourth step, and the iteration continues.

The procedure explained in step 2 should be repeated for $N_{\text{tot},i,j}$ future cycles, where

$$N_{\text{tot},i,j} = \max_{\ell \in \mathrm{G}_1(i,j)} N_{\ell,i,j} \quad (12)$$

with $N_{\text{tot},i,j}$ being the number of the future sampling windows on the same row as sampling window (i,j), for which at least one vehicle $\ell \in \mathrm{G}_1(i,j)$ remains on the same sampling road section. Moreover, $N_{\ell,i,j}$ is the minimum of the all future cycles, $n_{\text{cycles}} - i$, and the number of sampling cycles the vehicle $\ell \in \mathrm{G}_1(i,j)$ needs to reach the position $x = x_{j+1}$, i.e., for $\ell \in \mathrm{G}_1(i,j)$:

$$N_{\ell,i,j} = \min\left\{ \left\lceil \frac{L - v_\ell(i,j)\left(T - \displaystyle\sum_{r=0}^{\ell-1} h_r(i,j)\right)}{v_\ell(i,j)T} \right\rceil, n_{\text{cycles}} - i \right\} \quad (13)$$

The above calculations are done for $i = 1,\ 2,\ \ldots,\ n_{\text{cycles}}$.

Now, using the time-space definitions (9), (10), and (11) together with (3), (5), (6), (7), and (8) we will have:

$$\rho(i,j) = \frac{\displaystyle\sum_{\ell \in \mathrm{G}_{1,1}(i,j)} \frac{L}{v_\ell(i,j)} + \sum_{r \in \mathrm{G}_{1,2}(i,j)} t_r(i,j) + \sum_{m \in \mathrm{G}_{2,1}(i,j)} \frac{L - s_m(i-1,j)}{v_m(i,j)} + \sum_{p \in \mathrm{G}_{2,2}(i,j)} T}{TL} \quad (14)$$

$$q(i,j) = \frac{\displaystyle\sum_{k \in \mathrm{G}_{1,1}(i,j)} L + \sum_{r \in \mathrm{G}_{1,2}(i,j)} d_r(i,j) + \sum_{m \in \mathrm{G}_{2,1}(i-1,j)} (L - s_m(i-1,j)) + \sum_{p \in \mathrm{G}_{2,2}(i,j)} d_p(i,j)}{TL} \quad (15)$$

$$\mathrm{TSMS}(i,j) = \frac{\displaystyle\sum_{\ell \in \mathrm{G}_{1,1}(i,j)} L + \sum_{r \in \mathrm{G}_{1,2}(i,j)} d_r(i,j) + \sum_{m \in \mathrm{G}_{2,1}(i-1,j)} (L - s_m(i-1,j)) + \sum_{p \in \mathrm{G}_{2,2}(i,j)} d_p(i,j)}{\displaystyle\sum_{\ell \in \mathrm{G}_{1,1}(i,j)} \frac{L}{v_\ell(i,j)} + \sum_{r \in \mathrm{G}_{1,2}(i,j)} t_r(i,j) + \sum_{m \in \mathrm{G}_{2,1}(i,j)} \frac{L - s_m(i-1,j)}{v_m(i,j)} + \sum_{p \in \mathrm{G}_{2,2}(i,j)} T} \quad (16)$$

Remark: For the terms $\sum_{\ell \in \mathrm{G}_{1,1}(i,j)} L$ and $\sum_{p \in \mathrm{G}_{2,2}(i,j)} T$ in (14), (15), and (16), since L and T are constant, the terms can be evaluated more efficiently using the followings:

$$\sum_{\ell \in \mathrm{G}_{1,1}(i,j)} L = L.|\mathrm{G}_{1,1}(i,j)| \tag{17}$$

$$\sum_{\ell \in \mathrm{G}_{2,2}(i,j)} T = T.|\mathrm{G}_{2,2}(i,j)| \tag{18}$$

where by $|H|$, we mean the number of elements in the set H.

IV. Results

In order to evaluate the performance of the iterative procedure for estimation of the average speed, we have used the NGSIM real-life data set (available at `http://www.ngsim-community.org/`). This data set provides detailed information on a collection of vehicle trajectories on the I-880 highway in the San Francisco Bay Area. The data is available for limited time intervals. Therefore, we will have some distinct group of trajectories within the time-space plane.

For the simulation, we have selected two groups of these trajectories (see Figures 3(a) and 4(a)). Since the total time interval covered by each group of trajectories was relatively small, we had to define the width of the sampling windows small enough, i.e., $T = 5$ s, such that we could consider several windows. The length of the sampling windows was chosen to be $L = 200$ m. Note that the simulation results corresponding to cycle 1 in Figures 3 and 4 have not been presented. This is because the information corresponding to the trajectories that intersect the left edge of cycle 1 should be given to the procedure as the initial condition, and this information is not calculated by the procedure itself. Since we aimed to evaluate the performance of the procedure itself, we have therefore ignored this cycle and just used its data for the role they might have in future cycles.

Figures 3(b) and 3(c) represent the relative errors (in percentage) corresponding to the formulas given by Rakha and Zhang in [11], by Wardrop in [3] (together with the modifications proposed in [13]), by Jamshidnejad and De Schutter in [14], and the iterative procedure proposed in this paper for the top and bottom rows of the sampling windows shown in Figure 3(a). Similarly, Figures 4(b) and 4(c) illustrate the percentage of errors for the top and bottom rows of the windows in Figure 4(a).

These simulation results show that, in general, the iterative procedure performs better than the two formulas given by Rakha and Zhang [11] and by Wardrop [3]. In particular, the performance is improved for sampling windows that contain more trajectories that intersect the left edge of the given window. Compare, for instance, the results corresponding to cycle 2 in the top and bottom rows of Figure 4(a); for cycle 2 in the top row, the relative errors between the formulas by Rakha and Zhang [11] and by Wardrop [3] and Han et al. [13], and the new iterative procedure are larger. One reason is the higher number of trajectories that intersect the left edge of the sampling window corresponding to cycle 2 within the bottom row. These intersecting trajectories are corresponding to the vehicles that are already on the road section (but not within the detection zone of the upstream loop detector) at the beginning of the current sampling cycle. Note that these vehicles are considered by the iterative procedure proposed in this paper, while they are not taken into account by other formulas (i.e., formulas by Rakha and Zhang [11], by Wardrop [3], and by Jamshidnejad and De Schutter [14]).

As another example, for *cycle 5* in Figure 3(a), in the top row the relative errors produced by the formulas by Rakha and Zhang [11], by Wardrop [3] (together with the modifications proposed by Han et al. [13]), and by Jamshidnejad and De Schutter [14] with respect to the error produced by the new iterative procedure are more significant (see Figure 3(b)) compared with the bottom row (see Figure 3(c)). Furthermore, there are four trajectories in the top row that intersect the left edge of the sampling window 5, while in the bottom row only one trajectory intersects the left edge of the sampling window corresponding to cycle 5. Therefore, we expect to see a larger difference between the errors in the top row, as there are more vehicles that are ignored by the formulas of [11], [3], and [14] and are taken into account by the new iterative procedure.

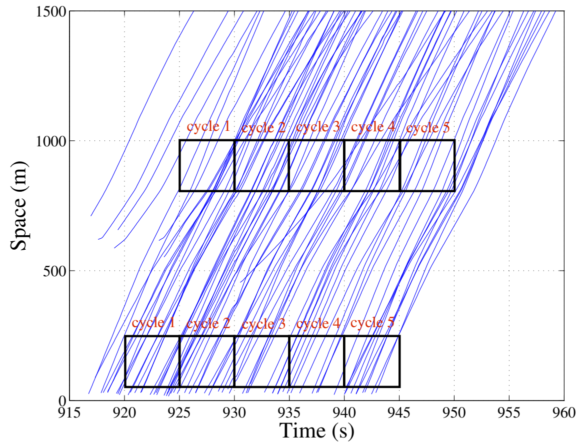
Considering the formula given in [14] and this new iterative procedure, it is observed that they both perform better than the formulas by Rakha and Zhang and by Wardrop in general (the exception is for cycle 5 in Figure 3(b)). This is while in some cases the formula given by Jamshidnejad and De Schutter [14] shows better performance, and in some cases the new iterative procedure. Analysis of the situations where each of the two methods, i.e., the formula given in [14] and the iterative procedure, shows better performance, and also the possibility of combining the two methods to produce an even better approach are the basis of the future work by the authors.

V. Discussion and future work

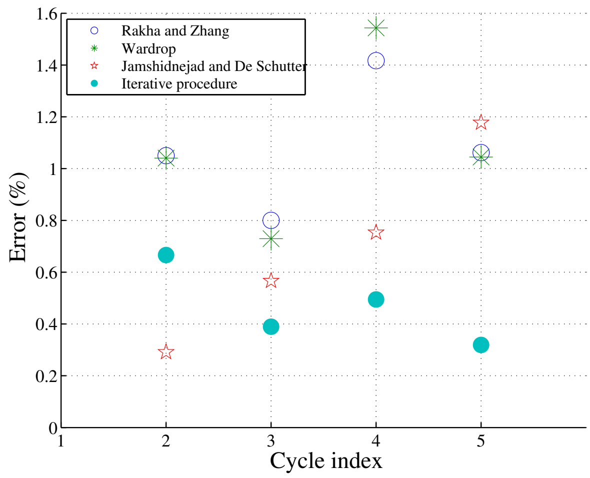
We have presented a new iterative procedure that estimates the time-space-mean speed (TSMS), i.e., an equivalent to the generalized average speed introduced by Edie in 1963 [4] for vehicles traveling on a road. The new iterative procedure utilizes microscopic traffic data (i.e., individual speeds and headways of the vehicles) and takes into account the vehicles that will stay on the same road section, i.e., a part of the road between two consecutive loop detectors, for several sampling cycles.

Our simulation results (using the NGSIM real-life data for the I-880 highway in the San Francisco Bay Area) show that the values produced by the new iterative procedure give more accurate estimates of the average speed than the formulas given by Rakha and Zhang [11] and by Wardrop [3,13]. In most cases the results are also better than the results by the formula proposed by Jamshidnejad and De Schutter [14].

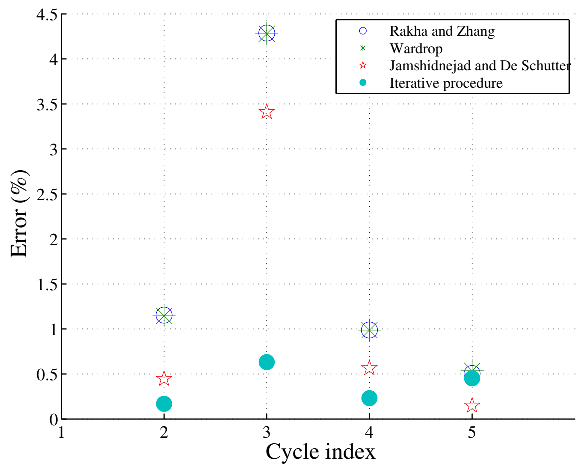
A topic of the future work is to combine the new iterative procedure proposed in this paper and the formula proposed by Jamshidnejad and De Schutter [14] to produce an even better approach for estimation of the current TSMS. Another



(a) Trajectories of the vehicles and the sampling windows

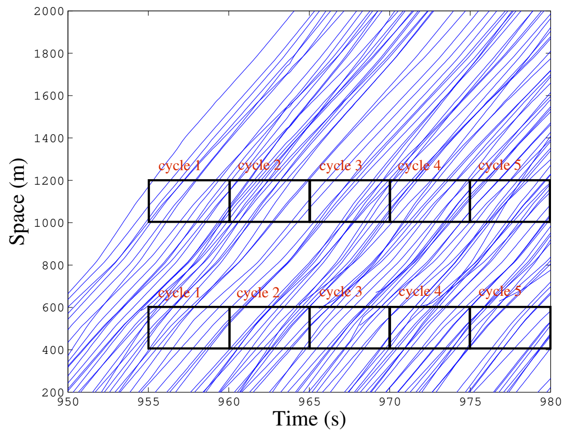


(b) Relative error for the top row of windows

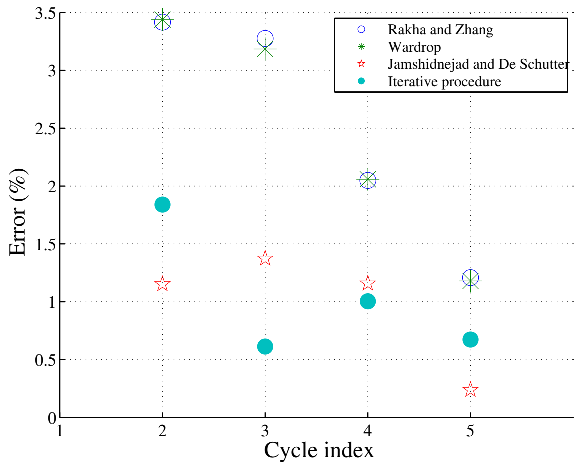


(c) Relative error for the bottom row of windows

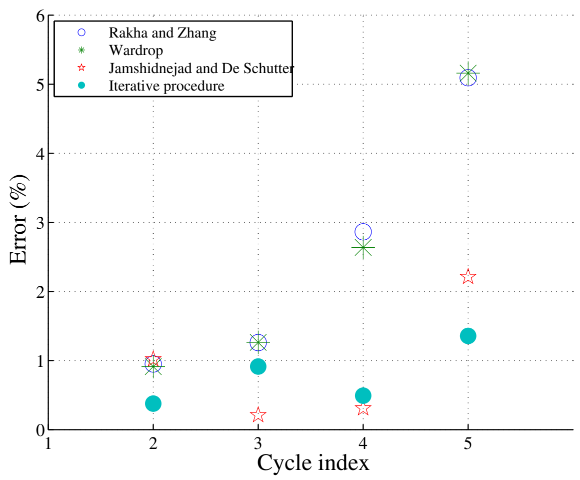
Fig. 3: Comparison of the relative errors (in percentage) for the formulas given by Rakha and Zhang [11], by Wardrop [3], by Jamshidnejad and De Schutter [14], and the new iterative procedure



(a) Trajectories of the vehicles and the sampling windows



(b) Relative error for the top row of windows



(c) Relative error for the bottom row of windows

Fig. 4: Comparison of the relative errors (in percentage) for the formulas given by Rakha and Zhang [11], by Wardrop [3], by Jamshidnejad and De Schutter [14], and the new iterative procedure

topic to take into account for future work is the application of the iterative procedure together with the microscopic approach proposed by Jamshidnejad and De Schutter in [14] for prediction of the TSMS for the current road section in the near future. In the current paper, we used the iterative algorithm to estimate the average speed on a road section within different sampling cycles. Yet, another topic for future work is to use the information on one road section to predict the upcoming situation for the succeeding road section.

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