Delft University of Technology

Delft Center for Systems and Control

Technical report 16-008a

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F. Alavi, E. Park Lee, N. van de Wouw, B. De Schutter, and Z. Lukszo

April 2016

Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: https://www.dcsc.tudelft.nl

* This report can also be downloaded via https://pub.bartdeschutter.org/abs/16_008a.html

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Abstract

This addendum contains the lemmas and their proofs that are used to simplify the optimization problem developed in Section 4 of the manuscript "Fuel cell cars in a microgrid for synergies between hydrogen and electricity networks" by F. Alavi, E. Park Lee, N. van de Wouw, B. De Schutter, and Z. Lukszo, *Applied Energy*, vol. 192, pp. 296-304, Apr. 2017.

Lemmas and proofs

Lemma 1: Defining

$$\tilde{\omega}_{\min} = \begin{bmatrix} \bar{\omega} & \dots & \bar{\omega} \end{bmatrix}^T_{N_\mathrm{p}\times 1} \tag{34}$$
$$\tilde{\omega}_{\max} = \begin{bmatrix} \underline{\omega} & \dots & \underline{\omega} \end{bmatrix}^T_{N_\mathrm{p}\times 1}, \tag{35}$$

the inequality (7) holds for all possible disturbances ω satisfying Assumption 1 if the following two inequalities hold:

$$G_1 X(k) \leq G_2 + G_3 \mathbf{x}(k) + G_4 \tilde{\omega}_{\min} \tag{36}$$
$$G_1 X(k) \leq G_2 + G_3 \mathbf{x}(k) + G_4 \tilde{\omega}_{\max}. \tag{37}$$

Proof: The existence of a maximum and minimum value for ω implies that:

$$\forall \omega \ \exists \lambda_1, \lambda_2 \in [0,1] \ : \ \lambda_1 \underline{\omega} + \lambda_2 \bar{\omega} = \omega \text{ and } \lambda_1 + \lambda_2 = 1. \tag{38}$$

Now assume an arbitrary realization of $\tilde{\omega}(k)$ as follows:

$$\tilde{\omega}(k) = \begin{bmatrix} \omega(k) & \omega(k+1) & \dots & \omega(k+N_\mathrm{p}-1) \end{bmatrix}^T.$$

The inequality constraint (7) consists of several inequalities belonging to each time step in the prediction horizon. Considering the structure of G_1, G_2, G_3, and G_4, it can be shown that (7) consists of the following inequalities, for $j \in \{0, \dots, N_\mathrm{p}-1\}$:

$$G_{1,k+j} X(k) \leq G_{2,k+j} + G_{3,k+j} \mathbf{x}(k) + g_{4,k+j} \omega(k+j) \tag{39}$$

where $G_{1,k+j}$, $G_{2,k+j}$, $G_{3,k+j}$, and $g_{4,k+j}$ are the $j+1$th row of G_1, G_2, G_3, and G_4, respectively. From (36) and (37), we have:

$$G_{1,k+j} X(k) \leq G_{2,k+j} + G_{3,k+j} \mathbf{x}(k) + g_{4,k+j} \underline{\omega} \tag{40}$$
$$G_{1,k+j} X(k) \leq G_{2,k+j} + G_{3,k+j} \mathbf{x}(k) + g_{4,k+j} \bar{\omega}, \tag{41}$$

Property (38) shows that for any realization of $\omega(k+j)$, there exists a pair $(\lambda_1(k+j), \lambda_2(k+j))$ such that $\lambda_1(k+j)\underline{\omega} + \lambda_2(k+j)\bar{\omega} = \omega(k+j)$ and $\lambda_1(k+j) + \lambda_2(k+j) = 1$. By multiplying these factors to (40) and (41), it can be easily seen that (39) holds. This reasoning can be done for all $j \in \{0, \ldots, N_\mathrm{p} - 1\}$ and, hence, (7) holds. $\square$

Lemma 2: The maximum of the cost function (9) over all the possible realizations of $\tilde{\omega}(k)$, i.e. $\max_{\tilde{\omega}(k)} \{J(k)\}$, can always be assumed to occur at one of the vectors $\tilde{\omega}_1(k)$, $\tilde{\omega}_2(k)$, ..., $\tilde{\omega}_N(k)$ defined as follows:

$$\begin{aligned}
\tilde{\omega}_1(k) &= \left[\begin{array}{cccc} \underline{\omega} & \underline{\omega} & \ldots & \underline{\omega} \end{array}\right]^T \\
\tilde{\omega}_2(k) &= \left[\begin{array}{cccc} \bar{\omega} & \underline{\omega} & \ldots & \underline{\omega} \end{array}\right]^T \\
&\vdots \\
\tilde{\omega}_N(k) &= \left[\begin{array}{cccc} \bar{\omega} & \bar{\omega} & \ldots & \bar{\omega} \end{array}\right]^T .
\end{aligned}$$

Proof: The first term of the cost function, $W_\mathrm{x}(k)X(k)$, can be written in the following from:

$$W_\mathrm{x}(k)X(k) = W_{\mathrm{x},1}(k)X(k) + W_{\mathrm{x},2}(\tilde{\omega}(k))X(k). \tag{42}$$

The first term in (42), $W_{\mathrm{x},1}(k)X(k)$, is not affected by $\tilde{\omega}(k)$. The expanded form of the second term, $W_{\mathrm{x},2}(\tilde{\omega}(k))X(k)$, is given by:

$$\sum_{j=0}^{N_\mathrm{p}-1} \Big(C_{\mathrm{e,imp}}(k+j)(1-\delta_{\mathrm{exp}}(k+j)) - C_{\mathrm{e,exp}}(k+j)\delta_{\mathrm{exp}}(k+j)\Big)\omega(k+j)$$

which is either equal to $C_{\mathrm{e,imp}}(k+j)\omega(k+j)$ or $-C_{\mathrm{e,exp}}(k+j)\omega(k+j)$ at each time step $k+j$, based on the value of $\delta_{\mathrm{exp}}(k+j)$. In these cases, the maximum value of $W_\mathrm{x}(\tilde{\omega}(k))X(k)$ at each time step k for all realizations of $\omega(k+j)$ occurs at $\bar{\omega}$ and $\underline{\omega}$, respectively.

The second term of the cost function, $W_\mathrm{d}(k)\tilde{\omega}(k)$, can be written in the form:

$$W_\mathrm{d}(k)\tilde{\omega}(k) = \sum_{j=0}^{N_\mathrm{p}-1} C_{\mathrm{e,imp}}(k+j)\omega(k+j).$$

It is assumed that the tariff of imported power, $C_{\mathrm{e,imp}}(k)$ is positive, and hence, the maximum value of $W_\mathrm{d}(k)\tilde{\omega}(k)$ at each time step k for all realizations of $\omega(k+j)$ occurs at $\bar{\omega}$. The total cost function contains the addition of $W_{\mathrm{x},2}(\tilde{\omega}(k))X(k)$ and $W_\mathrm{d}(k)\tilde{\omega}(k)$, and hence, the maximum of the total cost function at time step $k+j$ occurs on either $\bar{\omega}$ or $\underline{\omega}$. By following the same reasoning for all $j \in \{0, \ldots, N_\mathrm{p} - 1\}$, we can conclude that the maximum value of the cost, $J(k)$, would be realized when $\tilde{\omega}(k)$ is equal to one of the vectors $\tilde{\omega}_1$, ..., $\tilde{\omega}_N$. $\square$

Lemma 1 and 2 show that the optimization problem (10) subject to the constraints (7) can be formulated as a number of MILP problems. However, the number of MILP problems to be solved is $N = 2^{N_\mathrm{p}}$. In this case, an increase in the prediction horizon will create an exponential increase in the computational time needed for the controller, which is not desired. In order to reduce the number of MILP problems that need to be solved in each time step and therewith to reduce the computational burden of the proposed control strategy, we consider the following additional assumption:

Assumption 2: In the optimization problem (10), the value of $\delta_{\mathrm{exp}}(k+j)$ is the same for all $j \in \{0, \ldots, N_\mathrm{p} - 1\}$.

Assumption 2 is a restriction on the import or export of electricity in the future. It means that the control actions are determined based on the assumption of either exporting or importing electricity during the entire prediction period $[k, k + N_\mathrm{p}]$, but not a combination of them. This assumption may decrease the system performance, but it is used due to its influence on the reduction of computational burden.

Lemma 3: Considering Assumption 2, the maximum of the cost function (9) over all the possible realizations of $\tilde{\omega}(k)$, i.e. $\max_{\tilde{\omega}(k)} \{J(k)\}$, can always be assumed to occur at one of the vectors $\tilde{\omega}_\mathrm{min}$ or $\tilde{\omega}_\mathrm{max}$ defined in (34) and (35).
Proof: Similar to the proof of Lemma 2, we have the following term for the cost function that is related to $\tilde{\omega}(k)$:

$$\sum_{j=0}^{N_\mathrm{p}-1} \Big(C_\mathrm{e,imp}(k+j)(1-\delta_\mathrm{exp}(k)) - C_\mathrm{e,exp}(k+j)\delta_\mathrm{exp}(k)\Big)\omega(k+j)$$

which is either equal to $\sum_{j=0}^{N_\mathrm{p}-1} C_\mathrm{e,imp}(k+j)\omega(k+j)$ or $\sum_{j=0}^{N_\mathrm{p}-1} -C_\mathrm{e,exp}(k+j)\omega(k+j)$, based on the value of $\delta_\mathrm{exp}(k)$. In both cases, the maximum value for all realizations of $\omega(k+j)$ for $j \in \{0, \dots, N_\mathrm{p}-1\}$ occurs for extreme value of ω for all the prediction horizon. In the first case, this extreme case is $\bar{\omega}$ and in the second case, $\underline{\omega}$. Therefore, the maximum value of the cost, $J(k)$, would be realized when $\tilde{\omega}(k)$ is equal to either $\bar{\omega}$ or $\underline{\omega}$. $\square$

Lemma 1 and 3 show that the optimization problem (10) subject to the constraints (7) can be formulated as an MILP problem (11) subject to the constraints (12) and (13). The conversion of the optimization problem (10) into (11) reduces the complexity of the problem and as a result, the computation time in the model predictive controller is decreased.