Distributed model predictive control for cooperative synchromodal freight transport

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Abstract
This paper investigates cooperative synchromodal freight transport planning among multiple intermodal freight transport operators in different and interconnected service networks. The cooperative planning is formulated as a cooperative model predictive container flow control problem, and solved with three Distributed Model Predictive Flow Control (DMPFC) approaches: the parallel and serial Augmented Lagrangian Relaxation (ALR) based DMPFC approaches, and the Alternating Direction Method of Multipliers (ADMM) based DMPFC approach. The simulation results show that the serial ALR-based DMPFC approach requires the least iterations and information exchanges while the ADMM-based DMPFC approach takes the least amount of actual computation time.

Keywords: Cooperative synchromodal freight transport, distributed model predictive control, alternating direction method of multipliers

1. Introduction
Hinterland haulage among deep-sea terminals and inland terminals has become an important component in global logistics systems. Intermodal freight transport holds the promise of outperforming single-mode truck freight transport in port hinterland container transport in the aspects of cost efficiency, sustainability, robustness against planning uncertainties e.g., traffic congestion on freeways (Macharis and Bontekoning, 2004; Craig et al., 2013; SteadieSeifi et al., 2014). However, the integrated use of different modalities in intermodal freight transport also brings many planning challenges, among which the collaboration and cooperation among different stakeholders are central in practice (Macharis and Bontekoning, 2004; SteadieSeifi et al., 2014; van der Horst and de Langen, 2008; van der Horst and van der Lugt, 2011). These stakeholders include shippers, intermodal freight forwarders, intermodal freight transport operators, terminal operators, and port authorities. Therefore, cooperative planning among multiple intermodal freight transport operators is starting to receive more and more attention from both transport operation practitioners and academic researchers. To perform cooperative planning, efficient information...
and communication systems should be established and used in an integrated way for tracking vehicles and containers, and exchanging planning information among transport operators.

Synchromodal freight transport moves one step forward from intermodal freight transport by adopting the mode-free booking concept and allowing timely switching among available modalities according to the real-time information of the freight transport process (Groen et al., 2011; Europe Container Terminal, 2011; van Riessen et al., 2015a,b). In the mode-free booking, shippers make transport orders without specifying which mode of transport is going to be used and give transport operators the freedom to select the most suitable modalities on the basis of the up-to-date planning information. In this paper, we investigate cooperative planning among multiple intermodal freight transport operators that provide synchromodal freight transport services among deep-sea terminals and inland terminals in hinterland haulage in different but interconnected service networks. The service network of an intermodal freight transport operator consists of a set of terminals and a set of transport services connecting these terminals. This service network is pre-designed by the operator through solving its service network design problem (Crainic, 2000). These transport services are provided by the operator with its own or hired vehicles (e.g., trucks, trains, and barges), and are defined by routes (i.e., origin terminals, intermediate terminals, destination terminals), modalities (i.e., one modality or a combination of multiple modalities), transport times and costs, capacities, frequencies, and schedules. The current paper defines two service networks as non-overlapping when there is no vehicle sharing between any two transport services from these two service networks. Each operator has its own service network, and cooperates with the other operators in accommodating certain transport orders. The current paper considers that an operator is responsible for managing container flow in its own service network. Because this operator has or hires different types of vehicle (e.g., trucks, trains, and barges) that perform transport services in its own service network. These operators are either the customers or the service providers of other stakeholders (e.g., shippers, terminal operators) in the transport process.

The cooperative planning is done at the tactical flow level by all operators. The cooperation goal is to serve transport demands at the lowest overall freight delivery cost. Each operator aims to minimize its own container delivery cost while being willing to consider the interests of other operators in its planning process, although each operator holds independent planning authority in its own service network. The trade-off between the cooperation goal of all operators and the goal of each individual operator is obtained by negotiating with other operators about transport plans. For an operator, the transport plan consists of its route choices and flow assignments in the container delivery process. Operators cooperate to reach an agreement on the volumes of container flows that each operator will hand over to other operators during each planning interval. It is possible that some operators might sacrifice for reducing the freight delivery cost of other operators in order to achieve the cooperation goal. Therefore, proper profit distribution mechanisms should be designed and agreed upon to compensate for the sacrifice made by some operators. A feasible cooperation needs to guarantee a win-win situation and fairness among operators. Detailed discussions on the issue of fairness in cooperative planning in supply chain management are presented in Stadtler (2009). One way to obtain a feasible cooperation is to first find cooperative planning solutions to minimize the total operating cost, and then to distribute the profit among all operators in such a way that a fair distribution is reached. The work presented in this paper focuses on the important step of finding the cooperative transport plans that can minimize the total freight delivery cost (i.e., the total operating cost) of all operators. Calculation of side-payments, as part of a future fairness approach, could be based on the differences between the obtained cooperative planning solutions and the corresponding freight delivery costs.
of operators in the cooperative planning and the planning solutions and the corresponding freight delivery costs of operators in a situation where each operator performs its own transport planning without cooperation. This paper considers that each operator adopts an Model Predictive Control (MPC) strategy for the flow planning problem within an overall multi-level freight transport planning framework. This multi-level planning framework consists of the flow planning level and the container planning level, and integrates the information from different planning level with appropriate mapping approaches. We refer to our earlier work Li et al. (2015) for a detailed explanation of the multi-level freight planning framework. Therefore, the resulting cooperative planning problem can be abstracted as a Distributed Model Predictive Flow Control (DMPFC) problem.

Distributed model predictive control (DMPC) is a general control methodology that can cope with control problems arising in large-scale systems due to organizational couplings among different parties involved in a common task, limited measurement ability and control access of different parties, and different, possibly conflicting, objectives of different parties, etc. The papers and books Camponogara et al. (2002); Scattolini (2009); Christofides et al. (2013); Maestre and Negenborn (2014) review the basic concept of, the research results in, and future research directions on DMPC. DMPC approaches have been investigated in various controlled systems and applications (de Oliveira and Camponogara, 2010; Ghods et al., 2010; Frejo and Camacho, 2012; Zhou et al., 2015; De Souza et al., 2015; Negenborn et al., 2008; Mc Namara et al., 2013; del Real et al., 2014; Negenborn et al., 2009; Leirens et al., 2010; Maestre et al., 2009). The Augmented Lagrangian Relaxation based Distributed Model Predictive Control (ALR-based DMPC) approaches (Negenborn et al., 2008) and the Alternating Direction Method of Multipliers based Distributed Model Predictive Control (ADMM-based DMPC) approaches have been successively developed based on the method of multipliers and the Alternating Direction Method of Multipliers (ADMM) algorithm (Boyd and Vandenberghe, 2004; Boyd et al., 2010; Bertsekas, 1982). The ALR-based DMPC approaches have been used to successfully solve distributed control problems in various applications (Negenborn et al., 2008; Mc Namara et al., 2013; del Real et al., 2014; Negenborn et al., 2009; Leirens et al., 2010; Alvarado et al., 2011; Zhou et al., 2015). The ADMM-based DMPC approach is a counterpart of the ALR-based DMPC approaches and has effectively been used to determine cooperative control actions for multiple MPC controllers in many applications (Kögel and Findeisen, 2012; Farokhi et al., 2014; Summers and Lygeros, 2012; Costa et al., 2014; Mota et al., 2012; Spudić et al., 2015). To the best knowledge of the author, no work has been done in literature on applying ALR-based DMPC approaches or ADMM-based DMPC approaches for cooperative synchromodal freight transport planning. In this paper, we propose to use these two classes of DMPC approaches for cooperative synchromodal freight transport planning.

The current paper is an extended version of the authors’ earlier papers Li et al. (2014, 2015). The contributions of the current paper are as follows: 1) besides the parallel ALR-based DMPC approach we also investigates the serial ALR-based DMPC approach and the ADMM-based DMPC approach for solving cooperative synchromodal freight transport planning problems; 2) more importantly, we explain the basic concept of three DMPFC approaches, give their algorithm formulations, analyze their convergence properties, and discuss their practical implementation. In addition, we illustrate graphically the cooperation process of three DMPFC approaches, and evaluate their performance by conducting numerical simulations with a linear network model in an intermodal freight transport network connecting Rotterdam to Antwerp and Frankfurt.

The structure of this paper is as follows. Section 2 briefly reviews the literature on service network design problems, ALR-based DMPC approaches, ADMM-based DMPC approaches,
and the recent work on developing DMPC approaches in intermodal freight transport. Section 3 contains a detailed explanation of the considered cooperative synchronomodal freight transport planning problem and gives the Cooperative Model Predictive Flow Control (CMPCF) formulation. Section 4 presents three DMPFC approaches, discusses their implementation issues, and define their performance indicators. A comparison of the three DMPFC approaches is done by numerical simulation experiments in Section 5. Section 6 concludes the paper with some remarks and directions for future research.

2. Problem description and literature review on DMPC approaches

This section introduces first the Service Network Design (SND) problem in literature, and describes its similarities and differences between the work presented in the current paper. Next, the literature on DMPC approaches based on the ALR method and based on the ADMM algorithm is reviewed in Sections 2.2 and 2.3, respectively. Meanwhile, recent works on applying DMPC approaches in intermodal freight transport are also discussed in the end of this section.

2.1. Problem description

Planning models for freight transport should be formulated to address specific planning problems of specific stakeholders at specific decision making levels, i.e., strategic level, tactical level, and operational level (Crainic and Laporte, 1997). We refer to the review papers Crainic and Laporte (1997); Craig et al. (2013); Macharis and Bontekoning (2004); SteadieSeifi et al. (2014) and references therein for a detailed review of the main problems, planning models, and solution methods at each level of the freight transport. The cooperative planning problem considered in the current paper is built upon the operations research literature on the SND problems. This section reviews briefly the literature on the SND models, and explain the similarities and differences between the work presented in the current paper and the SND problems.

The SND problems generally involve the search for optimal decisions on the selection and scheduling of services, the specification of terminal operators, the routing of freight, and the repositioning of vehicles in freight transportation (Crainic, 2000; Armacost et al., 2002; Pedersen et al., 2009; Lium et al., 2009; Bai et al., 2012). SND models typically consist of both continuous variables and integer/binary variables. The continuous variables are used to represent the commodity flows in the network, while the integer/binary variables are typically used for determining whether to select a service or not (SteadieSeifi et al., 2014). SND models can be categorized into arc-based models and path-based models depending on whether the variables are used for representing flows on arcs or paths. SND models usually lead to mixed-integer network optimization problems, for which no exact and efficient solution methods are available, except for some special variants (Crainic, 2000). Many approaches have been developed to obtain tractable solutions to the SND problems for large-scale transport networks, e.g., deploying composite variable formulations of the SND problems (Armacost et al., 2002), using tabu search metaheuristics (Pedersen et al., 2009), and tabu assisted guided local search approaches (Bai et al., 2012). Moreover, recent research has also investigated the issue of stochasticity in the SND problems (Lium et al., 2009; Bai et al., 2012). In Lium et al. (2009), a stochastic service network design formulation has been used to take into account the stochasticity in the demand in service network design. The papers Bock (2010); Goel (2010) proposed replanning strategies to address the issue of uncertain and dynamically changing situations in transport networks. Following the same line of reasoning, a similar replanning strategy has also been used to solve the SND problems in an uncertain environment (Bai et al., 2012).
The current paper considers cooperative synchromodal freight transport planning among multiple transport operators, and proposes DMPC approaches for container flow control in pre-designed service networks under dynamic transport demands and dynamic traffic conditions in the network. We clarify the similarities and differences between the work presented in the current paper and the SND problems reviewed above as follows. On the one hand, a similarity is that the model predictive container flow control approach adopted by each operator in the current paper shares the same line of reasoning for coping with uncertainties as the approaches used in Bock (2010); Goel (2010); Bai et al. (2012). On the other hand, the main difference is that the cooperative synchromodal freight transport problem and the network models used in this paper require the input of or the availability of pre-determined service networks for all operators.

2.2. ALR-based DMPC approaches

The augmented Lagrangian relaxation method gains its name from employing two methods (i.e., auxiliary problem principle, and block coordinate descent) to decouple the quadratic terms in the augmented Lagrangian when the method of multipliers is directly applied to the original optimization problem with interconnecting constraints. These two methods will lead to two distributed optimization algorithms and consequently two ALR-based DMPC approaches, i.e., the parallel ALR-based DMPC approach and the serial ALR-based DMPC approach (Bertsekas, 1982; Royo, 2001; Negenborn et al., 2008). The paper Negenborn et al. (2008) gives a detailed explanation on these two DMPC approaches and compares their control performance on interconnected linear subsystems with an application to load-frequency control in a power network.

The parallel and serial ALR-based DMPC approaches have also been proposed and applied for frequency control in a multiple high-voltage-direct-current link power network (McNamara et al., 2013), power flow management of a mixed energy network that integrates renewable energy sources and storage (del Real et al., 2014), reference tracking for water levels in irrigation canals (Negenborn et al., 2009), controlling the loss coefficient of valves and pressure injection of pumps in urban water supply networks (Leirens et al., 2010), regulating the pneumatic valves in a three-tank benchmark (Alvarado et al., 2011), signal split control in large-scale urban traffic networks (Zhou et al., 2015), and container flow assignment in intermodal freight transport (Li et al., 2014). Most of the applications are for interconnected linear systems (Negenborn et al., 2008; McNamara et al., 2013; del Real et al., 2014; Negenborn et al., 2009; Leirens et al., 2010; Alvarado et al., 2011), while the papers Li et al. (2014) and Zhou et al. (2015) consider transport networks with nonlinear and non-convex dynamics.

2.3. ADMM-based DMPC approaches

The alternating direction method of multipliers algorithm aims to combine the efficient convergence property of the method of multipliers and the decomposability of the dual ascent method. It was originally introduced in Glowinski and Marroco (1975); Gabay and Mercier (1976). The paper Boyd et al. (2010) presents a recent review on applying the ADMM algorithm for distributed optimization and statistical machine learning problems. The ADMM algorithm and the method of multipliers share the same primal-variable-minimization and Lagrangian-multiplier-update structure in their iteration processes and both use the penalty parameter as the step size at the Lagrangian multiplier update steps. These two algorithms are different in the sense that the ADMM algorithm minimizes primal variables in an alternating fashion, while the method of multipliers minimizes them at the same time. Actually, the ADMM algorithm can be
interpreted as a special case of the method of multipliers where the primal variables are not minimized jointly, but in a single Gauss-Seidel procedure (Boyd et al., 2010; Summers and Lygeros, 2012).

Recently, some research works have been focused on developing DMPC strategies for a network of coupled subsystems based on the ADMM algorithm. The papers Kögel and Findeisen (2012); Farokhi et al. (2014); Summers and Lygeros (2012); Costa et al. (2014); Mota et al. (2012) consider linear systems while the papers Farokhi et al. (2014); Spudić et al. (2015) investigate nonlinear systems. The applications of the ADMM-based DMPC strategies cover various areas, e.g., a three tank system (Kögel and Findeisen, 2012), a formation acquisition problem of multiple nonholonomic vehicles (Farokhi et al., 2014), TCP/IP congestion control (Mota et al., 2012), distributed control of a water delivery canal (Costa et al., 2014), and wind farm (Spudić et al., 2015).

To the authors’ best knowledge, there is not yet any work reported in the literature on applying the ALR-based DMPC approaches and the ADMM approach for cooperative synchromodal freight transport planning.

2.4. DMPC for intermodal freight transport

Recently, research efforts in intermodal freight transport have been undertaken for developing hierarchical MPC schemes for intermodal container terminal operation (Nabais et al., 2013b,a), a cooperative MPC scheme for optimizing freight transport on multimodal corridors (Di Febbraro et al., 2013), and an DMPC approach for cooperative planning in intermodal container flow control (Li et al., 2014). The paper Nabais et al. (2013b) proposed to decompose the container terminal system into smaller subsystems, each of which is related to a transport connection available at the terminal. An MPC controller was adopted for the container flow assignment in each subsystem. A central coordinator was introduced to coordinate the use of limited handling resources at the terminal by all MPC controllers, and therefore a hierarchical MPC framework was established for optimizing terminal operations. Similarly, a multi-agent MPC system was also proposed to generate cooperative relations among intermodal terminals for using transport capacity in a seaport (Nabais et al., 2013a). The paper Di Febbraro et al. (2013) decomposed the overall freight transport planning problem on multimodal corridors into terminal operations at network nodes and transport operations on network links, each of which solves its own optimization problem with a particular planning goal and constraints. These operations interact two-by-two based on a cooperative receding horizon control scheme that uses Lagrangian relaxation for minimizing lateness of delivery of individual containers to end users.

Meanwhile, a multi-agent cooperative intermodal container transport planning approach for multiple transport operators was proposed using a parallel implementation of the DMPC approach based on augmented Lagrangian relaxation (Li et al., 2014). The differences between the paper Li et al. (2014) and the paper Di Febbraro et al. (2013) lie in the reasoning and the way that the intermodal freight transport network is partitioned. Instead of the node-link partition and the Lagrangian formulation of the cooperative planning problem used in Di Febbraro et al. (2013), the paper Li et al. (2014) considers that an intermodal freight transport network is partitioned into a group of non-overlapping subnetworks due to the involvement of multiple operators in the whole delivery process. Moreover, the paper Li et al. (2014) adopts the augmented Lagrangian formulation of the cooperative planning problem instead of the Lagrangian formulation to overcome strict requirements on convexity or finiteness of the objective function.

However, the paper Li et al. (2014) only investigates the parallel ALR-based DMPC approach and a simple network model. Therefore, the current paper will investigate and compare the
performance of two ALR-based DMPC approaches, and the ADMM-based DMPC approach in cooperative synchronod modal container flow control.

3. Cooperative synchronodal freight transport planning

This section explains our considered cooperative synchronod modal freight transport planning problem. Section 3.1 first introduces basic settings and main assumptions in cooperative planning. After introducing interconnecting variables and interconnecting constraints in Section 3.2, Section 3.3 formulates cooperative synchronodal freight transport planning as an CMPFC problem.

3.1. Cooperative planning setting and main assumptions

Cooperative transport planning happens among a group of \( N_{\text{sub}} \) operators in an intermodal freight transport network \( \mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M}) \). The network \( \mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{M}) \) is an integration of a group of non-overlapping subnetworks \( \mathcal{G}_n(\mathcal{V}_n, \mathcal{E}_n, \mathcal{M}_n), n = 1, \ldots, N_{\text{sub}}, \) i.e., \( \mathcal{V} = \bigcup_{n=1}^{N_{\text{sub}}} \mathcal{V}_n, \mathcal{E} = \bigcup_{n=1}^{N_{\text{sub}}} \mathcal{E}_n, \mathcal{M} = \bigcup_{n=1}^{N_{\text{sub}}} \mathcal{M}_n, \mathcal{V}_n \cap \mathcal{V}_m = \emptyset, \mathcal{E}_n \cap \mathcal{E}_m = \emptyset, n \in \{1, \ldots, N_{\text{sub}}\}, m \in \{1, \ldots, N_{\text{sub}}\}, n \neq m. \) Three sets \( \mathcal{V}_n, \mathcal{E}_n, \text{and } \mathcal{M}_n \) represent the set of all nodes in the network of operator \( n \), the set of all links in the network of operator \( n \), and the set of all modalities (e.g., trucks, trains, and barges) and modality changes (e.g., from trucks to trains) in the network of \( n \), respectively. The partition of the overall network is determined during the establishment of the cooperation among operators, and is assumed to be fixed during the cooperative planning. Figure 1 presents a cooperative synchronodal freight transport setting among three operators. Each of these three operators control container flows in a subnetwork indicated by dash-dotted black ellipses. A link connecting two subnetworks is an interconnecting link, e.g., the railway link from node 4 to node 5 in Figure 1. This railway link has two functionalities, an incoming interconnecting link for subnetwork/operator 1, and an outgoing interconnecting link for subnetwork/operator 2. This railway link is assumed to belong to subnetwork 1, from which it starts. The set of incoming interconnecting links of subnetwork \( n \) and that of outgoing interconnecting links are denoted as \( \mathcal{E}_n^{\text{in}} \) and \( \mathcal{E}_n^{\text{out}} \). For any two subnetworks, a subnetwork is considered as one neighboring subnetwork of the other subnetwork if there is at least one interconnecting link from this subnetwork to the other subnetwork. The set of all neighboring subnetworks of subnetwork \( n \) is denoted as \( \mathcal{N}_n^{\text{nei}} \).

Cooperative transport planning considers certain transport demands in the whole network, and their origin and destination pairs and their volumes for time interval \( k \) are given as the set \( \mathcal{O}_{\text{od}} \subseteq \mathcal{V} \times \mathcal{V}, \) and \( d_{o,d}(k), (o, d) \in \mathcal{O}_{\text{od}}. \) Operator \( n \) provides transport services and determines container transport plans in subnetwork \( n \). In cooperative planning, operator \( n \) plans intermodal freight transport in subnetwork \( n \) by solving an optimal container flow control problem with the objective of minimizing its own total delivery cost,

\[
\min_{x_n, u_n, y_n, v_n} J_n(x_n, u_n, y_n, v_n) \tag{1}
\]

and is subject to the dynamics and planning constraints of subnetwork \( n \):

\[
x_n(k+1) = f_1(n)(x_n(k), u_n(k), d_n(k), v_n(k)), \tag{2}
\]

\[
y_n(k+1) = f_2(n)(x_n(k+1), u_n(k), d_n(k), v_n(k)), \tag{3}
\]

\[
g_n(x_n(k+1), y_n(k+1), u_n(k), d_n(k), v_n(k)) \leq 0. \tag{4}
\]
where $x_n(k)$, $y_n(k)$, $d_n(k)$, $v_n(k)$, and $u_n(k)$ are subnetwork states, subnetwork outputs, disturbances, and the remaining variables that influence the dynamics of subnetwork $n$ and subsequently the container flow control actions of operator $n$ for time interval $k$, and $x_n$, $y_n$, $u_n$, $v_n$ include $x_n(k)$, $y_n(k)$, $u_n(k)$, $v_n(k)$ for the whole planning period, respectively. The disturbances include the volumes of container flows entering each node from the outside of the network and the traffic density on freeway links. The objective function (1), equations (2)–(3), and constraints (4) are derived from a linear discrete-time intermodal freight transport network model developed by the authors in Li et al. (2015). The objective function (1) consists of the total transport cost, the cost penalty on unfinished transport demands, and the equivalent monetary costs of the total transport time and the time penalty on unfinished transport demands. These equivalent monetary costs are calculated with the conversion factor or the value of time. The current paper considers aggregated container flows at tactical planning level, and is therefore essentially not possible to directly take into account the due time requirement of each individual container in the network model. The inclusion of these monetary costs in the objective function enables operators to take into account the due time requirements of containers in transport planning in an indirect way. The cost penalty and the time penalty on unfinished transport demands are calculated as the multiplications of the volumes of containers that have not reached their destinations at the end of the planning period with the typically transport time and the typically transport cost between two nodes or one node and one link in the network, respectively. Equations (2)–(3) capture the evolution of container flows in links and nodes of the network, and their interactions. Constraints (4) basically describe physical capacity limitations of the network, and timetables of trains and barges.

Before presenting the cooperative container flow control formulation, some important assumptions on cooperative planning made in this paper are listed as follows:

- The topology and properties of, transport capacities of and traffic conditions in service network $n$ are the operator-dependent information, and can only be measured/estimated by
- The typical transport time and the typical transport cost between two nodes or one node and one link in the whole network are obtained from the historical data or the working knowledge of operators, and are independent of the particular operators. For instance, if it typically takes 3 hours for trucks from any operators to travel from Euromax Container Terminal in the Port of Rotterdam to Venlo in The Netherlands, the typical transport time between Euromax Container Terminal and Venlo will be taken as 3 hours. Therefore, we assume the typical transport time and the typical transport cost between two nodes or one node and one link in the whole network are available for all operators.

- Transport demand information can be estimated with a certain accuracy and is shared by all operators.

- Operator $n$ only cooperates with its neighboring operators $m \in \mathcal{N}_n^{\text{nei}}$. Operator $n$ implements the cooperative planning by sharing its container flow information with its neighboring operators and by taking into account the information shared by its neighboring operators. The decisions of operator $n$ consist of three parts: 1) the volumes of container flows leaving each node through each of the node’s outgoing links in subnetwork $n$, 2) the volumes of container flows leaving subnetwork $n$ through all outgoing interconnecting links of subnetwork $n$, and 3) the expected volumes of container flows entering subnetwork $n$ through all incoming interconnecting links of subnetwork $n$. For operator $n$, the container flow information shared with its neighboring operators in the cooperative planning are typically the parts 2) and 3) of its decisions. For clarification, operator $n$ could optimize the volumes of the container flows entering subnetwork $n$ according to its own planning objective, and share the optimized volumes to its neighboring operators in the cooperative planning. But the optimized volumes are just something expected by operator $n$, and are actually determined by the neighboring operators of operator $n$. Therefore, we use the word “expected” to make a clarification.

Depending on a particular cooperative planning approach, the information that is provided by so-called Lagrange multipliers represents the preferences of operators on whether to enlarge or reduce the volumes of container flows exchanged among them, and this information also needs to be shared among neighboring operators. The Lagrange multipliers used by the presented DMPFC approaches will be explained in detail in Section 4.

- Transport plans in subnetwork $n$ are determined through a negotiation process among operator $n$ and its neighboring operators by considering both this operator’s objective and the cooperation goal. But the final transport plans will be made by operator $n$.

3.2. Interconnecting variables and interconnecting constraints

Incoming and outgoing container flows on interconnecting links create interactions with the states of neighboring subnetworks and further with the corresponding container flow control decisions made by the operators. These interactions are captured by including input and output interconnecting variables and interconnecting constraints in the container flow control problem of each operator. The input interconnecting variables of an operator represent the volumes of container flows that enter the subnetwork of this operator through all incoming interconnecting links of the subnetwork. The output interconnecting variables of an operator represent the volumes of container flows that leave the subnetwork of this operator through all outgoing interconnecting
links of the subnetwork. There are two sets of interconnecting variables respectively from two neighboring operators associated with an interconnecting link between the subnetworks of these two neighboring operators. For example, there is a railway link from node 4 in the subnetwork of operator 1 to node 5 in the subnetwork of operator 2 in Figure 1. On the one hand, a set of input interconnecting variables from operator 2 associated with this railway link represents the volumes of container flows that operator 2 is expected to receive from operator 1 at node 5 through this railway link. On the other hand, a set of outgoing interconnecting variables from operator 1 associated with this railway link represents the volumes of container flows that operator 1 hands over to operator 2 at node 5 through this railway link. Operator 1 and operator 2 will negotiate and search for an agreement on the values of these two sets of interconnecting variables during the cooperative planning process. For all incoming interconnecting links of subnetwork \( n \), input interconnecting variables \( w_{in,n}(k) \) represent the volumes of container flows that enter subnetwork \( n \) from its neighboring subnetworks for time interval \( k \). Output interconnecting variables \( w_{out,n}(k) \) are introduced in the same way. Specific to the optimal container flow control problem of operator \( n \), \( w_{in,n}(k) \) and \( w_{out,n}(k) \) are formulated as:

\[
\begin{align*}
    w_{in,n}(k) &= v_n(k), \\
    w_{out,n}(k) &= K_n y_n(k),
\end{align*}
\]

where the interconnecting output selection matrix \( K_n \) is constructed to select the output container flow variables on the outgoing interconnecting links of subnetwork \( n \). Interconnecting links between subnetwork \( n \) and subnetwork \( m \), \( m \in \mathcal{A}_n^{nei} \) function both as outgoing interconnecting links of subnetwork \( n \), and as incoming interconnecting links of subnetwork \( m \) simultaneously. Therefore, the following interconnecting constraints should be met:

\[
\begin{align*}
    w_{in,m,n}(k) &= w_{out,n,m}(k), & m \in \mathcal{A}_n^{nei}, \\
    w_{out,m,n}(k) &= w_{in,m,n}(k), & m \in \mathcal{A}_n^{nei},
\end{align*}
\]

where \( w_{in,m,n}(k) \) and \( w_{out,m,n}(k) \) represent the volumes of container flows that respectively enter or leave subnetwork \( n \) from or to its neighboring subnetwork \( m \). The collections of all \( w_{in,m,n}(k), m \in \mathcal{A}_n^{nei} \) and all \( w_{out,m,n}(k), m \in \mathcal{A}_n^{nei} \) are \( w_{in,n}(k) \) and \( w_{out,n}(k) \), respectively.

3.3. Cooperative model predictive container flow control

In cooperative planning, each operator adopts an MPC strategy for controlling container flows in its subnetwork. Therefore, cooperative synchromodal freight transport planning can be formulated as an CMPFC problem and solved with different DMPC approaches. The CMPFC problem for \( N_{sub} \) operators for time interval \( k \) is formulated as follows:

\[
\begin{align*}
    \min \quad & \quad \sum_{n=1}^{N_{sub}} J_n(\mathbf{x}_n(k+1), \mathbf{u}_n(k), \mathbf{y}_n(k+1)) \\
    \text{s.t.} \quad & \quad \mathbf{x}_{N_{sub}}(k+1), \mathbf{u}_{N_{sub}}(k), \mathbf{y}_{N_{sub}}(k+1)
\end{align*}
\]

for \( n = 1, \ldots, N_{sub} \), subject to
where

- The network states, network outputs, and disturbances of subnetwork $n$ and the container flow control actions of operator $m$ in a finite prediction horizon $N_{\text{pred}}$ are denoted as $\hat{x}_{n}(k+1) = \left[ x_{n}^{T}(k+1), \cdots, x_{n}^{T}(k+N_{\text{pred}}) \right]^T$, $\hat{y}_{n}(k+1) = \left[ y_{n}^{T}(k+1), \cdots, y_{n}^{T}(k+N_{\text{pred}}) \right]^T$, $\hat{d}_{n}(k) = \left[ d_{n}^{T}(k), \cdots, d_{n}^{T}(k+N_{\text{pred}}-1) \right]^T$, and $\hat{u}_{m}(k) = \left[ u_{m}^{T}(k), \cdots, u_{m}^{T}(k+N_{\text{pred}}-1) \right]^T$, respectively. The remaining variables that influence the dynamics of subnetwork $n$ in the prediction horizon are included in $\hat{v}_{n}(k) = \left[ v_{n}^{T}(k), \cdots, v_{n}^{T}(k+N_{\text{pred}}-1) \right]^T$. The initial states of subnetwork $n$ are given by $\hat{x}_{n,k}$ in (16). The disturbance information of subnetwork $n$ in the prediction horizon is given by $\hat{d}_{n,k}$ in (17).

- The interconnecting input and output variables of the model predictive container flow control problem of operator $n$ with respect to that of operator $m \in \mathcal{N}_{n}^{\text{nei}}$ in the prediction horizon $N_{\text{pred}}$ are expressed as $\hat{w}_{\text{in},n}(k) = \left[ w_{\text{in},n}^{T}(k), \cdots, w_{\text{in},n}^{T}(k+N_{\text{pred}}-1) \right]^T$ and $\hat{w}_{\text{out},n}(k) = \left[ w_{\text{out},n}^{T}(k), \cdots, w_{\text{out},n}^{T}(k+N_{\text{pred}}-1) \right]^T$. The interconnecting output selection matrix $\hat{K}_{n}$ is constructed to select the output container flow variables on the interconnecting outgoing links from subnetwork $n$ to all its neighboring subnetworks $m \in \mathcal{N}_{n}^{\text{nei}}$ in the prediction horizon.

Equations (10)–(11) are the dynamics of subnetwork $n$. All the planning constraints in the planning problem of subnetwork $n$ are included in inequalities (12). The equalities (13)–(14) correspond to the definition of interconnecting input and output variables in the model predictive container flow control problem of operator $n$. The interconnecting constraints between the planning problem of operator $n$ and that of subnetwork $m \in \mathcal{N}_{n}^{\text{nei}}$ are equalities (15). In the cooperative planning setting considered in this paper, the incoming container flow information of subnetwork $n$ from neighboring subnetwork $m \in \mathcal{N}_{n}^{\text{nei}}$, $\hat{w}_{\text{in,m,n}}(k)$, is not directly available for operator $n$ and has to be exchanged with neighboring operators. The value of $\hat{w}_{\text{in,m,n}}(k)$ can be negotiated by operator $n$ with its neighboring operator $m$ through an iterative process, but will finally be determined by operator $m$ after the negotiation even in case of no feasible agreements have been reached. This is because that for concerns of information privacy and independent operations each operator persists to have the independent power in planning freight transport in its subnetwork when participating in the cooperative planning. Therefore, the CMPFPC problem (9)-(17) has to be solved distributedly.
4. Distributed model predictive container flow control

In the CMPFC problem (9)–(17), the interconnecting variables from different Model Predictive Flow Control (MPFC) problems exist in the interconnecting constraints (15). Therefore, the problem (9)–(17) cannot be directly distributed as a group of MPFC problems, each of which can be solved by an operator independently. Three algorithms are first presented in this paper to address the issue of interconnecting constraints correspondingly resulting in three DMPFC approaches for solving the problem (9)–(17). These three algorithms are the parallel ALR algorithm, the serial ALR algorithm, and the ADMM algorithm. Moreover, this section discusses some implementation issues of and presents performance indicators for these three DMPFC approaches.

4.1. The ALR-based DMPFC approaches

The ALR-based DMPFC approaches cope with interconnecting constraints (15) by constructing an augmented Lagrangian formulation of the problem (9)–(17) that captures interconnecting constraints (15) in the control objective function as a combination of linear and quadratic terms:

\[
\begin{align*}
    \sum_{n=1}^{N_{\text{sub}}} & \left[ J_n(x_{n}(k+1), \bar{y}_n(k+1), \bar{u}_n(k), \bar{v}_n(k)) + \\
    & \sum_{m \in \mathcal{N}(n)} \left( \lambda^T_{\text{in},m,n}(k) (\bar{w}_{\text{in},m,n}(k) - \bar{w}_{\text{out},n,m}(k)) + \frac{\rho}{2} ||\bar{w}_{\text{in},m,n}(k) - \bar{w}_{\text{out},n,m}(k)||_2^2 \right) \right].
\end{align*}
\]

where \( \lambda_{\text{in},m,n}(k) \) and \( \rho > 0 \) are the Lagrangian multipliers associated with interconnecting constraints (15) and the penalty parameter. Due to the appearance of quadratic terms, the control objective function (18) cannot be separated over the operators. In order to solve the augmented Lagrangian formulation of the problem (9)–(13) in a distributed way, the non-separable quadratic terms have to be decoupled such that the control objective function (18) can be distributed across operators for a distributed implementation of multiple MPC controllers.

The parallel and serial DMPFC approaches are obtained by using the ALR-based DMPC approaches reviewed in Section 2.2, which apply the auxiliary problem principle, and block coordinate descent to decouple the quadratic terms in (18), respectively. Algorithm 1 presents the implementation of the parallel ALR-based DMPFC approach for time interval \( k \). In Algorithm 1 operators are ordered according to a predetermined cooperation and communication protocol, i.e., operator 1, ..., operator \( N_{\text{sub}} \). The distinction between the serial ALR-based DMPFC approach and the parallel ALR-based DMPFC approach mainly lies on the Iteration process for performing cooperation among operators. Therefore, Algorithm 2 only gives the Iteration process part of the implementation of the serial ALR-based DMPFC approach. In Algorithms 1 and 2, the initial states of subnetwork \( n \) for time interval \( k \) are denoted by \( x_{n,k} \). The disturbance information of subnetwork \( n \) in the prediction period \([kT_s, (k+N_{\text{prod}})T_s]\) is denoted by \( \bar{d}_{n,k} \). The notations \( \bar{u}_{n,k} \), \( \bar{w}_{\text{in},m,n,k} \), \( \bar{w}_{\text{out},m,n,k} \), and \( \lambda_{\text{in},m,n,k} \) denote the control actions of operator \( n \) at iteration \( s \) for time interval \( k \), the input and output interconnecting variables of the control problem of operator \( n \) related to its neighboring operators \( m \in \mathcal{N}_{\text{sub}}^n \), and the associated Lagrangian multipliers at iteration \( s \) for time interval \( k \).

In the current paper, each transport operator uses a linear discrete-time intermodal freight transport network model. For each ALR-based DMPFC approach, each operator repeatedly solves Quadratic Programming (QP) optimization problems. The QP optimization problem has a positive semi-definite quadratic matrix. Therefore, in general the QP optimization problem will
not have a unique solution. A regularization term is typically included in the objective functions (19) and (20) to obtain a *positive definite* quadratic matrix in the QP optimization problem. On the one hand, adding the regularization term might lead to a slightly increase in the value of the objective function, but it guarantees one unique solution for the regularized QP optimization problem in each DMPFC approach considered in this paper. On the other hand, the regularization term is typically of the form: $\rho_{\text{regular}} \| \mathbf{x}(k+1) \|^2_F$, where $\rho_{\text{regular}}$ is a small positive number. This implies that the corresponding flow control actions will be determined not only to minimize the freight delivery cost, but also to avoid having very large container flows at nodes or in links of the network. After the inclusion of the regularization term in the objective functions, the solutions from the DMPFC approaches converge theoretically to the solution resulting from solving the problem in a centralized way (Bertsekas, 1982; Royo, 2001). Moreover, the above convergence property holds also for the ADMM-based DMPFC approach presented in Section 4.2.

### 4.2. The ADMM-based DMPFC approach

The ADMM-based DMPFC approach introduces a set of global optimization variables to deal with the coupling issue of interconnecting constraints (15). The global optimization variables contain the outgoing container flow information on all interconnecting links among all subnetworks over the prediction horizon, denoted as $\tilde{\mathbf{z}}(k) = [\mathbf{z}_1(k), \ldots, \mathbf{z}_{N_{\text{sub}}}(k)]^T$.

The cooperative control problem (9)–(17) can then be reformulated as an extended version of the general form consensus optimization problem by replacing interconnecting constraints (15) with

$$\tilde{\mathbf{w}}_{\text{in},m,n}(k) = \mathbf{E}_{m,n} \tilde{\mathbf{z}}(k), \quad \forall m \in \mathcal{S}_{\text{nei}}$$

$$\tilde{\mathbf{w}}_{\text{out},m,n}(k) = \mathbf{E}_{n,m} \tilde{\mathbf{z}}(k), \quad \forall m \in \mathcal{S}_{\text{nei}},$$  \hspace{1cm} (21)

where $\mathbf{E}_{m,n}$ and $\mathbf{E}_{n,m}$ are constructed to select the outgoing container flow information on interconnecting outgoing links from subnetwork $m$ to subnetwork $n$ and from subnetwork $n$ to subnetwork $m$, i.e., $\tilde{\mathbf{z}}_{m,n}(k) = \mathbf{E}_{m,n} \tilde{\mathbf{z}}(k)$ and $\tilde{\mathbf{z}}_{n,m}(k) = \mathbf{E}_{n,m} \tilde{\mathbf{z}}(k)$, respectively. The variables $\tilde{\mathbf{z}}_{n}(k) = \left[ \mathbf{E}_{m_1,n}, \ldots, \mathbf{E}_{m_{N_{\text{nei}}},n} \right]^T \tilde{\mathbf{z}}(k)$ include operator $n$’s local copies of some components of global optimization variables. Operator $n$ updates and stores $\tilde{\mathbf{z}}_{n}(k)$ during each time interval such that there is no need for a central coordinator in the implementation. The ADMM-based DMPFC approach applies the ADMM algorithm to solve the augmented Lagrangian formulation of the resulting extended version of the problem (9)–(17). The detailed implementation of the ADMM-based DMPFC approach is presented in Algorithm 3. The notations defined in Section 4.1 for Algorithms 1 and 2 holds also for Algorithm 3. The additional notation $\tilde{\mathbf{z}}^s_{n}(k)$ represents operator $n$’s local copies of some components of global optimization variables at iteration $s$ for time interval $k$.

### 4.3. Implementation issues

For practical implementation of the DMPFC approaches, cooperative planning decisions should be made during each time interval even if agreement cannot be obtained by the operators. To achieve this, a fixed maximum computation time $T_{\text{allowed}}$ is allowed for all operators to achieve agreement on the container flow control actions for each time interval of the cooperative planning process. The maximum allowed computation time is typically set to be equal to or smaller than $T_s$. In the case that the operators cannot reach agreement within a period of length $T_{\text{allowed}}$ during a particular time interval, the cooperative planning will be done in a master-slave
Algorithm 1 The parallel ALR-based DMPFC approach using the auxiliary problem principle for time interval $k$

**Input**: $s_{n,t}, d_{n}(k)$, maximum allowed computation time $T_{\text{allowed}}$ (h), iteration stopping threshold $\varepsilon$, positive parameters $\rho$ and $b$, $b \geq 2\rho$

**Initialization**: iteration count $s \leftarrow 1$, maximum absolute difference among the values of the Lagrange multipliers at iteration $s$ and its previous iteration $\varepsilon^{s} \leftarrow \infty$, current computation time $t_{n}(k) \leftarrow 0$ (h) spent by operators $n = 1, \ldots, N_{\text{sub}}$ for time interval $k$, $\tilde{u}_{n}(k), \tilde{w}_{in,m,n}(k), \tilde{w}_{out,m,n}(k)$, and Lagrangian multipliers $\lambda_{in,m,n}, n = 1, \ldots, N_{\text{sub}}, m \in \mathcal{N}_{n}^{\text{nei}}$ in the prediction period $[kT_{n}, (k + N_{\text{pred}})T_{n})$ are initialized as zeros when $k = 1$, and are initialized by using a warm start strategy with their values computed during time interval $k - 1$ when $k > 1$.

**Iteration process**:

while $\varepsilon^{s} \geq \varepsilon$ and $\max_{n=1,\ldots,N_{\text{sub}}} t_{n}(k) \leq T_{\text{allowed}}$ do

for operators $n = 1, \ldots, N_{\text{sub}}$, in a parallel fashion do

Compute $\tilde{u}_{n}^{+1}(k), \tilde{w}_{in,m,n}^{+1}(k)$ and $\tilde{w}_{out,m,n}^{+1}(k)$ for a local MPFC problem (19) subject to subnetwork dynamics (10)–(17) as follows:

$$\begin{aligned}
\min_{\tilde{x}_{n}(k+1), \tilde{u}_{n}(k), \tilde{y}_{n}(k+1)} J_{\varepsilon}(\tilde{x}_{n}(k+1), \tilde{y}_{n}(k+1), \tilde{u}_{n}(k), \tilde{y}_{n}(k)) + \\
\sum_{m \in \mathcal{N}_{n}^{\text{nei}}} \left[ \frac{\lambda_{in,m,n}^{+1}}{\lambda_{in,m,m}^{+1}} \right] \left[ \begin{array}{c}
\tilde{w}_{in,m,n}(k) \\
\tilde{w}_{out,m,n}(k)
\end{array} \right] + \frac{\rho}{2} \left[ \begin{array}{c}
\tilde{w}_{in,m,n}(k) - \tilde{w}_{out,m,n}(k) \\
\tilde{w}_{out,m,n}(k) - \tilde{w}_{in,m,n}(k)
\end{array} \right]^{2} + \\
\frac{b - \rho}{2} \left[ \begin{array}{c}
\tilde{w}_{in,m,n}(k) - \tilde{w}_{in,m,n}(k) \\
\tilde{w}_{out,m,n}(k) - \tilde{w}_{out,m,n}(k)
\end{array} \right]^{2}
\end{aligned}$$

(19)

end for

Send $\tilde{w}_{in,m,n}^{+1}(k)$ and $\tilde{w}_{out,m,n}^{+1}(k)$ to the neighboring operators $m \in \mathcal{N}_{n}^{\text{nei}}$ and receive $\tilde{w}_{in,m,n}(k)$ and $\tilde{w}_{out,m,n}(k)$ from the neighboring operators correspondingly

Update $\lambda_{in,m,m}^{+1} \leftarrow \lambda_{in,m,m}^{+1} + \rho \left[ \tilde{w}_{in,m,n}^{+1}(k) - \tilde{w}_{out,m,n}(k) \right], n = 1, \ldots, N_{\text{sub}}, m \in \mathcal{N}_{n}^{\text{nei}}$

Send $\lambda_{in,m,n}^{+1}$ to the neighboring operators $m \in \mathcal{N}_{n}^{\text{nei}}$ and in parallel receive $\lambda_{in,m,n}^{+1}$ from the neighboring operators

$$\begin{aligned}
\lambda_{in,m,n}^{+1} = \left[ \begin{array}{c}
\lambda_{in,m,n}^{+1} \\
\lambda_{in,m,n_{1}^{\text{nei}}}^{+1} \\
\vdots \\
\lambda_{in,m,m_{1}^{\text{nei}}}^{+1} \\
\vdots \\
\lambda_{in,m,n_{\text{sub}}}^{+1} - \lambda_{in,m_{\text{sub}}}^{+1} - \lambda_{in,m_{\text{sub}}}^{+1}
\end{array} \right]_{m_{n}}
\end{aligned}$$

Compute $\varepsilon^{s+1} \leftarrow$

$s \leftarrow s + 1$

end while

**Output**: $\tilde{u}_{n}^{+1}(k), \tilde{w}_{in,m,n}^{+1}(k), \tilde{w}_{out,m,n}(k), \lambda_{in,m,n}^{+1}, n = 1, \ldots, N_{\text{sub}}, m \in \mathcal{N}_{n}^{\text{nei}}$. 

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Algorithm 2 The Iteration process part of a serial ALR-based DMPFC approach using block coordinate descent for time interval $k$

while $\epsilon^t \geq \epsilon$ and $\sum_{n=1}^{N_{\text{sub}}} t_n(k) \leq T_{\text{allowed}}$

for operators $n = 1, \ldots, N_{\text{sub}}$, in a serial fashion do

Compute $\tilde{u}_n^{t+1}(k), \tilde{w}_{\text{in},m,n}^{t+1}(k)$, and $\tilde{w}_{\text{out},m,n}^{t+1}(k)$ for a local MPFC problem (20) subject to subnetwork dynamics (10)–(17) as follows:

$$
\min_{\tilde{x}_n(k+1), \tilde{u}_n(k), \tilde{y}_n(k+1), \tilde{w}_{\text{in},n}(k), \tilde{w}_{\text{out},n}(k)} J_n(\tilde{x}_n(k+1), \tilde{y}_n(k+1), \tilde{u}_n(k), \tilde{v}_n(k)) +
\sum_{m \in \mathcal{N}_{\text{nei}}^{\text{in},n} \text{m} < n} \left[ \begin{array}{c}
\frac{\lambda}{\text{in},m,n} \\
-\frac{\lambda}{\text{in},m,n}
\end{array} \right]^T \left[ \begin{array}{c}
\tilde{w}_{\text{in},m,n}(k) \\
\tilde{w}_{\text{out},m,n}(k)
\end{array} \right] + \frac{\rho}{2} \left\| \left[ \begin{array}{c}
\tilde{w}_{\text{in},m,n}^{t+1}(k) - \tilde{w}_{\text{out},m,n}(k) \\
\tilde{w}_{\text{out},m,n}^{t+1}(k) - \tilde{w}_{\text{in},m,n}(k)
\end{array} \right] \right\|^2_2 +
\sum_{m \in \mathcal{N}_{\text{nei}}^{\text{out},n} \text{m} > n} \left[ \begin{array}{c}
\frac{\lambda}{\text{in},m,n} \\
-\frac{\lambda}{\text{in},m,n}
\end{array} \right]^T \left[ \begin{array}{c}
\tilde{w}_{\text{in},m,n}(k) \\
\tilde{w}_{\text{out},m,n}(k)
\end{array} \right] + \frac{\rho}{2} \left\| \left[ \begin{array}{c}
\tilde{w}_{\text{in},m,n}^{t+1}(k) - \tilde{w}_{\text{out},m,n}(k) \\
\tilde{w}_{\text{out},m,n}^{t+1}(k) - \tilde{w}_{\text{in},m,n}(k)
\end{array} \right] \right\|^2_2
$$

(20)

Send $\tilde{w}_{\text{in},m,n}^{t+1}(k)$ and $\tilde{w}_{\text{out},m,n}^{t+1}(k)$ to the neighboring operators $m \in \mathcal{N}_{\text{nei}}^{\text{in},n}$

end for

Update $\lambda_{\text{in},m,n}^{t+1} \leftarrow \lambda_{\text{in},m,n}^t + \rho \left( \tilde{w}_{\text{in},m,n}^{t+1}(k) - \tilde{w}_{\text{out},m,n}^{t+1}(k) \right)$, $n = 1, \ldots, N_{\text{sub}}, m \in \mathcal{N}_{\text{nei}}^{\text{in},n}$

Send $\lambda_{\text{in},m,n}^{t+1}$ to neighboring operators $m \in \mathcal{N}_{\text{nei}}^{\text{out},n}$ and in parallel receive $\lambda_{\text{out},m,n}^{t+1}$ from the neighboring operators

$$
\begin{bmatrix}
\lambda_{\text{in},m,1}^{t+1} - \lambda_{\text{in},m,1}^t \\
\vdots \\
\lambda_{\text{in},m,n_{\text{sub}}}^{t+1} - \lambda_{\text{in},m,n_{\text{sub}}}^t \\
\lambda_{\text{in},m,1}^{t+1} - \lambda_{\text{in},m,1}^t \\
\vdots \\
\lambda_{\text{in},m,n_{\text{sub}}}^{t+1} - \lambda_{\text{in},m,n_{\text{sub}}}^t
\end{bmatrix}
$$

Compute $\epsilon^{t+1} \leftarrow$

$$
\begin{bmatrix}
\lambda_{\text{in},m,1}^{t+1} - \lambda_{\text{in},m,1}^t \\
\vdots \\
\lambda_{\text{in},m,n_{\text{sub}}}^{t+1} - \lambda_{\text{in},m,n_{\text{sub}}}^t \\
\lambda_{\text{in},m,1}^{t+1} - \lambda_{\text{in},m,1}^t \\
\vdots \\
\lambda_{\text{in},m,n_{\text{sub}}}^{t+1} - \lambda_{\text{in},m,n_{\text{sub}}}^t
\end{bmatrix}_{n_{\text{sub}}}
$$

$s \leftarrow s + 1$

end while
Algorithm 3 The ADMM-based DMPFC approach for time interval $k$

**Input**: $\mathbf{s}_{n,k}$, $\mathbf{d}_{n}(k)$, maximum allowed computation time $T_{\text{allowed}}$ (h), iteration stopping threshold $\varepsilon$, positive parameters $\rho$

**Initialization**: iteration count $s \leftarrow 1$, maximum absolute difference among the values of the Lagrange multipliers at iteration $s$ and its previous iteration $\varepsilon^s \leftarrow \infty$, current computation time $t_{n}(k) = 0$ (h) spent by operators $n = 1, \cdots, N_{\text{sub}}$ for time interval $k$, $\mathbf{u}_{n}(k)$, $\mathbf{z}_{n}(k)$, and Lagrangian multipliers $\lambda_{\text{in},m,n}^s$, $\lambda_{\text{out},m,n}^s$, $\lambda_{\text{in},n,m}^s$, and $\lambda_{\text{out},n,m}^s$ corresponding to constraints (21)–(22) for $n = 1, \ldots, N_{\text{sub}}$ in the prediction period $[kT_{r}, (k+N_{\text{pred}})T_{r}]$ are initialized as zeros when $k = 1$, and are initialized by using a warm start strategy with their values computed during time interval $k-1$ when $k > 1$.

**Iteration process**: while $\varepsilon^s \geq \varepsilon$ and $\max_{n=1,\cdots,N_{\text{sub}}} \epsilon_{n}(k) \leq T_{\text{allowed}}$ do

for agents $i = 1, \ldots, N_{\text{sub}}$, in a parallel fashion do

Compute $\mathbf{u}_{n}^{s+1}(k)$, $\mathbf{w}_{\text{in},m,n}^{s+1}(k)$, and $\mathbf{w}_{\text{out},m,n}^{s+1}(k)$ for a local MPFC problem (23) subject to subnetwork dynamics (10)–(14), (16)–(17), and (21)–(22) as follows:

$$\min_{\mathbf{s}_{n}(k+1), \mathbf{u}_{n}(k), \mathbf{z}_{n}(k)} J_{n}(\mathbf{s}_{n}(k+1), \mathbf{u}_{n}(k), \mathbf{z}_{n}(k)) + \frac{1}{2} \sum_{m \in \mathcal{N}_{n}^{\text{ nei}}} \left[ \begin{array}{c} \lambda_{\text{in},m,n}^s \\ \lambda_{\text{out},m,n}^s \end{array} \right] \begin{bmatrix} \mathbf{w}_{\text{in},m,n}(k) - \mathbf{z}_{n,m}^s(k) \\ \mathbf{w}_{\text{out},m,n}(k) - \mathbf{z}_{n,m}^s(k) \end{bmatrix}^T + \frac{\rho}{2} \left\| \begin{bmatrix} \mathbf{w}_{\text{in},m,n}(k) - \mathbf{z}_{n,m}^s(k) \\ \mathbf{w}_{\text{out},m,n}(k) - \mathbf{z}_{n,m}^s(k) \end{bmatrix} \right\|^2$$

(23)

Send $\mathbf{w}_{\text{in},m,n}^{s+1}(k)$ and $\mathbf{w}_{\text{out},m,n}^{s+1}(k)$ to neighboring operator $m \in \mathcal{N}_{n}^{\text{ nei}}$ and in parallel receive $\mathbf{w}_{\text{in},m,n}^{s+1}(k)$ and $\mathbf{w}_{\text{out},m,n}^{s+1}(k)$ from the neighboring operators

Compute $\mathbf{z}_{n,m}^{s+1}(k)$ with the relation (24)

$$\mathbf{z}_{n,m}^{s+1}(k) \leftarrow \frac{1}{2} \left( \begin{bmatrix} \mathbf{w}_{\text{in},m,n}^{s+1}(k) \\ \mathbf{w}_{\text{out},m,n}^{s+1}(k) \end{bmatrix} \right)$$

(24)

end for

Update $\lambda_{\text{in},m,n}^{s+1}$, $\lambda_{\text{out},m,n}^{s+1}$ $\leftarrow \frac{1}{2} \left( \begin{array}{c} \lambda_{\text{in},m,n}^{s+1} \\ \lambda_{\text{out},m,n}^{s+1} \end{array} \right) + \rho \left( \begin{bmatrix} \mathbf{w}_{\text{in},m,n}^{s+1}(k) \\ \mathbf{w}_{\text{out},m,n}^{s+1}(k) \end{bmatrix} - \mathbf{z}_{n,m}^{s+1}(k) \right)$, $n = 1, \cdots, N_{\text{sub}}$,

$m \in \mathcal{N}_{n}^{\text{ nei}}$

Compute $\varepsilon^{s+1} \leftarrow \max_{n=1,\cdots,N_{\text{sub}},m \in \mathcal{N}_{n}^{\text{ nei}}} \left\| \begin{bmatrix} \lambda_{\text{in},m,n}^{s+1} - \lambda_{\text{in},m,n}^{s} \\ \lambda_{\text{out},m,n}^{s+1} - \lambda_{\text{out},m,n}^{s} \end{bmatrix} \right\|_{\infty}$

$s \leftarrow s + 1$

end while

**Output**: $\mathbf{u}_{n}(k)$, $\mathbf{w}_{\text{in},m,n}(k)$, and $\mathbf{w}_{\text{out},m,n}(k)$, $n = 1, \cdots, N_{\text{sub}}, m \in \mathcal{N}_{n}^{\text{ nei}}$.  

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fashion by operators in a given pre-defined order based on the distances between the main seaport and different subnetworks, i.e., operators 1, 2, and 3 in a sequence.

The implementation of different DMPFC approaches in practice is influenced by different properties. Three main properties are:

- The cooperation mechanism: Whether the mechanism operates in parallel or in a serial way.
- The degree of confidentiality of information exchange: We use the number of information types that are exchanged and the number of information exchanges among operators to indicate the degree of confidentiality.
- The way of information processing: How each type of information shared by one operator is used by its neighboring operators.

First of all, the serial ALR-based DMPFC approach performs cooperation in a serial way while the parallel ALR-based DMPFC approach and the ADMM-based DMPFC approach cooperate in a parallel fashion. In terms of the degree of confidentiality of information exchange, the ADMM-based DMPFC approach requires only to exchange the interconnecting variables among neighboring operators, while the two ALR-based DMPFC approaches need to exchange both the interconnecting variables and the Lagrange multipliers. Thirdly, the ADMM-based DMPFC approach first uses the information of interconnecting variables received from neighboring operators to update the local copy of a part of the global optimization variables, i.e., \( \tilde{z}_n^{k+1} \) in (24), and then indirectly to update the Lagrange multipliers and to implement the optimization (23) at the next iteration. The exchanged information is used by the two ALR-based DMPFC approaches for directly updating the Lagrange multipliers and performing the optimization (20) or (19) at the next iteration.

4.4. Performance indicators

To make a comparison of the performance of different DMPFC approaches, the following performance indicators are employed:

- The total delivery cost \( J_{\text{total}} (\mathcal{E}) \): the sum of the delivery costs incurred in the transport planning of all operators when the operators cooperate to complete the transport demands, i.e.,

\[
J_{\text{total}} = \sum_{k=1}^{N_{\text{planning}}} \sum_{n=1}^{N_{\text{s}}} J_n (s_n(k+1), y_n(k+1), u_n(k), v_n(k)).
\]  

The whole planning period is \( N_{\text{planning}} T_s (h) \), where \( T_s \) is the length of the planning time interval. This indicator corresponds to the cooperative planning goal of all \( N_{\text{s}} \) operators.

- The communication cost \( J_{\text{com}} \) (float): the total number of floating-point numbers transmitted between operators during the whole cooperative planning process.

- The computation time \( T_{\text{com}} \) (h): the total amount of time taken by operators to perform cooperative planning in the whole planning period.
5. Simulation study

The three DMPFC approaches for cooperative synchronomodal freight transport planning presented in Section 4 will be evaluated using an international intermodal freight transport network connecting Rotterdam with Antwerp and Frankfurt. Section 5.1 introduces the basic setting of the cooperative planning problem. The performance of the three DMPFC approaches is analyzed and evaluated in Section 5.2.

5.1. The cooperative planning problem

This section considers a group of three operators that cooperatively plan synchronomodal freight transport in an international intermodal freight transport network. The network topology and the corresponding virtual network representation are shown in Figure 2 and Figure 3, respectively. This network consists of 6 nodes, and 16 transport connections, i.e., 5 railway connections, 5 inland waterway connections, and 6 freeway connections. As indicated by the dotted black ellipsoids in Figure 2, the network consists of 3 subnetworks, in each of which one operator provides synchronomodal freight transport services. The numbers 1, 2, 3, 4, 5, and 6 in the labels of nodes in the network denotes Rotterdam, Venlo, Antwerp, Liege, Neuss, and Frankfurt, respectively. The distance-dependent transport costs and the links, determined by timetables of trains and barges for railway links and inland waterway links, other nodes is 1000 (TEU). The maximum entering container flow is 400 (TEU/h) on freeway for loading containers, then departure from node 3

Trains and barges are assumed to operate according to pre-determined timetables that plan regular train services and barge services on railway links and inland waterways links, respectively. On link $l_{3T,4T}$, a train is scheduled to be available at node 3$^T$ every 6 hours, spends 2 hours for loading containers, then departure from node 3$^T$ and run $l_{3T,4T} = 4$ hours to arrive at node 4$^T$. On other railway links and inland waterways links, services are planned in the same way but with

$\rho_{i,j}^{road} = 33.5$ (veh/km/lane) (Kotsialos et al., 2002). The speed-density relation model is one part of the intermodal freight transport network model proposed by the authors in Li et al. (2015). This model is used to calculate and predict the flow speed on freeway links in the network with the input of the current and estimated flow density and the model parameters listed above. The typical length of trucks is three times that of cars.
the following two differences: the service frequency on links from and to deep-sea ports (i.e., Rotterdam and Antwerp) is every 4 hours while that on other links is every 6 hours; the actual running times on links $t_{1T,2T}$, $t_{2T,6T}$, $t_{4T,6T}$, $t_{1W,3W}$, $t_{1W,5W}$, $t_{3W,4W}$, and $t_{5W,6W}$ are 6 hours, 2 hours, 8 hours, 6 hours, 12 hours, 18 hours, 10 hours, and 16 hours, respectively. The capacity of trains and the allocated container handling capacities are 100 (TEU) and 100 (TEU/h) and they are 200 (TEU) and 200 (TEU/h) for barges. For both the case of a central operator and the case of multiple operators, timetables of barges and trains and the capacities of the network are the same.

We consider cooperative planning for a period of 48 (h) with a planning time interval $T_s = 2$ (h). The densities of traffic flows on the freeway links are given in Table 2. A transport demand enters subnetwork 1 from node $1W$ with the destination node $6R$ in subnetwork 3. This transport demand has a piecewise constant volume as shown in Table 2. The value of time for the transport demand is taken as 25 (€/h). The use of this transport demand scenario in this simulation study is helpful for the graphical illustration of the cooperation process of and the comparison of the performance of our proposed cooperative planning approaches. It is noteworthy that in practical transport planning, there are multiple transport demands with different origin and destination pairs. The increase of the transport demand will augment both the number of decision variables that have to be determined by operators and the number of interconnecting variables that need to be shared by and negotiated among operators in cooperative planning problems. Consequently, a larger amount of computation time will be required by the DMPFC approaches to perform cooperative planning. Because the DMPFC approaches can theoretically converge when a regularization term is added to the objective functions (19), (20), and (23), the container
Figure 3: The corresponding virtual network representation of the network shown in Figure 2. Each double-headed arc in the figure represents two directed links with opposite directions.
kilometers in the intermodal freight network that we consider. We therefore assumed that traffic are initially considered to be empty (i.e., $\tilde{\gamma} = \gamma$), the same as the container flow control actions and the total delivery cost obtained by a central process of the investigated DMPFC approaches and assesses their performance using the cooperative synchromodal freight transport planning problem defined in Section 5.1. Based on initial empirical experiments carried out for this particular problem setting, the cooperation of the investigated DMPFC approaches and assesses their performance using the cooperative synchromodal freight transport planning problem defined in Section 5.1.

### Table 1: The typical transport time. The entity “–” indicates that there is no transport service from the corresponding row node to the corresponding column node in the intermodal freight transport network shown in Figure 2

| $r_{cd}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 1        | 0 | 1 | 1 | 1 | 8 | 1 | 11 | 11 | 10 | 10 | 4 | 5 | 3 | 13 | 14 | 6 | 14 | 11 | 6 | 10 | 7 | 20 | 9 | 15 |
| 2        | 0 | 0 | 2 | 6 | 9 | 9 | 8 | 8 | 3 | 5 | 6 | 12 | 12 | 7 | 12 | 9 | 7 | 10 | 8 | 18 | 10 | 15 |
| 3        | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 4 | 7 | 4 | 8 | 7 | 16 | 7 | 11 |
| 4        | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5        | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6        | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7        | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8        | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9        | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

### Table 2: Densities of traffic flows on the freeway links and transport demand

<table>
<thead>
<tr>
<th>Period (h)</th>
<th>0 – 8</th>
<th>9 – 20</th>
<th>21 – 32</th>
<th>33 – 40</th>
<th>41 – 48</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{gh,dl}$ (veh/km/lane)</td>
<td>35.0</td>
<td>45.0</td>
<td>35.0</td>
<td>30.0</td>
<td>30.0</td>
</tr>
<tr>
<td>$\rho_{gh,dt}$ (veh/km/lane)</td>
<td>20.0</td>
<td>45.0</td>
<td>20.0</td>
<td>45.0</td>
<td>20.0</td>
</tr>
<tr>
<td>$d_{x,gh}$ (TEU/h)</td>
<td>150</td>
<td>200</td>
<td>175</td>
<td>150</td>
<td>0</td>
</tr>
</tbody>
</table>

flow control actions and the total delivery costs resulting from the DMPFC approaches will be the same as the container flow control actions and the total delivery cost obtained by a central operator. The current paper considers that freight truck flows generated by operators can only slightly influence the traffic conditions on freeways with tens of kilometers or even hundreds of kilometers in the intermodal freight network that we consider. We therefore assumed that traffic conditions in the network are independent of the volume of transport demands. All subnetworks are initially considered to be empty (i.e., $\bar{x}_i(k) = 0$, $i = 1, \ldots, N, k \leq 0$).

#### 5.2. DMPFC approach evaluations

After introducing the controller and solver settings, this section illustrates the cooperation process of the investigated DMPFC approaches and assesses their performance using the cooperative synchromodal freight transport planning problem defined in Section 5.1.

##### 5.2.1. Controller and solver settings

Based on initial empirical experiments carried out for this particular problem setting, the prediction horizon of individual operators is taken as $N_{pred} = 16$. In the simulation, we assume
that the information of traffic density on freeway links and the transport demand in the prediction period \( [k \cdot T_c, (k+N_{\text{pred}}) \cdot T_c) \) can be predicted accurately at time \( kT_c \). The maximum allowed computation time for operators is \( T_{\text{allowed}} = 30 \) (min). The cooperation parameters are taken as \( \rho = 0.3 \) and \( b = 54 \) for the parallel DMPFC approach, and \( \rho = 0.3 \) for the other two DMPFC approaches. The iteration stopping threshold for the two ALR-based DMPFC approaches is set as \( \varepsilon = 3 \times 10^{-3} \), while the threshold for the ADMM-based DMPFC approach is set as \( \varepsilon = 1.5 \times 10^{-3} \). These iteration stopping thresholds are selected to make sure that an accuracy of \( \epsilon_{\text{variable}} = 10^{-2} \) on interconnecting constraints (7)-(8) is required for all the three DMPFC approaches.

We assume that a floating-point number is needed to transmit one interconnecting variable or one Lagrange multiplier between two operators. Therefore, the communication cost \( J_{\text{con}} \) can be calculated as \( N_{\text{iteration}} I_{\text{iteration}} \), where \( N_{\text{iteration}} \) is the total number of iterations during the whole simulation process and \( I_{\text{iteration}} \) is the number of information exchanges per iteration.

For comparison purposes, a central operator is first assumed to be able to obtain all necessary planning information and to plan synchronomodal freight transport in the whole network. Therefore, this central operator performs planning in a centralized way. The corresponding planning problem for the central operator is a linear programming optimization problem given by (9)–(12) and (16)–(17). This planning problem is solved using the simplex method implemented by the CPLEX solver of the TOMLAB Optimization Toolbox (Inc., 2014). Next, our presented DMPFC approaches are applied for the cooperative synchronomodal freight transport planning problem. The regularized QP optimization problems in each DMPFC approach are solved with the barrier method implemented by the CPLEX Barrier QP solver of the TOMLAB Optimization Toolbox (Inc., 2014). The simulation experiments are done with the use of a desktop computer with an Intel(R) Xeon(R) CPU W3690 with 3.47 GHz and 16 GB RAM.

5.2.2. Cooperation process illustration

We illustrate the cooperation process of the three DMPFC approaches by presenting the evolution of the differences between particular interconnecting variables in the MPFC problems of two neighboring operators and the evolution of the associated Lagrangian multipliers for a particular time interval. We choose two sets of interconnecting variables between the MPFC problem of operator 1 (providing transport services in The Netherlands) and the MPFC problem of operator 3 (providing transport services in Germany) for time interval \( k = 1 \). These two sets of interconnecting variables are output interconnecting variables \( w_{\text{out},1,\text{road}}^{l,k} (k+1) \) and the corresponding input interconnecting variables \( w_{\text{in},1,\text{road}}^{l,k} (k+1) \) associated with the freeway link \( l = 2 \text{R}_{2,R} \) for \( l = 1, 2, \ldots, N_{\text{pred}} \).

For the serial ALR-based DMPFC approach, Figure 4 shows the cooperation process on the values of \( w_{\text{out},1,\text{road}}^{l,k} (k+1) \) and the values of \( w_{\text{in},1,\text{road}}^{l,k} (k+1) \) between operator 1 and operator 3 at the first iteration of time interval \( k = 1 \). At the first iteration, operator 1 first solves its MPFC problem and sends the preferred values of output interconnecting variables \( w_{\text{out},1,\text{road}}^{l,k} (k+1) \) (given in Figure 4(a)) to operator 3; then operator 3 computes its preferred values of input interconnecting variables \( w_{\text{in},1,\text{road}}^{l,k} (k+1) \) (by solving its MPFC problem with the values of \( w_{\text{out},1,\text{road}}^{l,k} (k+1) \) received from operator 1), and sends these preferred values (given in Figure 4(b)) to operator 1; finally, operator 3 updates the values of the associated Lagrange multipliers \( \lambda_{\text{in},1,\text{road}}^{l,k} (k+1) \) (presented in Figure 4(c)) and sends them to operator 1 to
be used at the next iteration. The next iterations will repeat the same cooperation process of the first iteration until the iterations are stopped. It is noteworthy that a negative value of a Lagrange multiplier $\lambda^s_{\text{in},1,3;\text{road},s,k} (k+l)$ implies that larger values of $w^s_{\text{in},1,3;\text{road},s,k} (k+l)$ and smaller values of $w^s_{\text{out},3,1;\text{road},s,k} (k+l)$ are preferred in the next iteration by operator 3 and operator 1, respectively.

The evolution of the differences between $w^s_{\text{out},3,1;\text{road},s,k} (k+l)$ and $w^s_{\text{in},1,3;\text{road},s,k} (k+l)$ during the iteration process for time interval $k = 1$ in the serial ALR-based DMPFC approach are presented in Figure 5. Figure 6 presents the evolution of the values of the Lagrange multipliers $\lambda^s_{\text{in},1,3;\text{road},s,k} (k+l)$ associated with interconnecting variables $w^s_{\text{out},3,1;\text{road},s,k} (k+l)$ and $w^s_{\text{in},1,3;\text{road},s,k} (k+l)$ for time interval $k = 1$ in the serial ALR-based DMPFC approach. In general, the cooperation process between two operators involves multiple pairs of interconnecting variables and should be terminated when the absolute differences between the values of their associated Lagrange multipliers at two successive iterations are smaller than certain threshold, e.g., $\varepsilon = 3 \times 10^{-3}$. The relation $\varepsilon = \rho \varepsilon_{\text{variable}}$ holds in the serial ALR-based DMPFC approach (see Algorithm 2). Terminating the iteration process with a threshold $\varepsilon = 3 \times 10^{-3}$ will therefore guarantee that the absolute differences between the values of all pairs of interconnecting variables are not larger than $\varepsilon_{\text{variable}} = 10^{-2}$.

For the parallel ALR-based DMPFC approach, the cooperation process on the values of $w^s_{\text{out},3,1;\text{road},s,k} (k+l)$ and the values of $w^s_{\text{in},1,3;\text{road},s,k} (k+l)$ between operator 1 and operator 3 at the first iteration of time interval $k = 1$ is presented in Figure 7. This cooperation process has the same execution sequence, (i.e., first solving optimization problems, and next updating Lagrange multipliers), as that of the cooperation process of the serial ALR-based DMPFC approach given in Figure 4. The major difference is the sequence in which operator 1 and operator 3 solve their MPFC problems: for the serial ALR-based DMPFC approach, operator 1 first solves its MPFC problem (presented in Figure 4(a)), and next operator 3 solves its MPFC problem (presented in Figure 4(b)); for the parallel ALR-based DMPFC approach, operator 1 and operator 3 solve their MPFC problems simultaneously (presented in Figure 7(a)). The cooperation process of the iterations in the parallel ALR-based DMPFC approach is similar to the cooperation process of the serial ALR-based DMPFC approach shown in Figures 5 and 6.

For the ADMM-based DMPFC approach, Figure 8 shows the cooperation process on the values of $w^s_{\text{out},3,1;\text{road},s,k} (k+l)$ and the values of $w^s_{\text{in},1,3;\text{road},s,k} (k+l)$ between operator 1 and operator 3 at the first iteration of time interval $k = 1$. At the first iteration, operator 1 and operator 3 first solve their MPFC problems in parallel and send their preferred values of the input interconnecting variables $w^s_{\text{in},1,3;\text{road},s,k} (k+l)$, and $w^s_{\text{out},3,1;\text{road},s,k} (k+l)$ (see Figure 8(a)) to each other; next, each of two operators uses the values of $w^s_{\text{out},3,1;\text{road},s,k} (k+l)$ or $w^s_{\text{in},1,3;\text{road},s,k} (k+l)$ received from the other operator to update its own Lagrange multipliers $\lambda^s_{\text{in},1,3;\text{road},s,k} (k+l)$ or $\lambda^s_{\text{out},3,1;\text{road},s,k} (k+l)$ (see Figure 8(b)). Each operator will keep the Lagrange multipliers updated by itself to be used in the next iteration. The next iterations will perform the same cooperation process of the first iteration (Figure 8) until the stopping criteria are reached. Moreover, the cooperation process of the iterations in the ADMM-based DMPFC approach differs from the cooperation process of the parallel ALR-based DMPFC approach in terms of having locally updated and privately used Lagrange multipliers for operator 1 and operator 3, respectively.
At the first iteration: iteration starts

Output interconnecting variables computed by operator 1

At the first iteration: iteration proceeds

Input interconnecting variables computed by operator 3

At the first iteration: iteration finishes

The associated Lagrange multipliers updated by operator 3

Figure 4: The cooperation process on the values of interconnecting variables $w_{\text{out},3,l}^{l_1,k+l}$ and $w_{\text{in},3,l}^{l_1,k+l}$ over a prediction period of $N_{\text{pred}} = 16$ time intervals, hence, for $l = 1, 2, \ldots, 16$ between operator 1 and operator 3 at the first iteration of time interval $k = 1$ in the serial ALR-based DMPFC approach. The associated Lagrange multipliers $\lambda_{w_{\text{out},3,l}^{l_1,k+l}}^{l_1} (k+l)$ are shown in Figure 4(c).
Figure 5: The evolution of the differences between the values of output interconnecting variable of the MPFC problem of operator 1, i.e., $w^s_{\text{out},3,1,l}(k+l)$, and the values of the corresponding input interconnecting variable of the MPFC problem of operator 3, i.e., $w^d_{\text{in},1,3,l}(k+l)$ in the serial ALR-based DMPFC approach, for the time interval $k=1$ over a prediction period of $N_{\text{pred}}=16$ time intervals, hence, for $l=1,2,...,16$.

Figure 6: The evolution of the Lagrange multipliers $\lambda^s_{\text{in},1,3,l}(k+l)$ computed by operator 3 in the serial ALR-based DMPFC approach for the time interval $k=1$ over a prediction period of $N_{\text{pred}}=16$ time intervals, hence, for $l=1,2,...,16$. 

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Figure 7: The cooperation process on the values of interconnecting variables $w_{\text{out},1,3,\text{pred}}^{1}(k+l)$ and $w_{\text{in},1,3,\text{pred}}^{1}(k+l)$ over a prediction period of $N_{\text{pred}} = 16$ time intervals, hence, for $l = 1, 2, \ldots, 16$ associated with the freeway link $\mathcal{L}_{2k,5k}$ between operator 1 and operator 3 at the first iteration of time interval $k = 1$ in the parallel ALR-based DMPFC approach. The associated Lagrange multipliers $\lambda_{w_{\text{in},1,3,\text{pred}}^{1}(k+l)}^{1}$ are shown in Figure 4(b).
(a) At the first iteration: iteration starts

Output interconnecting variables computed by operator 1
Input interconnecting variables computed by operator 3

(b) At the first iteration: iteration finishes

The associated Lagrange multipliers updated by operator 1
The associated Lagrange multipliers updated by operator 3

Figure 8: The cooperation process on the values of interconnecting variables $w^{l}_{\text{out},3,1,3,l\text{road}_{2R},5R}(k+l)$ and $w^{l}_{\text{in},1,3,1,l\text{road}_{2R},5R}(k+l)$ over a prediction period of $N_{\text{pred}} = 16$ time intervals, hence, for $l = 1, 2, \ldots, 16$ associated with the freeway link $\text{road}_{2R,5R}$ between operator 1 and operator 3 at the first iteration of time interval $k = 1$ in the ADMM-based DMPFC approach. The associated Lagrange multipliers $\lambda^{l}_{\text{out},3,1,3,l\text{road}_{2R,5R}}(k+l)$ and $\lambda^{l}_{\text{in},1,3,1,l\text{road}_{2R,5R}}(k+l)$ updated by two operators are shown in Figure 8(b).
Table 3: The performance of three DMPFC approaches. The entities ‘sDMPFC’, ‘pDMPFC’ and ‘ADMM’ in the ‘DMPFC approaches’ column stands for the serial ALR-based DMPFC approach, the parallel ALR-based DMPFC approach, and the ADMM-based DMPFC approach, respectively.

<table>
<thead>
<tr>
<th>DMPFC approaches</th>
<th>Delivery cost</th>
<th>Communication cost</th>
<th>Computation time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N^{\text{iteration}}$</td>
<td>$J^{\text{cost}}$ (floats)</td>
<td></td>
</tr>
<tr>
<td>sDMPFC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>$5.8627 \times 10^6$</td>
<td>$1840$</td>
<td>$618240$</td>
</tr>
<tr>
<td>Operator 1</td>
<td>$3.9326 \times 10^6$</td>
<td>$336$</td>
<td></td>
</tr>
<tr>
<td>Operator 2</td>
<td>$1.9076 \times 10^6$</td>
<td>$336$</td>
<td></td>
</tr>
<tr>
<td>Operator 3</td>
<td>$2.2500 \times 10^4$</td>
<td>$336$</td>
<td></td>
</tr>
<tr>
<td>pDMPFC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>$5.8627 \times 10^6$</td>
<td>$5249$</td>
<td>$1763664$</td>
</tr>
<tr>
<td>Operator 1</td>
<td>$3.9326 \times 10^6$</td>
<td>$336$</td>
<td></td>
</tr>
<tr>
<td>Operator 2</td>
<td>$1.9076 \times 10^6$</td>
<td>$336$</td>
<td></td>
</tr>
<tr>
<td>Operator 3</td>
<td>$2.2500 \times 10^4$</td>
<td>$336$</td>
<td></td>
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<tr>
<td>ADMM</td>
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<td></td>
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<tr>
<td>Overall</td>
<td>$5.8627 \times 10^6$</td>
<td>$4652$</td>
<td>$1042048$</td>
</tr>
<tr>
<td>Operator 1</td>
<td>$3.9326 \times 10^6$</td>
<td>$224$</td>
<td></td>
</tr>
<tr>
<td>Operator 2</td>
<td>$1.9076 \times 10^6$</td>
<td>$224$</td>
<td></td>
</tr>
<tr>
<td>Operator 3</td>
<td>$2.2500 \times 10^4$</td>
<td>$224$</td>
<td></td>
</tr>
</tbody>
</table>

5.2.3. Performance evaluation

The total delivery cost obtained by this central operator is $5.8627 \times 10^6$ (€). The planning performance of the DMPFC approaches is presented in Table 3. As shown in Table 3 all three DMPFC approaches obtain the same total delivery cost, i.e., $5.8627 \times 10^6$ (€), the same as the total delivery cost attained by the central operator. The corresponding delivery costs of the three operators in their subnetworks are also the same for different DMPFC approaches. The control actions resulted from the three DMPFC approach are also the same as the control actions of the centralized operator. However, the corresponding performance of the DMPFC approaches is still different in terms of communication cost, and actual computation time. On the one side, the serial ALR-based DMPFC approach takes the minimum total number of iterations $N^{\text{iteration}} = 1840$ and also the least total communication cost $J^{\text{cost}} = 618240$ (floats). On the other side, the ADMM-based DMPFC approach requires the least amount of actual computation time i.e., 7.55 (min) while that for the serial ALR-based DMPFC approach and the parallel ALR-based DMPFC approach are 8.23 (min), and 8.26 (min), respectively. It is worthwhile to note that even though the serial ALR-based DMPFC approach requires the least number of iterations, it actually spends more computation time than the ADMM-based DMPFC approach due to its serial cooperation mechanism.

For the cooperative planning problem considered in this section, it can be concluded that all three DMPFC approaches can attain the same cooperation goal as that of the central operator. With respect to the other performance indicators and the properties of different DMPFC approaches listed in Section 4.4 two comments can be made: the ADMM-based DMPFC approach outperforms the parallel ALR-based DMPFC approach by exchanging fewer types of information and requiring fewer iterations and actual computation time; the serial ALR-based DMPFC approach takes fewer iterations than the ADMM-based DMPFC approach, but it needs a higher degree of confidentiality (or more types of information exchanged) in information exchanges and requires a larger amount of actual computation time.

The DMPFC approaches determine container flow control actions at the beginning of each time interval for a prediction period of $N^{\text{pred}}$ time intervals with the up-to-date planning information available at the beginning of the current time interval. The up-to-date planning information, which is the real-time information, consists of the real-time measurements and estimations on
the transport demand and the traffic conditions in the network. Only the control actions for the current time interval will be implemented. For next time intervals, the same procedure will be executed in a receding horizon way. Because the control actions are determined for each time interval with the current available planning information, the DMPFC approaches can perform timely changing among multiple modalities according to real-time information in the simulation study. In general, the implementation and performance of an DMPFC approach depends on the cooperation mechanism, the required degree of confidentiality in information exchanges, the quantity of information exchanges, and the way in which the received information is used by operators. Therefore, for a particular cooperative synchronomodal freight transport planning problem the appropriate DMPFC approach should be chosen considering the communication ability of transport operators, their accepted degree of confidentiality in information exchanges, and their preferences on performance indicators, e.g., less information exchanges or less computation time.

6. Conclusions and future work

This paper has investigated cooperative synchronomodal freight transport planning among multiple transport operators in the hinterland haulage among deep-sea ports and inland terminals. Each operator is assumed to adopt a model predictive control approach for controlling container flows in one of the multiple interconnected subnetworks at the tactical flow level. This paper has formulated the Cooperative Model Predictive Flow Control (CMPFC) problem by introducing interconnecting variables and interconnecting constraints among the planning problems of each of the operators. Three Distributed Model Predictive Flow control (DMPFC) approaches have been used to solve the CMPFC problem in a distributed way, either adopting the Augmented Lagrange Relaxation (ALR) method or the Alternating Direction of Multiplier Method (ADMM) algorithm to decouple the interconnecting constraints. The simulation results indicate that the ADMM based-DMPFC approach takes the smallest actual computation time while the serial ALR-based DMPFC approach requires the least iterations and information exchanges.

Future research will investigate more complex demand scenarios and different organizational structures of cooperative planning, e.g., the cooperative planning for providing synchronomodal freight transport services by multiple different unimodal freight transport operators. Moreover, since a fixed network partition has been considered in the current paper the investigation of having a dynamic network partition is one future research direction.

Acknowledgments

This research is supported by the China Scholarship Council under Grant 2011629027.

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