

Technical report 16-015

# **A multi-class ramp metering and routing control scheme to reduce congestion and traffic emissions in freeway networks\***

C. Pasquale, S. Sacone, S. Siri, and B. De Schutter

*If you want to cite this report, please use the following reference instead:*

C. Pasquale, S. Sacone, S. Siri, and B. De Schutter, "A multi-class ramp metering and routing control scheme to reduce congestion and traffic emissions in freeway networks," *Proceedings of the 14th IFAC Symposium on Control in Transportation Systems (CTS 2016)*, Istanbul, Turkey, pp. 329–334, May 2016.

Delft Center for Systems and Control  
Delft University of Technology  
Mekelweg 2, 2628 CD Delft  
The Netherlands  
phone: +31-15-278.51.19 (secretary)  
fax: +31-15-278.66.79  
URL: <http://www.dcsc.tudelft.nl>

---

\*This report can also be downloaded via [http://pub.deschutter.info/abs/16\\_015.html](http://pub.deschutter.info/abs/16_015.html)

# A multi-class ramp metering and routing control scheme to reduce congestion and traffic emissions in freeway networks

C. Pasquale\* S. Sacone\* S. Siri\* B. De Schutter\*\*

\* *Department of Informatics, Bioengineering, Robotics and Systems Engineering, University of Genova, Italy*

\*\* *Delft Center for Systems and Control, Delft University of Technology, The Netherlands*

---

**Abstract:** In this paper, a multi-class and multi-objective combined ramp metering and routing control strategy is proposed to improve performance in freeway traffic networks. The control strategy is of the multi-class type, i.e. different classes of vehicles are taken into account, so that specific control policies can be devised for each class. Moreover, ramp metering and route guidance are properly coordinated in order to reduce the total travel time and the total emissions in the freeway system in a balanced way. The controller proposed in the paper is of the predictive feedback type, i.e. the control computed at each time step depends on the measured system state and on the prediction of the system evolution, considering both a traffic and an emission model. Some simulation results are presented showing the effectiveness of the proposed control scheme.

---

## 1. INTRODUCTION

The problem of freeway traffic control has been studied by researchers for some decades. Different control strategies have shown their effectiveness, such as ramp metering, variable speed limits, route guidance, as well as combined and coordinated strategies. Most of the research works on traffic control in freeway networks have dealt with the definition of control schemes aiming at the minimization of the congestion and the total travel delay in the system (see e.g. [1, 2, 3]). Some more recent works focus on other objective functions to be minimized, such as fuel consumption and emissions. In [4, 5, 6], ramp metering or combined control strategies are studied to take into account the reduction of traffic emissions.

In the last years, several models have been developed to evaluate the emissions produced by vehicles. The average-speed emission model is a simple but widespread model, assuming that the average emission factor for a certain pollutant and a given type of vehicle only depends on the average speed. In particular, the average-speed emission model COPERT [7] is used in [6, 8] where a ramp metering freeway control strategy is proposed. Other more accurate emission models consider the dependence of the emissions both on the speed and on the acceleration of vehicles. Among them, the VT-macro emission model has been firstly proposed in [4] and then extended to the multi-class case in [5]. The more recent VERSIT+ emission model [9] computes the emissions for different categories of pollutants and many types of vehicles, considering acceleration, speed and driving behavior in different conditions. The macroscopic version of VERSIT+ has been adopted in [10, 11] for control purposes.

In this work, a multi-class and multi-objective combined ramp metering and routing control strategy is proposed, in order to reduce the total travel time and the total

emissions in the freeway system in a balanced way. Route guidance is a control technique which has been broadly analyzed in the literature [12]. In [13] a predictive feedback routing control strategy is proposed, in [14] a strategy combining route guidance with ramp metering and motorway-to-motorway control is developed by solving a discrete-time constrained nonlinear optimal control problem, while in [15] model predictive control techniques are applied for optimally combining route guidance and ramp metering. Only a few contributions on route guidance deal with environmental issues, such as [16] where a multi-objective approach is proposed to minimize the total travel time, the travel distance and the pollutant emissions, [17] where a traffic assignment problem is solved taking into account the vehicle emissions, and [18] in which two eco-routing algorithms based on feedback assignment are discussed.

The multi-class routing control strategy proposed in this paper is inspired by [13]; specifically, the present paper proposes a multi-class traffic assignment scheme based on the predicted total travel time and the predicted total emissions, whereas in [13] only the predicted total time is considered in presence of only one class of vehicles. Moreover, in the current paper a multi-class feedback ramp-metering strategy is added in the control scheme in order to further reduce emissions and congestion in freeway networks. It is worth noting that, since the control strategy is based on a multi-class freeway model and on a multi-class emission model, it is possible to specify different control policies for the different classes of vehicles, i.e. cars, trucks, and other types of vehicles that can be of interest for the considered application case.

The paper is organized as follows. In Section 2 the adopted traffic and emission models are addressed. The proposed predictive feedback control scheme is then discussed in Section 3. Some simulation results are proposed in Section 4, while concluding remarks are reported in Section 5.

## 2. THE TRAFFIC AND EMISSION MODELS

The model adopted to represent the traffic dynamics in freeway networks is the well-known METANET model, considering the destination oriented mode, properly extended to take into account the multi-class case. The free-way system is represented by means of a directed graph, with  $M$  links and  $N$  nodes; each link  $m$  is further divided into  $N_m$  sections with equal length. The time horizon consists of  $K$  time steps. In the paper, let  $k = 0, \dots, K$  denote the time step,  $m = 1, \dots, M$  the freeway link,  $i = 1, \dots, N_m$  the section of link  $m$ , and  $c = 1, \dots, C$  the vehicle class. Moreover, let  $T$  indicate the sample time interval,  $L_m$  the length of each section of link  $m$ ,  $\lambda_m$  the number of lanes of link  $m$ ,  $J_m$  the set of destinations reachable from link  $m$ .

The main variables of the considered model are:

- $\rho_{m,i}^c(k)$  is the traffic density of class  $c$  in section  $i$  of link  $m$  at time instant  $kT$  ([veh of class  $c$ /km/lane]);
- $\rho_{m,i,j}^c(k)$  is the partial traffic density of class  $c$  in section  $i$  of link  $m$  with destination  $j \in J_m$  at time instant  $kT$  ([veh of class  $c$ /km/lane]);
- $v_{m,i}^c(k)$  is the mean traffic speed of class  $c$  in section  $i$  of link  $m$  at time instant  $kT$  ([km/h]);
- $q_{m,i}^c(k)$  is the traffic volume of class  $c$  leaving section  $i$  of link  $m$  during time interval  $[kT, (k+1)T)$  ([veh of class  $c$ /h]);
- $\gamma_{m,i,j}^c(k)$  is the composition rate of class  $c$ , that is the portion of vehicles of class  $c$  in section  $i$  of link  $m$  that at time step  $k$  have destination  $j \in J_m$ .

The following types of links are considered:

- *freeway links*, modeling the traffic behavior in homogeneous freeway stretches;
- *origin links*, modeling the links which receive traffic volumes from outside the network and forward them into the mainstream;
- *on-ramp links*, modeling the ramps receiving the demand that wants to access the freeway network.

Let us start from the freeway links: the partial traffic density  $\rho_{m,i,j}^c(k+1)$ ,  $c = 1, \dots, C$ ,  $i = 1, \dots, N_m$ ,  $m = 1, \dots, M$ ,  $j \in J_m$ ,  $k = 0, \dots, K-1$  is given by

$$\rho_{m,i,j}^c(k+1) = \rho_{m,i,j}^c(k) + \frac{T}{L_m \lambda_m} \left[ \gamma_{m,i-1,j}^c(k) q_{m,i-1}^c(k) - \gamma_{m,i,j}^c(k) q_{m,i}^c(k) \right] \quad (1)$$

where  $\gamma_{m,i,j}^c(k) = \frac{\rho_{m,i,j}^c(k)}{\rho_{m,i}^c(k)}$  and  $\rho_{m,i}^c(k) = \sum_{j \in J_m} \rho_{m,i,j}^c(k)$ .

A fundamental relationship in macroscopic models is the steady-state speed-density relation  $V_{m,i}^c(k)$  expressed as

$$V_{m,i}^c(k) = v_{m,i}^{f,c} \cdot \left[ 1 - \left( \frac{\rho_{m,i}^c(k)}{\rho_{m,i}^{\max}} \right)^{l_c} \right]^{m_c} \quad (2)$$

where  $v_{m,i}^{f,c}$  is the free-flow speed in section  $i$  of link  $m$  for class  $c$ ,  $\rho_{m,i}^{\max}$  is the jam density in section  $i$  of link  $m$ , whereas  $l_c$ ,  $m_c$  are other model parameters.

The traffic volume and traffic mean speed are given by

$$q_{m,i}^c(k) = \rho_{m,i}^c(k) v_{m,i}^c(k) \lambda_m \quad (3)$$

$$v_{m,i}^c(k+1) = v_{m,i}^c(k) + \frac{T}{\tau_c} \left[ V_{m,i}^c(k) - v_{m,i}^c(k) \right] + \frac{T}{L_m} v_{m,i}^c(k) \left[ v_{m,i-1}^c(k) - v_{m,i}^c(k) \right] - \frac{\nu_c T [\rho_{m,i+1}^c(k) - \rho_{m,i}^c(k)]}{\tau_c L_m [\rho_{m,i}^c(k) + \chi_c]} \quad (4)$$

$c = 1, \dots, C$ ,  $i = 1, \dots, N_m$ ,  $m = 1, \dots, M$ ,  $k = 0, \dots, K-1$  and where  $\tau_c$ ,  $\nu_c$ ,  $\chi_c$ ,  $c = 1, \dots, C$ , are model parameters.

In (4) a further term can be added, to take into account the speed reduction caused by the merging of two links in a node. In particular, if  $\mu$  is the merging link and  $m$  the leaving link, in the first section of the leaving link there is a speed reduction related to the merging flow, i.e.

$$-\delta^c T \frac{v_{m,1}^c(k) q_{\mu,N_\mu}^c(k)}{L_m \lambda_m [\rho_{m,1}^c(k) + \chi_c]} \quad (5)$$

where  $\delta^c$  is a constant parameter defined for class  $c$ .

The total density in section  $i = 1, \dots, N_m$  of link  $m = 1, \dots, M$  at time step  $k = 0, \dots, K-1$  is computed as

$$\rho_{m,i}(k) = \sum_{c=1}^C \eta^c \rho_{m,i}^c(k) \quad (6)$$

where  $\eta^c$  has a meaning analogous to the definition of passenger car equivalents (PCE) [19].

Let us now consider the origin links in which  $d_{o,j}^{O,c}(k)$  denotes the partial demand, i.e. the portion of the traffic volume at the origin link  $o = 1, \dots, O$  which has destination  $j \in J_o^O$  (with  $J_o^O$  the set of destinations reachable from origin  $o$ ), defined as

$$d_{o,j}^{O,c}(k) = \theta_{o,j}^{O,c}(k) d_o^{O,c}(k) \quad (7)$$

The allowed entering flow of class  $c = 1, \dots, C$  at origin link  $o = 1, \dots, O$  at time step  $k = 0, \dots, K-1$  is given by

$$q_o^{O,c}(k) = \min \left\{ \sum_{j \in J_o^O} d_{o,j}^{O,c}(k) + \frac{l_{o,j}^{O,c}(k)}{T}, Q_o^{O,c}(k) \right\} \quad (8)$$

where, at time step  $k$ ,  $l_{o,j}^{O,c}(k)$  is the partial queue length at the origin link  $o$  for the flow with destination  $j$ , whereas  $Q_o^{O,c}(k)$  is the flow capacity for class  $c$ , link  $o$ , depending on the density of the primary downstream leaving link  $\mu$  and on the mainstream capacity  $q_o^{O,\max,c}$  for origin  $o$  and class  $c$ :

$$Q_o^{O,c}(k) = \begin{cases} q_o^{O,\max,c} & \text{if } \rho_{\mu,1}(k) < \rho_{\mu}^{\text{cr}} \\ q_o^{O,\max,c} \cdot \frac{\rho_{\mu}^{\max} - \rho_{\mu,1}(k)}{\rho_{\mu}^{\max} - \rho_{\mu}^{\text{cr}}} & \text{else} \end{cases} \quad (9)$$

At time step  $k$ ,  $k = 0, \dots, K-1$ , the dynamic evolution of the partial queue length  $l_{o,j}^{O,c}(k)$  in (8) is calculated as

$$l_{o,j}^{O,c}(k+1) = l_{o,j}^{O,c}(k) + T [d_{o,j}^{O,c}(k) - \gamma_{o,j}^{O,c}(k) q_o^{O,c}(k)] \quad (10)$$

with  $\gamma_{o,j}^{O,c}(k) = \frac{l_{o,j}^{O,c}(k)}{l_{o,j}^{O,c}(k)}$ ,  $c = 1, \dots, C$ ,  $o = 1, \dots, O$ ,  $j \in J_o^O$ .

The on-ramp links are modeled analogously to the origin links. Eqs. (7)-(10) are adapted by defining, at time step  $k$ :

- the partial demand  $d_{r,j}^{\text{R},c}(k)$ , i.e. the portion of the traffic demand of class  $c$  that wants to enter the on-ramp  $r$  at time step  $k$  with destination  $j \in J_r^{\text{R}}$  (with  $J_r^{\text{R}}$  the set of destinations reachable from on-ramp link  $r$ ), and defined by the rate  $\theta_{r,j}^{\text{R},c}(k)$ ;
- the on-ramp flow  $q_r^{\text{R},c}(k)$  that enters at the on-ramp  $r$ ;
- the queue length  $l_{r,c}^{\text{R},c}(k)$  at the on-ramp  $r$ , and the partial queue length  $l_{r,j}^{\text{R},c}(k)$  at the on-ramp  $r$  for the flow with destination  $j \in J_r^{\text{R}}$ ;
- the composition rate  $\gamma_{r,j}^{\text{R},c}(k)$ , that represents the portion of traffic volume  $q_r^{\text{R},c}(k)$  with destination  $j \in J_r^{\text{R}}$ .

In case the on-ramps are controlled via ramp metering, and letting  $\bar{q}_r^{\text{R},c}(k)$  denote the on-ramp flow computed by the controller, the allowed flow  $q_r^{\text{R},c}(k)$ ,  $c = 1, \dots, C$ ,  $r = 1, \dots, R$ ,  $k = 0, \dots, K - 1$  is given by

$$q_r^{\text{R},c}(k) = \min \left\{ \sum_{j \in J_r^{\text{R}}} d_{r,j}^{\text{R},c}(k) + \frac{l_{r,j}^{\text{R},c}(k)}{T}, \bar{q}_r^{\text{R},c}(k), Q_r^{\text{R},c}(k) \right\} \quad (11)$$

As for the node model, it is assumed that each node does not include more than three links (in case of more complex nodes, they are decomposed in nodes meeting such condition by introducing dummy links). Let  $Q_n^c(k)$ ,  $c = 1, \dots, C$ ,  $k = 0, \dots, K - 1$  denote the total flow entering node  $n = 1, \dots, N$ , directed to destination  $j \in J_n$ , being  $J_n$  the set of the possible destinations reachable from  $n$ . Let  $I_n^{\text{M}}$ ,  $I_n^{\text{O}}$  and  $I_n^{\text{R}}$  denote the set of freeway, origin and on-ramp links entering node  $n$ , and  $O_n$  the set of links leaving node  $n$ . The incoming traffic flow is

$$Q_n^c(k) = \sum_{\mu \in I_n^{\text{M}}} q_{\mu, N_\mu}^c(k) \cdot \gamma_{\mu, N_\mu, j}^c(k) + \sum_{o \in I_n^{\text{O}}} q_o^{\text{O},c}(k) \cdot \gamma_{o, j}^{\text{O},c}(k) + \sum_{r \in I_n^{\text{R}}} q_r^{\text{R},c}(k) \cdot \gamma_{r, j}^{\text{R},c}(k) \quad (12)$$

The outgoing traffic from node  $n$  that chooses link  $m$  to reach destination  $j$  is calculated as

$$q_{m,0}^c(k) = \sum_{j \in J_m} \beta_{m,n,j}^c(k) \cdot Q_{n,j}^c(k) \quad (13)$$

where  $\beta_{m,n,j}^c(k)$  represents the splitting rate. In presence of control actions, the splitting rates are modeled by

$$\beta_{m,n,j}^c(k) = (1 - \varepsilon_{m,n}^c) \beta_{m,n,j}^{\text{N},c}(k) + \varepsilon_{m,n}^c \beta_{m,n,j}^{\text{C},c}(k) \quad (14)$$

where  $\beta_{m,n,j}^{\text{N},c}(k)$  is the portion of vehicles choosing link  $m$  without route recommendation,  $\beta_{m,n,j}^{\text{C},c}(k)$  is the splitting rate defined with a suitable control approach, and  $\varepsilon_{m,n}^c$  is the compliance rate with the route recommendations ( $0 \leq \varepsilon^c \leq 1$ ),  $c = 1, \dots, C$ .

Finally, in (4) boundary conditions are used, i.e. the virtual density  $\rho_{m, N_{m+1}}^c(k)$  and the virtual speed  $v_{m,0}^c(k)$ . If  $n$  has more than one leaving link, the downstream density is

$$\rho_{m, N_{m+1}}^c(k) = \frac{\sum_{\mu \in O_n} (\rho_{\mu,1}^c(k))^2}{\sum_{\mu \in O_n} \rho_{\mu,1}^c(k)} \quad (15)$$

$c = 1, \dots, C$ ,  $m = 1, \dots, M$ ,  $k = 0, \dots, K - 1$ . Moreover, if node  $n$  has more than one entering link, the virtual speed may be computed as

$$v_{m,0}^c(k) = \frac{\sum_{\mu \in I_n^{\text{M}}} v_{\mu, N_\mu}^c(k) q_{\mu, N_\mu}^c(k)}{\sum_{\mu \in I_n^{\text{M}}} q_{\mu, N_\mu}^c(k)} \quad (16)$$

$c = 1, \dots, C$ ,  $m = 1, \dots, M$ ,  $k = 0, \dots, K - 1$ .

For traffic emissions, the adopted model is the multi-class macroscopic VERSIT+ emission model proposed in [11] and here briefly reported (we refer to [11] for further details). This model is based on the microscopic emission model VERSIT+ [9], according to which the emission factor depends both on the combination of acceleration  $a$  and speed  $v$ , included in the model via the variable  $w = a + 0.014v$ , and on the value of the speed  $v$ , divided in four categories corresponding to different driving conditions. The emission factor  $E$  is given by

$$E = \begin{cases} u_0 & \text{if } v < 5 \text{ and } a < 0.5 \\ u_1 + u_2 w_+ + u_3 (w - 1)_+ & \text{if } v \leq 50 \\ u_4 + u_5 w_+ + u_6 (w - 1)_+ & \text{if } 50 < v \leq 80 \\ u_7 + u_8 (w - 0.5)_+ + u_9 (w - 1.5)_+ & \text{if } v > 80 \end{cases} \quad (17)$$

where  $u_h$ ,  $h = 0, \dots, 9$ , are model coefficients, and  $(x)_+ = 0$  if  $x < 0$ ,  $(x)_+ = x$  otherwise.

In order to extend this microscopic model to a macroscopic context, properly defined for the multi-class network case, the emission factors in the mainstream and in the on-ramps/origin links are distinguished and separately computed. Specifically, in the mainstream, two types of acceleration, the segmental acceleration considering the speed variation within a section, and the cross-segmental acceleration representing the speed variation of vehicles moving from one section to the consecutive one between time steps  $k$  and  $k + 1$  [4, 5] are computed together with the corresponding number of vehicles. In this way, the emission factors  $E_{m,i}^{\text{seg},c}(k)$  and  $E_{m,i,i+1}^{\text{cross},c}(k)$ ,  $c = 1, \dots, C$ ,  $m = 1, \dots, M$ ,  $i = 1, \dots, N_m$ ,  $k = 0, \dots, K - 1$ , are calculated.

For on-ramps and origin links, four groups of vehicles (i.e. arriving, waiting, leaving with stop and leaving without stop) are considered. Again the acceleration and number of vehicles in each group are computed thus leading to the computation of the emission factors  $E_r^{\text{ramp},y,c}(k)$ ,  $c = 1, \dots, C$ ,  $k = 0, \dots, K - 1$ ,  $r = 1, \dots, R$ , and  $E_o^{\text{origin},y,c}(k)$ ,  $c = 1, \dots, C$ ,  $k = 0, \dots, K - 1$ ,  $o = 1, \dots, O$ , where  $y = \{\text{a, w, ls, lns}\}$  identifies the group of vehicles.

### 3. THE PROPOSED CONTROL SCHEME

This section describes the proposed combined ramp metering and routing strategy, specifically devised to reduce traffic emissions and travel times in a freeway network where many classes of vehicles are explicitly considered.

In order to correctly define the assignment choices in the controller, the predicted travel times and the predicted emissions must be calculated for each alternative path. Inspired by the works [15] and [20], such evaluation is

carried out considering some virtual test vehicles, which leave from the origin nodes until they reach the chosen destination through alternative paths. In particular, for each pair of nodes  $(n, j)$ , the  $P$  most likely (shortest) paths are considered and a number of virtual vehicles equal to the number of such paths is introduced. The exact position of the vehicle in the network  $s_{n,j}^{z,c}(k)$  is updated as follows

$$s_{n,j}^{z,c}(k+1) = s_{n,j}^{z,c}(k) + v_{m,i}^{c,z}(k) \cdot T \quad (18)$$

with  $c = 1, \dots, C$ ,  $k = 0, \dots, K-1$ ,  $z \in J_{n,j}^c$ , where  $J_{n,j}^c$  is the set of test vehicles that leave node  $n$  to reach node  $j$ , whereas  $m$  and  $i$  are the freeway link and section in which the vehicle is located at time step  $k$ .

The travel time  $\tau_{n,j}^{z,c}$  and the emissions  $\xi_{n,j}^{z,c}$  for each test vehicle are computed based on its current location in the network by using the following relations:

$$\tau_{n,j}^{z,c}(k+1) = \tau_{n,j}^{z,c}(k) + T \quad (19)$$

$$\begin{aligned} \xi_{n,j}^{z,c}(k+1) = & \xi_{n,j}^{z,c}(k) + T \cdot [E_{m,i}^{\text{seg},z,c}(k) \\ & + E_{m,i,i+1}^{\text{cross},z,c}(k) + E_r^{\text{ramp},y,z,c}(k) + E_o^{\text{origin},y,z,c}(k)] \quad (20) \end{aligned}$$

In (20), only one of the emission factors is used, depending on the location of the vehicle between time steps  $k$  and  $k+1$ . Then, it is possible to define the *predicted travel time* and the *predicted emissions*, i.e. the travel time and the emissions that the virtual vehicle will actually experience along the route. In order to limit the computational effort, for each pair of nodes  $(n, j)$ ,  $P = 2$  shortest paths connecting them will be identified (note that the case  $P = 2$  is chosen for illustration purposes; the proposed approach can be also extended to the case  $P > 2$ ). On this basis, the predicted travel time (emissions) for the primary direction  $\tau_{n,j}^{P,c}(k)$  ( $\xi_{n,j}^{P,c}(k)$ ) and for the secondary direction  $\tau_{n,j}^{S,c}(k)$  ( $\xi_{n,j}^{S,c}(k)$ ) can be calculated according to (19) ((20)).

The predicted travel time difference is then obtained as

$$\Delta\tau_{n,j}^c(k) = \tau_{n,j}^{S,c}(k) - \tau_{n,j}^{P,c}(k) \quad (21)$$

whereas the predicted emissions difference is given by

$$\Delta\xi_{n,j}^c(k) = \xi_{n,j}^{S,c}(k) - \xi_{n,j}^{P,c}(k) \quad (22)$$

As regards route choice models, the conditions of Dynamic User Equilibrium (DUE), also taking into account the dynamic nature of the traffic conditions, are adopted. These conditions consider that traffic flows with the same origin and destination are distributed in the network so that the travel times on the recommended routes are the same, and therefore the users who do not follow these suggestions are penalized. Analogously to the travel time, and based on the same considerations, we propose a new eco-routing model named Dynamic Emissions Equilibrium (DEE), aimed at balancing the pollutant emissions along the suggested routes.

Given the predicted travel time difference  $\Delta\tau_{n,j}^c(k)$ , the conditions DUE may be formulated as follows:

$$\Delta\tau_{n,j}^c(k) > 0 \quad \Rightarrow \quad \beta_{m,n,j}^{\tau,c}(k) = 1 \quad (23)$$

$$\Delta\tau_{n,j}^c(k) = 0 \quad \Rightarrow \quad 0 < \beta_{m,n,j}^{\tau,c}(k) < 1 \quad (24)$$

$$\Delta\tau_{n,j}^c(k) < 0 \quad \Rightarrow \quad \beta_{m,n,j}^{\tau,c}(k) = 0 \quad (25)$$

with  $c = 1, \dots, C$ ,  $n = 1, \dots, N$ ,  $k = 0, \dots, K-1$ ,  $j \in J_n$ , and where  $m$  is the first link of the primary direction.

With the same notation, given the predicted emission difference  $\Delta\xi_{n,j}^c(k)$ , the conditions DEE may be formulated as

$$\Delta\xi_{n,j}^c(k) > 0 \quad \Rightarrow \quad \beta_{m,n,j}^{\xi,c}(k) = 1 \quad (26)$$

$$\Delta\xi_{n,j}^c(k) = 0 \quad \Rightarrow \quad 0 < \beta_{m,n,j}^{\xi,c}(k) < 1 \quad (27)$$

$$\Delta\xi_{n,j}^c(k) < 0 \quad \Rightarrow \quad \beta_{m,n,j}^{\xi,c}(k) = 0 \quad (28)$$

In order to balance travel times and emissions along both directions, conditions (24) and (27) will be considered, with the objective of maintaining the differences as close as possible to zero.

As regards the structure of the proposed predictive feedback control scheme, the controller is composed of two blocks, one for route guidance and the other for ramp metering. The feedback routing regulator computes the splitting rates  $\beta_{m,n,j}^{C,c}(k)$ , on the basis of the predicted travel time differences  $\Delta\tau_{n,j}^c(k)$  and the predicted traffic emission differences  $\Delta\xi_{n,j}^c(k)$  produced by the prediction models. The prediction models are updated with the current state of the freeway network, working similarly to a rolling-horizon scheme. The feedback ramp metering controller, instead, computes the on-ramp flow on the basis of the occupancy measurements obtained from the real system.

In particular, the PI-controllers adopted to define the splitting rates  $\beta_{m,n,j}^{\tau,c}(k)$  and  $\beta_{m,n,j}^{\xi,c}(k)$  are designed in order to approximate conditions (24) and (27). Given the predicted travel time difference, it is possible to calculate the splitting rate  $\beta_{m,n,j}^{\tau,c}(k)$  as

$$\begin{aligned} \beta_{m,n,j}^{\tau,c}(k) = & \beta_{m,n,j}^{\tau,c}(k-1) \\ & + K_P^{\tau,c}[\Delta\tau_{n,j}^c(k) - \Delta\tau_{n,j}^c(k-1)] + K_I^{\tau,c}\Delta\tau_{n,j}^c(k) \quad (29) \end{aligned}$$

Similarly, given the predicted emission difference, the splitting rates  $\beta_{m,n,j}^{\xi,c}(k)$  are obtained as

$$\begin{aligned} \beta_{m,n,j}^{\xi,c}(k) = & \beta_{m,n,j}^{\xi,c}(k-1) \\ & + K_P^{\xi,c}[\Delta\xi_{n,j}^c(k) - \Delta\xi_{n,j}^c(k-1)] + K_I^{\xi,c}\Delta\xi_{n,j}^c(k) \quad (30) \end{aligned}$$

with  $c = 1, \dots, C$ ,  $n = 1, \dots, N$ ,  $k = 0, \dots, K-1$ ,  $j \in J_n$ , and where  $m$  is the first link of the primary direction, whereas  $K_P^{\tau,c}$ ,  $K_I^{\tau,c}$ ,  $K_P^{\xi,c}$  and  $K_I^{\xi,c}$  are the gain parameters of the PI-controllers. It is worth noting that the resulting splitting rates  $\beta_{m,n,j}^{\tau,c}(k)$  and  $\beta_{m,n,j}^{\xi,c}(k)$  should be truncated to the admissible interval  $[0, 1]$ .

The splitting rates are given by the following weighted sum:

$$\beta_{m,n,j}^{C,c}(k) = \alpha \cdot \beta_{m,n,j}^{\tau,c}(k) + (1 - \alpha) \cdot \beta_{m,n,j}^{\xi,c}(k) \quad (31)$$

where  $\alpha$  is a weighing parameter.

For the feedback ramp metering controller, the multi-class PI-ALINEA is adopted (for major details see [8]). Let us first of all define the variable indicating the ratio of the occupancy of each class over the entire occupancy (including both the mainstream and the queues)

$$f_r^c(k) = \frac{\eta^c \cdot [o_{\mu,1}^c(k)L_\mu + l_r^c(k)]}{\sum_{c=1}^C \eta^c \cdot [o_{\mu,1}^c(k)L_\mu + l_r^c(k)]} \quad (32)$$

with  $c = 1, \dots, C$ ,  $r = 1, \dots, R$ ,  $k = 0, \dots, K - 1$ , where  $o_{\mu,1}^c(k)$  is the occupancy measurement in the first section of the primary downstream leaving link  $\mu$  for class  $c$  at time step  $k$ .

The on-ramp flow of class  $c = 1, \dots, C$ , for on-ramp  $r = 1, \dots, R$  and time step  $k = 0, \dots, K - 1$ , is computed as follows

$$\bar{q}_r^c(k) = \max \left\{ q_r^{\min,c}, q_r^c(k-1) - K_P^c \cdot [o_{\mu,1}^c(k-1) - o_{\mu,1}^c(k-2)] + K_R^c \cdot f_r^c(k-1)[\hat{o} - o_{\mu,1}^c(k-1)] \right\} \quad (33)$$

where  $\hat{o}$  is the occupancy set-point,  $K_P^c$  and  $K_R^c$ ,  $c = 1, \dots, C$ , are suitable parameters for the considered regulators,  $q_r^{\min,c}$  is the minimum on-ramp traffic volume for class  $c$  and on-ramp  $r$  and the total occupancy is obtained as  $o_{\mu,1}(k) = \sum_{c=1}^C \eta^c \cdot o_{\mu,1}^c(k)$ .

#### 4. SIMULATION RESULTS

The performance of the proposed multi-class control scheme is evaluated via simulation and compared with the uncontrolled scenario. The freeway network considered in the simulation test is composed of  $M = 8$  freeway links, each one with sections of 500 [m], one on-ramp link, one origin link and one destination (see Fig. 1). In this network, the origin and the destination are connected by two alternative paths: the primary direction is composed by the links M1, M2, M3, M4, M8, with a total length of 9.5 [km]; the secondary direction is given by the links M1, M5, M6, M7, M8, with a total length of 12 [km].

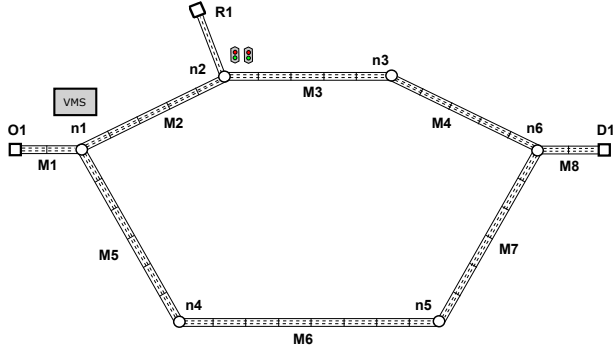


Fig. 1. Layout of the case study freeway network.

The sample time is  $T = 10$  [s] and a total time horizon of 2 and half hours ( $K = 900$ ) is considered for the simulation tests, whereas the routing strategy is calculated and applied every  $3T$  time steps. For the sake of simplicity,  $C = 2$  types of vehicles, namely cars and trucks, are distinguished in this case of study. Trapezoidal demand profiles for both vehicle classes at the on-ramp are taken into account, and a constant mainstream flow of 5000 [cars/hour] and 460 [trucks/hour] is considered. For this case study the following traffic model parameters are selected:  $v_{m,i}^{f,1} = 120$  [km/h],  $v_{m,i}^{f,2} = 90$  [km/h],  $\rho_{m,i}^{\max} = 200$  [veh/km/lane], for all  $m$ , for all  $i$ ,  $q_r^{\max,1} = 1800$  [veh/h],  $q_r^{\max,2} = 450$  [veh/h], for all  $r$ , and the conversion factors

are  $\eta^1 = 1$  and  $\eta^2 = 4$ . As for the VERSIT+ emission model, the on-ramp speed is considered constant and set equal to 30 [km/h] for both vehicle classes, and the speed of the vehicles moving within the queue, considered constant as well, is set equal to 5 [km/h] for the two classes.

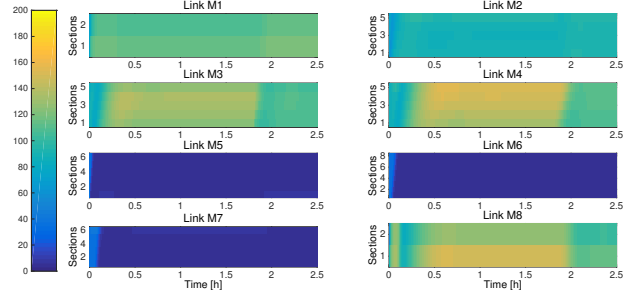


Fig. 2. Mainstream density [veh/km]: uncontrolled case.

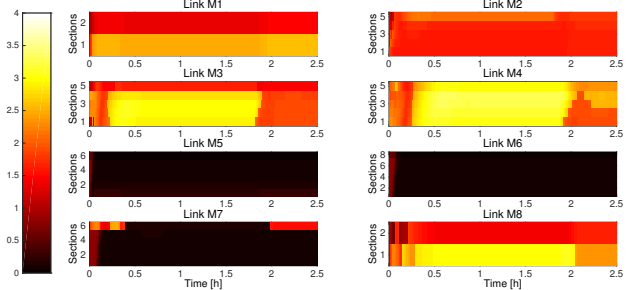


Fig. 3. CO<sub>2</sub> emissions [kg]: uncontrolled case.

Fig. 2 and Fig. 3 report, respectively, the profile of the mainstream density and the CO<sub>2</sub> emissions for the uncontrolled case. Referring to Fig. 2, it is possible to observe that, in the uncontrolled case, the primary direction is rather congested, especially the links M3, M4, and M8. Indeed, in the uncontrolled scenario most of the vehicles choose this path, which is the shortest alternative, while the secondary route results completely uncongested. In particular, the portion of vehicles choosing the main route is  $\beta_{2,1,1}^{N,c}(k) = 0.9$ ,  $c = 1, 2$ ,  $k = 0, \dots, K - 1$ . Analogously, referring to Fig. 3, the highest CO<sub>2</sub> emissions are in the links of the primary direction. In the uncontrolled case, the resulting Total Time Spent is TTS = 3266 [veh·h], whereas the Total Emissions are TE = 41170 [kg].

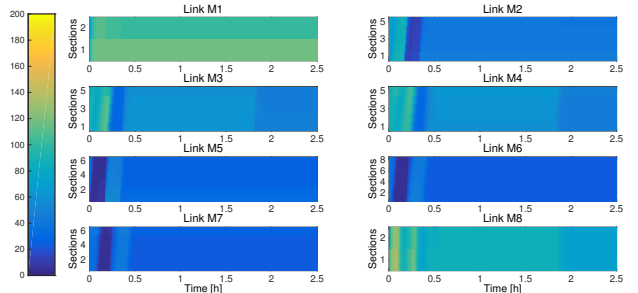


Fig. 4. Mainstream density [veh/km]: controlled case.

Fig. 4 and Fig. 5 report, respectively, the profile of the mainstream density and the CO<sub>2</sub> emissions for the controlled case. Comparing these profiles with those of the

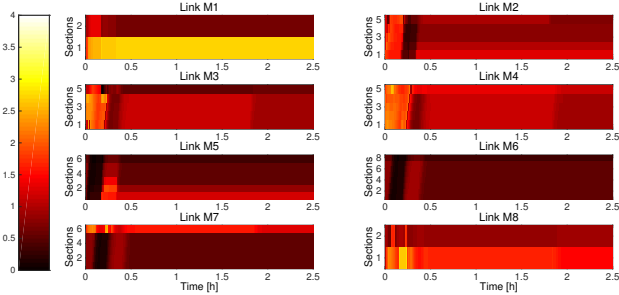


Fig. 5. CO<sub>2</sub> emissions [kg]: controlled case.

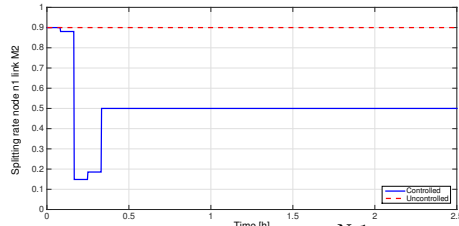


Fig. 6. The splitting rate values  $\beta_{2,1,1}^{N,1}(k)$  (red line) and  $\beta_{2,1,1}^C(k)$  (blue line) for cars.

uncontrolled case, a strong reduction in both congestion and CO<sub>2</sub> emissions is observed. In particular, the TTS is reduced to 2071 [veh·h], which is a 36.56% reduction compared with the uncontrolled case, whereas TE is reduced to 31148 [kg], corresponding to a 24.34% reduction.

Moreover, Fig. 6 shows the values of the splitting rates for cars obtained by applying the proposed control strategy (choosing the weight parameter  $\alpha$  equal to 0.5), compared to the uncontrolled case. The proposed routing strategy reduces at the beginning the splitting rate for the main route (lower than 0.2) and then assigns it the value of 0.5, i.e. balancing vehicles between the two paths.

## 5. CONCLUSION

In this paper, a multi-class combined ramp metering and routing control strategy has been proposed for reducing, in a balanced way, the total travel time and the total emissions in freeway traffic networks. The proposed control scheme is predictive and based on feedback. The presented simulation results show very high improvements of the freeway network performance, in terms of reduction of the total time spent and reduction of the total emissions, in case the proposed control strategy is applied compared with the uncontrolled case.

## REFERENCES

- [1] M. Papageorgiou and A. Kotsialos. Nonlinear optimal control applied to coordinated ramp metering. *IEEE Trans. on Control Syst. Techn.*, 12(6): 920–933, 2004.
- [2] A. Hegyi, B. De Schutter, and H. Hellendoorn. Model predictive control for optimal coordination of ramp metering and variable speed limits. *Transp. Research C*, 13(3): 185–209, 2005.
- [3] A. Ferrara, S. Sacone, and S. Siri. Event-triggered model predictive schemes for freeway traffic control. *Transp. Research C*, 58: 554–567, 2015.
- [4] S.K. Zegaye, B. De Schutter, J. Hellendoorn, E.A. Breunese, and A. Hegyi. Integrated macroscopic traffic flow, emission, and fuel consumption model for control purposes. *Transp. Res. C*, 31: 158–171, 2013.
- [5] S. Liu, B. De Schutter, and H. Hellendoorn. Integrated traffic flow and emission control based on FASTLANE and the multi-class VT-macro model. In *Proc. European Control Conf.*, 2908–2913, 2014.
- [6] C. Pasquale, I. Papamichail, C. Roncoli, S. Sacone, S. Siri, and M. Papageorgiou. Two-class freeway traffic regulation to reduce congestion and emissions via nonlinear optimal control. *Transp. Research C*, 55: 85–99, 2015.
- [7] L. Ntziachristos and C. Kouridis. Road transport emission chapter of the EMEP/CORINAIR emission inventory guidebook. *European Environment Agency Technical Report*, 16, 2007.
- [8] C. Pasquale, S. Sacone, and S. Siri. Two-class emission traffic control for freeway systems. In *Proc. 19th IFAC World Congress*, 936–941, 2014.
- [9] N. E. Ligterink, R. De Lange, and E. Schoen. Refined vehicle and driving-behavior dependencies in the VERSIT+ emission model. In *Proc. ETAPP Symposium*, 177–186, 2009.
- [10] S. Liu, B. De Schutter, and H. Hellendoorn. Model predictive control based on multi-class traffic models. Technical report, Delft Center for Systems and Control, Delft University of Technology, 2014.
- [11] C. Pasquale, S. Liu, S. Siri, S. Sacone, B. De Schutter. A new emission model including on-ramps for two-class freeway traffic control. In *Proc. 18th IEEE International Conf. on Intelligent Transportation Systems*, 1143–1149, 2015.
- [12] M. Ben-Akiva, J. Bottom, and M.S. Ramming. Route guidance and information systems. *Journal of Systems and Control Engineering*, 215: 317–324, 2001.
- [13] Y. Wang, M. Papageorgiou, and A. Messmer. A predictive feedback routing control strategy for freeway network traffic. In *Proc. American Control Conf.*, 3606–3611, 2002.
- [14] A. Kotsialos, M. Papageorgiou, M. Mangeas, and H. Haj-Salem. Coordinated and integrated control of motorway networks via non-linear optimal control. *Transp. Research C*, 10(1): 65–84, 2002.
- [15] A. Karimi, A. Hegyi, B. De Schutter, H. Hellendoorn, and F. Middelham. Integration of dynamic route guidance and freeway ramp metering using model predictive control. In *Proc. American Control Conf.*, 5533–5538, 2004.
- [16] G.H. Tzeng and C.H. Chen. Multiobjective decision making for traffic assignment. *IEEE Transactions on Engineering Management*, 40(2): 180–187, 1993.
- [17] M.M. Venigalla, A. Chatterjee, and M.S. Bronzini. A specialized equilibrium assignment algorithm for air quality modeling. *Transp. Res. D*, 4(1): 29–44, 1999.
- [18] H. Rakha, K. Ahn, and K. Moran. Integration framework for modeling eco-routing strategies: logic and preliminary results. *International Journal of Transp. Science and Technology*, 1(3): 259–274, 2012.
- [19] *Highway Capacity Manual*. TRB special report 209. National Research Council, Washington, D.C., 1994.
- [20] M. Cremer. On the calculation of individual travel times by macroscopic models. In *Proc. 6th International Vehicle Navigation and Information Systems Conf.*, 187–193, 1995.