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Scenario-Based Distributed Model Predictive Control for Freeway Networks

Shuai Liu, Anna Sadowska, Hans Hellendoorn, and Bart De Schutter

Abstract— **In this paper we develop a scenario-based Distributed Model Predictive Control (DMPC) approach for large-scale freeway networks. The uncertainties in a large-scale freeway network are categorized into global uncertainties for the overall network and local uncertainties for subnetworks. A reduced scenario tree is proposed, consisting of global scenarios and a reduced local scenario tree. For handling uncertainties in the scenario-based DMPC problem, a min-max setting is considered. A case study is implemented for investigating the scenario-based DMPC approach, and the results show that in the presence of uncertainties it is effective in improving the control performance with the queue length constraint being satisfied.**

I. Introduction

In Model Predictive Control (MPC), the uncertainties that affect the accuracy of the system predictions for determining the optimal control actions will affect the control performance and the satisfaction of constraints. For handling uncertainties in MPC, some approaches have been developed according to the literature [1–5]. In [1], a min-max scheme was considered for handling uncertainties in nonlinear Robust MPC, i.e. the worst case of the control objective function among all possible uncertainties is optimized, with constraints defined for all possible uncertainties. Based on a min-max scheme and a linear model for urban networks, Tettamanti et al. [2] proposed a robust MPC approach with constraints defined for all possible uncertainties. For a nonlinear system, Mayne et al. [3] proposed a tube-based MPC approach, which forces the trajectories of the perturbed system within a tube around a reference trajectory that is obtained by a nominal control approach based on tightened constraints. The scenario approach for robust control design was proposed by Calafiore and Campi [4] for systems with linear objective functions and convex constraints. For the scenario approach in [4], only a limited number of uncertainty scenarios are considered for handling the robust control problem; thus the computational load can be effectively reduced w.r.t. the case that all possible realizations of uncertainties are considered. Liu et al. [5] have developed a scenario-based receding horizon parameterized control approach for freeway networks with queue length constraints being penalized in the control objective function.

Considering the computational load, large-scale traffic networks can be controlled by Distributed Model Predictive Control (DMPC), with a large-scale network being partitioned into subnetworks and controlled by local agents. In DMPC, in general a local agent needs to communicate with other local agent about coupling variables; thus the uncertainties for other local agents will also affect the control effectiveness. For handling uncertainties in the DMPC problem, some robust DMPC approaches are available in the literature [6–8]. Giselsson et al. [6] proposed a robust DMPC approach for linear systems by considering the global constraint set as the Cartesian product of tightened local constraint sets, with robustness being ensured in the presence of small disturbances. For continuous-time decoupled nonlinear subsystems, Li and Shi [7] developed a robust DMPC approach, with coupling in a global control objective function being distributed to local control objective functions; the robustness w.r.t. external bounded disturbances is ensured by a robustness constraint, which makes local cost functions (i.e. Lyapunov functions) decrease. Maestre et al. [8] developed a scenario-based DMPC approach with uncertainty scenarios being distributed to local agents; however, the scenario-based DMPC approach in [8] is not for multiple subsystems, but for a single system.

Based on the scenario approach for uncertainties, in this paper, we propose a scenario-based DMPC approach for a large-scale freeway network, which is divided into multiple subnetworks. We distinguish uncertainties in a large-scale networks into global uncertainties for the overall network and local uncertainties for a single subnetwork. These uncertainties are assumed to be described by finite sets of scenarios. For a complete local scenario tree, all the combinations of the local scenarios are considered, leading to large computational load. We propose to construct a reduced scenario tree consisting of global scenarios and a reduced local scenario tree, in which the dynamics of a subnetwork depend on the local scenarios for that subnetwork, not on local scenarios for other subnetworks. Moreover, we consider an expected-value setting and a min-max setting for handling uncertainties in the scenario-based control problem. The new scenario-based DMPC approach is developed based on the dual decomposition method and the augmented Lagrangian relaxation method. The Alternating Direction Method of Multipliers (ADMM) [9] is chosen as the illustrative DMPC algorithm in this paper; however other DMPC algorithms based on the dual decomposition method can also be used.

The paper is organized as follows. In Section II, we describe the general DMPC problem. In Section III, we distinguish global uncertainties for the overall network

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from local uncertainties for subnetworks. After that, we develop a new scenario-based DMPC approach in Section IV. In Section V, we investigated the effectiveness of the scenario-based DMPC approach by a case study for a freeway network. At last, in Section VI we conclude this paper and give some recommendations for future work.

II. Distributed Model Predictive Control

In Model Predictive Control (MPC) [10, 11], for a given control step the performance of the considered network is predicted over the prediction period, and the predicted performance is optimized, leading to a optimal control input sequence over the control period. Then, the first element of the optimal control input sequence is applied to the considered network, and the prediction period is shifted one step ahead for next control step.

In Distributed Model Predictive Control (DMPC), a large-scale traffic network is controlled by local controllers, with the traffic network being divided into several subnetworks. In this paper, we consider additive performance criteria, for which the sum of all the local performance criteria equals to the performance criterion for the overall network.

Similar to [12], we describe the centralized Model Predictive Control (MPC) problem as follows:

$$\min_{\tilde{u}_1(k),\ldots,\tilde{u}_{N_{\text{sub}}}(k)} \sum_{s=1}^{N_{\text{sub}}} J_s\big(\tilde{x}_s(k),\tilde{y}_s(k),\tilde{u}_s(k)\big) \tag{1}$$

$$\text{s.t. } x_s(k+z+1) = f_s\big(x_s(k+z),u_s(k+z),D_s^{\text{in}}(k+z), E_s^{\text{in}}(k+z)\big) \quad \text{for } z=0,\ldots,N_{\text{p}}-1 \tag{2}$$

$$y_s(k+z) = h_s\big(x_s(k+z)\big) \quad \text{for } z=1,\ldots,N_{\text{p}} \tag{3}$$

$$x_s(k) = x_s^k \tag{4}$$

$$u_s(k+z) = u_s(k+N_{\text{c}}-1) \quad \text{for } z=N_{\text{c}},\ldots,N_{\text{p}}-1 \tag{5}$$

$$F_s(\tilde{x}_s(k),\tilde{y}_s(k),\tilde{u}_s(k)) \leq 0 \tag{6}$$

$$E_{j,s}^{\text{in}}(k+z) - E_{s,j}^{\text{out}}(k+z) = 0 \ \text{ for } j\in S_s^{\text{nb}},\ z=0,\ldots,N_{\text{p}}-1 \tag{7}$$

$$\text{for } s=1,\ldots,N_{\text{sub}}$$

where k is the control time step counter, N_{p} is the prediction horizon, N_{c} is the control horizon, s is the index for subnetworks, N_{sub} is the number of subnetworks, J_s is the local control objective function of subnetwork s, $\sum_{s=1}^{N_{\text{sub}}} J_s\big(\tilde{x}_s(k),\tilde{y}_s(k),\tilde{u}_s(k)\big)$ is the overall control objective function, x_s is the state vector of subnetwork s, y_s is the output vector of subnetwork s, u_s is the control input vector for subnetwork s, x_s^k is the measured state vector of subnetwork s at time step k, f_s is the dynamic function of subnetwork s, and $\tilde{x}_s(k)$, $\tilde{y}_s(k)$, and $\tilde{u}_s(k)$ are as follows:

$$\tilde{x}_s(k) = [{x_s}^T(k+1),\ldots,{x_s}^T(k+N_{\text{p}})]^T \tag{8}$$

$$\tilde{y}_s(k) = [{y_s}^T(k+1),\ldots,{y_s}^T(k+N_{\text{p}})]^T \tag{9}$$

$$\tilde{u}_s(k) = [{u_s}^T(k),\ldots,{u_s}^T(k+N_{\text{c}}-1)]^T \tag{10}$$

Moreover, h_s is the output function of subnetwork s, F_s is a general constraint function on the states, outputs, and control inputs for subnetwork s, $S_s^{\text{nb}} = \{j_{s,1},\ldots,j_{s,N_s^{\text{nb}}}\}$ is the set of all the neighbors of subnetwork s, N_s^{nb} is the number of the neighbors for subnetwork s, D_s^{in} is the external uncontrollable input vector for subnetwork s, E_s^{in} is the interconnecting input vector from all neighbors to subnetwork s: $E_s^{\text{in}}(k) = [(E_{j_{s,1},s}^{\text{in}}(k))^T,\ \ldots,(E_{j_{s,N_s^{\text{nb}}},s}^{\text{in}}(k))^T]^T$, $E_{j,s}^{\text{in}}$ is the interconnecting input vector for subnetwork s from neighbor j, $E_{s,j}^{\text{out}}$ is the interconnecting output vector from neighbor j to subnetwork s, (7) describes the couplings between subnetwork s and all neighbors, $E_{s,j}^{\text{out}}(k) = K_{s,j}[{x_j}^T(k),\ {y_j}^T(k),{u_j}^T(k)]^T$, and $K_{s,j}$ is the interconnecting output selection matrix from j to s.

By defining an augmented Lagrangian function L, (7) can be incorporated into the overall control objective function [12, 13]:

$$L\big(\tilde{x}_1(k),\tilde{y}_1(k),\tilde{u}_1(k),\ldots,\tilde{x}_{N_{\text{sub}}}(k),\tilde{y}_{N_{\text{sub}}}(k),\tilde{u}_{N_{\text{sub}}}(k),\tilde{\Lambda}^{\text{in}}(k)\big) = \sum_{s=1}^{N_{\text{sub}}}\Big(J_s\big(\tilde{x}_s(k),\tilde{y}_s(k),\tilde{u}_s(k)\big) + \sum_{j\in S_s^{\text{nb}}}\Big((\tilde{\lambda}_{j,s}^{\text{in}}(k))^T\big(\tilde{E}_{j,s}^{\text{in}}(k) - \tilde{E}_{s,j}^{\text{out}}(k)\big) + \frac{c}{2}\left\|\tilde{E}_{j,s}^{\text{in}}(k) - \tilde{E}_{s,j}^{\text{out}}(k)\right\|_2^2\Big)\Big) \tag{11}$$

with c a positive constant, $\lambda_{j,s}^{\text{in}}$ the Lagrange multiplier vector determined by agent s for (7), and $\Lambda^{\text{in}}(k) = [(\lambda_{j_{1,1},1}^{\text{in}}(k))^T,\ldots,(\lambda_{j_{N_{\text{sub}},N_{N_{\text{sub}}}^{\text{nb}}},N_{\text{sub}}}^{\text{in}}(k))^T]^T$. In addition, $\tilde{\Lambda}^{\text{in}}(k)$, $\tilde{\lambda}_{j,s}^{\text{in}}(k)$, $\tilde{E}_{j,s}^{\text{in}}(k)$, and $\tilde{E}_{s,j}^{\text{out}}(k)$ are defined similarly to $\tilde{x}_s(k)$ over control steps $k+z$, $z=0,\ldots,N_{\text{p}}-1$.

On the basis of duality theory [12–14], the dual problem for the original problem ((1)-(7)) is defined as follows:

$$\max_{\tilde{\Lambda}^{\text{in}}(k)}\ \min_{\tilde{u}_1(k),\ldots,\tilde{u}_{N_{\text{sub}}}(k)} L\big(\tilde{x}_1(k),\tilde{y}_1(k),\tilde{u}_1(k),\ldots, \tilde{x}_{N_{\text{sub}}}(k),\tilde{y}_{N_{\text{sub}}}(k),\tilde{u}_{N_{\text{sub}}}(k),\tilde{\Lambda}^{\text{in}}(k)\big) \tag{12}$$

$$\text{s.t. } (2)-(6) \quad \text{for } s=1,\ldots,N_{\text{sub}}$$

For the original problem with convex local control objective functions and inequality constraints and affine equality constraints, the solutions can be obtained by iteratively solving the dual problem [12, 13, 15]. To solve the dual problem, the Lagrange multipliers are fixed within one iteration, and are estimated based on the solution for the previous iteration for a given iteration.

According to [9, 12, 16], some approaches are available for decomposing the quadratic terms in (11), such as auxiliary problem principle (e.g. [16]), block coordinate descent (e.g. [12]), and dual ascent (e.g. [9]). By means of these approaches, the overall control problem can be decomposed into subproblems:

$$\min_{\substack{\tilde{u}_s(k)\\ \tilde{E}_{j_{s,1},s}^{\text{in}}(k),\ldots,\tilde{E}_{j_{s,N_s^{\text{nb}}},s}^{\text{in}}(k)\\ \tilde{E}_{j_{s,1},s}^{\text{out}}(k),\ldots,\tilde{E}_{j_{s,N_s^{\text{nb}}},s}^{\text{out}}(k)}} \Big(J_s\big(\tilde{x}_s(k),\tilde{y}_s(k),\tilde{u}_s(k)\big) + \sum_{j\in S_s^{\text{nb}}} J_s^{\text{inter}}\big(\tilde{\lambda}_{j,s}^{\text{in}}(k), \tilde{\lambda}_{j,s}^{\text{out}}(k),\tilde{E}_{j,s}^{\text{in}}(k),\tilde{E}_{j,s}^{\text{out}}(k)\big)\Big) \tag{13}$$

$$\text{s.t. } (2)-(6) \quad \text{for } s=1,\ldots,N_{\text{sub}}$$

where $\lambda_{j,s}^{\text{out}}$ is the Lagrange multiplier vector corresponding to the outputs from subnetwork s to neighbor j, $E_{j,s}^{\text{out}}$ is the interconnecting output vector from subnetwork s to neighbor j, and J_s^{inter} is the function dealing with the interconnecting variables determined by agent s. In addition, $\tilde{\lambda}_{j,s}^{\text{out}}(k)$ is defined similarly to $\tilde{x}_s(k)$ over control steps $k+z$, $z=0,\ldots,N_{\text{p}}-1$.

III. GLOBAL AND LOCAL UNCERTAINTIES FOR LARGE-SCALE FREEWAY NETWORKS

Uncertainties in freeway networks can be introduced via traffic measurements, uncontrollable inputs, and model parameters, etc. These uncertainties can e.g. be described by bounded sets including all the possible values of the uncertainties, or by a library of the possible uncertainty scenarios with the scenario possibilities being estimated. In this paper, we assume that the uncertainties are described by a library of the possible uncertainty scenarios.

For a large-scale network, we divide the uncertainties into global uncertainties for the overall network (e.g. weather condition) and local uncertainties for subnetworks (e.g. local demands at origins). The set of all the global scenarios is denoted as Ω_{glob}, and the set of all the local scenarios for a subnetwork s is denoted as Ω_s^{loc}. Compared to the case with all the uncertainties being considered in the same way, the size of the scenario tree can be reduced by distinguishing global scenarios from local scenarios. More specifically, there will be ${N_{\text{glob}}}^{N_{\text{sub}}}\prod_{s=1,\dots,N_{\text{sub}}} N_s^{\text{loc}}$ combinations of uncertainty scenarios when global scenarios and local scenarios are considered in the same way. Note that, N_{glob} represents the number of all the possible global scenarios, and N_s^{loc} is the number of all the possible local scenarios for subnetwork s. However, when global scenarios are considered to be the same for the overall network, there will be $N_{\text{glob}}\prod_{s=1,\dots,N_{\text{sub}}} N_s^{\text{loc}}$ combinations of uncertainty scenarios.

IV. SCENARIO-BASED DMPC WITH GLOBAL AND LOCAL UNCERTAINTIES

In this section, we propose a new scenario-based DMPC approach on the basis of global uncertainties and local uncertainties. First, we merge global uncertainties into the control problem. Then, we include local uncertainties into the control problem by proposing a reduced scenario tree. Moreover, we consider a min-max setting for handling uncertainties in scenario-based DMPC.

In the reminder of this paper, $x_{s,g}$, $y_{s,g}$, $J_{s,g}$, $f_{s,g}$, $F_{s,g}$, $J_{s,g}^{\text{inter}}$, $D_{s,g}^{\text{in}}$, $E_{s,g}^{\text{in}}$, $E_{j,s,g}^{\text{in}}$, $E_{s,j,g}^{\text{out}}$, $E_{j,s,g}^{\text{out}}$, Λ_g^{in}, $\lambda_{j,s,g}^{\text{in}}(k)$, and $\lambda_{j,s,g}^{\text{out}}$ describes quantities for the case with global uncertainties, and $x_{s,g,l}$, $y_{s,g,l}$, $J_{s,g,l}$, $f_{s,g,l}$, $F_{s,g,l}$, $D_{s,g,l}^{\text{in}}$, and $E_{j,s,g,l}^{\text{out}}$ describes quantities for the case with both global uncertainties and local uncertainties. Note that all the above variables have similar meanings to the corresponding variables without subscripts g and l in Section II. Additionally, $\tilde{x}_{s,g}(k)$, $\tilde{y}_{s,g}(k)$, $\tilde{x}_{s,g,l}(k)$, and $\tilde{y}_{s,g,l}(k)$ are defined similarly to $\tilde{x}_s(k)$ over the control steps $k+z$, $z=1,\dots,N_{\text{p}}$; $\tilde{E}_{j,s,g}^{\text{in}}(k)$, $\tilde{E}_{s,j,g}^{\text{out}}(k)$, $\tilde{E}_{j,s,g}^{\text{out}}(k)$, $\tilde{\Lambda}_g^{\text{in}}(k)$, $\tilde{\lambda}_{j,s,g}^{\text{in}}(k)$, $\tilde{\lambda}_{j,s,g}^{\text{out}}(k)$, and $\tilde{E}_{j,s,g,l}^{\text{out}}(k)$ are defined in a similar way to $\tilde{x}_s(k)$ over the control steps $k+z$, $z=0,\dots,N_{\text{p}}-1$.

A. Scenario-Based DMPC with Global Uncertainties

On the basis of a min-max setting, the centralized scenario-based MPC problem with global uncertainties can be described as follows:

$$\min_{\tilde{u}_1(k),\dots,\tilde{u}_{N_{\text{sub}}}(k)} \max_{g=1\dots N_{\text{glob}}} \sum_{s=1}^{N_{\text{sub}}} J_{s,g}\big(\tilde{x}_{s,g}(k),\tilde{y}_{s,g}(k),\tilde{u}_s(k)\big) \quad (14)$$

$$\text{s.t. } x_{s,g}(k+z+1) = f_{s,g}\big(x_{s,g}(k+z),u_s(k+z),D_{s,g}^{\text{in}}(k+z), E_{s,g}^{\text{in}}(k+z),\omega_g(k+z)\big) \quad \text{for } z=0,\dots,N_{\text{p}}-1 \quad (15)$$

$$y_{s,g}(k+z) = h_s\big(x_{s,g}(k+z)\big) \quad \text{for } z=1,\dots,N_{\text{p}} \quad (16)$$

$$x_{s,g}(k) = x_s^k \quad (17)$$

$$\omega_g(k+z) \in \Omega_{\text{glob}}(k+z) \quad \text{for } z=0,\dots,N_{\text{p}}-1 \quad (18)$$

$$F_{s,g}\big(\tilde{x}_{s,g}(k),\tilde{y}_{s,g}(k),\tilde{u}_s(k)\big) \le 0 \quad (19)$$

$$E_{j,s,g}^{\text{in}}(k+z) - E_{s,j,g}^{\text{out}}(k+z) = 0 \quad (20)$$

$$\text{for } j \in S_s^{\text{nb}},\ z=0,\dots,N_{\text{p}}-1$$

Equation (5)

for $s=1,\dots,N_{\text{sub}}$, and $g=1,\dots,N_{\text{glob}}$

where g is the index for global uncertainty scenarios, ω_g describes scenario g for global uncertainties, and $\max_{g=1\dots N_{\text{glob}}} \sum_{s=1}^{N_{\text{sub}}} J_{s,g}\big(\tilde{x}_{s,g}(k),\tilde{y}_{s,g}(k),\tilde{u}_s(k)\big)$ is the overall control objective function for the case with global uncertainties.

In order to deal with uncertainties in constraints on the states, outputs, and control inputs, we merged (19) into the local control objective functions by means of penalty terms. Based on the DMPC approach in Section II, we define local control problems of subnetworks for the case with global uncertainties as follows:

$$\min_{\substack{\tilde{u}_s(k)\\ \tilde{E}_{j_{s,1},s,g}^{\text{in}}(k),\dots,\tilde{E}_{j_{s,N_s^{\text{nb}}},s,g}^{\text{in}}(k)\\ \tilde{E}_{j_{s,1},s,g}^{\text{out}}(k),\dots,\tilde{E}_{j_{s,N_s^{\text{nb}}},s,g}^{\text{out}}(k)}} \max_{g=1\dots N_{\text{glob}}} \Big(J_{s,g}\big(\tilde{x}_{s,g}(k),\tilde{y}_{s,g}(k),\tilde{u}_s(k)\big) + \gamma \max\big(F_{s,g}(\tilde{x}_{s,g}(k),\tilde{y}_{s,g}(k),\tilde{u}_s(k)),0\big) + \sum_{j\in S_s^{\text{nb}}} J_{s,g}^{\text{inter}}\big(\tilde{\lambda}_{j,s,g}^{\text{in}}(k),\tilde{\lambda}_{j,s,g}^{\text{out}}(k), \tilde{E}_{j,s,g}^{\text{in}}(k),\tilde{E}_{j,s,g}^{\text{out}}(k)\big)\Big) \quad (21)$$

$$\text{s.t. } (5),(15)-(18)$$

where γ is a positive weight that makes the constraint penalty term for subnetwork s (i.e. the second term of (21)) dominate when (19) is violated.

B. Scenario-Based DMPC with Global Uncertainties and Local Uncertainties

Traffic variables of subnetwork s are affected by interconnecting inputs from neighboring subnetworks, which are affected by local scenarios of the neighboring subnetworks. For *a complete local scenario tree*, all the combinations of the local scenarios for all subnetworks are considered, and the number of these combinations is $N_{\text{com}} = \prod_{s=1,\dots,N_{\text{sub}}} N_s^{\text{loc}}$. Aiming at reducing the computational load w.r.t. that for the complete local scenario tree, we propose *a reduced local scenario tree* by assuming that the interconnecting inputs for a subnetwork from its neighbors are independent of the local scenarios of its neighbors, i.e. the interconnecting inputs from a neighboring subnetwork are combined for all the local scenarios of that neighboring subnetwork. Note that for a given subnetwork, the interconnecting inputs from a neighbor are equivalent to the interconnecting outputs for that neighbor to the given subnetwork.

The *reduced scenario tree* is defined as a scenario tree consisting of the combinations of all global scenarios and the reduced local scenario tree, and the *complete scenario tree* is defined as a scenario tree consisting of the combinations of all global scenarios and the complete local scenario tree. For the reduced scenario tree, the number of combinations of global and local scenarios is $N_{\text{glob}}N_s^{\text{loc}}$, and this is smaller than that for the complete scenario: $N_{\text{glob}}N_{\text{com}}$. Accordingly, using the reduced scenario tree can reduce the computational load w.r.t. using the complete scenario tree.

Based on the min-max setting, we combine the interconnecting outputs for a subnetwork as follows:

$$\tilde{E}^{\text{out}}_{j,s,g}(k) = \max_{l=l_{s,1},\ldots,l_{s,N_s^{\text{loc}}}} \left\| \tilde{E}^{\text{out}}_{j,s,g,l}(k) - \tilde{E}^{\text{in}}_{s,j,g}(k) \right\|_2^2 \tag{22}$$

where l is the index for local scenarios, and $\tilde{E}^{\text{out}}_{j,s,g}$ is the maximum distance between $\tilde{E}^{\text{out}}_{j,s,g,l}(k)$ and $\tilde{E}^{\text{in}}_{s,j,g}(k)$ for all local scenarios of subnetwork s.

On the basis of the reduced scenario tree, we formulate the scenario-based DMPC problem for the case with global and local uncertainties as follows:

$$\begin{aligned}
\min_{\substack{\tilde{u}_s(k)\\ \tilde{E}^{\text{in}}_{j_{s,1},s,g}(k),\ldots,\tilde{E}^{\text{in}}_{j_{s,N_s^{\text{nb}}},s,g}(k)\\ \tilde{E}^{\text{out}}_{j_{s,1},s,g}(k),\ldots,\tilde{E}^{\text{out}}_{j_{s,N_s^{\text{nb}}},s,g}(k)}} \max_{g=1,\ldots,N_{\text{glob}}} \Bigg(& \max_{l=l_{s,1},\ldots,l_{s,N_s^{\text{loc}}}} \Big(J_{s,g,l}\big(\tilde{x}_{s,g,l}(k), \tilde{y}_{s,g,l}(k), \tilde{u}_s(k)\big) \\
& + \gamma \max\big(F_{s,g,l}(\tilde{x}_{s,g,l}(k), \tilde{y}_{s,g,l}(k), \tilde{u}_s(k)), 0\big)\Big) \\
& + \sum_{j\in S_s^{\text{nb}}} J^{\text{inter}}_{s,g}\big(\tilde{\lambda}^{\text{in}}_{j,s,g}(k), \tilde{\lambda}^{\text{out}}_{j,s,g}(k), \tilde{E}^{\text{in}}_{j,s,g}(k), \tilde{E}^{\text{out}}_{j,s,g}(k)\big)\Bigg)
\end{aligned} \tag{23}$$

$$\begin{aligned}
\text{s.t. } & x_{s,g,l}(k+z+1) = f_{s,g,l}\big(x_{s,g,l}(k+z), u_s(k+z), \\
& \quad D^{\text{in}}_{s,g,l}(k+z), E^{\text{in}}_{s,g}(k+z), \omega_g(k+z), \omega_{s,l}(k+z)\big) && (24)\\
& \quad \text{for } z = 0,\ldots,N_{\text{p}}-1 \\
& y_{s,g,l}(k+z) = h_s\big(x_{s,g,l}(k+z)\big) \quad \text{for } z = 1,\ldots,N_{\text{p}} && (25)\\
& x_{s,g,l}(k) = x_s^k && (26)\\
& \omega_{s,l}(k+z) \in \Omega_s^{\text{loc}}(k+z) \quad \text{for } z = 0,\ldots,N_{\text{p}}-1 && (27)\\
& \text{Equations (5) and (18)} \\
& \text{for } g = 1,\ldots,N_{\text{glob}} \text{ and } l = 1,\ldots,N_s^{\text{loc}}
\end{aligned}$$

where $\omega_{s,l}$ is local uncertainty scenario l for subnetwork s.

V. Case Study

In this section, we compare the scenario-based DMPC approach with nominal DMPC by a case study. A macroscopic traffic flow model METANET [17, 18] is used as both the process model and the prediction model. Since the METANET model is nonlinear and nonconvex, the optimization problem for DMPC is nonlinear and nonconvex. In the case study, the parameters for METANET are taken from [18]. The performance criterion considered in the case study is the Total Time Spent (TTS), which represents the time that all vehicles spent in the considered network. The Alternating Direction Method of Multipliers (ADMM) algorithm stated in Chapter 7.2 of [9] is chosen as the algorithm for solving the DMPC problem.

A. Network

For the case study, we consider a freeway network as shown in Fig. 1. More specifically, this freeway network consists of 10 double-lane links, 1 origin (O_0), 1 destination (D_0), 2 single-lane on-ramps (O_1 and O_2), and 2 single-lane off-ramps (O_3 and O_4). The links are divided into 18 segments with equal length (L_m=1 km). The destination and the off-ramps are unrestricted, while the queue lengths at on-ramps are restricted within 100 veh for avoiding spillback. The turning fractions for off-ramps are considered to be constant: 5% of the mainstream flow. The network is decomposed into 3 subnetworks, which are controlled by variable speed limits and ramp metering.

The weights γ is chosen as 100. The simulation time step is $T = 10\,\text{s}$, and the control time step is $T_{\text{c}} = 180\,\text{s}$. We choose the prediction horizon to be $N_{\text{p}} = 3$, corresponding the average time needed for a vehicle to go through the network. We set the control horizon to be smaller than N_{p} for reducing the computational load: $N_{\text{c}} = 2$. The simulation period is 2.5 h, and the nominal demands are shown in Figure 2.

B. Uncertainty Scenarios

1) Uncertainty Scenarios for the Simulations: As an illustration, global uncertainties are considered to be uncertainties in weather condition: sunny (the probability is 0.8) or rainy (the probability is 0.2). In particular, for sunny days the model parameters τ and the free-flow speed for METANET are considered to be nominal values. For rainy days τ is 5% smaller than the nominal value, and the free-flow speed is 5% larger than the nominal value.

Uncertainties in demands for the origins and on-ramps are considered as local uncertainties. For constructing simulation scenarios, three base scenarios are considered, i.e. base scenario 1: nominal demands with a probability of 0.7, base scenario 2: 90% of nominal demands with a probability of 0.1, and base scenario 3: 110% of nominal demands with a probability of 0.2.

In total, 10 demand scenarios are considered for simulations, with the sampling interval equal to the control sampling interval. Each simulation scenario is constructed by randomly setting the demand for each origin to be one of the base scenarios with the corresponding probabilities (0.7, 0.1, or 0.2) at every sampling step.

2) Uncertainty scenarios for scenario-based DMPC: For applying scenario-based DMPC, global scenarios are considered to be sunny days and rainy days, and local demand scenarios are developed based on the above base scenarios. For a given control step, there are $(3 \text{ base scenarios})^{N_{\text{p}}} = 27$ demand scenarios over the prediction period for each origin. With the demand scenarios with probabilities smaller that 0.02 being ignored, there are 10 demand scenarios over the

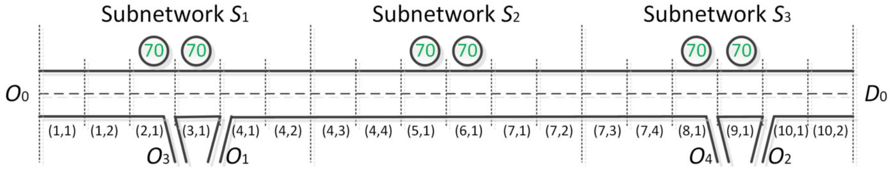

Fig. 1: The freeway network used for the case study

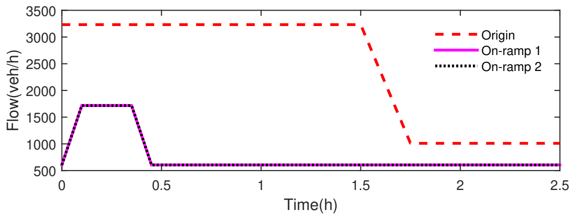

Fig. 2: Nominal demands for the mainstream origin and on-ramps

prediction period left for each origin, and the total probability of the 10 demand scenarios is 0.868. Therefore the number of scenarios for scenario-based DMPC is obviously reduced, while the total probability is large.

C. Control Approaches

Two control approaches are considered for comparison, i.e. *nominal DMPC* based on nominal parameters and nominal demands, and *scenario-based DMPC* based on the reduced scenario tree and the min-max setting.

We implement nominal DMPC and scenario-based DMPC in a serial scheme, see e.g. [12]. Within one negotiation iteration, one after another each local controller solves a local optimization problem, using the latest information of neighbors. The METANET model is coded in C, and the optimization problems are solved by a multi-start sequential quadratic programming, i.e. “fmincon” (“active-set” algorithm [19]) in MATLAB. For a given control step, the number of negotiation iterations is fixed as 10, and the number of starting points for “fmincon” is fixed as 20.

D. Results and Analysis

For different random starting points of “fmincon”, the simulation with control is repeated 10 times for a given simulation scenario, and the average of the 10 repeated simulations with control is seen as the results for the given simulation scenario.

The averages and the standard deviations of the results for all simulation scenarios are listed in Table I. The symbol $J_{\mathrm{TTS}}^{\mathrm{imp}}$ describes the relative improvement in TTS over the entire simulation period (denoted as $\mathrm{TTS}_{\mathrm{total}}$) w.r.t. the no-control case. The symbol J_{pen} describes the maximum relative queue constraint violation over the entire simulation period, which is defined as $\max\limits_{o\in\{O_1,O_2\}} \max\left(\max\limits_{k=1,\dots,k_{\mathrm{end}}} (w_o(k)/w_o^{\max}-1),0\right)$, with k_{end} the last simulation time step, w_o the queue length for on-ramp o, and $w_o^{\max}$ the maximum permitted queue length for on-ramp o. The total performance over the entire simulation period is defined as $J_{\mathrm{tot}} = \frac{\mathrm{TTS}_{\mathrm{total}}}{\mathrm{TTS}_{\mathrm{nom}}} + \gamma J_{\mathrm{pen}}$, with $\mathrm{TTS}_{\mathrm{nom}}$ a predefined nominal TTS value. The symbol $J_{\mathrm{tot}}^{\mathrm{imp}}$ describes the relative improvement of J_{tot} w.r.t. the no-control case.

TABLE I: Simulation results of nominal DMPC and scenario-based DMPC

Approaches		Nominal DMPC	Scenario-based DMPC
Average	$J_{\mathrm{TTS}}^{\mathrm{imp}}$	4.3%	3.3%
	J_{pen}	10.9%	0%
	$J_{\mathrm{tot}}^{\mathrm{imp}}$	-36.7%	3.3%
Standard deviation	$J_{\mathrm{TTS}}^{\mathrm{imp}}$	0.8%	0.5%
	J_{pen}	4.6%	0%
	$J_{\mathrm{tot}}^{\mathrm{imp}}$	17.4%	0.5%

Based on Table I, it is shown that the nominal DMPC approach leads to a worse total performance w.r.t. the no-control case. However, the scenario-based DMPC approach improves the total performance by 3.3% compared to the no-control case. In particular, the TTS is improved by both the nominal DMPC approach and the scenario-based DMPC approach. The queue length constrains are violated for the nominal DMPC approach, but are satisfied for the scenario-based DMPC approach. For the nominal DMPC approach, the queue length constraint violations result in a worse total performance in comparison with the no-control case. Note, however, that there is a sacrifice in the improvement for the TTS for scenario-based DMPC w.r.t. nominal DMPC.

For scenario-based DMPC, the standard deviations of $J_{\mathrm{TTS}}^{\mathrm{imp}}$, J_{pen}, and $J_{\mathrm{tot}}^{\mathrm{imp}}$ are small, showing that scenario-based DMPC can lead to a stable total performance. For nominal DMPC, the standard deviations of $J_{\mathrm{tot}}^{\mathrm{imp}}$ is large, showing that the total performance for nominal DMPC is less stable than that for scenario-based DMPC.

VI. Conclusions

A scenario-based DMPC approach has been developed in this paper, and in this new approach global uncertainties for the overall network are distinguished from local uncertainties for subnetworks. For a complete scenario tree, all the combinations of the local scenarios for all subnetworks are considered. For reducing the computational load w.r.t. the complete scenario tree, we have proposed a reduced scenario tree, where the dynamics of a subnetwork are considered to be independent of local uncertainty scenarios for other subnetworks. The scenario-based DMPC approach is based on the reduced scenario tree and a min-max setting. Particularly, a local controller optimizes the worst case of the sum of the control performance index, the constraint violation penalty, and the interconnecting term of the corresponding subnetwork for all the considered uncertainty scenarios. We have showed by a case study for freeway networks that scenario-based DMPC can improve the total performance compared to the no-control case, but nominal DMPC cannot improve the total performance due to queue length constraint violations.

For future research, multiple layouts and traffic scenarios, and uncertainties with different skewness and distributions can be considered for further investigating the effectiveness of the scenario-based DMPC approach. In addition, other schemes can be considered for handling uncertainties in the scenario-based control problem, such as an expected value scheme based on a probabilistic setting.

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