Delft University of Technology

Delft Center for Systems and Control

Technical report 16-022

Model predictive control for rail condition-based maintenance: A multilevel approach*

Z. Su, A. Núñez, S. Baldi, and B. De Schutter

If you want to cite this report, please use the following reference instead:
Z. Su, A. Núñez, S. Baldi, and B. De Schutter, "Model predictive control for rail condition-based maintenance: A multilevel approach," *Proceedings of the 2016 IEEE 19th International Conference on Intelligent Transportation Systems*, Rio de Janeiro, Brazil, pp. 354–359, Nov. 2016.

Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: https://www.dcsc.tudelft.nl

Model Predictive Control for Rail Condition-Based Maintenance: A Multilevel Approach

Zhou Su[1]   Alfredo Núñez[2]   Simone Baldi[1]   Bart De Schutter[1]

Abstract— **This paper develops a multilevel decision making approach based on model predictive control (MPC) for condition-based maintenance of rail. We address a typical railway surface defect called "squat", in which three maintenance actions can be considered: no maintenance, grinding, and replacement. A scenario-based scheme is applied to address the uncertainty in the deterioration dynamics of the key performance indicator for each track section, and a piecewise-affine model is used to approximate the expected dynamics, which is to be optimized by a scenario-based MPC controller at the high level. A static optimization problem involving clustering and mixed integer linear programming is solved at the low level to produce an efficient grinding and replacing schedule. A case study using real measurements obtained from a Dutch railway line between Eindhoven and Weert is performed to demonstrate the merits of the proposed approach.**

I. INTRODUCTION

Maintenance is crucial for the proper functioning of a railway network, which consists of several assets like tracks, switches, power lines, stations, signaling systems, etc. Condition-based maintenance, where decision making is based on the observed "condition" of an asset, has received growing attention in literature [1], [2], [3]. Compared with cyclic preventive maintenance prevailing in current practice, condition-based maintenance is more efficient, as it is able to suggest timely and crucial actions by predicting the evolution of the deterioration process [4]. To apply condition-based maintenance, performance indicators have been developed to indicate the condition of an asset. A new indicator using the concept of dynamic infrastructure occupation is developed in [5] to asses the capacity of four different railway signaling systems of the Dutch Utrecht-Den Bosch corridor. In [6], a general hierarchical multi-criteria framework is proposed, that is used to benchmark railway infrastructure maintenance [7]. A fuzzy global indicator combining different performance indicators is developed for a track section to facilitate decision making in rail grinding and replacement [8].

In this paper, we focus on track maintenance, which takes more than 40% of the yearly maintenance budget for the whole Dutch railway network [9]. In particular, we consider the maintenance of a typical track defect called "squat", which is a type of rolling contact fatigue [10]. Squats first appear on the rail surface, and evolve into a network of cracks beneath the surface over time. When not treated in time, severe squats can lead to rail breakage. Early-stage squats can be efficiently treated by grinding, which removes the irregularities in the rail geometry [11], [12], while the only remedy for late-stage squats is replacement, which can be very costly.

Model Predictive Control (MPC), an advanced design methodology for control systems, has been applied in [13] to optimal planning of maintenance operations, considering generic track defects. The advantage of MPC in maintenance planning is that the resulting strategy is flexible, as the degradation model is regularly updated by new measurements, enabling the maintenance plan to be adapted dynamically. However, only nominal degradation dynamics is considered in [13], while in practice, various uncertainties exist in the deterioration process. In particular, squats can grow at different rates, according to experimental results [14].

Similar to railway timetables, maintenance plans should also be robust to deal with real time perturbations [15]. Robust control, which maintains the control performance within a specific range and guarantees constraint satisfactions in the presence of uncertainties, is introduced to MPC to handle uncertainties. Robust control is computationally demanding for nonlinear systems. Many approaches, like the min-max approach [16], [17], have been developed. A scenario-based MPC approach optimizing the expectation of the deterioration dynamics and the maintenance cost is developed in this paper for the optimal planning of rail condition-based maintenance. This is inspired by the tractable robust approaches developed recently for traffic networks, like the scenario-based min-max scheme for multi-class freeway networks [18].

The major contribution of this paper includes a novel multi-level scheme for optimal condition-based maintenance planning. The proposed multilevel scheme is able to determine optimal maintenance actions for a long track over a long prediction horizon, and provides an efficient work plan based on individual defects for the recommended maintenance actions at each time step. Uncertainties are addressed by a scenario-based MPC controller at the high level. Nonlinear deterioration dynamics is considered, which is approximated by Piecewise-affine (PWA) functions.

This paper is organized as follows: first, the model of the deterioration process is described in Section II. Based on

[1]Z. Su, S. Baldi and B. De Schutter are with the Delft Center for Systems and Control, Delft University of Technology, Delft, the Netherlands {Z.Su-1, S.Baldi, B.DeSchutter}@tudelft.nl

[2]A. Núñez are with the Section of Railway Engineering, Delft University of Technology, Delft, the Netherlands {A.A.NunezVicencio}@tudelft.nl

Research sponsored by the NWO/ProRail project "Multi-party risk management and key performance indicator design at the whole system level (PYRAMIDS)", project 438-12-300.

the dynamic model, in Section III we develop a multilevel approach for optimal condition-based maintenance planning, including an MPC controller at the high level and a scheduling problem at the low level. The proposed approach is then applied to a case study including a Dutch railway line in Section IV; conclusions and directions of future work are provided in Section V.

II. Model Description

Consider railway track consisting of n sections, excluding all bridges, insulated joints, switches and crossings[1]. Let $u_j(k) \in \{1, 2, 3\}$ denote the action applied to section j at time step k, with 1, 2, 3 representing "no maintenance", "grinding", and "replacing", respectively. Let $x_j(k) \in [m, M]$ denote the rail quality of the j-th section at time step k. In the treating of squats, the rail quality is a continuous scalar determined by the number of the squats in a track section, as well as the severity of each squat. The rail quality can be viewed as a global indicator of the deterioration level of a track section, and the deterioration dynamics of a track section can be expressed in the following generic model:

$$
\begin{aligned}
x_j(k+1) &= f_j(x_j(k), u_j(k), \theta(k)) \\
&= \begin{cases} f_{\mathrm{Deg},j}(x_j(k), \theta(k)) & \text{if } u_j(k)=1 \text{ (no maintenance)} \\ f_{\mathrm{Maint},j}(x_j(k), \theta(k)) & \text{if } u_j(k)=2 \text{ (grinding)} \\ \varepsilon_{\mathrm{R}}(\theta(k)) & \text{if } u_j(k)=3 \text{ (replacing)} \end{cases} \qquad (1) \\
&\forall j \in \{1,\dots,n\}
\end{aligned}
$$

where $\theta(k) \in \Theta$ represents the realization of the uncertainties related to the deterioration dynamics $f_{\mathrm{Deg},j}$ and the effect of grinding $f_{\mathrm{Maint},j}$ at time step k. The random variable ε_{R} describes the rail quality after a replacement, and we have

$$
\varepsilon_{\mathrm{R}}(\theta) \in [m, m+\Delta] \quad \forall \theta \in \Theta
$$

where Δ is usually a very small positive number.

In general, the set of all the possible realizations of the uncertainties Θ is huge. Due to the concern of tractability, we select a finite set of representative scenarios from Θ. Define $\mathscr{H}$ as the set of representative scenarios of any track section. The representative scenarios can be fast/average/slow growth of squat length, and heavy/medium/light load, etc. We assume that the scenario set $\mathscr{H}$ is the same for every section, but the probability of occurrence of each scenario is in general different for different sections. Instead of addressing all possible uncertainties as in (1), we consider the following scenario-based model for any $h \in \mathscr{H}$:

$$
\begin{aligned}
x_j(k+1) &= f_j^h(x_j(k), u_j(k)) \\
&= \begin{cases} f^h_{\mathrm{Deg},j}(x_j(k)) & \text{if } u_j(k)=1 \\ f^h_{\mathrm{Maint},j}(x_j(k)) & \text{if } u_j(k)=2 \\ \varepsilon^h_{\mathrm{R}} & \text{if } u_j(k)=3 \end{cases} \qquad (2) \\
&\forall j \in \{1,\dots,n\}
\end{aligned}
$$

[1]Bridges, insulated joints, switches and crossings have very different deterioration dynamics, and are maintained in a different way.

Denote $\mathrm{E}_{\mathscr{H},j}$ as the expectation taken over the scenario set $\mathscr{H}$ for section j, then the expected degradation dynamics of the j-th section can be approximated by the following PWA function:

$$
\bar{f}_{\mathrm{Deg}}(x_j) = \mathrm{E}_{\mathscr{H},j}[f^h_{\mathrm{Deg}}(x_j)] = a_{jp}x_j + b_{jp} \quad \text{if } x_j(k) \in \mathscr{X}_p \qquad (3)
$$

where $\{\mathscr{X}_p\}_{p=1}^P$ is a partition of the state space $\mathscr{X} = [m, M]$. In the treatment of squats, three stages are usually considered: early, middle, and late. The corresponding three intervals are denoted by:

$$
\mathscr{X}_1 = [m, x^{\mathrm{eff}}], \quad \mathscr{X}_2 = (x^{\mathrm{eff}}, x^{\mathrm{sev}}], \quad \mathscr{X}_3 = (x^{\mathrm{sev}}, M]
$$

where the parameters x^{eff} and x^{sev} are the threshold values for early and late stage deterioration. Moreover, since grinding is only effective for early-stage deterioration, x^{eff} is also the threshold value for effective grinding. Then the expected rail quality after grinding can be represented by the following PWA functions:

$$
\begin{aligned}
\bar{f}_{\mathrm{Maint},j}(x_j) &= \mathrm{E}_{\mathscr{H},j}[f^h_{\mathrm{Maint},j}(x_j)] \\
&= \begin{cases} \mathrm{E}_{\mathscr{H},j}[\varepsilon^h_{\mathrm{G}}] & \text{if } x_j \le x^{\mathrm{eff}} \\ \mathrm{E}_{\mathscr{H},j}[z^h_j(x_j - x^{\mathrm{eff}}) + \varepsilon_{\mathrm{G}}] & \text{if } x_j > x^{\mathrm{eff}} \end{cases} \qquad (4)
\end{aligned}
$$

where $\varepsilon_{\mathrm{G}} \in [m, m+\Delta]$ is also a very small positive random variable, and the parameter z^h_j represents the inefficiency of grinding when the degradation level exceeds the threshold value for scenario h. From (4) we can see that for early-stage deterioration, grinding is almost as efficient as replacement. Constraints must be considered for each individual section. Typical constraints include upper/lower bound on the degradation model, operational constraints, budget limit, etc. Here we assume that all the constraints are linear in the state x and input u, and can be expressed in the following general form:

$$
R_j x_j(k) + K_j u_j(k) + l_j \le 0 \quad \forall j \in \{1,\dots,n\} \qquad (5)
$$

which must be satisfied at every time step.

To avoid disturbance of normal rail traffic, each maintenance action must be performed within the pre-allocated time slots, so it might not be possible for the maintenance agent to perform the suggested maintenance actions for all the n sections within one time step. Denote n_{G} and n_{R} as the maximum number of sections that can be grinded and replaced at each time step, respectively. Then we have the following global constraints on the maintenance actions:

$$
\sum_{j=1}^{n} I_{u_j(k)=2} \le n_{\mathrm{G}} \qquad (6)
$$

$$
\sum_{j=1}^{n} I_{u_j(k)=3} \le n_{\mathrm{R}} \qquad (7)
$$

where the binary-valued indicator function I_X equals to one when the statement X is true, and zero otherwise. Constraint (6) and (7) are the workload bound for grinding and replacing, respectively.

As only one type of maintenance action (grinding or replacing) can be performed within one time slot, two situations must be considered:

(1) only one maintenance slot is available within one time step;
(2) more than one maintenance slot is available within one time step.

For the first situation, grinding and replacing exclude each other, i.e. there can be no replacing in any section if grinding is suggested for at least one section, and vice versa. This can be expressed by the following constraint:

$$\max_{j\in\{1,\ldots,n\}} I_{u_j(k)=2} + \max_{j\in\{1,\ldots,n\}} I_{u_j(k)=3} \leq 1 \tag{8}$$

Constraint (8) is not needed for the second situation, as the maintenance agent can schedule grinding and replacing actions to different time slots in this case.

The objective of the maintenance agent at the high level is to optimize the expected rail quality for the entire track over a planning horizon, while minimizing the accumulated maintenance cost. This can be expressed by the following objective function:

$$J_{\text{end}} = \sum_{j=1}^{n} \sum_{k=0}^{k_{\text{end}}-1} \mathrm{E}_{\mathscr{H},j}[x_j(k+1)] + \lambda(c_{\text{G}} I_{u_j(k)=2} + c_{\text{R}} I_{u_j(k)=3}) \tag{9}$$

where k_{end} is the last time step in the planning horizon, and c_{G} and c_{R} are the cost of one grinding and replacing cost of one section, respectively. The parameter λ captures the trade-off between rail quality and maintenance cost.

III. Multilevel Approach

A multi-level decision making scheme is applied to the optimal planning of maintenance and replacement actions for a track, using the treatment of squats as an example. The high-level decision making problem is solved to produce the optimal long-term maintenance plan for the entire track. Based on the deterioration model of rail quality for each section, an MPC controller is developed to determine the optimal maintenance actions that optimize the aggregated degradation and maintenance cost of all the sections over a given prediction horizon. The low-level problem is solved to produce an efficient schedule to execute the grinding and replacing actions suggested by the high-level controller. For this a two-step procedure consisting of clustering and mixed integer linear programming (MILP) is proposed, trying to cover as many severe squats as possible within the pre-allocated time slots. The whole procedure of the proposed multilevel approach is illustrated by pseudo-code in Algorithm 1.

The major motivation to adopt a multilevel scheme is due to computational tractability. The maintenance agent wants to design a long-term plan for a long track (25 km in the case study) which specifies the treatment of each squat. However, the number of squats in a fairly short stretch (about 10 km long) is already huge, and so to obtain the optimal maintenance plan for each independent squat, considering individual degradation dynamics is in general not tractable. Partitioning the track into several sections and applying one maintenance action to one section significantly reduces the number of discrete decision variables. This high-level decision-making problem is solved to obtain the optimal long-term section-wise maintenance plan for the entire track. The resulting rough maintenance plan can then be refined by solving the low-level problem, which considers the location and severity of each squat that need to be treated according to the high-level controller.

Another motivation for applying a multilevel approach are the different time scales in planning. Long-term planning of maintenance actions for the entire track considers a very slow process with a large sampling time (usually larger than one month), while non-severe squats demonstrate no discernible growth within the time slot allocated for one maintenance action (usually no more than 8 hours). Such clear separation of slow dynamics and static setting naturally leads to a hierarchical decision-making scheme.

Data: Initial squat lengths L_0
Result: Cluster positions ξ , work plan δ
Function HighLevel_MPC (L_0)
    $k = 1$;
    $L(k) = L_0$;
    $x(k)$=**Aggregate**($L(k)$);
    while $k \leq k_{end}$ **do**
        $u(k)$=**MPC_Optimize**($x(k)$);
        if *any* $u_j(k)$ = *replacing* **then**
            (ξ, δ)=**LowLevel_Replace**($L(k)$);
            Replace(ξ, δ);
        else if *any* $u_j(k)$ = *grinding* **then**
            (ξ, δ)=**LowLevel_Grind**($L(k)$);
            Grind(ξ, δ);
        else
            do nothing;
        if *New measurements available* **then**
            $L(k+1)$=New measurements;
        else
            $L(k+1)$=**Simulate**($L(k)$, ξ, δ);
        $x(k+1)$=**Aggregate**($L(k+1)$);
        $k = k+1$;
    end

Algorithm 1: Procedure of the multilevel approach.

A. High-level MPC

An MPC controller is implemented at the higher level to optimize the expected value of the trade-off between the aggregated degradation and total maintenance cost for the entire track over a given prediction horizon, where the expectation is taken over the set of all representative scenarios. First we rewrite the dynamic model introduced in Section II as a standard PWA state space model. Then we convert the n local PWA models together with the global constraints into a global Mixed Logic Dynamic (MLD) system for the entire track. An MPC controller is then designed for the resulting

MLD system.

Recall that $\mathscr{X} = [m, M]$ represents the state space of each section, and let $\mathscr{U} = \{1, 2, 3\}$ denote the action space available for each section. Define $\Omega = \mathscr{X} \times \mathscr{U}$, which is partitioned into a finite number of convex[2] polyhedra Ω_i, $i \in \mathscr{I} = \{1, \ldots, I\}$. Then we can rewrite the system described in (2),(5) as a standard PWA system[3]

$$x_j(k+1) = A_{ji} x_j(k) + f_{ji} \tag{10}$$
$$R_{ji} x_j(k) + g_{ji} \leq 0 \tag{11}$$
$$\text{if } [x_j^{\mathrm{T}}(k)\, u_j^{\mathrm{T}}(k)]^{\mathrm{T}} \in \Omega_i,\, i \in \mathscr{I}$$

The values of the parameter matrices A_{ji}, R_{ji} and vector f_{ji} and g_{ji} can be derived from (2),(5).

Let the vector $x(k) = [x_1^{\mathrm{T}}(k) \ldots x_n^{\mathrm{T}}(k)]^{\mathrm{T}}$ denote the state for the whole system ($\delta(k)$ and $z(k)$ can be defined in the same manner). Following the procedure described in [19], the whole system consisted of the n local PWA model (10),(11) and the global constraints (6)-(8) can be converted to the following standard MLD system:

$$x(k+1) = Ax(k) + B_1\delta(k) + B_2 z(k) + f \tag{12}$$
$$E_1 x(k) + E_2 u(k) + E_3 \delta(k) + E_4 z(k) \leq g \tag{13}$$

Let $\hat{x}(k+l|k)$ denote the estimated state at time step $k+l$ with the information available at time step k, and N_{P} and N_{C} the prediction and control horizons[4], respectively. Define:

$$\tilde{x}(k) = [\hat{x}^{\mathrm{T}}(k+1|k) \ldots \hat{x}^{\mathrm{T}}(k+N_{\mathrm{P}}|k)]^{\mathrm{T}}$$
$$\tilde{\delta}(k) = [\delta^{\mathrm{T}}(k) \ldots \delta^{\mathrm{T}}(k+N_{\mathrm{P}}-1)]^{\mathrm{T}}$$

We can define $\tilde{z}(k)$ in the same manner as $\tilde{\delta}(k)$. Grouping the binary and auxiliary variables together and defining $\tilde{V}(k) = [\tilde{\delta}^{\mathrm{T}}(k)\, \tilde{z}^{\mathrm{T}}(k)]^{\mathrm{T}}$, we can write the whole-system dynamics in the following compact form after successive substitution of (12):

$$\tilde{x}(k) = M_1 \tilde{V}(k) + M_2 x(k) \tag{14}$$

The objective of the high-level problem is to minimize the expectation of the deterioration levels of all sections at the lowest possible total maintenance cost. This can be captured by the following objective function:

$$\begin{aligned} J(k) &= J_{\mathrm{Deg}}(k) + \lambda J_{\mathrm{Maint}}(k) \\ &= \|P\tilde{x}(k)\|_1 + \lambda \left\|Q\tilde{V}(k)\right\|_1 \end{aligned} \tag{15}$$

where P and Q are positive definite and positive semidefinite weighting matrices, respectively, and the notation $\|\cdot\|_1$ denote 1-norm. The terms $J_{\mathrm{Deg}}(k)$ and $J_{\mathrm{Maint}}(k)$ represent the predicted degradation and maintenance cost within the prediction window with length N_{P} at time step k, respectively. The parameter λ captures the trade-off between rail quality and maintenance cost.

[2]Although $\mathscr{U}$ is discrete, the nonconvex space Ω can still be partitioned into convex polyhedra in the form of $\mathscr{X}_p \times \{q\}$, with $p, q \in \{1, 2, 3\}$.

[3]There is no terms including the discrete action u in Equation (10), as the partition Ω_i already uniquely determines the actions.

[4]We have $u_j(k+l) = 1, \forall l \in \{N_{\mathrm{C}}, \ldots, N_{\mathrm{P}}-1\}$ if $N_{\mathrm{C}} < N_{\mathrm{P}}$

Substituting (14) into (15) we obtain the following MILP with decision variable $\tilde{V}(k)$:

$$\min_{\tilde{V}(k)} S_1 \tilde{V}(k) + S_2 x(k) \tag{16}$$
$$\text{such that: } F_1 \tilde{V}(k) - F_2 x(k) - F_3 \leq 0 \tag{17}$$

where the parameter matrices S_1, S_2, F_1, F_2, F_3 can be derived from (13)(14). This MILP must be solved at each time step k, where the value of $x(k)$ can be obtained by new measurements or simulation.

B. Low-level Optimizer

The results of the high-level MPC controller only gives a rough indication. In practice, the maintenance agent has only a limited time (usually 8 hours at night) to perform all the maintenance actions for the entire track considered. Priorities must be given to the most severe squats, so that the worst defects are properly treated within the limited time. Moreover, the decision made by the high-level controller is based on a global indicator aggregated from measurements of independent squats in the whole section. A section suggested to be grinded/replaced by the high-level controller might only have several severe squats located closed to each other. In this case, it is not efficient to grind/replace the entire section. Furthermore, it is important to note that:

- The low-level problem does not need to be solved at each time step, and is only activated when a grinding or replacing action is suggested for at least one section by the high-level controller.
- Only squats in sections that received “grinding” or “replacing” actions from the high-level controller are considered, and all the squats in the sections that received “no maintenance” actions are excluded from the lower-level problem.

The following two-step procedure is proposed for the low-level problem:

(1) Depending on the location and severity, the squats that need to be grinded at time step k are clustered into clusters with different aggregated severity. The K-means algorithm [20] is applied to minimize the within-cluster sum of squares distance of each point to the center.

(2) A MILP is solved to produce an efficient work plan to execute the maintenance action in order to cover as many severe clusters as possible, while restricting to a working time limit.

Note that although the low-level problems for grinding and replacing share the similar two-step procedure, they are different problems with different parameters.

First we explain the clustering of squats. Let N_k be the total number of squats that needed to be treated at time step k, and N_{cl} the number of clusters used to cover all the N_k squats. Let w_i denote the aggregated severity of the i-th cluster, calculated from the number of squats in the cluster and the length of each squat. Let $\underline{\xi}_i$ and $\overline{\xi}_i$ denote the beginning and ending position of cluster i, respectively. The allowed minimum and maximum length of one cluster is denoted by $\Delta\xi_{\min}$ and $\Delta\xi_{\max}$, respectively. The minimum length $\Delta\xi_{\min}$ is usually given by the minimum grinding/replacing length, while the maximum length $\Delta\xi_{\max}$ is determined by the maximum length that can be grinded/replaced within one time slot. The clustering should consider an algorithm that guarantees

$$\Delta\xi_{\min} \leq \overline{\xi}_i - \underline{\xi}_i \leq \Delta\xi_{\max} \quad i \in \{1,\ldots,N_{\text{cl}}\}$$

and there is no overlapping between clusters, namely:

$$\underline{\xi}(i+1) > \overline{\xi}(i) \quad \forall i \in \{1,\ldots,N_{\text{cl}}-1\}$$

After the clustering of squats, a MILP will be solved to produce a work plan for the resulting clusters. Let v_{G}^{on} and $v_{\text{G}}^{\text{off}}$ denote the speed of the grinding machine while grinding and not grinding. Similarly, Let v_{R}^{on} denote how many kilometers can be replaced within one hour, and $v_{\text{R}}^{\text{off}}$ the speed of transporting the machinery and personnel. Let t_{G}^{S} denote the time needed for the grinding machine to switch between grinding and non-grinding mode, and t_{R}^{S} the time needed to prepare/finish one replacement. Let $t_{\text{G}}^{\max}$ and $t_{\text{R}}^{\min}$ denote the length of the time slot for grinding and replacing, respectively. We introduce the binary variable δ_i to indicate whether cluster i is grinded ($\delta_i = 1$) or bypassed ($\delta_i = 0$). Then we have the following MILP for optimal grinding:

$$\max_{\{\delta_i\}_{i=1}^{N_{\text{cl}}}} \sum_{i=1}^{N_{\text{cl}}} w_i \delta_i \tag{18}$$

$$\text{such that: } \sum_{i=1}^{N_{\text{cl}}} \delta_i \left(\frac{\overline{\xi}_i - \underline{\xi}_i}{v_{\text{G}}^{\text{on}}} + 2t_{\text{G}}^{\text{S}} \right) + (1-\delta_i) \frac{\overline{\xi}_i - \underline{\xi}_i}{v_{\text{G}}^{\text{off}}} + \sum_{i=1}^{N_{\text{cl}}-1} \frac{\underline{\xi}_{i+1} - \overline{\xi}_i}{v_{\text{G}}^{\text{off}}} \leq t_{\text{G}}^{\max} \tag{19}$$

The optimal planing for replacing can be formulated in a similar MILP as (18),(19).

IV. Case Study

A. Settings

For the case study, we consider the treatment of squats for an open track between the line Eindhoven-Weert in the Dutch railway network. This 25-km-long track is divided into five equidistant sections. The rail quality x_j (ranged between 0 and 65 mm) is defined as the average length of all the squats located in section j, and the initial track condition $x(0) = [39.22\ 38.92\ 39.49\ 38.63\ 35.84]^{\text{T}}$ mm. We consider three representative scenarios: fast, average, and slow squat growth. The threshold value for early and late stage squats, x^{eff} and x^{sev}, are 30 mm and 50 mm, respectively. The dynamic model (2) for each section and scenario is fitted with real measurements from the track considered. The sampling time is one month, and every month there is one separate time slot allocated to grinding and replacing, respectively. The maintenance agent can grind at most three sections and replace at most one section each time step. The total planning time is 36 months, and we consider the situation where $N_{\text{P}} = N_{\text{C}}$. The trade-off between performance and cost (parameter λ) is 2, and we set $c_{\text{R}} = 65$ and $c_{\text{G}} = 65/30$ in the objective function (9).

For illustration purpose we only trigger the low-level problem associated with grinding. The monthly time slot for grinding is 4 hours. The speed of the grinding machine is 1.5 km/h and 80 km/h at grinding and non-grinding mode, respectively. Five clusters are used, where the minimum and maximum length for one cluster is 100 m and 5 km, respectively. The weight for each cluster is the average length of all the squats within the cluster.

The simulation is performed in Matlab R2015b, and the Gurobi Optimizer 5.6.3 is used as the MILP solver. The clustering algorithm is implemented using the Matlab build-in function *kmeans*.

B. Discussions of Results

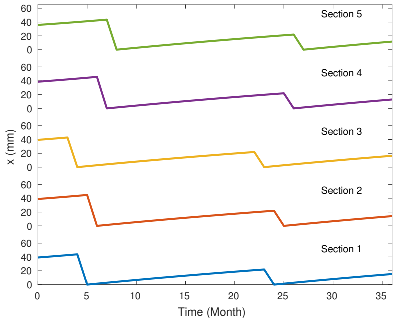
The high-level MPC controller is simulated with three different prediction horizons. The performance (value of the closed-loop objective function) and computational effort (maximum CPU time to solve the optimization problem at each time step for the whole simulation time) of each resulting maintenance plan is shown in Table I. A conclusion can be made that longer prediction horizon results in a more optimal maintenance plan, at the cost of higher computational effort. Note that even the maximum CPU time per step for the longest prediction horizon (309.92 s) is much smaller than the sampling time (one month), thus guaranteeing real time implementability. Moreover, since the low-level problem is not time consuming (can be solved within 1 s), we see the potential of applying our methodology to a larger railway network divided into more sections. The simulated states and control actions of the high-level controller with $N_{\text{P}} = 6$ is shown in Figure 1. Replacing is suggested for a section when the average squat length is around 45 mm, and grinding is suggested when the average squat length is around 20 mm. An insight can be obtained that for condition-based predictive maintenance, it is more cost-efficient to perform maintenance or replacement when the deterioration is not yet severe.

The results of one corresponding low-level problem is shown in Figure 2. This problem is triggered at time step 27, when grinding is suggested for Section 5 by the high-level controller. Due to the 4-hour working time limit, the third cluster must be bypassed, as it is not as severe as the other clusters.

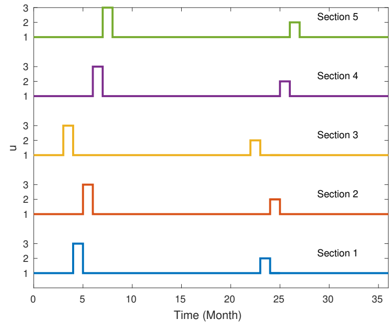
TABLE I

Performance and computational effort of the MPC controller with different prediction horizons

N_{P}	J_{end}	Maximum CPU time (seconds)
6	2760.7	0.74
9	1805.9	32.41
12	1676.5	309.92



(a)



(b)

Fig. 1. State and control actions of each section obtained from the high-level MPC controller with $N_{\mathrm{P}} = 6$

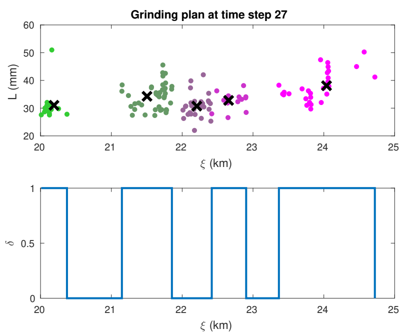


Fig. 2. Resulting clustering and solution for the MILP (18),(19) for one low-level problem. Squats in different clusters are marked with different colors, and the X-mark represent the centroid location and weight (average squat length) of each cluster.

V. Conclusions and Future Work

A multilevel decision making approach is developed for the optimal planning of rail condition-based maintenance. The MPC controller at the high level provides an optimal long-term maintenance plan of each track section, and the low-level problem produces the optimal short-term work plan whenever a replacing/grinding action is suggested. A case study with real measurements is performed to demonstrate the proposed approach. In the future, the high-level MPC can be improved by adopting a more robust approach that guarantees control performance and constraints sanctification for a wide range of uncertainties. Furthermore, the proposed two-level scheme can be extended by adding a middle-level problem, which determines the optimal allocation of time slots.

References

[1] V. Karlsson, "An overall view of maintenance," *European Railway Review*, vol. 11, no. 3, pp. 11–17, 2005.

[2] S. Fararooy and J. Allan, "Condition-based maintenance of railway signalling equipment," in *International Conference on Electric Railways in a United Europe*. IET, 1995, pp. 33–37.

[3] Y. Al-Douri, P. Tretten, and R. Karim, "Improvement of railway performance: a study of Swedish railway infrastructure," *Journal of Modern Transportation*, vol. 24, no. 1, pp. 22–37, 2016.

[4] A. Jardine, D. Lin, and D. Banjevic, "A review on machinery diagnostics and prognostics implementing condition-based maintenance," *Mechanical Systems and Signal Processing*, vol. 20, no. 7, pp. 1483–1510, 2006.

[5] R. Goverde, F. Corman, and A. D'Ariano, "Railway line capacity consumption of different railway signalling systems under scheduled and disturbed conditions," *Journal of Rail Transport Planning & Management*, vol. 3, no. 3, pp. 78–94, 2013.

[6] A. Parida and G. Chattopadhyay, "Development of a multi-criteria hierarchical framework for maintenance performance measurement (MPM)," *Journal of Quality in Maintenance Engineering*, vol. 13, no. 3, pp. 241–258, 2007.

[7] T. Åhrén and A. Parida, "Maintenance performance indicators (MPIs) for benchmarking the railway infrastructure: a case study," *Benchmarking: An International Journal*, vol. 16, no. 2, pp. 247–258, 2009.

[8] A. Jamshidi, A. Núñez, Z. Li, and R. Dollevoet, "Fuzzy maintenance decision support for treating squats in railway infrastructures," in *Joint Rail Conference 2015, San Jose CA, USA, 23-26 March*. DOI: 10.1115/JRC2015-5676, 2015.

[9] A. Zoeteman, Z. Li, and R. Dollevoet, "Dutch research results in wheel rail interface management: 2001-2013 and beyond," *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, vol. 228, no. 6, pp. 642–651, 2014.

[10] C. Esveld, *Modern Railway Track*. MRT-productions Zaltbommel, 2001.

[11] Z. Li, M. Molodova, A. Núñez, and R. Dollevoet, "Improvements in axle box acceleration measurements for the detection of light squats in railway infrastructure," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 7, pp. 4385–4397, 2015.

[12] M. Molodova, Z. Li, A. Núñez, and R. Dollevoet, "Automatic detection of squats in railway infrastructure," *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, no. 5, pp. 1980–1990, 2014.

[13] Z. Su, A. Núñez, A. Jamshidi, S. Baldi, Z. Li, R. Dollevoet, and B. De Schutter, "Model predictive control for maintenance operations planning of railway infrastructures," in *Computational Logistics* (Proceedings of the 6th International Conference on Computational Logistics (ICCL'15), Delft, The Netherlands, Sept. 2015), 2015, pp. 673–688.

[14] A. Jamshidi, A. Núñez, Z. Li, and R. Dollevoet, "Maintenance decision indicators for treating squats in railway infrastructures," in *94th Annual Meeting Transportation Research Board, Washington, USA, 11-15 January 2015*. TRB, 2015.

[15] F. Corman, A. D'Ariano, and I. Hansen, "Evaluating disturbance robustness of railway schedules," *Journal of Intelligent Transportation Systems*, vol. 18, no. 1, pp. 106–120, 2014.

[16] P. Campo and M. Morari, "Robust model predictive control," in *American Control Conference*. IEEE, 1987, pp. 1021–1026.

[17] J. Gruber, D. Ramirez, D. Limon, and T. Alamo, "Computationally efficient nonlinear min-max model predictive control based on Volterra series models –application to a pilot plant," *Journal of Process Control*, vol. 23, no. 4, pp. 543–560, 2013.

[18] S. Liu, A. Sadowska, J. Frejo, A. Núñez, E. Camacho, H. Hellendoorn, and B. De Schutter, "Robust receding horizon parameterized control for multi-class freeway networks: A tractable scenario-based approach," *International Journal of Robust and Nonlinear Control*, vol. 26, no. 6, pp. 1211–1245, Apr. 2016.

[19] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.

[20] S. Lloyd, "Least squares quantization in PCM," *IEEE Transactions on Information Theory*, vol. 28, no. 2, pp. 129–137, 1982.