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# **INTEGRATING DYNAMIC SIGNALLING COMMANDS UNDER FIXED-BLOCK SIGNALLING SYSTEMS INTO TRAIN DISPATCHING OPTIMIZATION MODELS**

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**ABSTRACT**

In real operations, train dispatching works aim at adjusting the impacted schedule and reducing negative consequences during disruptions. As one of the most important components of the railway system, railway signals are used to guarantee the safety of train service. We study the train dispatching problem with consideration of railway signalling commands under the fixed-block signalling system. In such a system, signalling commands dynamically depend on the movement of the preceding train in the network. We clarify the impact of the signalling commands on train schedules, which has been so far neglected in the railway train dispatching literature, and then we innovatively propose a set of signalling constraints to implement the impact. The determination of the signal indicators is presented “if-then” constraints, which are further transformed into linear inequalities by applying two transformation properties. Activation of the train speed limits that result from the signalling commands are the core purpose of the signalling constraints, and this is implemented by using the signal indicators. Moreover, we formulate the Greenwave (GW) policy, which requires that trains always proceed under green signals, and we further investigate the impact of the GW policy on the results (delay). In the numerical experiments, the proposed signalling constraints are employed by a time-instant optimization model, which is a mixed-integer linear programming (MILP) model. The experimental results demonstrate the effectiveness of the proposed signalling constraints and show the impact of the signalling commands and GW policy on the train dispatching solution.

*Keywords:* Train dispatching, Signalling commands, Train speed limit, Mixed-integer linear programming (MILP)

## INTRODUCTION

Railways are crucial to address the ever-growing mobility of population and goods, due to their positive characteristics of high capacity, high speeds, and eco-friendliness. A still negative characteristic of the railway services is their limited reliability and punctuality, which hinders reaching the modal share sought by policy makers and researchers. Train movements on a railway network are regulated by official timetables. In daily operations, perturbations (caused by bad weather, infrastructure failures, extra passenger flows, etc.) unavoidably happen, which may affect the normal operations and cause a deviation from the planned timetable. In such cases, the task of a train dispatcher (controller) is to take proper measures to adjust the impacted schedules for reducing the negative consequences (delays). This is the train dispatching problem. Due to the high interdependency between trains for the available capacity, the train dispatching work is usually complex, especially when the railway networks are operated close to saturation, in densely urbanized zones, and during peak hours. An ineffective train dispatching decision could result in a snowball influence with consecutive delays, downgrading the reliability and punctuality of train services. Fast and effective decisions for the train dispatching problem are always desired.

Railway signals are one of the most important parts of the railway system. There is a wide variety of railway signals and many signalling systems with different principles all over the world, e.g., the moving-block signalling systems commonly used on high-speed railway networks and the fixed-block signalling systems commonly used by conventional railways. However, the core function of the signalling systems is same, i.e., indicating the state of the block section(s) ahead for the train drivers in order to guarantee the safety of train services. A signalling command is usually indicated by a single-aspect or a multi-aspects. In the United States and in many European countries, the signalling command provides an additional requirement for train operations, namely indicating the maximum allowed speed to the driver, and the driver has to control the train to not exceed this speed for ensuring safety (otherwise, a worse situation may happen where the available distance is not enough to stop the train).

An extensive body of studies is available in the literature that addresses the train dispatching problem, having different focuses, e.g., considering multi-classes of running traffic (1), passenger connections (2), speed management (3), and maintenance plans (4), by using different approaches (e.g., linear/nonlinear optimization and heuristics). However, a gap still exists with regards to the signalling commands. Train speed limits that result from the signalling commands have been neglected in the literature (5), although they are indeed required in real train operations. To the best of our knowledge, no study is available now for generating optimal train dispatching solutions, which integrate precisely the actual signal aspect shown to train drivers, and that would allow guarantee of no violation of the signalling commands, based on the fixed-block signalling system. The reality of train operations and the gap in the scientific literature have motivated us to include the signalling commands while addressing the train dispatching problem.

We therefore study the train dispatching problem with consideration of railway signalling commands, focusing on conventional railways with the fixed-block signalling system, as they are common in the United States and in many European countries. As the signalling commands dynamically depend on the movement of the preceding trains in the network, we use binary variables (namely signal indicators) to indicate the commands. These signal indicators are determined by a set of “if-then” constraints, which could be generalized to other signalling systems and further transformed into linear inequalities by applying two transformation properties. Train speed limits that result from the signalling commands are restricted in the signalling constraints by employing

the signal indicators. In addition, we formulate the Greenwave (GW) policy and explore its impact on the train dispatching solution (train delay). Basically, the aim of the GW policy is to impose trains to exactly follow their scheduled speed profile, thus avoiding the need for speed profile adjustments. Therefore, the result with GW policy can be regarded as an effective solution for real operations without activating the speed limits. In the numerical experiments, a time-instant optimization model (mixed-integer linear programming, MILP) proposed in our previous study (6) is used to apply the proposed signalling constraints, aiming at delivering a train dispatching solution with minimization of train delays. The experimental results demonstrate the effectiveness of the proposed model, including signalling constraints, and show the impact of the signalling commands and GW policy on the train dispatching solution.

## LITERATURE REVIEW

An extensive body of literature is available for the railway train dispatching problem; interested readers might refer to the surveys (5, 7, 8). This section briefly reviews the state of the art for the train dispatching problem, especially focusing on the studies that use operations research based techniques and on how signalling issues are treated.

Advances on scheduling theory made it possible to solve real-life train dispatching instances, in which not only departure/arrival times (9), but also train orders, routes, speeds, and further operational freedom were considered as variables (10, 11, 12, 13, 14).

Several operations research based techniques are available now for addressing the train dispatching problem. A particularly popular model is the alternative graph based model, which uses a combination of job shop and alternative graph techniques (9). This alternative graph based formulation method considers microscopic details and is further employed in many studies, e.g., dealing with the train rerouting problem by developing meta-heuristics, including a Tabu Search algorithm in (11) and a variable neighborhood search algorithm in (15); investigating the impact of the details and the number of operational constraints on the applicability of models, in terms of solution quality and computational efficiency (16).

Another stream of studies addresses the train dispatching problem at a macroscopic level, which allows for faster resolution and larger geographical scope. Schöbel (17) proposed an event-activity based integer programming model to solve the delay management problem. The model was further extended to address a discrete time/cost trade-off problem of maintaining service quality and reducing passengers' inconvenience (18); and by including headways and capacity constraints and testing multiple pre-processing heuristics in order to fix integer variables and to speed up the computation (19). In the proposed models, connections are decided to be maintained or dropped by minimizing the number of missed connections, while minimizing the delays of all events.

Other approaches have also been proposed to solve this problem. Meng and Zhou (14) investigated the benefits of simultaneous train rerouting and rescheduling. Network-wide flag variables were used to implicitly model capacity constraints, which enabled an easy problem decomposition mechanism. The decomposed sub-problems were then solved by an adapted time-dependent least-cost path algorithm. Luan et al. (4) employed the flag variables based formulation method to address the integration of train scheduling and preventive maintenance planning. In (20), a heuristic algorithm, named RECIFE-MILP, was developed based on an extended version of the MILP formulation proposed in (21). Samà et al. (22) further investigated how to select the most promising train routes among all possible alternatives, through developing an ant colony optimization meta-heuristic.

Some dispatching decision support systems have been developed by researchers, and a few of them have been used in practice. The authors of (23) developed a traffic management support system, which is able to optimize traffic fluency in large railway networks equipped with either fixed or moving block signalling system. D'Ariano et al. (24) developed an advanced decision support system, known as ROMA (railway traffic optimization by means of alternative graphs), for dispatching trains based on microscopic details. Decomposition technologies were used for handling large areas in this system. It is verified that the system is able to find feasible and efficient schedules quickly. An exact approach and a master-slave solution algorithm (based on decomposition) were presented by (25), based on which a dispatching decision support system was developed. This system has been in operation in Norway since February 2014 and represents one of the first operative applications of mathematical optimization to train dispatching.

Most approaches in the literature neglect the impact of signalling issues while rescheduling the traffic. A recent study (3) takes the signalling impact into account, focusing on the high-speed traffic based on a quasi-moving block system. The authors proposed an alternative graph based optimization model to reschedule the high-speed traffic, which integrates the modeling of traffic management measures and the supervision of speed, braking, and headway. The safety headway was considered as a variable space, depending on the train speed.

This paper fills the knowledge gap regarding the signalling issues in the train dispatching problem, which are mostly neglected in the literature. The main contributions is thus the consideration of different signalling commands, not only “Clear” and “Stop” (commonly satisfied by track capacity constraint), but also “Approach (Limited)” (mostly neglected in the literature), depending dynamically on the traffic state. The proposed approach is suitable for inclusion in optimization schemes, in time-instant (namely time-continuous in the terminology of (5)) formulations, and can be extended to virtually any other fixed-block signalling system.

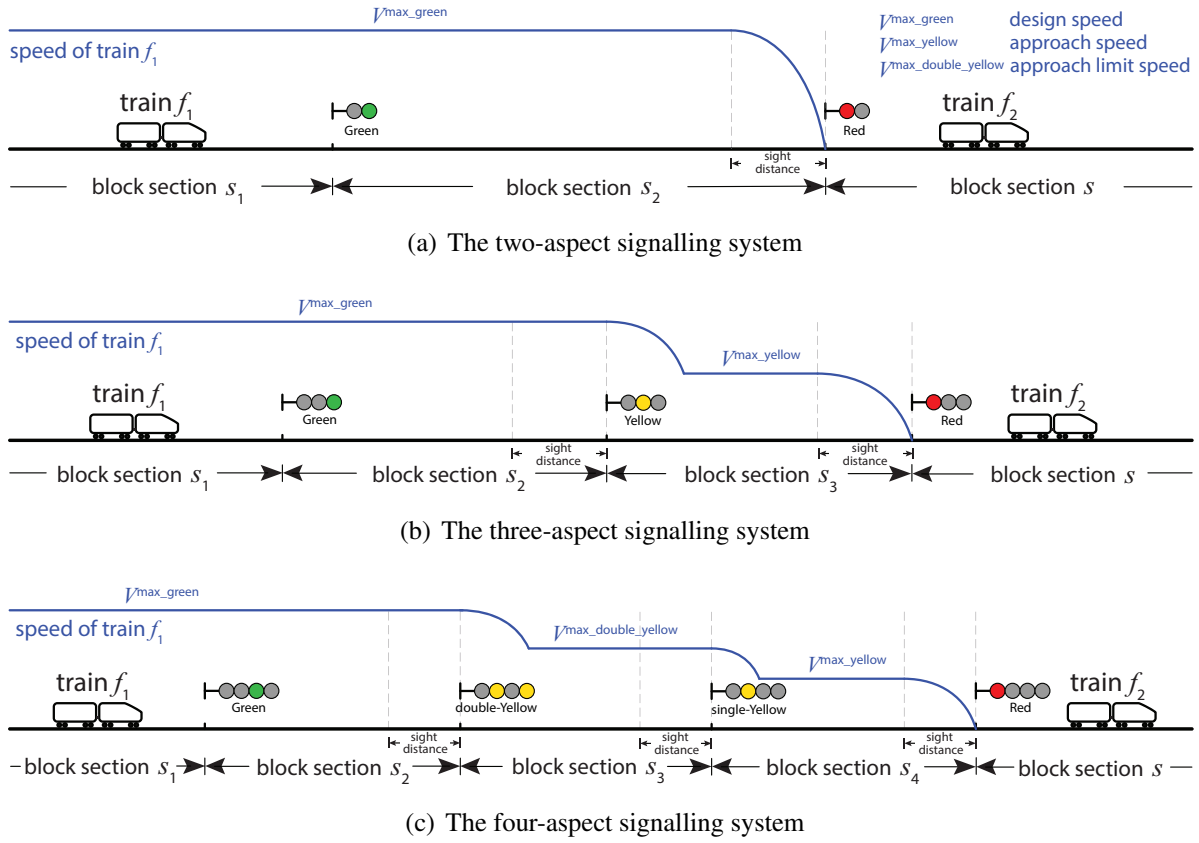
## PROBLEM DESCRIPTION

This section introduces the fixed-block signalling systems, followed by the relevant signalling commands that should be respected during train operations. We then describe the formulation method of the time-instant optimization model used in our previous study (6) for addressing the train dispatching problem. This formulation method will be used to construct the signalling constraints later on.

### Railway signalling systems

The railway signalling system is used to direct railway traffic and to keep trains clear of each other at all times (26, 27). A railway signal shows whether the track is clear ahead and also indicates to train drivers how far ahead the track is clear. Figure 1 illustrates three fixed-block signalling systems that are basic and widespread all over the world. In such fixed-block signalling systems, each track is divided into a sequence of block sections, and each block section is protected by a fixed signal placed at its entrance and displayed to the driver for an approaching train.

Figure 1(a) illustrates a two-aspect signalling system, which has a basic signal with a red and a green aspect. A green aspect indicates that the block section is accessible for trains, and a red aspect rejects the access of trains. As state in (28), this two-aspect signalling system works well for the operations of trains with speeds less than 50 km/h; however, for a train having a higher speed (like over 50 km/h), the train driver needs a warning of a red aspect ahead to give him/her room to stop. Therefore, multi-aspect signals have appeared, as shown in Figure 1(b)-Figure 1(c)

**FIGURE 1 : Three fixed-block signalling systems**

for a three-aspect signal and a four-aspect signal respectively. The three-aspect signal has a red, a yellow, and a green aspect. If a block section is occupied by a train, the signal placed at its entrance will display a red aspect, and the two signals behind will show a yellow aspect and a green aspect respectively. The yellow aspect provides an advanced warning of the red aspect ahead, and the driver then knows that there is only one clear block section ahead. The green aspect indicates that there are at least two clear block sections ahead, and the driver can maintain the design speed until facing a yellow aspect. The four-aspect signalling system in Figure 1(c) works similarly as the three-aspect signalling system, except that two advanced warnings are provided before a red aspect, i.e., a single-yellow and a double-yellow aspect. One purpose of doing this is to provide an earlier warning for higher-speed trains, and another purpose is to allow better track occupancy by shortening the length of the block sections.

In the United States and in most European countries, the signal indicates the maximum allowed speed to the driver, and the driver has to control the train to not exceed this speed for ensuring safety (otherwise, the driver will be penalized). The blue curves in Figure 1 indicate the maximum speed profiles of train  $f_1$  at the current moment, in the case of different signal aspects at the sight distance.

Let us consider the three-aspect signalling system as an example to explain. Figure 2 presents the time-space-speed graphs for a train  $f_1$  on five consecutive block sections. Different scenarios are illustrated, depending on the movement of the preceding train  $f_2$ . The orange hori-

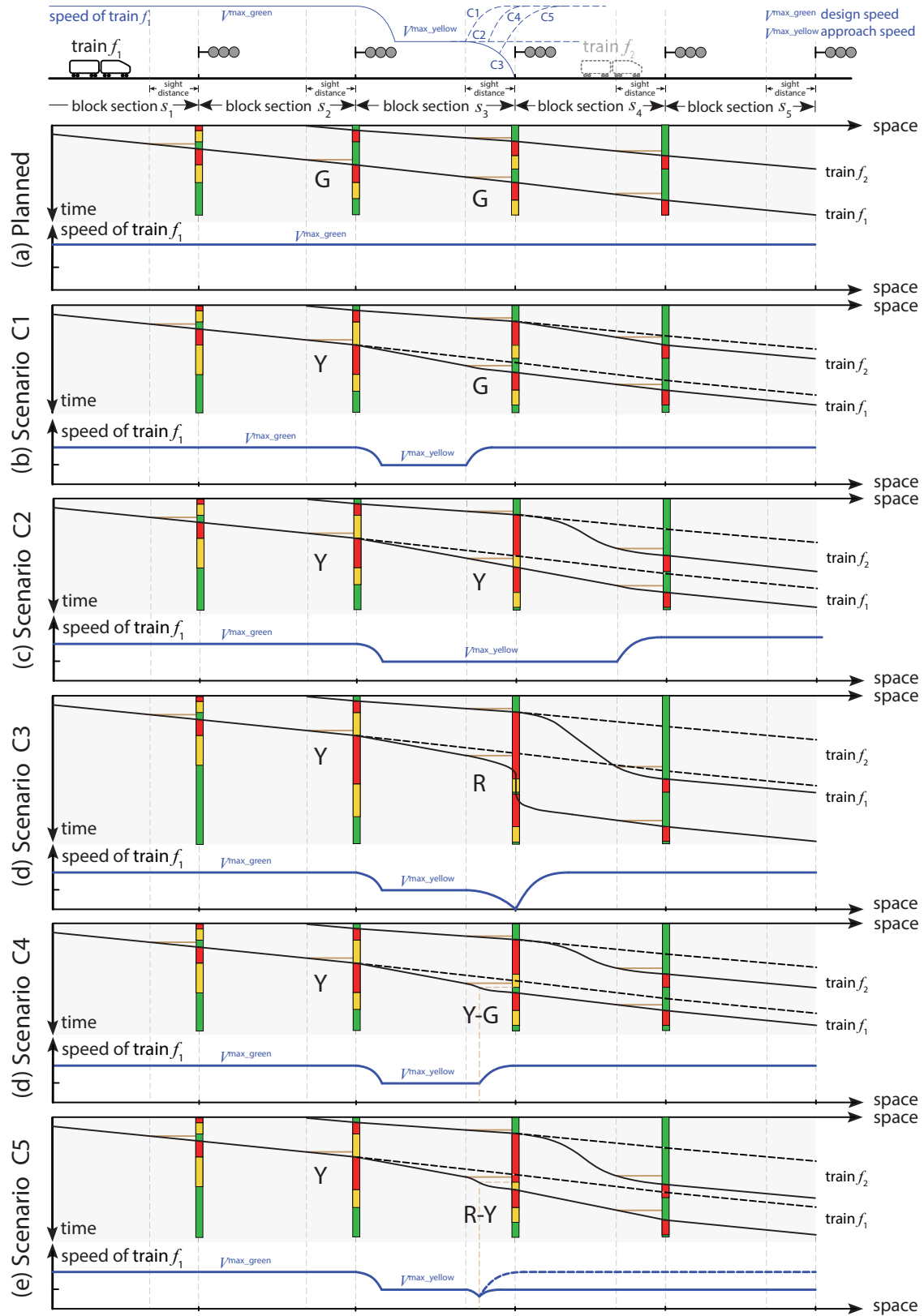


FIGURE 2 : Time-space-speed graphs of the possible scenarios



zontal line marked for each train on each block section directs the signal aspect faced by the train at the corresponding sight distance. In the case of always facing a green aspect, the drives can proceed with the design speed ( $V^{\max\_green}$ ), as illustrated in the planned scenario of Figure 2(a). The pre-planned train paths are further indicated by the dashed black lines in the cases of disruptions, illustrated in Figure 2(b)-Figure 2(f), and the train paths after adjusting are indicated by the solid black lines. In the upper portion of each sub-figure, we sketch the train paths and the signal aspects (in colors) displayed along the time; in the lower portion, we present the maximum allowed speed for train  $f_1$ . In the case of facing a yellow aspect at the sight distance of block section  $s_2$ , the driver of train  $f_1$  has to decrease the train speed from the design speed ( $V^{\max\_green}$ ) to the approach speed ( $V^{\max\_yellow}$ ), which is maintained until the following signal becomes visible. When the train reaches the next sight point, there are five possible scenarios shown in Figure 2(b)-Figure 2(f), labeled as scenario C1, ..., C5 respectively:

- In scenario C1, the signal placed at the sight distance of block section  $s_3$  shows a green aspect, and the driver is allowed to accelerate the train from the approach speed to the design speed.
- In scenario C2, the signal stays yellow, and the train is controlled to enter the next block section at the approach speed.
- In scenario C3, the signal stays red until the train completely stops, and the signal becomes green only after a certain waiting time.
- In scenario C4, the signal is yellow and then switches to green in the sight time (i.e., the running time of a train over the sight distance on a block section). The driver possibly accelerates the train to the design speed after a small reaction time.
- In scenario C5, the signal is red and then switches to yellow (or green directly) during the braking phase. In this scenario, the driver stops braking and possibly accelerates the train to the approach speed (or the design speed) after a small reaction time.

We can conclude that the maximum allowed speed of a train on a block section depends on the signalling command (i.e., the signal aspect displayed), and the signalling command dynamically depends on the movement of the preceding train in the railway network. Note that here after we make an assumption of considering the most conservative case in terms of safety: the movement of each train on each block section respects the signalling command shown at the moment that the train is reaching the sight distance, i.e., the possibilities of scenarios C4 and C5 in Figure 2 are neglected.

### The time-instant formulation method for the train dispatching problem

In a time-instant (namely time-continuous in the terminology of (5)) optimization model for addressing the train dispatching problem, we use arrival time variables  $a$  and departure time variables  $d$  to describe train movements on block sections. More specifically,  $a_{f,s}$  indicates the arrival time of train  $f$  at block section  $s$ , and  $d_{f,s}$  indicates the departure time of train  $f$  from block section  $s$ . The arrival and departure safety headway time intervals  $g_{f,s}$  and  $h_{f,s}$  can be either pre-determined as parameters (6) or considered as variables (10). For determining the section blocking time, the

occupancy time of block section  $s$  for the arrival of train  $f$  is formulated as

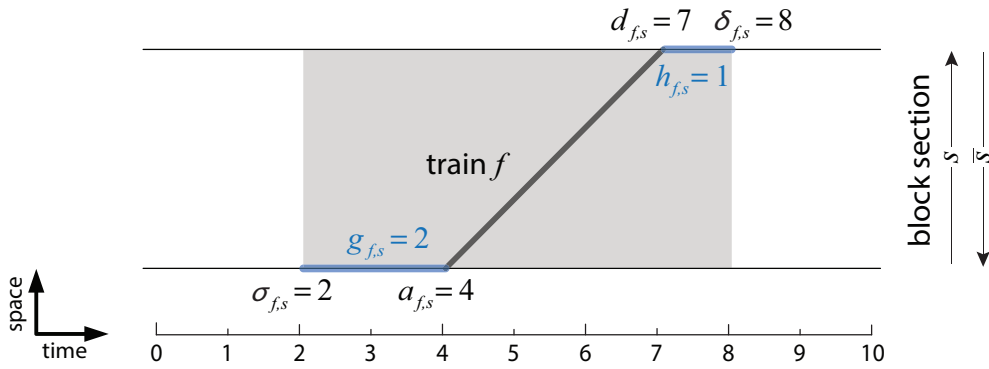
$$\sigma_{f,s} = a_{f,s} - g_{f,s}, \quad \forall f \in F, s \in E_f, \quad (1)$$

and the release time of block section  $s$  for the departure of train  $f$  is formulated as

$$\delta_{f,s} = d_{f,s} + h_{f,s}, \quad \forall f \in F, s \in E_f, \quad (2)$$

where  $F$  is the set of trains,  $E_f$  is the set of block sections that train  $f$  may use, and  $\sigma_{f,s}$  and  $\delta_{f,s}$  indicate the starting and ending time of blocking section  $s$  for train  $f$ .

Figure 3 illustrates the movement of train  $f$  on block section  $s$  by using arrival and departure time variables. More specifically, train  $f$  arrives at time  $a_{f,s} = 4$  and departs at time  $d_{f,s} = 7$ . As we have the safety headway times  $g_{f,s} = 2$  and  $h_{f,s} = 1$ , block section  $s$  is blocked for train  $f$  from time  $\sigma_{f,s} = 2$  to time  $\delta_{f,s} = 8$ .



**FIGURE 3 : The time-instant optimization method**

For generating a conflict-free train dispatching solution, the block section capacity constraint is proposed by avoiding the overlap between any pair of trains on the same block section, formulated as follows:

$$\sigma_{f_2,s} + (1 - \theta_{f_1,f_2,s}) \cdot M \geq \delta_{f_1,s}, \quad \forall f_1 \in F, f_2 \in F, s \in E_{f_1} \cap E_{f_2} \quad (3)$$

$$\sigma_{f_2,\bar{s}} + (1 - \theta_{f_1,f_2,s}) \cdot M \geq \delta_{f_1,s}, \quad \forall f_1 \in F, f_2 \in F, s \in E_{f_1}, \bar{s} \in E_{f_2}. \quad (4)$$

where  $\theta_{f_1,f_2,s}$  is a binary train order variable, with  $\theta_{f_1,f_2,s} = 1$  if train  $f_2$  arrives at block section  $s$  or block section  $\bar{s}$  after train  $f_1$ , and otherwise  $\theta_{f_1,f_2,s} = 0$ , and  $M$  is a sufficiently large positive number. Note that we indicate bi-directional block section on a single-track segment as  $s$  and  $\bar{s}$ , which refer to one physical block section in opposite direction. Thus, the model can be applied to single-track, double-track, or  $N$ -track networks. Interested readers may refer to (6, 10, 14) for more details.

## MATHEMATICAL FORMULATION OF THE SIGNALLING COMMANDS

This section formulates the signalling constraints to implement the signalling commands, which are innovative in comparison with the previous studies. We first clarify the impact of the signalling commands on the train schedules, revealed by the train travel times. Then, we formulate signalling constraints by applying signal indicators to represent signalling commands, which dynamically depend on the condition of the block sections ahead, i.e., the relative position of the preceding train

in the network. The time-instant formulation method is considered for constructing the signalling constraints. Moreover, we consider the three-aspect signalling system in Figure 1(b) as an example to formulate the signalling constraints. For the four-aspect signalling system and other signalling systems that are not detailed in this paper, a similar approach can be followed, as far as a set of unique rules can be derived.

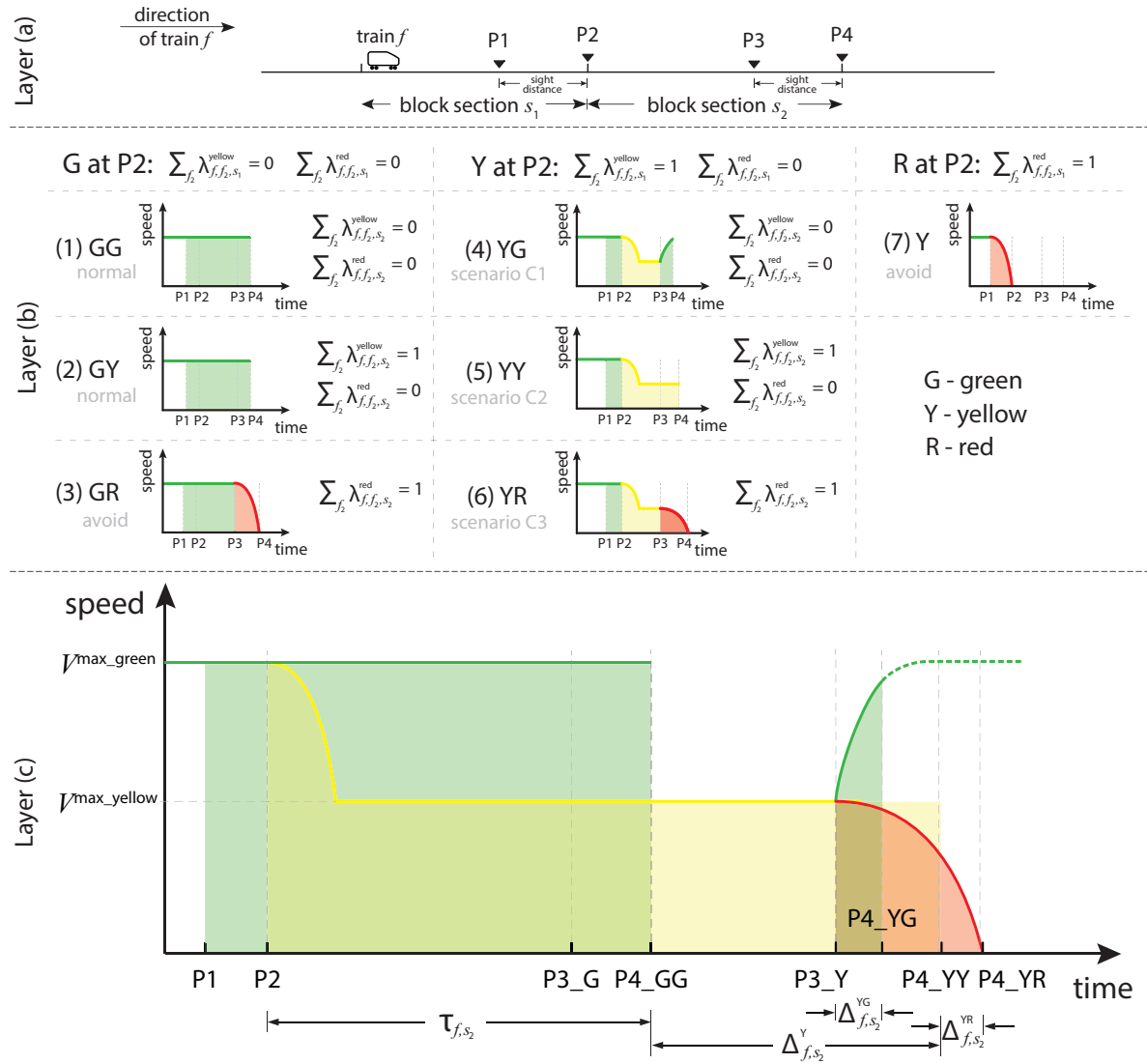
### Impact of signalling commands on train schedules: additional train travel time

A non-green signal requires a train speed reduction. To implement the train speed reduction, a direct way is to restrict train speeds. However, few train dispatching models consider the train speeds as variables; as a result, we cannot add restrictions to the train speeds. As the train speed reduction can be reflected by an increased travel time of a train on a block section, we can formulate the train speed reduction by requiring additional train travel time. Therefore, this section interprets the impact of the signalling commands on train schedules, revealed by train travel time.

Figure 4 is a three-layer figure, which illustrates the possible scenarios of a case that train  $f$  traverses two adjacent block sections  $s_1$  and  $s_2$ . The sight point and the end point of block sections  $s_1$  and  $s_2$  are labeled as P1, ..., P4 respectively, as shown in Layer (a) of Figure 4. We use a speed-time graph to show the relation between the speed reduction and the additional travel time. In Layer (b), seven speed-time graphs for train  $f$  from the position P1 to P4 are sketched. A single or double capital letter(s) is used to indicate the signal aspects shown at position P2 and P4 respectively, e.g., “GG” means two green aspects. The X-axis indicates the time at which train  $f$  arrives at the positions P1, ..., P4. An integrated graph of these separate speed-time graphs is provided in Layer (c), in which the effects of the signalling commands (speed reduction) on the train travel times are graphically interpreted. In this layer, the X-axis indicates the time at which train  $f$  arrives at a position under a scenario, e.g., the time point labeled “P4\_YG” is the arrival time of train  $f$  at position P4, if the two signals at positions P2 and P4 show the yellow and green aspects respectively.

In Layer (b), the colored area of the graphs (1)-(6) implies the distance from position P1 to P4, i.e., the distance traveled by a train equals its speed multiplied by the elapsed time; thus these areas should be equal to each other. As sketched in the graphs (3) and (7), train  $f$  faces the red aspect directly after the green aspect, which should be prevented in the three-aspect signalling system for safety purpose; therefore, these two scenarios will not occur. The other five graphs correspond to the scenarios C1, ..., C3 presented in Figure 2. Recall that we make an assumption of considering the worst case: the possibilities of scenarios C4 and C5 in Figure 2 are neglected. Moreover, we use two binary variables (speed indicators)  $\lambda_{f,f_2,s}^{\text{red}}$  and  $\lambda_{f,f_2,s}^{\text{yellow}}$  to indicate whether train  $f_2$  results in a red aspect and a yellow aspect for train  $f$  on block section  $s$ . More specifically,  $\lambda_{f,f_2,s}^{\text{red}} = 1$  if train  $f_2$  is occupying the block section one ahead block section  $s$ , which makes train  $f$  faces the red aspect on block section  $s$ , otherwise,  $\lambda_{f,f_2,s}^{\text{red}} = 0$ . Thus, the scenarios presented in Layer (b) of Figure 4 can be represented by using these speed indicators, as marked.

Let us focus on Layer (c) of Figure 4. Note that  $\tau_{f,s}$  indicates the free-flow travel time of train  $f$  on block section  $s$  under the normal condition (i.e., a green aspect), and  $\Delta_{f,s}^Y$ ,  $\Delta_{f,s}^{YG}$ , and  $\Delta_{f,s}^{YR}$  indicate the additional/decreased travel time caused by the train speed reduction (i.e., by the “Approach” and “Stop” signalling commands). If the signal placed at position P2 is green, as in graphs (1)-(2) in Layer (b), no train speed reduction is required. If the signal placed at position P2 is yellow, as in graphs (4)-(6) in Layer (b), then train  $f$  reduces its speed by following the yellow



**FIGURE 4 : Seed-time graphs to illustrate the relation between the speed reduction and the additional travel time**

curve, and its travel time on block section  $s_2$  increases. The amount of the increased travel time further depends on the display of the signal placed at position P4:

- If the signal at position P4 is green, i.e., graph (4) in Layer (b), then train  $f$  is allowed to accelerate by following the green curve, and the additional travel time in this case is shorter than the case of facing a yellow aspect, calculated by  $\Delta_{f,s_2}^Y - \Delta_{f,s_2}^{YG}$ .
- If the signal at position P4 is yellow, i.e., graph (5) in Layer (b), then train  $f$  maintains its speed by following the yellow line, and the additional travel time on block section  $s_2$  is denoted as  $\Delta_{f,s_2}^Y$ , compared with the normal scenario of facing green aspects. We consider that a second yellow aspect allows train to pass at the approaching speed.
- In the remaining case that train  $f$  faces a red aspect at position P4, corresponding to graph

(6) in Layer (b), train  $f$  has to decelerate by following the red curve until it stops completely, and the additional travel time is measured by  $\Delta_{f,s_2}^Y + \Delta_{f,s_2}^{YR}$ .

### Formulations of the signalling constraints

This section formulates the signalling constraints, dynamically determining the signal indicators and implementing the train speed reduction by requiring additional travel time. As discussed in Section 3.1, for each train on each block section, the signalling command received by the driver depends on the movement of the preceding train in the network. Therefore, we need to identify the signalling commands by determining the condition (i.e., being occupied or released) of the block section(s) ahead.

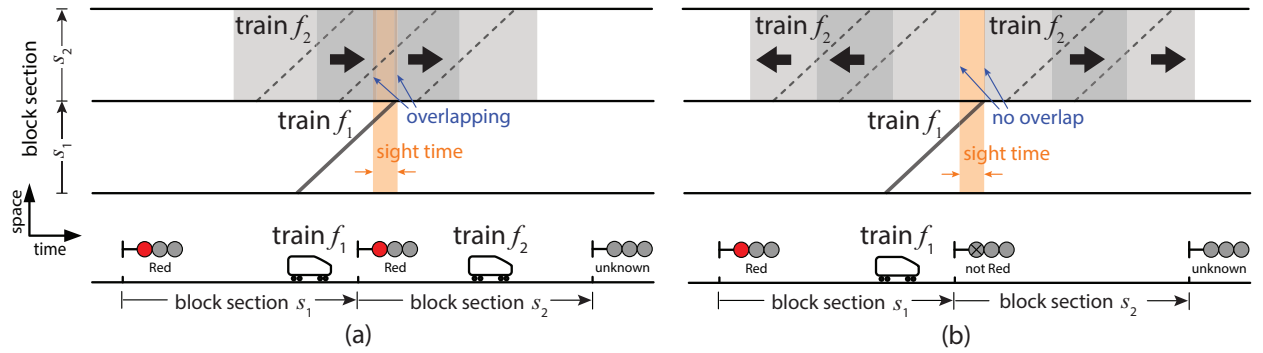


FIGURE 5 : Identifying the “Stop” signalling command

Figure 5 illustrates how can we identify the “Stop” signalling command; for the “Approach” command, we can follow a similar approach. In Figure 5(a), train  $f_1$  meets a red aspect on block section  $s_1$  at the sight distance, because train  $f_2$  is occupying block section  $s_2$  at that moment. In such cases, the blocking time of train  $f_1$  on block section  $s_1$  and the blocking time of train  $f_2$  on block section  $s_2$  are “overlapping” in the sight time as shown in the timetable; we then have  $\lambda_{f_1,f_2,s_1}^{\text{red}} = 1$ . For other cases shown in Figure 5(b), train  $f_1$  faces a non-red aspect on block section  $s_1$ , and no “overlap” happens in the sight time between the blocking time of train  $f_1$  and train  $f_2$ . As a result, we have  $\lambda_{f_1,f_2,s_1}^{\text{red}} = 0$ .

To determine the red signal indicator  $\lambda_{f_1,f_2,s}^{\text{red}}$ , let us consider the binary variables  $\gamma_{f_1,f_2,s}^{r1}$  and  $\gamma_{f_1,f_2,s}^{r2}$  to satisfy the conditions

$$[d_{f_1,s_1} \leq \sigma_{f_2,s_2}] \Leftrightarrow [\gamma_{f_1,f_2,s_1}^{r1} = 1], \quad \forall f_1 \in F, f_2 \in F, f_1 \neq f_2, s_1 \in E_{f_1}, s_2 \in E_{f_2} \cap E_{s_1}^{\text{adj}2}, \quad (5)$$

$$[\delta_{f_2,s_2} \leq d_{f_1,s_1} - \iota_{f_1,s_1}^{\text{sight}}] \Leftrightarrow [\gamma_{f_1,f_2,s_1}^{r2} = 1], \quad \forall f_1 \in F, f_2 \in F, f_1 \neq f_2, s_1 \in E_{f_1}, s_2 \in E_{f_2} \cap E_{s_1}^{\text{adj}2} \quad (6)$$

if train  $f_1$  and train  $f_2$  run in the same direction, or

$$[d_{f_1,s_1} \leq \sigma_{f_2,\bar{s}_2}] \Leftrightarrow [\gamma_{f_1,f_2,s_1}^{r1} = 1], \quad \forall f_1 \in F, f_2 \in F, f_1 \neq f_2, s_1 \in E_{f_1}, \bar{s}_2 \in E_{f_2} \cap E_{s_1}^{\text{adj}2}, \quad (7)$$

$$[\delta_{f_2,\bar{s}_2} \leq d_{f_1,s_1} - \iota_{f_1,s_1}^{\text{sight}}] \Leftrightarrow [\gamma_{f_1,f_2,s_1}^{r2} = 1], \quad \forall f_1 \in F, f_2 \in F, f_1 \neq f_2, s_1 \in E_{f_1}, \bar{s}_2 \in E_{f_2} \cap E_{s_1}^{\text{adj}2} \quad (8)$$

if train  $f_1$  and train  $f_2$  run in the opposite direction. Note that  $\iota_{f_1,s_1}^{\text{sight}}$  indicates the running time of train  $f_1$  over the sight distance on block section  $s_1$ , and  $E_{s_1}^{\text{adj}2}$  indicates the set of the adjacent

block sections of block section  $s_1$ . As a result, we have  $\gamma_{f_1, f_2, s_1}^{r1} = 0$  and  $\gamma_{f_1, f_2, s_1}^{r2} = 0$  for the case in Figure 5(a), and we have either  $\gamma_{f_1, f_2, s_1}^{r1} = 1$  or  $\gamma_{f_1, f_2, s_1}^{r2} = 1$  for the case in Figure 5(b). Then, the red signal indicator  $\lambda_{f_1, f_2, s}^{\text{red}}$  can be formulated as

$$\lambda_{f_1, f_2, s}^{\text{red}} = (1 - \gamma_{f_1, f_2, s}^{r1}) \cdot (1 - \gamma_{f_1, f_2, s}^{r2}), \quad \forall f_1 \in F, f_2 \in F, f_1 \neq f_2, s \in E_{f_1}. \quad (9)$$

By applying the transformation properties proposed in (29), the if-then constraints (5)-(8) and the nonlinear constraint (9) can be reformulated as linear inequalities. Moreover, formulations similar to (5)-(9) can also be constructed for the yellow signal indicator  $\lambda_{f_1, f_2, s}^{\text{yellow}}$ . For the sake of compactness, we do not provide those details here.

According to analysis of the additional train travel time in Section 4.1, which reflects the train speed reduction required by the signalling commands, the travel time constraint is given as follows:

$$d_{f, s_2} - a_{f, s_2} \geq w_{f, s_2}^{\min} + \tau_{f, s_2} + \Delta_{f, s_2}^Y \cdot \sum_{f_2 \in F} \lambda_{f, f_2, s_1}^{\text{yellow}} + \Delta_{f, s_2}^{YR} \cdot \sum_{f_2 \in F} \lambda_{f, f_2, s_2}^{\text{red}} - \Delta_{f, s_2}^{YG} \cdot (1 - \sum_{f_2 \in F} \lambda_{f, f_2, s_2}^{\text{red}}) \cdot (1 - \sum_{f_2 \in F} \lambda_{f, f_2, s_2}^{\text{yellow}}), \quad \forall f \in F, s_1 \in E_f, s_2 \in E_f \cap E_{s_1}^{\text{adj2}}, \quad (10)$$

where  $w_{f, s_2}^{\min}$  indicates the minimum dwell time of train  $f$  on block section  $s_2$ . Constraint (10) ensures that the actual travel time of train  $f$  on block section  $s$  is not less than the sum of the minimum dwell time, the free-flow travel time, and the additional travel time caused by the non-“Clear” signalling commands.

Moreover, the GW policy is proven to be effective to saving energy consumption and dealing with delays in (30), and the GW result would not violate the speed limits that result from the signalling commands. Therefore, we employ the GW policy and formulate it by setting all signal indicators to be zero, as follows:

$$\sum_{f \in F} \sum_{s \in E_f} \sum_{f_2 \in F: f_2 \neq f} (\lambda_{f, f_2, s}^{\text{yellow}} + \lambda_{f, f_2, s}^{\text{red}}) = 0. \quad (11)$$

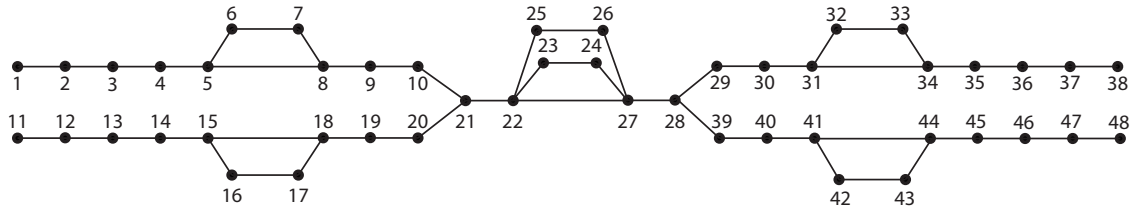
As a result, all trains will always encounter green aspects when traversing the network.

As the signalling constraints proposed here can be transformed into linear inequalities, we can still use the solution approach of the original models to solve the dispatching problem that integrates the signalling commands. We consider the train dispatching model proposed in (6), which is an MILP model; therefore, a standard MILP solver can be used. For more details, interested readers may refer to the description in (6).

## NUMERICAL EXPERIMENTS

We conduct the experiments on a rail network with two main tracks, composed of 48 nodes and 53 block sections, as depicted in Figure 6. The two tracks in different directions are independent, so only one direction is considered. We consider 1.5 hours of traffic, with 20 trains in 3 train categories, i.e., 8 intercity, 8 sprinter, and 4 freight trains. Sprinter trains stop at all stations; intercity and freight trains stop only at the origin and destination stations. We consider ten delay cases of the primary delays following a 3-parameter Weibull distribution, as investigated in (31). We adopt the CPLEX solver version 12.6.3 with default settings to solve the MILP problems. The following experiments are all performed on a computer with an Intel® Core™ i7 @ 2.00 GHz processor and 16GB RAM.

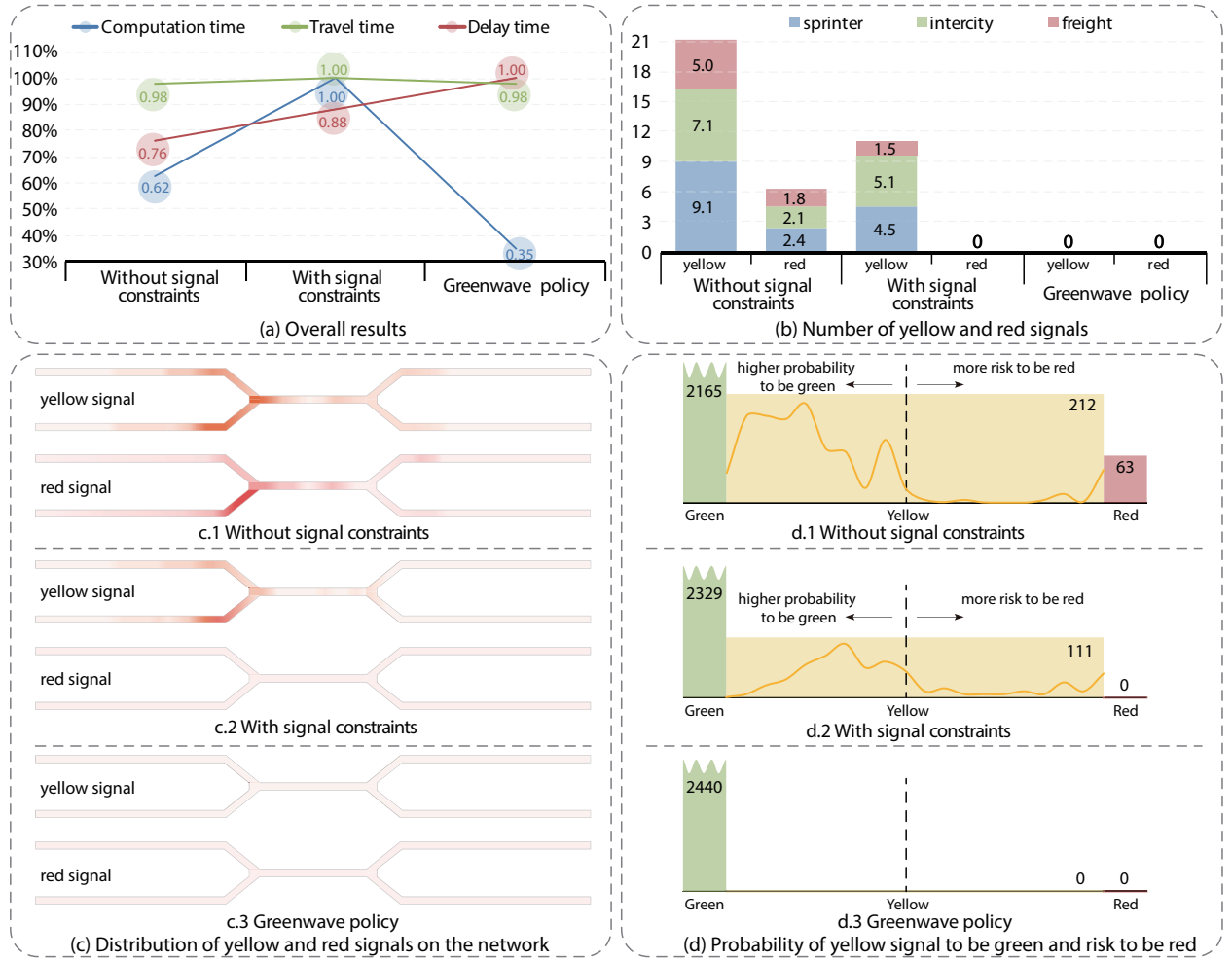


**FIGURE 6 : A rail network**

In Figure 7(a)-(b), the average results of the ten delay cases are given, in terms of the train delay times, the train travel times, and the numbers of the red and yellow signals faced by the trains. The results in Figure 7(a) are normalized in the range [0,1]. As shown, when integrating the signalling constraints, the total delay time and the total travel time increase by 16% and 2% respectively, and the number of yellow aspects that the trains meet decreases by 47%. As the speed limits required by the signalling commands are activated, the model optimizes the train departure and arrival times by avoiding trains to face the non-green aspects (as much as possible). However, some trains still face the yellow aspects on some block sections, i.e., 11 times on average. In such cases, the speed reduction (i.e., the additional travel time) is enforced in our model; therefore, the train travel time becomes larger, which further results in the increased total delay time and travel time. *Considering the signalling constraints enables us to avoid the full stop that caused by the red signal, as the times that the trains confront the red signal are reduced to zero.* By applying the GW policy, where the trains can always proceed under green signals, the total delay time further increases by 13%, compared to the results with the signalling constraints; however, the total travel time decreases to become similar as the results without the signalling constraints. The computation time for obtaining the results with consideration of the signalling constraints is longer, but still within 300 seconds (5 minutes).

We further explore more details regarding the signals. In Figure 7(c)-(d), we present the results of all the ten delay cases by summing up the numbers of the red/yellow signals that faced by the trains. Figure 7(c) illustrates the distribution of the red and yellow signals on the network. The network layout origins from Figure 6. A darker color implies a larger number of red/yellow signals occurred at the position. As shown, the red and yellow signals are mostly faced by the trains before entering the merging area of the two lines and also in the merging area. This results from the increased train interactions in these areas. Figure 7(d) presents the chance of the yellow signals to become green and the risk to be red, which is obtained by analyzing the overlapping time of the two blocking times, as interpreted in Figure 5. The bars indicate the number of the green, yellow, and red signals, labeled with the values. The Y-axis of the yellow line (within the yellow bar) indicates the number of the yellow signals, and the X-axis is the probability of becoming green or red. The black dashed line is a benchmark line, indicating that the chance to become green equals the risk to be red. When neglecting the signalling constraints, lots yellow signals occurred have big chance to become green. By considering the signalling constraints, the total number of the yellow signals decreases from 212 to 111, and the number of the yellow signals that have larger chance to become green is reduced. *Considering the signalling constraints enables us to reduce the number of the yellow signals faced by the trains, i.e., taking the chance of letting the yellow signals become green as much as possible.*

We can conclude that the consideration of the signalling commands leads to larger delay



**FIGURE 7 : Illustration of the experimental results**

times and larger travel times, but increases the realism of the dispatching solution obtained and the safety of the train services. The GW policy hardly affects the train travel time, but results in a significant increase of train delays. The benefits of considering the signalling constraints are demonstrated, i.e., reducing the yellow signals and avoiding red signals that faced by the trains.

## CONCLUSIONS

This paper integrates the dynamic signalling commands under the fixed-block signalling system into train dispatching optimization models. We have investigated the impact of the signalling commands on train schedules, and we have implemented this impact in our model by presenting the signalling constraints. In these constraints, the signalling commands (represented by the signal indicators) are dynamically determined by the train movements, and the train speed limits are activated according to the signal indicators. In our experimental results, the consideration of the signalling commands results in larger delay times (16%) and larger travel times, but reduces the times of meeting the non-green signals and increases the realism of the train dispatching solution and the safety of the train services, as the real operational requirements (speed limits) caused by the signalling commands are included. In fact, the 16% gap can be thought of as an approximation er-



ror of currently available fixed speed dispatching models neglecting signalling aspects. Moreover, we consider the GW policy to dispatch trains, which results in larger train delay times; however, the total train travel time does not change much, similar to the result that neglects the signalling impact.

Our future research will focus on the following main extensions. First, as the proposed approach for constructing the signalling constraints can be extended to other signalling systems, we might study different signalling systems and compare their performance, in terms of delay, energy, capacity, robustness, etc., from the planning and operational perspective(s). Second, when applying the GW policy, the energy consumption is expected to be reduced. We may evaluate the energy consumption by applying the model proposed in (10), instead of the model in (6). Finally, in order to extend the applicability of the model to large-scale networks, we could focus on developing heuristic algorithms and distributed optimization methods for improving the computational efficiency of the model.

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