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# Optimal scheduling of track maintenance activities for railway networks<sup>\*</sup>

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**Abstract:** We consider optimal scheduling of track maintenance activities for a railway network divided into sections. The goal is to find an optimal time schedule for the maintenance activities and optimal routes for the maintenance crew (including all necessary equipment and technicians) that minimize the total setup costs and the travel costs over the whole planning horizon. The maintenance time budget, which can be the same, different, or flexible for each period, and the minimum time to maintain a section are also taken into account. We recast the track maintenance scheduling problem with three different settings as three variants of the Capacitated Arc Routing Problem with Fixed cost (CARPF), which are solved by transforming them into three node routing problems. The proposed approach is demonstrated using a case study of a part of the Dutch regional network.

*Keywords:* maintenance, railway infrastructure, arc routing problem, logistics, scheduling

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## 1. INTRODUCTION

Maintenance is essential for the reliability, availability, and safety of railway networks. We focus on track maintenance activities. One typical example is grinding, which is applied to a rail surface defect called squats (Jamshidi et al., 2017). Due to the high cost of track maintenance activities, and the limited resources regarding machinery, technicians, and available track possession time for maintenance (usually less than 10 hours for one maintenance activity), cost-efficient scheduling of maintenance activities is of great concern for railway infrastructure managers. The track maintenance scheduling problem is usually formulated as a Mixed Integer Linear Programming (MILP) problem. Some papers (Wen et al., 2016; Higgins et al., 1999; Budai et al., 2006) focus on maintenance scheduling for one single track, e.g., the MILP model developed in (Wen et al., 2016) for the optimal scheduling of condition-based tamping for a Danish railway corridor, considering several technical and economic factors like track quality and train speed limit. Optimal scheduling of different routine maintenance activities to minimize traffic disruption and maintenance completion time for a single railway line is formulated as an integer programming problem in (Higgins et al., 1999). A similar problem is addressed in (Budai et al., 2006), which formulates a MILP problem to schedule both routine maintenance activities and projects like grinding to minimize track possession cost and maintenance cost. A maintenance crew contains all the necessary equipment (e.g. grinding machine) and technicians for one specific maintenance activities. The

Vehicle Routing Problem (VRP) (Dantzig and Ramser, 1959) is the most popular approach for optimal scheduling and routing of track maintenance activities for a railway network. In Heinicke et al. (2015), the optimal scheduling of maintenance tasks with different priorities for a railway network is formulated as a VRP with customer costs. The optimal clustering of track maintenance jobs into major projects is formulated as a VRP in Peng and Ouyang (2014) to minimize the total duration of all projects. Another popular approach for maintenance scheduling and routing is the time-space network model. A time-space network model with side constraints is developed in Peng and Ouyang (2012) for the optimal scheduling and routing of major track maintenance projects like rail replacement. The time-space network formulation and a VRP-based formulation are applied to rail maintenance scheduling in Gorman and Kanet (2010), with a discussion on their relative merits. Other approaches include the mixed integer programming formulation based on network flow proposed in Boland et al. (2013) to schedule a variety of maintenance tasks, and the mixed integer nonlinear programming problem formulated in Zhang et al. (2013) to minimize the total traveling costs, maintenance costs, as well as costs associated with condition deterioration.

This paper contributes to the state-of-the-art in the following ways. Unlike other VRP-based approaches, we model the track maintenance scheduling problem as a Capacitated Arc Routing Problem with Fixed cost (CARPF). Three settings, namely, homogeneous, heterogeneous, and flexible time periods are considered. In particular, the flexible maintenance time periods setting allows the maintenance contractor to minimize total maintenance costs within the planning horizon, including travel costs, setup costs, and overtime penalties. The three variants of the CARPF are transformed into node routing problems and solved by compact MILP formulations using 2-index bi-

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nary variables.

## 2. PROBLEM DESCRIPTION

We consider optimal scheduling of one type of track maintenance activities (e.g. grinding or tamping) for a railway network divided into multiple sections. A section is defined as the track<sup>1</sup> between two major stations<sup>2</sup>. A Key Performance Indicator (KPI), which is obtained by considering a broad set of measurements from various sources, is defined for each section to represent its health condition. Whether a section is to be maintained within the given finite planning horizon, which is usually equal to the maintenance cycle (e.g. six months) is determined by the value of its KPI. In practice, a maintenance threshold is usually applied to maintenance decision making, and sections with a KPI exceeding this threshold must be maintained within the planning horizon. An estimated minimum maintenance time, which is obtained empirically, is also assigned to each section that requires maintenance. We define a maintenance operation as a tour of the maintenance crew. A tour must start and end at a maintenance base, where the machine can be stored. The length of one maintenance time period is defined as the smallest time unit a maintenance operation is planned. This ensures that at most one operation can be performed per maintenance time period. A fixed setup cost, including the machinery, personnel, etc., is associated with each maintenance operation. This setup cost incentivizes the maintenance agent to cover the sections that requires maintenance in as few operations as possible. A maintenance time budget (usually less than 10 hours), which specifies the maximum track possession time available for maintenance, is defined for each period.

The track maintenance scheduling problem can then be defined as finding the optimal schedule for maintenance operations, and the optimal routes for the maintenance crews, that minimize the total setup costs and travel costs over the entire planning horizon, guaranteeing that all the sections that needed to be maintained are treated by exactly one operation, and the total track possession time for each maintenance operation does not exceed the maintenance time budget for that period. Furthermore, we consider the following three settings:

- Setting 1 (*homogeneous maintenance time periods*): the maintenance time budget and setup cost are the same for each maintenance time period in the planning horizon;
- Setting 2 (*heterogeneous maintenance time periods*): maintenance time budgets for different time periods in the planning horizon are in general different;
- Setting 3 (*flexible maintenance time periods*): in addition to the given maintenance time budgets, additional track possession time can be required for each period with a fine per extra hour.

## 3. ARC ROUTING PROBLEM

In this section we formulate the maintenance scheduling and routing problem described in Section 2 as an undi-

<sup>1</sup> In this paper, we consider only single track line.

<sup>2</sup> A major station is either a start/end station or a transfer station.

rected Capacitated Arc Routing Problem with Fixed cost (CARPF), which consists of the following elements:

- a connected undirected graph  $G = (\mathcal{V}, \mathcal{E})$ ;
- a cost matrix  $C$  defining the travel cost associated with each edge  $\{i, j\} \in \mathcal{E}$ ;
- a subset of required edges  $\mathcal{R} \subseteq \mathcal{E}$ ;
- a fleet  $\mathcal{T}$  of vehicles;
- a depot node (denoted by 0);
- a fixed setup cost  $c_{\text{Setup},t}$  for each vehicle  $t \in \mathcal{T}$ ;
- a demand  $q_{ij}$  for each required edge  $\{i, j\} \in \mathcal{R}$ ;
- and a capacity  $Q_t$  associated with each vehicle  $t \in \mathcal{T}$ .

The undirected CARPF can then be defined as finding the optimal set of routes of the fleet starting and ending at the depot, minimizing the total travel costs and setup costs, and guaranteeing that each required edge is serviced exactly once, and the demand of each required edge is satisfied without exceeding the capacity of the vehicle visiting it.

In this paper, we map the physical railway network into the virtual graph  $G$ , in which the node set  $\mathcal{V}$  contains all the major stations, and the edge set  $\mathcal{E}$  contains all the railway sections. The maintenance base is mapped to the depot node 0. The sections to be maintained within the planning horizon correspond to the required edges in  $\mathcal{R}$ . Furthermore, we consider each maintenance time period in the planning horizon as a virtual vehicle, and the set of maintenance time periods is mapped to the set of vehicles  $\mathcal{T}$ . The capacity  $Q_t$  for each vehicle, i.e. the maximum quantity of goods the vehicle can handle, is interpreted as the maintenance time budget of period  $t$ , while the demand of each required edge, i.e. the minimum quantity of goods that must be delivered to the edge, refers to the minimum maintenance time for the corresponding section. The setup cost of a vehicle is equivalent to the setup cost of a maintenance operation in the corresponding period. Finally, the three settings of the maintenance scheduling problem result in three variants of the CARPF, namely, the homogeneous CARPF for Setting 1, the heterogeneous CARPF for Setting 2, and the CARPF with flexible vehicle capacity for Setting 3.

## 4. NODE ROUTING PROBLEM

### 4.1 Arc-to-Node Transformation

Several arc-to-node transformations have been proposed in literature, resulting in a node routing instance with  $3|\mathcal{R}|$  customer nodes. Recently a compact transformation is proposed in Foulds et al. (2015), which results in only  $|\mathcal{R}|$  customer nodes. However, this transformation is equivalent only when the resulting node routing problem is solved by a specific branch-and-price process (Barnhart et al., 1998). We adopt the transformation proposed in Baldacci and Maniezzo (2006), which uses both endpoints of the required edges to create a node routing instance with  $2|\mathcal{R}|$  customer nodes. The resulting node routing problem can be solved directly by state-of-the-art MILP solvers like CPLEX or Gurobi. Here we briefly sketch the transformation from Baldacci and Maniezzo (2006).

Let the undirected complete graph  $\hat{G} = (\hat{\mathcal{V}}, \hat{\mathcal{E}})$  denote the transformed graph, where the node set  $\hat{\mathcal{V}}$  contains the

depot 0 in the original graph, and  $2|\mathcal{R}|$  customers corresponding to the endpoints of the required edges in  $G$ . In particular, for each  $\{i, j\} \in \mathcal{R}$ , we denote the transformed customer nodes  $s_{ij}$  and  $s_{ji}$  as the endpoints on the  $i$  and  $j$  side of the original required edge, respectively<sup>3</sup>. Let  $\hat{\mathcal{C}}$  and  $\hat{\mathcal{R}}$  denote the set of customers and required edges in the transformed graph, respectively. In particular, a required edge  $\{s_{ij}, s_{ji}\} \in \hat{\mathcal{R}}$  corresponds to a required edge  $\{i, j\} \in \mathcal{R}$ . The demand of a required arc in the original graph is equally divided between its two endpoints in the new graph, i.e.

$$\hat{q}_{s_{ij}} = \hat{q}_{s_{ji}} = \frac{1}{2}q_{ij} \quad \forall \{s_{ij}, s_{ji}\} \in \hat{\mathcal{R}}. \quad (1)$$

The cost of each edge  $\{s_{ij}, s_{kl}\} \in \hat{\mathcal{E}}$  is defined as:

$$\hat{c}_{s_{ij}s_{kl}} = \begin{cases} -f_{\text{UB}} & \text{if } \{s_{ij}, s_{kl}\} \in \hat{\mathcal{R}} \\ \text{dist}(0, k) + 0.5 \cdot c_{kl} & \text{if } s_{ij} = 0 \\ \text{dist}(i, k) + 0.5 \cdot c_{ij} & \text{otherwise,} \\ +0.5 \cdot c_{kl} & \end{cases} \quad (2)$$

where the upper bound  $f_{\text{UB}}$  is the objective function value of a feasible solution of the original arc routing problem, and  $\text{dist}(i, j)$  represents the shortest path distance between node  $i$  and  $j$  in the original graph. The upper bound  $f_{\text{UB}}$  causes inconvenience in the numerical implementation as a feasible solution of the original arc routing problem is needed before the transformation. As suggested in Baldacci and Maniezzo (2006), we replace  $-f_{\text{UB}}$  in (2) with 0, and add the following constraints:

$$x_{ij} = 1 \quad \forall \{i, j\} \in \hat{\mathcal{R}} \quad (3)$$

to the resulting node routing problem.

#### 4.2 Homogeneous Capacitated Vehicle Routing Problem with Fixed Cost

In this section, we present the MILP formulation of the homogeneous Capacitated Vehicle Routing Problem with Fixed cost (CVRPF), the node-routing counterpart of the homogeneous CARPF corresponding to Setting 1 of the track maintenance scheduling problem. In CVRPF,  $|\mathcal{T}|$  homogeneous vehicles are available at the single depot 0. Let  $Q$  and  $c_{\text{Setup}}$  denote the capacity and fixed setup cost of each vehicle, respectively. First we define

$$x_{ij} = \begin{cases} 1 & \text{if node } j \text{ is visited directly after node } i; \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

for each  $\{i, j\} \in \hat{\mathcal{E}}$  in the transformed graph.

We define the continuous variable  $u_i$  as the node potential in the Miller-Tucker-Zemlin (MTZ) subtour elimination constraints (Miller et al., 1960) for each customer  $i \in \hat{\mathcal{C}}$ . The node potential  $u_i$  can be interpreted as the maintenance time spent after leaving customer  $i$ . The CVRPF can then be formulated as an MILP problem:

$$\min \sum_{\{i, j\} \in \hat{\mathcal{E}}} \hat{c}_{ij} x_{ij} + \sum_{j \in \hat{\mathcal{C}}} c_{\text{Setup}} x_{0j} \quad (5)$$

subject to

$$\sum_{j \in \hat{\mathcal{V}}} x_{ij} = \sum_{j \in \hat{\mathcal{V}}} x_{ji} = 1 \quad \forall i \in \hat{\mathcal{C}} \quad (6)$$

<sup>3</sup> This is equivalent to defining a customer as a job performed on a required edge  $\{i, j\} \in \mathcal{R}$  from one direction ( $i$  to  $j$  or  $j$  to  $i$ ), as done in Peng and Ouyang (2014).

$$\sum_{i \in \hat{\mathcal{V}}} x_{i0} = \sum_{j \in \hat{\mathcal{V}}} x_{0j} \leq |\mathcal{T}| \quad (7)$$

$$u_i - u_j + Qx_{ij} + (Q - \hat{q}_i - \hat{q}_j)x_{ji} \leq Q - \hat{q}_i \quad (8)$$

$$\forall i, j \in \hat{\mathcal{C}}, i \neq j$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in \hat{\mathcal{V}} \quad (9)$$

$$\hat{q}_i \leq u_i \leq Q \quad \forall i \in \hat{\mathcal{C}} \quad (10)$$

The first term in the objective function (5) corresponds to the total travel costs, while the second term computes the total setup costs of all the routes. Constraints (6) and (7) are degree constraints for customers and depots, respectively. A strengthened formulation of MTZ (Desrochers and Laporte, 1991) is used for subtour elimination constraints (8), which also ensure the satisfaction of each customer's demand. Finally, constraints (9) are the integrality constraints for the binary variable, and constraints (10) are bounds for the continuous node potential variable.

#### 4.3 Heterogeneous Capacitated Vehicle Routing Problem

In this section, we present the MILP formulation of the Heterogeneous Capacitated Vehicle Routing Problem with Fixed cost (HCVRPF), the node-routing counterpart of the Heterogeneous CARPF corresponding to Setting 2 of the track maintenance scheduling problem. In HCVRPF, the types of  $|\mathcal{T}|$  vehicles are in general different. Let  $\mathcal{D}$  denote the set of vehicle types, and we have  $|\mathcal{D}| \leq |\mathcal{T}|$ . The capacity and setup cost of a type  $d$  vehicle are denoted by  $Q_d$  and  $c_{\text{Setup},d}$ , respectively. Let  $m_d$  denote the number of vehicles of type  $d$ . Furthermore, we consider each type  $d \in \mathcal{D}$  as a virtual depot that stores only vehicles of type  $d$ . The original depot 0 is then replaced by  $|\mathcal{D}|$  duplicates of virtual depots, resulting in a new complete graph  $\tilde{G} = (\tilde{\mathcal{V}}, \tilde{\mathcal{E}})$ , where the new node set  $\tilde{\mathcal{V}} = \mathcal{D} \cup \hat{\mathcal{C}}$  contains all the virtual depots and the customers of the node routing instance. A new cost matrix  $\tilde{C}$  is also defined, where the travel cost between any two customers or any customer and a depot remains the same in as in  $\hat{C}$ , and the travel costs between depots are 0. In addition to the binary variable  $x_{ij}$  defined in (4), we define another set of binary variables

$$z_{id} = \begin{cases} 1 & \text{if customer } i \text{ is visited by a vehicle} \\ & \text{from depot } d \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

as selection variables to associate each customer  $i \in \hat{\mathcal{C}}$  with some depot  $d \in \mathcal{D}$ .

The node potential variable  $u_i$  for the MTZ subtour elimination constraints is defined the same way as in Section 4.2. Since a virtual depot only stores one type of vehicle, a route starting from one depot must end at the same depot. Additional cycle imposition constraints are needed to ensure that each resulting route starts and ends at one depot. In this paper, we use the node current-based cycle imposition constraints (Burger et al., 2017), which were originally designed for fixed-destination multi-depot traveling salesman problems, and define the continuous decision variable  $k_i$  as the node current variable for each node  $i \in \tilde{\mathcal{V}}$ . Finally, the MILP formulation of the HCVRPF can be written as:

$$\min \sum_{\{i, j\} \in \tilde{\mathcal{E}}} \tilde{c}_{ij} x_{ij} + \sum_{d \in \mathcal{D}} \sum_{j \in \tilde{\mathcal{V}}} c_{\text{Setup},d} x_{dj} \quad (12)$$

subject to

$$\sum_{j \in \tilde{\mathcal{V}}} x_{ij} = \sum_{j \in \tilde{\mathcal{V}}} x_{ji} = 1 \quad \forall i \in \hat{\mathcal{C}} \quad (13)$$

$$\sum_{i \in \tilde{\mathcal{V}}} x_{id} = \sum_{j \in \tilde{\mathcal{V}}} x_{dj} \leq m_d \quad \forall d \in \mathcal{D} \quad (14)$$

$$z_{id} - z_{jd} \leq 1 - x_{ij} - x_{ji} \quad (15)$$

$$z_{jd} - z_{id} \leq 1 - x_{ij} - x_{ji} \quad (16)$$

$$\forall d \in \mathcal{D}, i, j \in \hat{\mathcal{C}}, i \neq j$$

$$x_{dj} + x_{jd} - z_{jd} \leq 0 \quad \forall d \in \mathcal{D}, j \in \hat{\mathcal{C}} \quad (17)$$

$$u_i - u_j + \bar{Q}x_{ij} + (\bar{Q} - \hat{q}_i - \hat{q}_j)x_{ji} \leq \bar{Q} - \hat{q}_i \quad (18)$$

$$\forall i, j \in \hat{\mathcal{C}}, i \neq j$$

$$k_d = d \quad \forall d \in \mathcal{D} \quad (19)$$

$$k_i - k_j \leq (|\mathcal{D}| - 1)(1 - x_{ij}) \quad (20)$$

$$k_j - k_i \leq (|\mathcal{D}| - 1)(1 - x_{ij}) \quad (21)$$

$$\forall i, j \in \tilde{\mathcal{V}}, i \neq j$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in \tilde{\mathcal{V}} \quad (22)$$

$$z_{id} \in \{0, 1\} \quad \forall i \in \hat{\mathcal{C}}, \forall d \in \mathcal{D} \quad (23)$$

$$\hat{q}_i \leq u_i \leq \sum_{d \in \mathcal{D}} Q_d z_{id} \quad \forall i \in \hat{\mathcal{C}} \quad (24)$$

$$1 \leq k_i \leq |\mathcal{D}| \quad \forall i \in \tilde{\mathcal{V}} \quad (25)$$

Similar to the homogeneous CVRPF, the first term in the objective function (12) corresponds to the total travel costs, while the second term computes the total setup costs of all the maintenance operations from all the virtual depots. Constraints (13) and (14) are degree constraints for customers and depots, respectively. Constraints (15) and (16) together ensure that any two customers precede or succeed each other are visited by the same type of vehicle. Constraints (17) guarantee that a customer first or last visited by a vehicle from a depot is associated with the same depot. The subtour elimination constraints (18) are the same as (8), except that  $\bar{Q} = \max_{d \in \mathcal{D}} Q_d$  is an upper bound of a vehicle's capacity, instead of the capacity of each homogeneous vehicle. Constraints (19)-(21) are the cycle imposition constraints based on node current Burger et al. (2017). Each depot node is assigned with a unique node current by constraints (19). Constraints (20) and (21) together ensure that the same node current value propagates along a path, just like the current in an electrical circuit. Finally, constraints (22) and (23) are the integrality constraints for the binary variables, and constraints (24) and (25) are bounds for the continuous node potential and node current variables, respectively. Note that unlike homogeneous VRP, in (24) the upper bound of the node potential of a customer is adjusted to the capacity of the vehicle that visits it.

#### 4.4 Capacitated Vehicle Routing Problem with Flexible Capacity

In this section, we present the MILP formulation of the Capacitated Vehicle Routing Problem with Flexible Capacity (CVRPFC), the node-routing counterpart of the CARPF with flexible vehicle capacity corresponding to Setting 3 of the track maintenance scheduling problem. The only difference between the CVRPFC and the HCVRPF in Section 4.3 is that the capacity of each vehicle is no longer

a fixed parameter, but a continuous decision variable. Similar to the HCVRPF, we also introduce a virtual depot for each vehicle type and create an extended transformed graph  $\tilde{G}$  to incorporate the virtual depots. Note that since every vehicle can have a distinctive type in the CVRPFC, we have  $\mathcal{D} = \mathcal{T}$  and  $m_d = 1$  for any  $d$ . Similarly, denote  $Q_d \in [\underline{Q}, \bar{Q}]$  as the capacity and  $c_{\text{Setup},d}$  as the setup cost of vehicle  $d$ . Different setup costs are assigned to different vehicles, and the following affine function is used to compute the setup cost of each vehicle from its capacity:

$$c_{\text{Setup},d} = v(Q_d - \underline{Q}) + \underline{c}_{\text{Setup}} \quad (26)$$

where  $v$  is positive parameter. In practice,  $v$  can be interpreted as the hourly fine that must be paid by the maintenance contractor for additional track possession time over the given maintenance time budget, while  $\underline{Q}$  and  $\underline{c}_{\text{Setup}}$  correspond to the length and setup cost of the existing maintenance time budget, respectively.

The optimization problem of the CVRPFC is the same as that of the HVRPF (12)-(25). However, since  $Q_d$  is a decision variable in the CVRPFC, the upper bound of the node potential in (24) becomes nonlinear. Moreover, substituting (26) into (12) we obtain the following objective for the CVRPFC:

$$\begin{aligned} \min \quad & \sum_{\{i,j\} \in \tilde{\mathcal{E}}} \tilde{c}_{ij} x_{ij} \\ & + \sum_{d \in \mathcal{D}} \sum_{j \in \tilde{\mathcal{V}}} v Q_d x_{dj} + (\underline{c}_{\text{Setup}} - v \underline{Q}) x_{dj} \end{aligned} \quad (27)$$

which gives rises to another nonlinear term  $Q_d x_{dj}$ . To eliminate these nonlinear terms, we apply the procedure described in Bemporad and Morari (1999) and introduce the following continuous variables:

$$y_{id} = z_{id} Q_d, \quad f_{dj} = x_{dj} Q_d \quad (28)$$

$$\forall d \in \mathcal{D}, i \in \hat{\mathcal{C}}, j \in \tilde{\mathcal{V}}.$$

The nonlinear variables are equivalent to the following linear constraints Bemporad and Morari (1999):

$$y_{id} \leq \bar{Q} z_{id}, \quad f_{dj} \leq \bar{Q} x_{dj} \quad (29)$$

$$y_{id} \geq \underline{Q} z_{id}, \quad f_{dj} \geq \underline{Q} x_{dj} \quad (30)$$

$$y_{id} \leq Q_d - \underline{Q}(1 - z_{id}), \quad f_{dj} \leq Q_d - \underline{Q}(1 - x_{dj}) \quad (31)$$

$$y_{id} \geq Q_d - \bar{Q}(1 - z_{id}), \quad f_{dj} \geq Q_d - \bar{Q}(1 - x_{dj}) \quad (32)$$

$$\forall d \in \mathcal{D}, i \in \hat{\mathcal{C}}, j \in \tilde{\mathcal{V}}.$$

The CVRPFC can then be expressed as:

$$\min \quad \sum_{\{i,j\} \in \tilde{\mathcal{E}}} \tilde{c}_{ij} x_{ij} + \sum_{d \in \mathcal{D}} \sum_{j \in \tilde{\mathcal{V}}} v f_{dj} + (\underline{c}_{\text{Setup}} - v \underline{Q}) x_{dj} \quad (33)$$

subject to

$$\hat{q}_i \leq u_i \leq \sum_{d \in \mathcal{D}} y_{id} \quad \forall i \in \hat{\mathcal{C}} \quad (34)$$

$$\text{and constraints (13)-(23), (25), (29)-(32),}$$

which is now a MILP.

## 5. CASE STUDY

### 5.1 Settings

We consider optimal scheduling of grinding, a typical track maintenance activity to treat a typical rolling contact fatigue called a squat. In practice, a cyclic preventive

Table 1. Capacities and setup costs of maintenance time periods for the three settings of the maintenance scheduling problem.

Setting	Capacity (h)	Setup Cost (k€)
1	7	110
2	8 for 2 long periods, 6 for 4 short periods	120 for a long period, 100 for a short period
3	$Q = 6, \bar{Q} = 10$	$v = 10, c_{\text{Setup}} = 100$

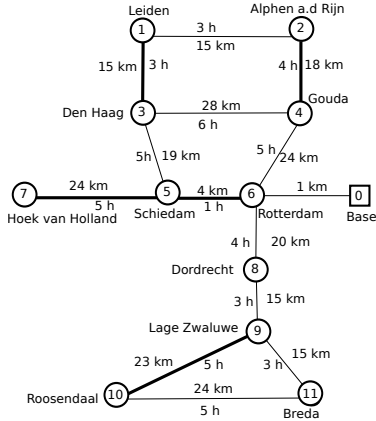


Fig. 1. Representation of a part of the Dutch railway network including Randstad Zuid and the middle-south region. The sections that must be maintained within the planning horizon are marked by thick lines. The length (rounded to kilometers) and the minimum maintenance time (in hours) estimated by expert judgments, of each section are also provided.

grinding strategy is used in the treatment of squats in the Dutch railway network. A rail inspection is performed every six months, and the sections to be ground within the next six months are selected according to their KPI values updated by the latest measurements. We set the planning horizon to six months, which corresponds to the current inspection/maintenance cycle. At most one grinding operation can be performed every month, so one period equals to one month. The capacities and setup costs of each period for the 3 settings in Section 2 are given in Table 1.

The part of the Dutch railway network considered in the case study is shown in Figure 1. The travel cost of each edge is defined as the length (in kilometers) of the corresponding section. The required edges (marked by thick lines in Figure 1) are  $\{1, 3\}$ ,  $\{2, 4\}$ ,  $\{5, 6\}$ ,  $\{5, 7\}$ , and  $\{9, 10\}$ . As stated in Section 3, the demand of each required edge is defined as the minimum maintenance time (in hours), while the demand of an unrequired edge is set to 0.

The proposed approach is implemented in Matlab R2016b, on a desktop computer with an Intel Xeon E5-1620 eight-core CPU and 64 GB of RAM. We use CPLEX 12.5 as the MILP solver.

## 5.2 Results and Discussion

The resulting optimal routes and node potentials of the three node routing problems for the three settings are provided in Table 2, where the transformed nodes are rep-

Table 2. Solutions of the CVRP (Setting 1), the HCVRP (Setting 2), and the CVRPF (Setting 3).

Setting	Period	Route	Node Potential
1	1	0, $s_{31}$ , $s_{13}$ , $s_{24}$ , $s_{42}$ , 0	1.5, 3, 5, 7
	2	0, $s_{65}$ , $s_{56}$ , $s_{57}$ , $s_{75}$ , 0	0.5, 1, 3.5, 6
	3	0, $s_{910}$ , $s_{109}$ , 0	2.5, 5
2	Long	0, $s_{42}$ , $s_{24}$ , $s_{13}$ , $s_{31}$ , 0	2, 4, 5.5, 7
	Short	0, $s_{65}$ , $s_{56}$ , $s_{57}$ , $s_{75}$ , 0	0.5, 1, 3.5, 6
3	$Q_1 = 8$	0, $s_{65}$ , $s_{56}$ , $s_{31}$ , $s_{13}$ , $s_{24}$ , $s_{42}$ , 0	0.5, 1, 2.5, 4, 6, 8
	$Q_4 = 10$	0, $s_{57}$ , $s_{75}$ , $s_{910}$ , $s_{109}$ , 0	2.5, 5, 7.5, 10

resented in the same way as in Section 4.1. In particular, the node potential of the last visited customer of a route represents the total track possession time spent for the corresponding maintenance operation. As shown in Table 2, the maintenance operations in Period 2 and 3 of Setting 1, as well as the long period and the first short period of Setting 2 do not use all their corresponding maintenance time budgets, leading to a waste of track possession time. Instead of three operations, only two operations are needed in Setting 3 to service all the five sections that needed to be maintained, saving setup costs. Moreover, because of the flexible maintenance time budgets/vehicle capacity, there is no waste of track possession time in the solution of the CVRPF.

The solutions of the node routing problems are transformed back to the solutions of the corresponding arc routing problems in Table 3 for easier interpretation. The path between any two transformed customers  $s_{ij}$  and  $s_{kl}$  or a customer  $s_{ij}$  and the depot is the shortest path between node  $i$  and  $k$  or  $i$  and 0. The maintenance time spent on a required edge  $\{i, j\} \in \mathcal{R}$  equals to the node potential of  $s_{ji}$  minus the node potential of the last visited customer of the preceding required edge, or minus 0, if there is no preceding required edge.

The performance and computational effort of the node routing problems of the three settings are presented in Table 4. The total maintenance costs (indicated by the objective function value) of Setting 3 are significantly lower than those of Setting 1 and 2, where the maintenance time budget is fixed for each time period. However, Setting 3 is also the most computationally demanding one, as a more difficult variant of the CVRPF must be solved.

## 6. CONCLUSIONS AND FUTURE WORK

In this paper we have considered the optimal scheduling of track maintenance activities for a railway network. We have recast three main settings, namely, homogeneous, heterogeneous, and flexible maintenance time periods, of the track maintenance scheduling problem as three variants of the capacitated arc routing problem with fixed cost, which are solved by transforming them into three node routing problems. Simulation results of the case study of a part of the Dutch railway network show that flexible maintenance time budgets leads to lower total maintenance costs, including the travel costs and setup costs, taking into account the penalties associated with additional track possession time.

Table 3. Solutions of the three arc routing problem instances. The required edges are marked in bold in the resulting routes.

Setting	Period	Route	Maintenance Time (h)
1	1	{0, 6}, {6, 5}, {5, 3}, <b>{3, 1}</b> , {1, 2}, <b>{2, 4}</b> , {4, 6}, {6, 0}	3, 4
	2	{0, 6}, <b>{6, 5}</b> , <b>{5, 7}</b> , {7, 5}, {5, 6}, {6, 0}	1, 5
	3	{0, 6}, {6, 8}, {8, 9}, <b>{9, 10}</b> , {10, 9}, {9, 8}, {8, 6}, {6, 0}	5
2	Long	{0, 6}, {6, 4}, <b>{4, 2}</b> , {2, 1}, <b>{1, 3}</b> , {3, 5}, {5, 6}, {6, 0}	4, 3
	Short	{0, 6}, <b>{6, 5}</b> , {5, 7}, <b>{7, 5}</b> , {5, 6}, {6, 0}	1, 5
	Short	{0, 6}, {6, 8}, {8, 9}, {9, 10}, <b>{10, 9}</b> , {9, 8}, {8, 6}, {6, 0}	5
3	$Q_1 = 8$	{0, 6}, <b>{6, 5}</b> , {5, 3}, <b>{3, 1}</b> , {1, 2}, <b>{2, 4}</b> , {4, 6}, {6, 0}	1, 3, 4
	$Q_4 = 10$	{0, 6}, {6, 5}, <b>{5, 7}</b> , {7, 5}, {5, 6}, {6, 8}, {8, 9}, <b>{9, 10}</b> , {10, 9}, {9, 8}, {8, 6}, {6, 0}	5, 5

Table 4. Objective function value and mean CPU time (obtained from five consecutive runs) of the three settings.

Setting	Setup cost	Travel cost	Mean CPU time (s)
1	330	273	0.046
2	320	273	0.113
3	260	271	6.2529

In the future, we would like to test the proposed approach by conducting computational experiments on an extensive test bench to find the size limit of test instances that can be solved by state-of-the-art MILP solvers like CPLEX. It is also worthwhile to develop dedicated exact methods for the arc routing problems resulting from the three settings of the maintenance scheduling problem. Furthermore, the proposed static crew scheduling problem can be extended to dynamic crew scheduling problem with time-dependent maintenance costs related to train dispatching. Finally, negative impact to normal train operation should also be incorporated in the maintenance crew scheduling problem.

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