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Corrections to "Integrated Urban Traffic Control for the Reduction of Travel Delays and Emissions"

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***Abstract*—This short paper provides corrections and modifications to the urban traffic flow model, called S-model, and the integrated flow and emission model given in [1]. These corrections involve some formulas in [1] that have been derived either based on a wrong reasoning or assumption, or that could be improved by the aid of the proposed corrections and modifications to obtain a higher level of accuracy in the simulations.**

CORRECTIONS

THE corrections given in this erratum involve Equations (4), (5), (9) in the S-model and Equations (13)–(16) in the integrated flow and emission model introduced in [1]. For details and extensive proofs of the proposed corrections and modifications, we refer the readers to [2], [3]. Moreover, we propose some extensions/modifications to Eq. (9) of [1] that result in a simple formula that can be used to compute the integral in (9). Since the S-model was originally introduced in [4], the corrections and modifications given for the S-model in this paper also hold for [4].

The mathematical notations and concepts that are frequently used in this short paper, are represented and defined in Table I.

A. Corrections to Eq. (4) of [1]

We first represent a remark that corrects two of the frequently used mathematical notations in [1].

Remark 1: The parameters $\tau(k_d)$ and $\gamma(k_d)$ in [1] should be corrected as $\tau_{u,d}(k_d)$ and $\gamma_{u,d}(k_d)$, as they are link-dependent. Therefore, in the remainder of the erratum, u and d are used as subscripts for these parameters (and their equivalent continuous-time versions $\tilde{\tau}_{u,d}(t)$ and $\tilde{\gamma}_{u,d}(t)$).

Eq. (4) given in [1] for computation of $\alpha^{\text{arriv}}_{u,d}(k_d)$ is not correct for the following two main reasons. First, although the two parameters $\tau_{u,d}$ and $\gamma_{u,d}$ in [1] have been considered to be time-varying, (4) has been derived based on a reasoning that assumes fixed values for these parameters in time. This can create major inaccuracies in the simulations when the S-model is used. Second, simulating the original S-model using Eq. (4), conservation of the vehicles is not always satisfied, and some vehicles arriving at the tail of the waiting queue

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TABLE I
FREQUENTLY USED MATHEMATICAL NOTATIONS IN THE PAPER.

(u,d)	An urban traffic link in the S-model with upstream intersection u and downstream intersections d
c_d	Cycle time of traffic signal at intersection d, which, in the S-model, is the simulation sampling time of all links with downstream intersection d
k_d	Cycle counter of traffic signal at intersection d, which, in the S-model, is the simulation time step counter of all links with downstream intersection d
$\Delta c^{\text{init}}_{u,d}$	Initial offset between cycles of traffic signals at upstream and downstream intersections of link (u,d), i.e., the time span from k_d=0 (start of simulation of link (u,d)) until beginning of the first upcoming cycle at intersection u
$\Delta c_{u,d}(k_d)$	Offset between cycles of traffic signals at intersections u and d at simulation time step k_d
$\Delta\tilde{x}_{u,d}(t)$	Distance between the entrance of link (u,d) and the tail of the waiting queue on this link at time instant t (this distance has been traveled by a vehicle that arrives at the tail of the waiting queue on link (u,d) at time instant t)
$\tilde{\delta}_{u,d}(t)$	Travel time (in the continuous-time domain) required for a vehicle that arrives at the tail of the waiting queue on link (u,d) at time instant t, to travel the distance $\Delta\tilde{x}_{u,d}(t)$
$\delta_{u,d}(k_d)$	Discrete-time version of $\tilde{\delta}_{u,d}(t)$
$\tilde{\tau}_{u,d}(t)$	Quotient of $\tilde{\delta}_{u,d}(t)$ when divided by c_d, described in the continuous-time domain
$\tau_{u,d}(k_d)$	Discrete-time version of $\tilde{\tau}_{u,d}(t)$
$\tilde{\gamma}_{u,d}(t)$	Remainder of $\tilde{\delta}_{u,d}(t)$ when divided by c_d, described in the continuous-time domain
$\gamma_{u,d}(k_d)$	Discrete-time version of $\tilde{\gamma}_{u,d}(t)$
$\alpha^{\text{arriv}}_{u,d}(k_d)$	Rate of the vehicles that arrive at the tail of the waiting queue on link (u,d) in the time span $[k_d c_d,(k_d{+}1)c_d)$
$\alpha^{\text{enter}}_{u,d}(k_d)$	Rate of the vehicles that enter link (u,d) in the time span $[k_d c_d,(k_d{+}1)c_d)$
$\alpha^{\text{leave}}_{i,u,d}(k_d)$	Rate of the vehicles that leave link (i,u) towards link (u,d) in the time span $[k_d c_d,(k_d{+}1)c_d)$
$v^{\text{free}}_{u,d}$	Free-flow speed of vehicles on link (u,d)
$v^{\text{low}}_{u,d}$	Idling speed of vehicles in waiting queue on link (u,d)
$a^{\text{acc}}_{u,d}$	Acceleration rate of the vehicles on link (u,d)
$a^{\text{dec}}_{u,d}$	Deceleration rate of the vehicles on link (u,d)
$C_{u,d}$	Capacity of link (u,d) (i.e., the maximum number of vehicles that can be positioned on link (u,d))
$\tilde{q}_{u,d}(t)$	Length of the waiting queue on link (u,d) at time instant t in the continuous-time domain
$q_{u,d}(k_d)$	Discrete-time version of $\tilde{q}_{u,d}(t)$
$N^{\text{lane}}_{u,d}$	Number of lanes on link (u,d)
l^{veh}	Average length of vehicles in the urban traffic network
$E^{\text{acc}}_{\theta,i,u,d}(k_d)$	Average emission rate of pollutant θ for vehicle i traveling on link (u,d) within the acceleration time interval that corresponds to simulation time step k_d (i.e., the time interval of the accelerating behavior that occurs in the time span $[k_d c_d,(k_d{+}1))c_d)$
$E^{\text{dec}}_{\theta,i,u,d}(k_d)$	Average emission rate of pollutant θ for vehicle i traveling on link (u,d) within the deceleration time interval that corresponds to simulation time step k_d

may be missed or counted twice. Therefore, we propose to substitute Eq. (4) of [1] by the following general formulation:

$$\alpha_{u,d}^{\text{arriv}}(k_d)=\sum_{i=0}^{|\tau_{u,d}(k_d)+1-\tau_{u,d}(k_d+1)|} B_{u,d,i}(k_d)\cdot \alpha_{u,d}^{\text{enter}}\Big(k_d-\max\Big(\tau_{u,d}(k_d+1),\tau_{u,d}(k_d)+1\Big)+i\Big) \tag{4-corrected}$$

where, in general, for $i\in\mathbb{N}$ s.t. $i<|\tau_{u,d}(k_d)+1-\tau_{u,d}(k_d+1)|$:

$$B_{u,d,i}(k_d)=1.$$

Moreover, for $i=0$ and $i=|\tau_{u,d}(k_d)+1-\tau_{u,d}(k_d+1)|$:

- In case $\tau_{u,d}(k_d)+1>\tau_{u,d}(k_d+1)$ holds (**case 1**), then:

$$B_{u,d,0}(k_d)=\frac{\gamma_{u,d}(k_d)}{c_d},$$
$$B_{u,d,|\tau_{u,d}(k_d)+1-\tau_{u,d}(k_d+1)|}(k_d)=\frac{c_d-\gamma_{u,d}(k_d+1)}{c_d}.$$

- In case $\tau_{u,d}(k_d)+1<\tau_{u,d}(k_d+1)$ holds (**case 2**), then:

$$B_{u,d,0}(k_d)=-\frac{\gamma_{u,d}(k_d+1)}{c_d},$$
$$B_{u,d,|\tau_{u,d}(k_d)+1-\tau_{u,d}(k_d+1)|}(k_d)=-\frac{c_d-\gamma_{u,d}(k_d)}{c_d}.$$

- In case $\tau_{u,d}(k_d)+1=\tau_{u,d}(k_d+1)$ holds (**case 3** for $\gamma_{u,d}(k_d)>\gamma_{u,d}(k_d+1)$, **case 4** for $\gamma_{u,d}(k_d)<\gamma_{u,d}(k_d+1)$, and **case 5** for $\gamma_{u,d}(k_d)=\gamma_{u,d}(k_d+1)$), then since $|\tau_{u,d}(k_d)+1-\tau_{u,d}(k_d+1)|=0$, the only term that will appear on the right-hand side of (4-corrected) is $B_{u,d,0}(k_d)\cdot\alpha_{u,d}^{\text{enter}}(k_d-\tau_{u,d}(k_d+1))$, for which we have

$$B_{u,d,0}(k_d)=\frac{\gamma_{u,d}(k_d)-\gamma_{u,d}(k_d+1)}{c_d}.$$

The main idea behind (4-corrected) is to find for simulation time step k_d the cumulation of all the inflows of the vehicles that have entered link (u,d) for simulation time steps prior to k_d, and that reach the tail of the waiting queue on the link during the time interval $[k_dc_d,(k_d+1)c_d)$ (this gives the arrival rate $\alpha_{u,d}^{\text{arriv}}(k_d)$). For the following two reasons, we should switch from the discrete-time to the continuous-time domain:

- First, the S-model is a discrete-time model, and all the states are updated at discrete time steps. Hence, the computations for the state values for simulation time step k_d should include the dynamics of the urban traffic network until the next update, i.e., from simulation time step k_d, until simulation time step k_d+1, in order to include all the vehicle inflows that influence $\alpha_{u,d}^{\text{arriv}}(k_d)$. Therefore, we should look at the dynamics of the urban traffic network within the time interval $[k_dc_d,(k_d+1)c_d)$ in the continuous-time domain.
- Second, for derivation of $\alpha_{u,d}^{\text{arriv}}(k_d)$, we should look backwards in time (for all $t\in[k_dc_d,(k_d+1)c_d)$) to find the influential time interval $I(k_d)=[t^{\text{s}}(k_d),t^{\text{e}}(k_d))$ (with the start-point at $t^{\text{s}}(k_d)$ and the endpoint at $t^{\text{e}}(k_d)$) for which $\forall t\in I(k_d)$, the time step $t/c_d+\delta_{u,d}(k_d)$ lies in the time interval $[k_dc_d,(k_d+1)c_d)$. The time delay $\delta_{u,d}(k_d)$ is in general a real value composed of an integer quotient $\tau_{u,d}(k_d)$, and a remainder $\gamma_{u,d}(k_d)$ (non-negative real value), i.e.,

$$\delta_{u,d}(k_d)=\tau_{u,d}(k_d)c_d+\gamma_{u,d}(k_d), \tag{1}$$

where $\tau_{u,d}(k_d)$ is an integer and $0\leq\gamma_{u,d}(k_d)<c_d$. Since the remainder is in general non-zero, moving backwards in time from any point in the time interval $[k_dc_d,(k_d+1)c_d)$, the resulting point may lie in between two consecutive simulation time steps of the S-model.

Next, we will provide the proof of (4-corrected). According to the S-model, the corresponding continuous-time inflow function is assumed to be piecewise constant between every two consecutive discrete time steps k_d and k_d+1 (corresponding to time instants t and $t+c_d$ in the continuous-time domain; see Figure 1). Correspondingly, the influential inflow $\bar{\alpha}_{u,d}^{\text{enter}}(t)$ of the vehicles that contributes to the arriving flow at the tail of the waiting queue within the time interval $[t,t+c_d)$, is the cumulative inflow of the vehicles in between time instants $t-\tilde{\delta}_{u,d}(t)$ and $t+c_d-\tilde{\delta}_{u,d}(t+c_d)$. So we have[1]:

$$\bar{\alpha}_{u,d}^{\text{enter}}(t)=\frac{1}{c_d}\int_{t-\tilde{\delta}_{u,d}(t)}^{t+c_d-\tilde{\delta}_{u,d}(t+c_d)}\tilde{\alpha}_{u,d}^{\text{enter}}(\theta)\text{d}\theta, \tag{2}$$

with $\tilde{\alpha}_{u,d}^{\text{enter}}(\cdot)$ the continuous-time inflow function of link (u,d). Figure 1 shows the corresponding continuous-time inflow function of link (u,d) versus time for the five cases given before for (4-corrected) (i.e., **case 1-5**), and the corresponding influential inflow (indicated with a light/yellow colored curve extended across the influential time interval $I(k_d)$ in between time instants $t^{\text{s}}(k_d)$ and $t^{\text{e}}(k_d)$) at time instant t.

For **case 1** (see the top plot of Figure 1), Equation (2) can be expanded as:

$$\begin{aligned}
\bar{\alpha}_{u,d}^{\text{enter}}(t)=&\frac{1}{c_d}\int_{t-\tilde{\tau}_{u,d}(t)c_d-\tilde{\gamma}_{u,d}(t)}^{t-\tilde{\tau}_{u,d}(t)c_d}\tilde{\alpha}_{u,d}^{\text{enter}}(\theta)\text{d}\theta\\
&+\frac{1}{c_d}\int_{t-\tilde{\tau}_{u,d}(t)c_d}^{t-\tilde{\tau}_{u,d}(t)c_d+c_d}\tilde{\alpha}_{u,d}^{\text{enter}}(\theta)\text{d}\theta\\
&+\ldots\\
&+\frac{1}{c_d}\int_{t+c_d-\tilde{\tau}_{u,d}(t+c_d)c_d-2c_d}^{t+c_d-\tilde{\tau}_{u,d}(t+c_d)c_d-c_d}\tilde{\alpha}_{u,d}^{\text{enter}}(\theta)\text{d}\theta\\
&+\frac{1}{c_d}\int_{t+c_d-\tilde{\tau}_{u,d}(t+c_d)c_d-c_d}^{t+c_d-\tilde{\tau}_{u,d}(t+c_d)c_d-\tilde{\gamma}_{u,d}(t+c_d)}\tilde{\alpha}_{u,d}^{\text{enter}}(\theta)\text{d}\theta\\
=&\frac{\tilde{\gamma}_{u,d}(t)}{c_d}\tilde{\alpha}_{u,d}^{\text{enter}}(t-(\tilde{\tau}_{u,d}(t)+1)c_d)\\
&+\frac{1}{c_d}\cdot c_d\sum_{i=1}^{\tilde{\tau}_{u,d}(t)-\tilde{\tau}_{u,d}(t+c_d)}\tilde{\alpha}_{u,d}^{\text{enter}}(t-(\tilde{\tau}_{u,d}(t)+1)c_d+ic_d)\\
&+\frac{c_d-\tilde{\gamma}_{u,d}(t+c_d)}{c_d}\tilde{\alpha}_{u,d}^{\text{enter}}(t+c_d-\tilde{\tau}_{u,d}(t+c_d)c_d-c_d).
\end{aligned}$$

Substituting t and $t+c_d$ by their discrete-time equivalents k_d and k_d+1, (4-corrected) is obtained, noting that in this case $\max(\tau_{u,d}(k_d+1),\tau_{u,d}(k_d)+1)=\tau_{u,d}(k_d)+1$.

[1]Note that in this paper, we use a tilde symbol for the continuous-time version of a discrete-time variable/function in the S-model.

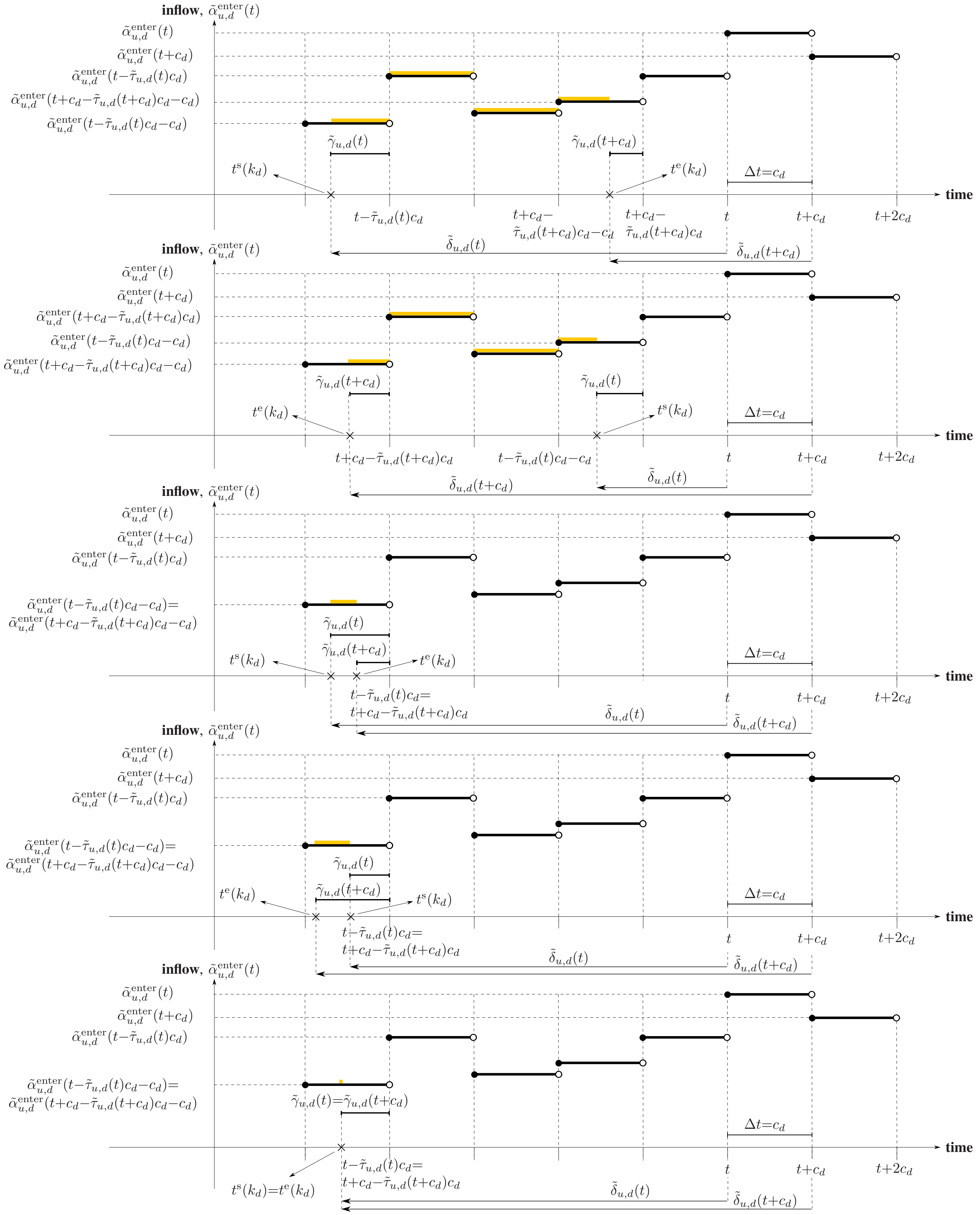

Fig. 1. Effective inflow ($\tilde{\alpha}_{u,d}^{\text{enter}}(t)$ within the influential time interval $I(k_d)$=[$t^{\text{s}}(k_d)$,$t^{\text{e}}(k_d)$)) during [t,t+c_d) for **cases 1-5** from top to bottom respectively.

Similarly, for **case 2** (see the plot in the second row of Figure 1), we have[2]:

$$\begin{aligned}
\bar{\alpha}^{\text{enter}}_{u,d}(t)=&-\frac{1}{c_d}\int_{t+c_d-\tilde{\tau}_{u,d}(t+c_d)c_d-\tilde{\gamma}_{u,d}(t+c_d)}^{t+c_d-\tilde{\tau}_{u,d}(t+c_d)c_d}\tilde{\alpha}^{\text{enter}}_{u,d}(\theta)\mathrm{d}\theta\\
&-\frac{1}{c_d}\int_{t+c_d-\tilde{\tau}_{u,d}(t+c_d)c_d}^{t+c_d-\tilde{\tau}_{u,d}(t+c_d)c_d+c_d}\tilde{\alpha}^{\text{enter}}_{u,d}(\theta)\mathrm{d}\theta\\
&-\ldots\\
&-\frac{1}{c_d}\int_{t-\tilde{\tau}_{u,d}(t)c_d-2c_d}^{t-\tilde{\tau}_{u,d}(t)c_d-c_d}\tilde{\alpha}^{\text{enter}}_{u,d}(\theta)\mathrm{d}\theta\\
&-\frac{1}{c_d}\int_{t-\tilde{\tau}_{u,d}(t)c_d-c_d}^{t-\tilde{\tau}_{u,d}(t)c_d-\tilde{\gamma}_{u,d}(t)}\tilde{\alpha}^{\text{enter}}_{u,d}(\theta)\mathrm{d}\theta\\
=&-\frac{\tilde{\gamma}_{u,d}(t+c_d)}{c_d}\tilde{\alpha}^{\text{enter}}_{u,d}(t+c_d-(\tilde{\tau}_{u,d}(t+c_d)+1)c_d)\\
&-\frac{1}{c_d}\cdot c_d\sum_{i=1}^{\substack{\tilde{\tau}_{u,d}(t+c_d)-\\ \tilde{\tau}_{u,d}(t)-2}}\tilde{\alpha}^{\text{enter}}_{u,d}(t+c_d-(\tilde{\tau}_{u,d}(t+c_d)+1)c_d+ic_d)\\
&-\frac{c_d-\gamma_{u,d}(t)}{c_d}\tilde{\alpha}^{\text{enter}}_{u,d}(t-\tilde{\tau}_{u,d}(t)c_d-c_d).
\end{aligned}$$

The discrete-time equivalent of the above expression is (4-corrected), as $\max(\tau_{u,d}(k_d+1),\tau_{u,d}(k_d)+1)=\tau_{u,d}(k_d+1)$.

For **case 3** (see the plot in the third row of Figure 1), Eq. (2) will be expanded as:

$$\bar{\alpha}^{\text{enter}}_{u,d}(t)=\frac{1}{c_d}\int_{t-\tilde{\tau}_{u,d}(t)c_d-\tilde{\gamma}_{u,d}(t)}^{t+c_d-\tilde{\tau}_{u,d}(t+c_d)c_d-\tilde{\gamma}_{u,d}(t+c_d)}\tilde{\alpha}^{\text{enter}}_{u,d}(\theta)\mathrm{d}\theta,$$

which, since in this case $\tau_{u,d}(k_d)+1=\tau_{u,d}(k_d+1)$, or equivalently $\tilde{\tau}_{u,d}(t)c_d+c_d=\tilde{\tau}_{u,d}(t+c_d)c_d$, simplifies to

$$\bar{\alpha}^{\text{enter}}_{u,d}(t)=\frac{\gamma_{u,d}(t)-\gamma_{u,d}(t+c_d)}{c_d}\tilde{\alpha}^{\text{enter}}_{u,d}(t-\tilde{\tau}_{u,d}(t)c_d-c_d). \quad (3)$$

Similarly, for **case 4** (see the plot in the fourth row of Figure 1), we obtain the same formulation as (3) (noting that the coefficient $(\gamma_{u,d}(t)-\gamma_{u,d}(t+c_d))/c_d$ in **case 3** is positive, while in **case 4** it is negative.

Finally, for **case 5** (see the plot in the last row of Figure 1) the area below the curve of $\tilde{\alpha}^{\text{enter}}_{u,d}$ in between time instants $t-\tilde{\delta}_{u,d}(t)$ and $t+c_d-\tilde{\delta}_{u,d}(t+c_d)$ is zero, and hence the influential inflow, using (2), will become zero.

Remark 2: Eq. (4) in [1] is derived based on a method given in [5]. The assumption in [5], however, in addition to a time-invariant time delay is that the time delay is less than or equal to the simulation sampling time c_d. Referring to the proof given above, Eq. (4-corrected) proposed in this short paper is general and is not based on this restrictive assumption.

B. Corrections to Eq. (5) of [1]

Remark 3: In Eq. (5) of [1], the free-flow speed is defined per link, while later on the free-flow and idling speeds, and the acceleration and deceleration rates are considered to be fixed for all the links in the entire urban traffic network. In this short paper, for consistency and for generalization, we consider these parameters as functions of links, and use the notations $v^{\text{free}}_{u,d}$, $v^{\text{low}}_{u,d}$, $a^{\text{acc}}_{u,d}$, and $a^{\text{dec}}_{u,d}$, correspondingly.

Eq. (5) in [1] has been derived based on the assumption that all vehicles enter link (u,d) with the free-flow speed $v^{\text{free}}_{u,d}$, and will reach the tail of the waiting queue with the same speed (i.e., vehicles follow a uniform speed motion all the time). However, since the speed of the waiting queue on link (u,d) is assumed to be equal to the idling speed $v^{\text{low}}_{u,d}$, the assumption given in [1] is unrealistic, resulting in errors up to 50% for the estimated values of $\delta_{u,d}(k_d)$, and hence $\tau_{u,d}(k_d)$ and $\gamma_{u,d}(k_d)$. In reality, vehicles should decelerate at the right time in order to reach the speed of the waiting queue at the time instant they arrive at the tail of the queue. Moreover, depending on the distance $\Delta\tilde{x}_{u,d}(t)$ that has been traveled by a vehicle that reaches the tail of the waiting queue at time instant t, there may be cases where vehicles should enter the link with an initial speed that is lower than the free-flow speed $v^{\text{free}}_{u,d}$.

In order to correct (5) in [1], we propose a different reasoning: we assume that the kinematics of a vehicle that arrives at the tail of the waiting queue on link (u,d) at time instant t, follows either of the three speed-position curves shown in the first column of Figure 2, and their corresponding speed-time curves in the second column of the figure. We define $\bar{X}_{u,d}$ as the distance traveled by a vehicle on link (u,d) that decelerates with the constant rate $a^{\text{dec}}_{u,d}$ from the free-flow speed $v^{\text{free}}_{u,d}$ to the idling speed $v^{\text{low}}_{u,d}$. The corresponding travel time is $\left(v^{\text{low}}_{u,d}-v^{\text{free}}_{u,d}\right)/a^{\text{dec}}_{u,d}$. Therefore,

$$\bar{X}_{u,d}=\frac{1}{2}a^{\text{dec}}_{u,d}\left(\frac{v^{\text{low}}_{u,d}-v^{\text{free}}_{u,d}}{a^{\text{dec}}_{u,d}}\right)^2+v^{\text{free}}_{u,d}\frac{v^{\text{low}}_{u,d}-v^{\text{free}}_{u,d}}{a^{\text{dec}}_{u,d}}. \quad (4)$$

- In case $\Delta\tilde{x}_{u,d}(t)>\bar{X}_{u,d}$ (**case A**), which is illustrated in the first row of Figure 2, the position $x^{\text{enter}}_{u,d}$ of the link entrance and the position $x^{\text{Qtail}}_{u,d}(t)$ of the tail of the queue at time instant t is large enough so that the vehicle first moves a distance of $X^{\text{free}}_{u,d}(t)$ within $T^{\text{free}}_{u,d}(t)$ time units with the free-flow speed $v^{\text{free}}_{u,d}$, and then switches to a constant-deceleration movement with $a^{\text{dec}}_{u,d}$ to travel the remainder $X^{\text{dec},1}_{u,d}$ of the distance within $T^{\text{dec},1}_{u,d}$ time units. Note that $X^{\text{dec},1}_{u,d}$ in Figure 2 is equal to $\bar{X}_{u,d}$.
- In case $\Delta\tilde{x}_{u,d}(t)=\bar{X}_{u,d}$ (**case B**), which is illustrated in the second row of Figure 2, immediately after entering link (u,d), the vehicle should decelerate with $a^{\text{dec}}_{u,d}$ from the free-flow speed $v^{\text{free}}_{u,d}$ so that in $T^{\text{dec},2}_{u,d}$ time units, when it reaches the tail of the waiting queue on the link, its speed has reached $v^{\text{low}}_{u,d}$. Note that $X^{\text{dec},2}_{u,d}$ in Figure 2 is equal to $\bar{X}_{u,d}$.
- Finally, in case $\Delta\tilde{x}_{u,d}(t)<\bar{X}_{u,d}$ (**case C**), which is illustrated in the third row of Figure 2, the vehicle should enter the link with an initial speed $v^{\text{init}}_{u,d}(t)$ lower than the free-flow speed, and in $T^{\text{dec},3}_{u,d}(t)$ time units reaches the tail of the waiting queue, after traveling a distance of $X^{\text{dec},3}_{u,d}(t)$.

[2]Note that for **case 2** and **case 4**, $t^{\text{s}}(k_d)>t^{\text{e}}(k_d)$ holds (see Figure 1), which results in a negative value for the integral of (2).

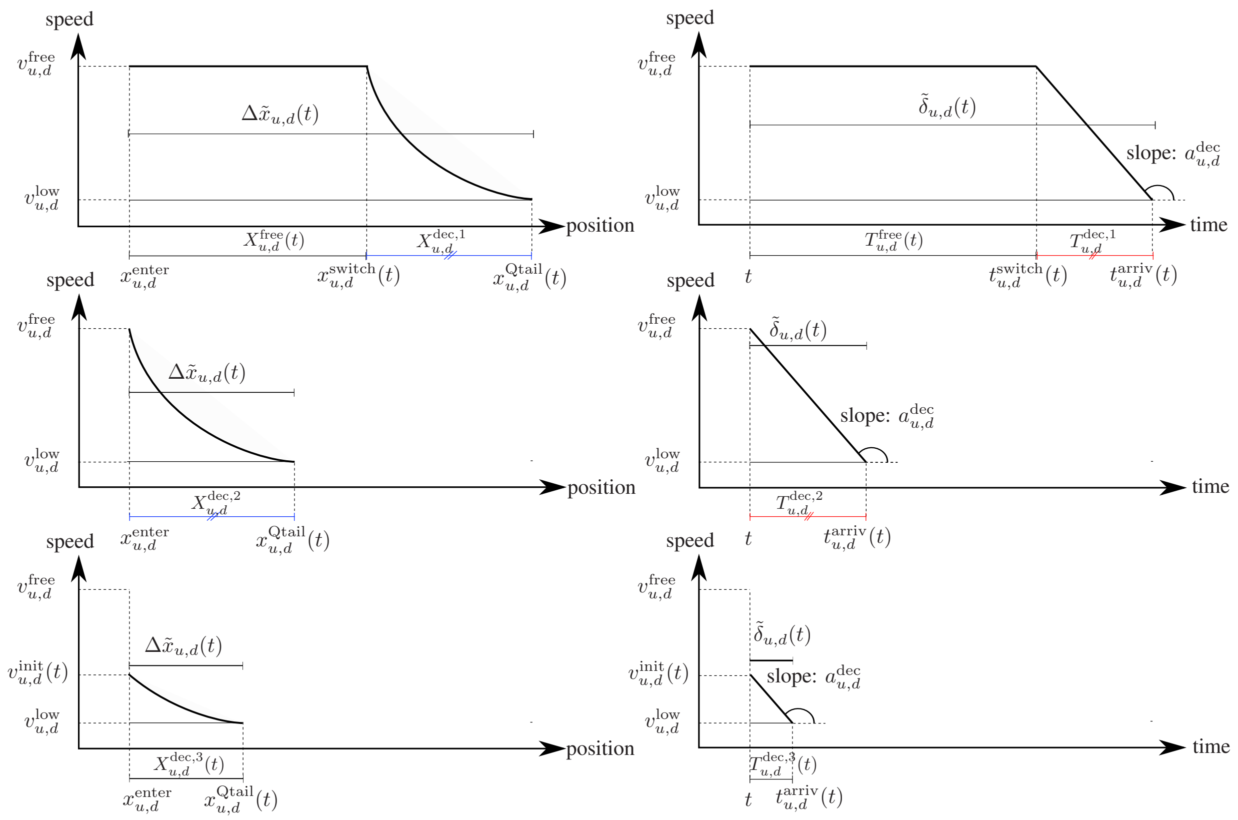

Fig. 2. Speed-position and corresponding speed-time curves for a vehicle that enters link (u,d) at time instant t, for **cases A-B** from top to bottom respectively.

For all these three cases, we have:

$$\Delta\tilde{x}_{u,d}(t)=\frac{(C_{u,d}-\tilde{q}_{u,d}(t))}{N^{\mathrm{lane}}_{u,d}}l^{\mathrm{veh}}. \tag{5}$$

For **case A**, we can write:

$$\begin{aligned}\Delta\tilde{x}_{u,d}(t)&=X^{\mathrm{free}}_{u,d}(t)+X^{\mathrm{dec},1}_{u,d}\\&=v^{\mathrm{free}}_{u,d}T^{\mathrm{free}}_{u,d}(t)+\frac{1}{2}a^{\mathrm{dec}}_{u,d}\left(T^{\mathrm{dec},1}_{u,d}\right)^2+v^{\mathrm{free}}_{u,d}T^{\mathrm{dec},1}_{u,d},\end{aligned} \tag{6}$$

with $T^{\mathrm{dec},1}_{u,d}=\left(v^{\mathrm{low}}_{u,d}-v^{\mathrm{free}}_{u,d}\right)/a^{\mathrm{dec}}_{u,d}$. Moreover, for this case $\tilde{\delta}_{u,d}(t)=T^{\mathrm{free}}_{u,d}(t)+T^{\mathrm{dec},1}_{u,d}$ (where $T^{\mathrm{free}}_{u,d}(t)$ is obtained solving (6) for this variable). Therefore:

$$\tilde{\delta}_{u,d}(t)=\frac{(C_{u,d}-\tilde{q}_{u,d}(t))}{N^{\mathrm{lane}}_{u,d}v^{\mathrm{free}}_{u,d}}l^{\mathrm{veh}}-\frac{\left(v^{\mathrm{low}}_{u,d}-v^{\mathrm{free}}_{u,d}\right)^2}{2a^{\mathrm{dec}}_{u,d}v^{\mathrm{free}}_{u,d}}. \tag{7}$$

For **case B**, $T^{\mathrm{dec},2}_{u,d}=\left(v^{\mathrm{low}}_{u,d}-v^{\mathrm{free}}_{u,d}\right)/a^{\mathrm{dec}}_{u,d}$. Moreover, for this case we have $\tilde{\delta}_{u,d}(t)=T^{\mathrm{dec},2}_{u,d}$. Hence,

$$\tilde{\delta}_{u,d}(t)=\left(v^{\mathrm{low}}_{u,d}-v^{\mathrm{free}}_{u,d}\right)/a^{\mathrm{dec}}_{u,d}. \tag{8}$$

Finally, for **case C**, where $v^{\mathrm{init}}_{u,d}(t)<v^{\mathrm{free}}_{u,d}$, we obtain

$$\Delta\tilde{x}_{u,d}(t)=\frac{1}{2}a^{\mathrm{dec}}_{u,d}\left(T^{\mathrm{dec},3}_{u,d}(t)\right)^2+v^{\mathrm{init}}_{u,d}(t)T^{\mathrm{dec},3}_{u,d}(t), \tag{9}$$

where, from the speed-time curve in the last row of Figure 2, we have $v^{\mathrm{init}}_{u,d}(t)=v^{\mathrm{low}}_{u,d}-a^{\mathrm{dec}}_{u,d}T^{\mathrm{dec},3}_{u,d}(t)$. Therefore, (9) becomes a quadratic equation that can be solved to determine the variable $T^{\mathrm{dec},3}_{u,d}(t)$. Moreover, in this case $\tilde{\delta}_{u,d}(t)=T^{\mathrm{dec},3}_{u,d}(t)$. Therefore,

$$\begin{aligned}\tilde{\delta}_{u,d}(t)=&v^{\mathrm{low}}_{u,d}/a^{\mathrm{dec}}_{u,d}+\\&\left(\left(v^{\mathrm{low}}_{u,d}/a^{\mathrm{dec}}_{u,d}\right)^2-2(C_{u,d}-\tilde{q}_{u,d}(t))l^{\mathrm{veh}}/(a^{\mathrm{dec}}_{u,d}N^{\mathrm{lane}}_{u,d})\right)^{0.5}.\end{aligned} \tag{10}$$

Equations (7), (8), and (10) have been derived considering the motion of a single vehicle that arrives at the tail of the waiting queue on link (u,d) at time instant t (i.e., a microscopic point-of-view). In order to transfer these equations into macroscopic ones, $\tilde{q}_{u,d}(t)$ can be substituted by $q^{\mathrm{ave}}_{u,d}(k_d)$, which is the average queue length computed within the time interval $[k_dc_d,(k_d+1)c_d)$. We refer the readers to [3] for estimation of $q^{\mathrm{ave}}_{u,d}(k_d)$. Finally, Eq. (5) in [1] should be corrected correspondingly, using $\delta_{u,d}(k_d)$ obtained for either of the three given cases, via the discrete-time version of Equations (7), (8), or (10), where $\tilde{q}_{u,d}(t)$ in (7) and (10) has been substituted by $q^{\mathrm{ave}}_{u,d}(k_d)$.

Remark 4: The conditions that define **cases A-C** have been defined based on the continuous-time variable $\Delta\tilde{x}_{u,d}(t)$, which is computed via (5). In order to transfer this continuous-time variable into its discrete-time version, $\tilde{q}_{u,d}(t)$ in (5) should be substituted by the discrete-time variable $q^{\mathrm{ave}}_{u,d}(k_d)$ (see the explanations given above).

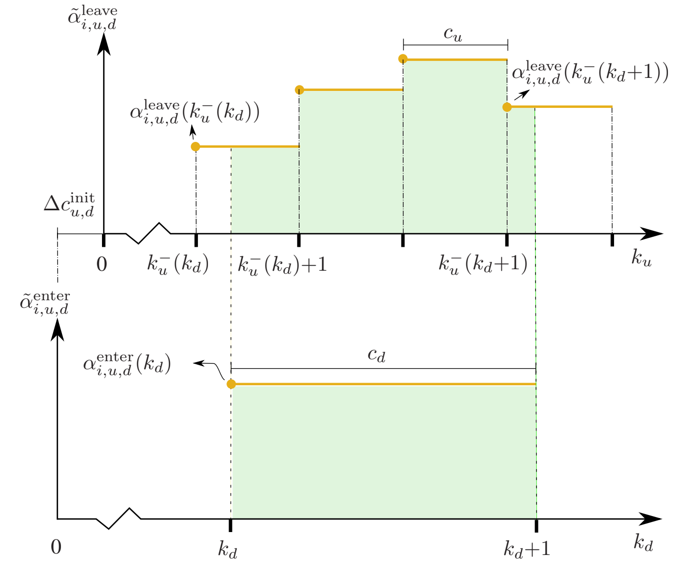

Fig. 3. Synchronization of the joint variables for connected links.

C. Corrections and extensions to Eq. (9) of [1]

Eq. (9) in [1] should be corrected as

$$\alpha^{\text{enter}}_{i,u,d}(k_d)=\frac{1}{c_d}\int_{k_d c_d}^{(k_d+1)c_d}\tilde{\alpha}^{\text{leave}}_{i,u,d}(t)\mathrm{d}t. \qquad \text{(9-corrected)}$$

Note that $\alpha^{\text{enter}}_{i,u,d}$, i.e., the flow rate of the vehicles that enter link (u,d) (from link (i,u)) is updated at simulation time step k_d. Furthermore, $\alpha^{\text{enter}}_{i,u,d}$ is equivalent to the flow rate $\alpha^{\text{leave}}_{i,u,d}$ of the vehicles that leave link (i,u) (towards link (u,d)), which is updated at simulation time step k_u. In the S-model, $\alpha^{\text{leave}}_{i,u,d}$ is first computed (see Eq. (2) in [1]) and $\alpha^{\text{enter}}_{i,u,d}$ is computed afterwards via $\alpha^{\text{leave}}_{i,u,d}$ (see Eq. (6) in [1]). The main issue is that, in general, k_d and k_u may not always be synchronized. We use the notation $\Delta c^{\text{init}}_{u,d}$ for the offset between the cycle times of the traffic signals at the upstream and downstream intersections of link (u,d) measured at simulation time step $k_d{=}0$ (see Figure 3). More specifically, $\Delta c^{\text{init}}_{u,d}$ is the time span from the start of the simulation of link (u,d) until the beginning of the first upcoming cycle of link (i,u). Note that the corresponding simulation time step of link (i,u) is indicated by $k_u{=}0$ in Figure 3. Furthermore, we define $k_u^-(k_d)$ as the most recent simulation time step of link (i,u) prior to simulation time step k_d of link (u,d). We have (see Figure 3):

$$\begin{aligned}&k_u^-(k_d)c_u+\Delta c^{\text{init}}_{u,d}\le k_d c_d<\left(k_u^-(k_d)+1\right)c_d+\Delta c^{\text{init}}_{u,d} \Rightarrow\\ &k_u^-(k_d)\le k_d c_d/c_u-\Delta c^{\text{init}}_{u,d}/c_u<k_u^-(k_d)+1 \Rightarrow\\ &k_u^-(k_d)=\left\lfloor k_d c_d/c_u-\Delta c^{\text{init}}_{u,d}/c_u\right\rfloor.\end{aligned}$$

If $\alpha^{\text{enter}}_{i,u,d}(\cdot)$ is a piecewise constant function in the continuous-time domain (see Figure 3), the average entering flow rate of the vehicles $\alpha^{\text{enter}}_{i,u,d}(k_d)$ within $[k_d c_d,(k_d+1)c_d)$ is computed via (9-corrected), which can be simplified as

$$\begin{aligned}\alpha^{\text{enter}}_{i,u,d}(k_d)=&\frac{1}{c_d}\alpha^{\text{leave}}_{i,u,d}\left(k_u^-(k_d)\right)\cdot\\ &\left(\min\left((k_d+1)c_d,\left(k_u^-(k_d)+1\right)c_u+\Delta c^{\text{init}}_{u,d}\right)-k_d c_d\right)+\\ &\frac{1}{c_d}\sum_{k_u=k_u^-(k_d)+1}^{k_u^-(k_d+1)-1}\alpha^{\text{leave}}_{i,u,d}(k_u)\cdot c_u+\\ &\frac{1}{c_d}\alpha^{\text{leave}}_{i,u,d}(k_u^-(k_d+1))\cdot\\ &\left((k_d+1)c_d-\min\left((k_d+1)c_d,k_u^-(k_d+1)c_u+\Delta c^{\text{init}}_{u,d}\right)\right).\end{aligned} \qquad (11)$$

The mathematical expression given by (11) has been derived based on the following reasoning. The first term of the summation in (11) is $1/c_d$ times the area of the first (from left) rectangle in the top plot of Figure 3. The right-hand side edge of this rectangle is positioned at $k_u{=}k_u^-(k_d){+}1$. Therefore, in case $(k_u^-(k_d)+1)c_u+\Delta c^{\text{init}}_{u,d}<(k_d+1)c_d$, then, from Figure 3, the width of this rectangle is $(k_u^-(k_d)+1)c_u+\Delta c^{\text{init}}_{u,d}-k_d c_d$; otherwise, in case $(k_u^-(k_d)+1)c_u+\Delta c^{\text{init}}_{u,d}\ge(k_d+1)c_d$, which indicates that this rectangle covers the entire time interval $[k_d c_d,(k_d+1)c_d)$, then the width of the rectangle is $(k_d+1)c_d-k_d c_d$. This explains the use of the minimum function in the first term of the right-hand side expression of (11).

The second term is $1/c_d$ times the summation of the areas of all those rectangles that entirely lie within $[k_d c_d,(k_d+1)c_d)$, and their left-hand side edge is necessarily positioned at the right-hand side of the dashed line crossing k_d in Figure 3.

Finally, the last term of (11) is $1/c_d$ times the area of the last rectangle in the top plot of Figure 3. Note that this term is non-zero only in case $k_u{=}k_u^-(k_d{+}1)$ lies at the left-hand side of the dashed line crossing $k_d{+}1$ in the figure, where the width of the rectangle equals $(k_d+1)c_d-\left(k_u^-(k_d+1)c_u+\Delta c^{\text{init}}_{u,d}\right)$. Otherwise, the area of this rectangle has already been computed (see the above explanations). This explains the use of the minimum function in the last term of (11).

Note that in [1], instead of the initial offset $\Delta c^{\text{init}}_{u,d}$ of cycles c_d and c_u, the offset $\Delta c_{u,d}(k_d)$ of these cycles at simulation time step k_d is used. Note that $\Delta c^{\text{init}}_{u,d}$ is a fixed value, initially known at $k_d{=}0$, while $\Delta c_{u,d}(k_d)$ is time-varying. Therefore, in addition to correcting the formulation of Eq. (9) in [1] using (9-corrected), by introducing and using $\Delta c^{\text{init}}_{u,d}$, we have also eliminated the need for computation of $\Delta c_{u,d}(k_d)$ at every simulation time step in order to synchronize $\alpha^{\text{leave}}_{i,u,d}$ and $\alpha^{\text{enter}}_{i,u,d}$.

D. Corrections to Eq. (13) of [1]

In Eq. (13) in [1], there is a mismatch between the units on two sides of the equality sign (i.e., the left-hand side of the equation gives the average emission rate with the unit [kg/s], while the right-hand side of the equation has the unit [kg·m/s^3])). This issue may originate from a mistake in substituting the integral's variable from time to speed. Eq. (13) in [1] should be substituted by

$$E^{\text{dec}}_{\theta,i,u,d}(k_d)=\frac{1}{v^{\text{low}}_{u,d}-v^{\text{free}}_{u,d}}\int_{v^{\text{free}}_{u,d}}^{v^{\text{low}}_{u,d}}E_{\theta,i,u,d}(v,a^{\text{dec}}_{u,d})\mathrm{d}v. \qquad \text{(13-corrected)}$$

The reason is given next. Define $t_{i,u,d}^{\mathrm{free}}$ and $t_{i,u,d}^{\mathrm{low}}$ as the time instants at which the speed of the vehicle i is $v_{u,d}^{\mathrm{free}}$ and $v_{u,d}^{\mathrm{low}}$, respectively. Since $E_{\theta,i,u,d}\left(v,a_{u,d}^{\mathrm{dec}}\right)$ denotes the instantaneous emission rate of vehicle i, the average emission rate of vehicle i within the deceleration time interval $\left[t_{i,u,d}^{\mathrm{free}},t_{i,u,d}^{\mathrm{low}}\right]$ that corresponds to the simulation time step k_d (more specifically, $k_d c_d \le t_{i,u,d}^{\mathrm{free}} \le t_{i,u,d}^{\mathrm{low}} \le (k_d+1)c_d$) is computed by

$$E_{\theta,i,u,d}^{\mathrm{dec}}(k_d)=\frac{1}{t_{i,u,d}^{\mathrm{low}}-t_{i,u,d}^{\mathrm{free}}}\int_{t_{i,u,d}^{\mathrm{free}}}^{t_{i,u,d}^{\mathrm{low}}} E_{\theta,i,u,d}\left(v_i(t),a_{u,d}^{\mathrm{dec}}\right)\mathrm{d}t,$$

with $v_i(t)$ the speed of vehicle i at time instant t. If the vehicle decelerates with a constant rate $a_{u,d}^{\mathrm{dec}}$ from $v_{u,d}^{\mathrm{free}}$ to $v_{u,d}^{\mathrm{low}}$, we have

$$v_i(t)=a_{u,d}^{\mathrm{dec}}\left(t-t_{i,u,d}^{\mathrm{free}}\right)+v_{u,d}^{\mathrm{free}}.$$

Hence, $\mathrm{d}t=\frac{1}{a_{u,d}^{\mathrm{dec}}}\mathrm{d}v$. Substitution of the integral's limits and the integral's variable give (13-corrected).

Remark 5: Since the speeds and the acceleration and deceleration rates are link-dependent in this erratum, the average emission rate is also link-dependent.

E. Corrections to Eq. (14)–(16) of [1]

With a similar reasoning as for (13), we can see that Eq. (14)–(16) should be substituted by

$$E_{\theta,i,u,d}^{\mathrm{acc}}(k_d)=\frac{1}{v_{u,d}^{\mathrm{free}}-v_{u,d}^{\mathrm{low}}}\int_{v_{u,d}^{\mathrm{low}}}^{v_{u,d}^{\mathrm{free}}} E_{\theta,i,u,d}\left(v,a_{u,d}^{\mathrm{acc}}\right)\mathrm{d}v, \tag{14-corrected}$$

$$E_{\theta,i,u,d}^{\mathrm{acc}}(k_d)=\frac{1}{v_{u,d}^{\mathrm{agg}}-v_{u,d}^{\mathrm{free}}}\int_{v_{u,d}^{\mathrm{free}}}^{v_{u,d}^{\mathrm{agg}}} E_{\theta,i,u,d}\left(v,a_{u,d}^{\mathrm{acc}}\right)\mathrm{d}v, \tag{15-corrected}$$

$$E_{\theta,i,u,d}^{\mathrm{dec}}(k_d)=\frac{1}{v_{u,d}^{\mathrm{con}}-v_{u,d}^{\mathrm{free}}}\int_{v_{u,d}^{\mathrm{free}}}^{v_{u,d}^{\mathrm{con}}} E_{\theta,i,u,d}\left(v,a_{u,d}^{\mathrm{dec}}\right)\mathrm{d}v. \tag{16-corrected}$$

F. Further extensions (origin queues)

Further extensions to the S-model have been proposed in [2] and [3] to allow the S-model to model the origin queues. With the inclusion of origin queues, those vehicles that intend to enter an urban traffic network, but cannot do so due to the fully occupied origin links, are stored in queues at the origins of the traffic network (instead of being inserted in the link anyway, as in [1], even if the capacity of the origin links is reached). For more details, we refer the readers to [2] and [3].

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