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Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: https://www.dcsc.tudelft.nl

A Multiple-Model Reliability Prediction Approach for Condition-Based Maintenance

K. Verbert, B. De Schutter, R. Babuška

Abstract—**Numerous prognostic methods have been developed, aiming at predicting future system reliability with the highest possible accuracy. It is striking that the relation with the subsequent maintenance optimization process is generally overlooked, while it is important in practice. Additionally, almost all existing methods are based on a single degradation measure, and focus on systems with only one degradation and failure mode. In practice, however, multiple degradation measures are often available and needed to adequately predict future system degradation. Moreover, systems may suffer from various kinds of faults, all resulting in different degradation behaviors. To accommodate these properties, we establish a link between failure prognosis and maintenance optimization, and accordingly propose a multivariate multiple-model approach to system reliability prediction. We conclude that in the presence of multiple degradation modes and provided they are correctly identified, a multiple-model approach outperforms a single-model approach with respect to prediction accuracy. Moreover, in the presence of multiple degradation and failure modes, overall predictions of the remaining useful life as generated by common prognostic approaches are not directly suited for maintenance decision making, as different kinds of system failures and maintenance activities are associated with different costs. In contrast, our approach yields conditional predictions of future system reliability, which much better suit the maintenance optimization process.**

Index Terms—**Reliability prediction; condition-based maintenance; degradation modeling; prognostics; multivariate time series analysis; recursive Bayesian filtering.**

I. Introduction

CONDITION-BASED maintenance is an increasingly popular maintenance strategy in which maintenance activities are planned based on information collected through real-time condition monitoring. Its promise is twofold [1]: first, unnecessary maintenance can be eliminated, reducing maintenance costs and downtime; second, failures can be avoided, improving safety and reducing unscheduled downtime.

Condition-based maintenance comprises[1]:

1) fault diagnosis;
2) failure prognosis;
3) maintenance optimization.

Fault diagnosis concerns the detection of faulty behavior and the determination of its cause(s). *Failure prognosis* refers to the prediction of future degradation behavior and the estimation of the associated failure time. Finally, *maintenance optimization* comprises the determination of the optimal time and type of maintenance. The focus of this work is on prognostics, i.e. modeling and forecasting of degradation behavior.

A. Motivation

Although in recent years a lot of attention has been devoted to failure prognosis, failure prognosis is still an emerging research area with a number of open challenges [3]–[5]. In this paper, we address two of them. *The first challenge* is to establish a link between failure prognosis and maintenance optimization [3], [5]. Most existing prognostic methods have been developed without explicitly considering how the method is going to be used for maintenance optimization [5]. Accordingly, most existing methods for condition-based maintenance optimization base their maintenance decisions solely on diagnostic information, without considering prognostic information [6]. Although the link to the subsequent maintenance planning process is often overlooked, it is important in practice [3], [5]. *The second challenge* is the development of methods that can deal with multiple degradation modes and multiple degradation measures [4]. Most existing methods are based on a single degradation measure and account for just one degradation mode. In practice, multiple degradation measures are often available. Moreover, systems may suffer from various kinds of faults, all resulting in different degradation behaviors. Therefore, improvement in prediction accuracy is expected when considering multiple degradation measures and accounting for variability due to different fault causes.

B. Literature review

Over the past few years, various prognostic methods have been proposed, ranging from model-based approaches to artificial neural networks and stochastic filtering approaches. Overviews of the various prognostic methods can be found in the review papers [3], [4], [7], [8].

Especially statistical approaches have received a lot of attention in the literature thanks to their ability to handle the uncertainty inherent to the degradation forecasting process. For instance, (hidden) Markov models [9]–[11] and models based on gamma [12], [13] and Wiener processes [14]–[16] have frequently been proposed for prognostic purposes. Nevertheless, most of the existing methods take only part of the uncertainty into account. For example, the authors of [15], [17] omit measurement variability, while the authors of [18], [19] consider measurement variability, but omit the case-to-case or temporal variability. Recently, in [16], [20] methods have been proposed that take both measurement uncertainty, temporal variability, and case-to-case variability into account.

K. Verbert, B. De Schutter, and R. Babuška are with the Delft Center for Systems and Control (DCSC), Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands. Email: {k.a.j.verbert, b.deschutter, r.babuska}@tudelft.nl.

[1]According to [2], we define a fault as an unpermitted deviation in the system operation that does not hinder the execution of the system tasks, whereas a failure indicates that at least one system task can no longer be executed properly.

These methods are however developed for the univariate case and do not account for *modeling uncertainty*.

Almost all existing prognostic methods are based on a single degradation measure. An exception is the approach proposed in [21]. However, this method is not suited for systems subject to different degradation modes. Moreover, in [21] no distinction is made between different types of possible relationships among the degradation measures (e.g. between redundant and complementary measures).

C. Contributions

To address the aforementioned challenges, we propose a multivariate multiple-model approach to degradation forecasting and reliability prediction. *Multiple models* are considered because degradation can be caused by various system faults. In general, for different system faults, system degradation evolves differently over time. So, to adequately model degradation behavior in the presence of multiple degradation modes, a distinct (multivariate) degradation model is needed for each degradation mode.

A *multivariate approach* is considered because in practice often multiple degradation measures are available and required to adequately model the degradation process. In the case of a single degradation measure, system reliability[2] follows from comparing the predicted degradation signal with a predefined threshold value. In the case of multiple degradation measures, the system reliability is less straightforward to determine.

To manage and represent the uncertainty inherent to degradation forecasting, a statistical approach is considered for degradation forecasting. More specifically, we consider stochastic state space models, which can include most common uncertainty sources inherent to the forecasting process, i.e., temporal uncertainty, case-to-case (or sampling) variability, and measurement uncertainty [16], [22]. In addition, Bayesian filtering and prediction are used to estimate and forecast system degradation.

In summary, the contributions of this paper are:

- We establish a link between failure prognosis and the subsequent maintenance optimization (Section III);
- We propose a multiple-model approach to multivariate degradation forecasting, taking both temporal, sampling, and measurement uncertainty into account (Section IV);
- For the multivariate case, we provide definitions of the (conditional) system reliability that are in line with the subsequent maintenance optimization process, together with a framework to determine these prognostic measures (Section V).

Before we elaborate on the proposed method, in Section II we introduce the terminology, the adopted assumptions, and the research goals and discuss two motivating cases.

II. Problem formulation

A. Terminology

In the sequel, the following terminology is used (see Figure 1). First, we made a distinction between three types of system behavior:

1) healthy behavior;
2) faulty behavior;
3) system failure.

Healthy behavior refers to the situation in which the system functions as desired. *Faulty behavior* describes the situation in which the system exhibits some aberrant behavior, but is still functional. When at least one of the system tasks can no longer be executed properly, we talk about a *system failure*.

[2]With the system reliability at time t we refer to the probability that the system does not fail at time t.

Notation

$P(\cdot)$	probability mass function
$p(\cdot)$	probability density function
$u(\cdot)$	utility function
$B(\cdot)$	standard Brownian motion
h	healthy mode
$f_1, \dots, f_\ell$	fault modes
$H(\tau)$	health state at time τ, $H(\tau) \in \Theta_H = \{h, f_1, \dots, f_\ell\}$
$d_1, \dots, d_{\mathrm{r}}$	degradation modes
$c_1, \dots, c_p$	failure modes
$\mathbf{X}$	(multivariate) degradation process
$\mathbf{Y}$	noise-disturbed measurement process of degradation process $\mathbf{X}$
$m_1(\cdot), ..., m_\ell(\cdot)$	parametrized models
θ_j	vector of stochastic parameters associated with degradation model d_j
ϕ_j	vector of deterministic parameters associated with degradation mode d_j
λ_i	failure threshold for failure mode c_i
τ_{eol}	failure time
$\mathbf{S}_k^j$	degradation state at τ_k according to degradation model j
$g_i(\cdot)$	function of $\mathbf{X}$ defining failure in mode c_i
$P_{\mathrm{func},i}$	system reliability with respect to failure mode c_i
P_{func}	overall system reliability
$P_{\mathrm{func},i}^j$	prediction of the system reliability with respect to failure mode c_i conditional to degradation mode d_j
$\mathcal{F}_i$	binary variable indicating whether the system fails in failure mode c_i
A	discrete set of possible maintenance actions
a	maintenance action, $a \in A$
T	discrete set of possible maintenance times
t	maintenance time, $t \in T$
$C_{\mathrm{m}}(\cdot)$	lifetime-averaged direct costs of maintenance as function of a and t
$C_{\mathrm{i}}(\cdot)$	lifetime-averaged indirect costs of maintenance as function of a and t
C_{c_i}	costs of failure in mode c_i
$C_{f_j}(a)$	penalty costs of (wrong) maintenance action a in case of fault f_j
$C_{\mathrm{r}}(\cdot)$	costs of risk as function of a and t
$C_{\mathrm{total}}(\cdot)$	total maintenance costs, $C_{\mathrm{total}}(a,t) = C_{\mathrm{m}}(a,t) + C_{\mathrm{i}}(a,t) + C_{\mathrm{r}}(a,t)$

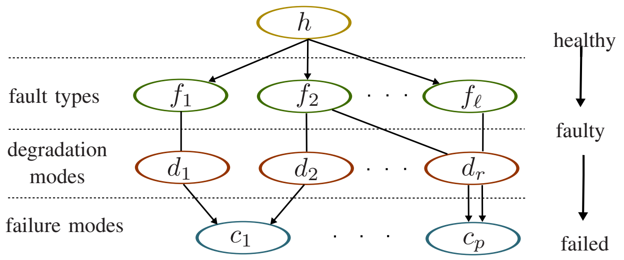


Fig. 1: Relationships between fault types, degradation modes, and failure modes.

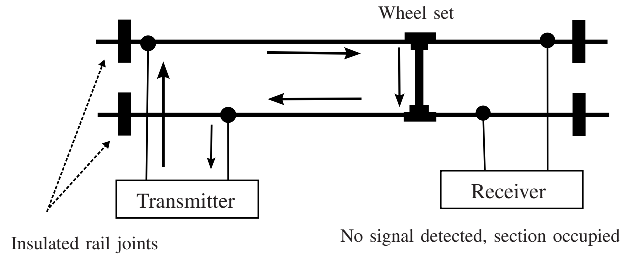


Fig. 2: Flow of current in a railway track circuit.

The transition from healthy behavior via faulty behavior to a system failure can take different forms, which we call *degradation modes*. So, the degradation modes d_1 till d_r (see Figure 1) describe possible time behaviors of system degradation.

Generally, a system can suffer from different kinds of faults f_1 through f_ℓ. The type of fault present determines to a large extent the temporal degradation behavior. Finally, a distinction is made between different failure modes c_1 till c_p. The failure mode indicates which system function is no longer executed properly.

In summary, the fault type indicates what is causing the faulty behavior, the degradation mode indicates how the system degrades over time, and the failure mode indicates which system function is (going to be) lost.

B. Assumptions

We assume that the different possible fault types f_1 till f_ℓ, the different possible degradation modes d_1 till d_r, and the different possible failure modes c_1 till c_p are given. Moreover, the temporal degradation behavior corresponding to each fault and with respect to each failure mode is assumed to be known (i.e. the connecting lines in Figure 1 are given).

As the focus of this paper is on failure prognosis, we assume the availability of a diagnostic result in the form of a probability mass function over the current (i.e. for time $\tau = \tau_c$) health state: $P(H(\tau_c))$, where the health state $H(\tau_c)$ takes a value in the set $\{h, f_1, f_2, \ldots, f_\ell\}$.

For failure prognosis, we assume the availability of z degradation measures $X_\xi \in \mathbb{R}$, $\xi = 1, \ldots, z$. A *degradation measure* is a continuous variable that can be computed from sensor information and that is highly correlated with system degradation [1], [21]. The set of degradation measures $\mathbf{X} = \{X_1, \ldots, X_z\}$ captures the system's degree of degradation. The degradation measures are linked to the failure modes and fault types as follows: The values of the degradation measures indicate to what extent the system is healthy, faulty, or in a specific failure mode. The evolution of the degradation measures over time is characteristic for the type of fault present.

C. Goals

As a *first goal*, we aim to explicitly make the link to the subsequent maintenance optimization process. Failure prognosis is not an isolated task, but a task within the condition-based maintenance process. We therefore analyze the dependencies between diagnosis, prognosis, and maintenance optimization. Moreover, we investigate how the prognostic result should be specified to support maintenance optimization in the case of multiple degradation and failure modes.

The first step to determine the prognostic result comprises the forecasting of the degradation measure(s) over time. As the degradation behavior varies for different fault types and from case to case (e.g. due to different environmental or operating conditions), the use of a fixed model for degradation forecasting is undesired [1], [16]. To handle case-to-case variability, methods have been proposed that model the degradation by a parametric model with stochastic parameters [1], [16], [23]. Our *second goal* is to augment these single-model approaches to a multiple-model approach, where a distinct model is defined for each degradation mode. The aim is to explicitly model variability due to different fault types, so as to reduce modeling error.

The second step is to predict, based on the forecasted degradation measures, future system reliability. In univariate, single-model approaches, failure is defined as the degradation measure being larger than or equal to a predefined threshold value. In the case of multiple models and multivariate degradation measures, the definition of a failure and the associated computation of the system reliability are less trivial. Our *third goal* is to extend the threshold-based approaches to the case that we have multiple degradation measures.

D. Motivating cases

To motivate the need for multivariate prognostic methods accounting for multiple degradation and failure modes two practical cases are described.

1) Failure prognosis for railway tracks: A key component of a railway network is the track. Besides that the track provides trains with a dependable surface for their wheels to roll on, it is an essential part of the train detection process using track circuits. For the purpose of train detection, at one end of each railway section, an electrical current is transmitted. In the absence of a train, this current flows via the rails to the other end of the section, where it is measured by a receiver. When the current measured at the receiver exceeds a certain threshold, the section is reported as free. When a train is present, the circuit is shorted by the wheels of the train and the current measured at the receiver is close to zero, in which case the section is reported as occupied (see Figure 2).

To guarantee reliable train detection, it needs to be ensured that the conductance properties of the rails and the shunting properties of track and trains are of high quality.

All together, the railway track serves the following purposes:

1) safe and comfortable guidance of trains;
2) correct detection of a free track;
3) correct detection of an occupied track.

Accordingly, three failure modes are defined (see Figure 3).

The proper execution of the aforementioned tasks may be impaired by different faults, four common ones of which are:

f_{rc} : rail contamination;
f_{rd} : rail surface defect;
f_{ed} : electrical disturbance;
f_{ij} : insulated joint defect.

The faults are related to the failure modes c_1 till c_3 as follows (see Figure 3): Contamination between the rail surface and the wheels (e.g. rust films, sand, and leafs) may hamper both the safe and comfortable guidance of trains and the correct detection of an occupied track (because the contamination hinders passing trains to shorten the circuit). Both rail surface defects and insulated joint defects may impair the safe and comfortable guidance of trains, as well as the correct detection of a free track. Finally, electrical disturbances may hamper the correct detection of a free track.

Faults f_{rc}, f_{rd}, f_{ed}, and f_{ij} are all associated with different time behaviors of degradation (see Figure 3), where a distinction is made between the following three types of qualitative degradation behavior [24]:

d_{l}: linear;
d_{e}: exponential;
d_{i}: intermittent.

From the above description, it follows that adequate degradation modeling for railway tracks requires a multivariate multiple-model approach. First, the railway track is subject to different degradation modes. For example, a false positive train detection (i.e. failure mode c_2) can be caused by both a rail defect, an insulated joint defect, or an electrical disturbance. How the degradation evolves over time depends to a large extent on the type of fault present. Therefore, multiple models are required to forecast degradation behavior. Second, multiple degradation measures are needed to adequately forecast degradation behavior. The system's ability to detect a free track is expressed in the current flowing through the track circuit receiver when the track is vacant [24]. The system's ability to detect an occupied track is reflected in the current not flowing through the track circuit receiver when the track is occupied by a train [24]. Among other things, the vertical axle box accelerations [25], [26] provide information about the system's ability to safely and comfortably guide vehicles. So, for this application it is not possible to adequately model all degradation behaviors using just one measure, e.g. the guidance abilities cannot be assessed adequately using just electrical information, whereas the detection abilities cannot be assessed adequately based on just mechanical information.

2) Failure prognosis in buildings: Heating, Ventilation, and Air Conditioning (HVAC) systems are another example of systems that fulfill multiple tasks and that are subject to different degradation modes. Without going into detail, an HVAC system serves the following purposes:

1) temperature control;
2) humidity control;
3) ventilation.

Accordingly three failure modes can be defined. Multiple faults can be identified that hinder the proper execution of one or more of these tasks. Just a few examples are [27]:

f_{mb}: malfunctioning boiler;
f_{sv}: stuck heating/cooling coil valve;
f_{df}: deteriorating supply fan;
f_{dd}: deteriorating damper (controlling the mixing ratio between outside and re-circulation air).

Like for the railway example, multiple degradation measures are needed to model degradation behavior; the system's ability the control zone air temperature is expressed in zone temperature measures, while the system's ability to regulate humidity is reflected in humidity (correlated) measures, and the ventilation quality is reflected in CO_2 (correlated) measures. Because some of the faults affect multiple system goals (e.g. a deteriorating supply fan may affect all goals) the degradation behavior of the different measures may be correlated. Therefore, it is advantageous to consider multivariate degradation modeling at the system level, rather than looking at the individual tasks.

III. Prognostics within the condition-based maintenance process

Condition-based maintenance aims to optimize maintenance planning by making use of real-time monitoring data. The path from the monitoring data to an optimal maintenance decision includes data pre-processing, fault diagnosis, failure prognosis, and maintenance optimization. Besides the proper implementation of the individual processes, adequate incorporation of the dependencies between the individual processes is crucial for the success of condition-based maintenance. With respect to failure prognosis, the following dependencies are relevant:

1) dependencies between the diagnosis and prognosis processes;
2) dependencies between failure prognosis and maintenance optimization.

A. Dependencies between diagnosis and prognosis

Although fault diagnosis and failure prognosis concern different tasks, and are often treated individually, exploiting their mutual dependence is valuable for both diagnosis and prognosis. As outlined before, different fault types are associated with different degradation behaviors. So, information regarding the type of fault present (diagnostic result) provides information about future degradation behavior. Vice versa, information about degradation trends (prognostic result) provides information regarding the type of fault present. We propose to exploit this dependence by using the diagnostic result to determine the likelihood of each prognostic (fault-specific) model (see Section III-B).

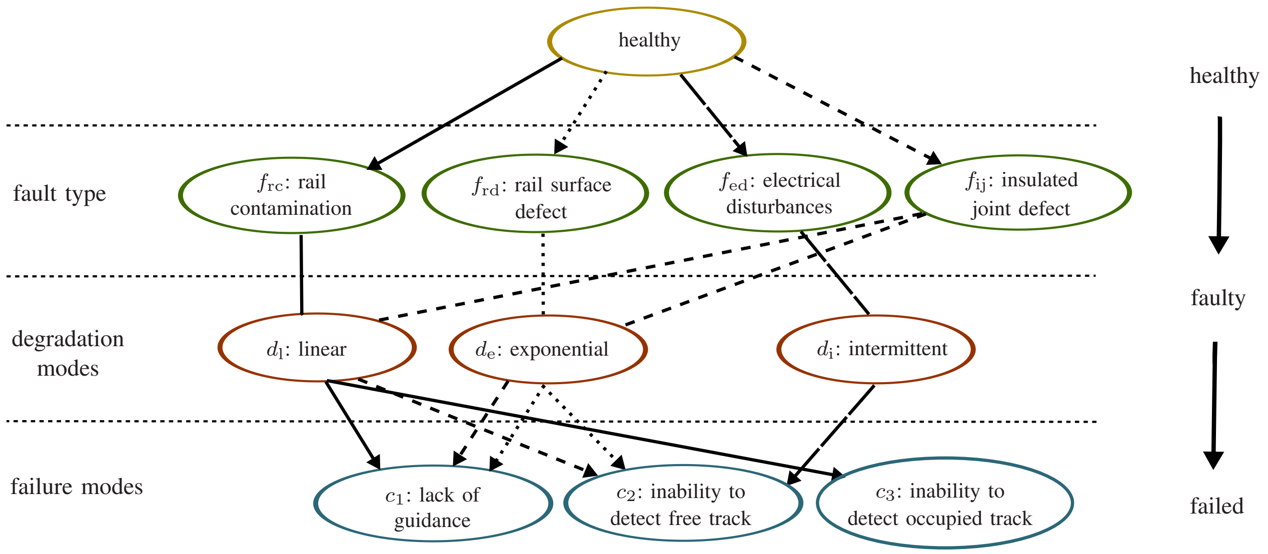


Fig. 3: Visualization of the relationships between the faults, degradation modes, and failure modes of a railway track. The different line types indicate possible degradation behaviors for the different fault types.

B. Dependencies between prognosis and maintenance optimization

The prognostic result serves (together with the diagnostic result) as an input for the maintenance optimization process. It is therefore important to ensure that the prognostic result is specified such that it facilitates maintenance optimization. An adequate specification of the prognostic result requires an understanding of the maintenance optimization process. Therefore, before defining an adequate specification of the diagnostic result, background information on the maintenance optimization process is given.

1) Maintenance optimization process: Maintenance optimization is a typical example of a decision task subject to risk and uncertainty: we have uncertainty regarding the current and future system health, and consequently, we have the risk of making non-optimal maintenance decisions. In the presence of risk and uncertainty, decisions are commonly made based on the expected utility theory [28], which is a framework for determining the best (maintenance) decision given probabilistic information regarding the actual situation[3]. In the sequel, we assume that maintenance decision are made based on the expected utility theory (see Appendix A for more details).

In contrast to common maintenance optimization methods, which limit the maintenance optimization task to deciding whether or not to perform preventive maintenance at a particular time instant, we augment this task by deciding on the following items [29] :

1) the required type of maintenance;
2) the optimal time to perform maintenance.

So, the possible maintenance decisions are:

$d_{0,\infty}$: do nothing;
$d_{a,t}$: perform maintenance activity $a \in A$ at time $t \in T$,

[3]For the maintenance optimization case, the situation is defined by the current and future system health.

with a and t, in turn, decision variables, A the finite set of possible maintenance activities, and T the discrete set of available maintenance time instants. The goal is to find the combination of type a and time t of maintenance that minimizes the total maintenance costs $C_{\mathrm{tot}}(a,t)$. The expected total maintenance costs can be computed as [29]:

$$\mathbb{E}(C_{\mathrm{total}}|a,t) = C_{\mathrm{m}}(a,t) + C_{\mathrm{i}}(a,t) + C_{\mathrm{r}}(a,t) \tag{1}$$

with:

$C_{\mathrm{m}}(\cdot)$: function of a and t, expressing the lifetime-averaged direct costs associated with maintenance action a at time t;
$C_{\mathrm{i}}(\cdot)$: function of a and t, expressing the lifetime-averaged indirect costs of maintenance (e.g. related to downtime) associated with action a at time t;
$C_{\mathrm{r}}(\cdot)$: function of a and t expressing the costs associated with the risk of action a being inadequate or time t being too late.

The risk costs $C_{\mathrm{r}}(a,t)$ can be further specified as:

$$\begin{aligned} C_{\mathrm{r}}(a,t) = & \sum_{i=1}^{p}\sum_{j=1}^{\ell} P(H(t)=f_j)P\big(\mathcal{F}_i(t)=1|H(t)=f_j\big)C_{c_i} \\ & + \sum_{j=1}^{\ell} P(H(t)=f_j)C_{f_j}(a) \end{aligned} \tag{2}$$

with:

C_{c_i}: additional costs of a failure in mode c_i;
$C_{f_j}(\cdot)$: function of a expressing the penalty costs of preparing a (wrong) maintenance activity a in the case of fault type f_j

The first term expresses the costs related to the risk of maintenance time t being too late to avoid a particular failure, with $\mathcal{F}_i(t)$ a binary variable indicating whether the system fails in mode i at maintenance time t. The second term expresses the costs related to the risk of maintenance action a being not appropriate to repair the system.

Based on the expected costs, the expected utilities can be defined as:

$$\begin{aligned}\mathbb{E}(u|a,t) &= -\,\mathbb{E}(C_{\text{total}}|a,t)\\ \mathbb{E}(u|a,t) &= -\left(\sum_{i=1}^{p}\sum_{j=1}^{\ell} P(H(t)=f_j)P\big(\mathcal{F}_i(t)=1|H(t)=f_j\big)C_{c_i}\right.\\ &\qquad \left. + \sum_{j=1}^{\ell} P(H(t)=f_j)C_{f_j}(a) + C_{\mathrm{m}}(a,t) + C_{\mathrm{i}}(a,t)\right)\end{aligned} \tag{3}$$

To compute $\mathbb{E}(u|a,t)$, next to the cost functions, the probabilities $P(H(t)=f_j)$ and $P(\mathcal{F}_i(t)=1|H(t)=f_j)$ need to be known for $j=1,\ldots,\ell$ and $i=1,\ldots,p$. The probability that a certain fault is present, i.e. $P(H(t)=f_j)$, reflects the diagnostic result. The probability that the system fails at a particular maintenance time given the system health state, i.e. $P(\mathcal{F}_i(t)=1|H(t)=f_j)$, refers to prognostic information.

2) Specification of the prognostic result: From the analysis of the maintenance optimization process, we conclude that, in the case of multiple degradation and failure modes, the prognosis process should output conditional predictions of the system reliability, i.e. the functions $P^j_{\text{fail},i}(\cdot)$ defined by:

$$\begin{aligned} P^j_{\text{fail},i}(\tau) = P\big(\mathcal{F}_i(\tau)=1|H(\tau)=f_j\big),\\ \forall j\in\{1,\ldots,\ell\},\, \forall i\in\{1,\ldots,p\}\end{aligned} \tag{4}$$

where $P^j_{\text{fail},i}(\tau)$ indicates the probability of a failure in mode c_i at time τ given that the system degrades according to mode d_j.

Remark 1: In agreement with the above result, we propose the set of conditional system reliabilities as prognostic measure. Considering conditional system reliabilities allows to account for different costs associated with different failure modes and maintenance activities in the subsequent decision making process. Consequently, in the case of multiple degradation and failure modes, this measure is more valuable than one overall estimation of the remaining useful life.

To determine the conditional predictions of the (future) system reliability we consider a two-step approach: First we propose a multivariate multiple-model approach for degradation modeling and forecasting (Section IV); Second, we consider how these predictions can be used to determine (future) system reliability (Section V).

IV. Degradation modeling and forecasting

In this section, we propose a multivariate multiple-model stochastic filtering approach based on Wiener processes for modeling and forecasting of degradation behavior. Note that the strategies presented in the rest of this paper are also valid when another forecasting model (e.g. one based on gamma processes [12], [13]) is used as long as the model outputs a distribution of the degradation measure (and not just the expected value).

A. Multivariate, multiple-model degradation modeling

For each fault f_j, $j=1,\ldots,\ell$, the corresponding time behavior of a z-dimensional degradation process $\{\mathbf{X}(\tau) = [X_1(\tau),\ldots,X_z(\tau)]^\top,\ \tau\geq 0\}$ is described by a Wiener process[4] plus (nonlinear) drift, i.e.

$$\mathbf{X}(\tau) = m_j\big(\tau,\theta_j(\tau)\big) + \sigma_j B(\tau) \tag{5}$$

with $\sigma_j B(\tau) = [\sigma_{j,1},\ldots,\sigma_{j,z}]^\top B(\tau)$ a Wiener process, i.e. $B(\tau)$ represents a standard Brownian motion with $\sigma_{j,\xi}B(\tau)\sim N(0,\sigma^2_{j,\xi}\tau)$). Models m_1 till m_ℓ are z-dimensional vectors the elements of which are (nonlinear) mappings expressing non-decreasing degradation trends (e.g. linear [16], exponential [1], quasi-linear/asymptotic [23]) associated with the corresponding fault mode f_j. The vector $\theta_j(\tau)\in\mathbb{R}^{n_{\theta_j}}$ denotes the model parameters, which might be stochastic. Here we assume $\theta_j(\tau)\sim N(\mu_{\theta_j},\Sigma_{\theta_j})$. Information regarding the degradation process is obtained through noise-disturbed measurements, i.e.:

$$\mathbf{Y}(\tau) = \mathbf{X}(\tau) + \epsilon(\tau) \tag{6}$$

with $\{\mathbf{Y}(\tau) = [Y_1(\tau),\ldots,Y_z(\tau)]^\top, \tau\geq 0\}$ the process describing the time behavior of the measurements, and $\epsilon(\tau) = [\epsilon_1(\tau),\ldots,\epsilon_z(\tau)]^\top$, with $\epsilon_\xi(\tau)\sim N(0,\gamma^2_\xi)$. It is assumed that the random variables ϵ, θ_j, and $B(\tau)$ are mutually statistically independent.

The proposed degradation model (5)-(6) can describe a wide range of degradation trends, and captures both temporal, sampling, and measurement uncertainty [16]. *Temporal uncertainty*, which is the uncertainty associated with the progression of the degradation over time, is characterized by the dynamics of the Brownian motion $\{B(\tau),\tau\geq 0\}$. *Sampling (or case-to-case) variability* characterizes the heterogeneity among the degradation paths of different systems under different operation conditions, and is represented in (5) by the stochastic parameters $\theta_j(\tau)$. Finally *measurement uncertainty* is reflected by the error term $\epsilon(\tau)$ in (6), and reflects the fact that the degradation cannot be perfectly measured, i.e. the measurements are disturbed by measurement errors arising e.g. from non-ideal measurement instruments. Moreover, in our modeling framework, *modeling uncertainty* is minimized by considering a separate model for each fault cause.

B. Online updating and forecasting

Suppose the degradation process is monitored at times $\tau_1<\tau_2<\tau_3<\ldots$ and let $\mathbf{Y}_k=\mathbf{Y}(\tau_k)$ denote the observation vector at time τ_k. The sequence of measurement vectors $\mathbf{Y}_1,\mathbf{Y}_2,\ldots,\mathbf{Y}_k$ is represented by $\mathbf{Y}_{1:k}$ and the corresponding sequence of degradation measures is represented by $\mathbf{X}_{1:k}$, with $\mathbf{X}_k=\mathbf{X}(\tau_k)$. At time τ_k the goal is to estimate the current degradation state $\mathbf{X}(\tau_k)$ and to predict the evolution of the degradation measure $\mathbf{X}(\tau_q)$ for $\tau_q>\tau_k$ based on the model (5)-(6) and observations $\mathbf{Y}_{1:k}$. For that purpose, we rewrite the model (5)-(6) as a discrete-time stochastic state space model:

[4]Wiener processes are considered because they can model non-monotonic degradation behavior, which is often encountered in practice [1], [16], [30]. In case of monotonic degradation behavior, gamma or compound Poisson processes can be used instead.

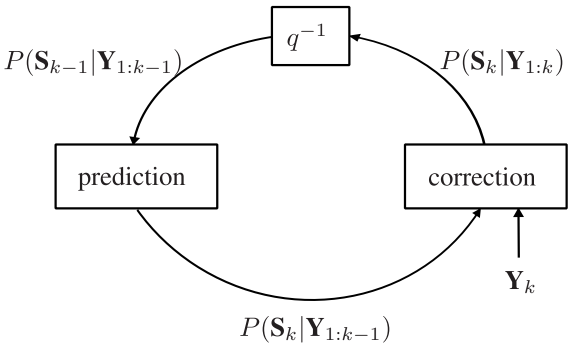


Fig. 4: Bayesian filtering. At each time step k, first, the state is estimated based on the model (prediction step). Next, this estimate is updated based on the current measurements $\mathbf{Y}_k$ (correction step).

$$\underbrace{\begin{pmatrix} \mathbf{X}_k^j \\ \theta_{j,k} \end{pmatrix}}_{\mathbf{S}_k^j} = \begin{pmatrix} \mathbf{X}_{k-1}^j + m_j(\tau_k, \theta_{j,k-1}) - m_j(\tau_{k-1}, \theta_{j,k-1}) + v_k^j \\ \theta_{j,k-1} \end{pmatrix} \tag{7a}$$

$$\mathbf{Y}_k = \mathbf{X}_k^j + \epsilon_k, \quad \text{for system in degradation mode } j \tag{7b}$$

with $\mathbf{S}_k^j \in \mathbb{R}^{z+n_{\theta_j}}$ the state vector at τ_k according to model j, which is composed of the degradation measure $\mathbf{X}_k^j \in \mathbb{R}^z$ and the parameter vector $\theta_{j,k} \in \mathbb{R}^{n_{\theta_j}}$, $\mathbf{Y}_k \in \mathbb{R}^z$ the measurements, $v_k^j \sim N(0, \mathrm{diag}(\sigma_{j,1}^2(\tau_k - \tau_{k-1}), \ldots, \sigma_{j,z}^2(\tau_k - \tau_{k-1})))$, and ϵ_k the realization of ϵ at τ_k. Equation (7a) is the transition equation, which specifies how each element of the state vector evolves over time according to degradation model j. Equation (7b) is the output equation, specifying how the measurements are linked to the system states. In this formulation, degradation forecasting can be considered as a state estimation and prediction problem, where the goal is to estimate and predict the state, so to statistically minimize the state error. This is a common problem that can be solved using Bayesian filtering [31]. The Bayesian approach to statistics attempts to utilize all available information, i.e. it combines new information with existing knowledge, in order to reduce uncertainty. The formal mechanism to combine new information with existing knowledge is known as Bayes' theorem [31]. Roughly, this information fusion consists of two steps: a prediction step based on the state transition equation, and a correction step based on new measurements (see Figure 4).

Different types of Bayesian filters have been proposed, among which the Kalman filter [32], its nonlinear extensions, i.e. the extended and unscented Kalman filter, and particle filters [33]. The choice for a filter depends on the exact form of the model (7) and on computational constraints. When the transition and output equation are linear in the state, and the process and measurement noise are additive and Gaussian, the Kalman filter is the optimal filter. When the linearity assumptions are violated (but the noise assumptions are satisfied), an extended or unscented Kalman filter can be used. Another possibility is to use a particle filter, a Monte Carlo methodology, which can also be used when the noise is non-Gaussian or non-additive. The performance of a particle filter depends on the number of particles used. In general, when enough particles are used, a particle filter outperforms the extended and unscented Kalman filter in terms of estimation accuracy and robustness, but at the costs of higher computational demands [34]–[36].

Because of the attractive properties of the Kalman filter (e.g. computational efficiency, analytic solutions), work has been devoted to transform nonlinear degradation data to an approximate linear form. Examples of such transformations are the log transformation [1] and the time-scale transformation [37]. This way, analytic solutions can be obtained for the approximate linear degradation process in a computationally efficient way, however, at the cost of modeling error. So, for nonlinear degradation processes a trade-off needs to be made between modeling accuracy and solution accuracy. This trade-off is application-specific and a further elaboration is beyond the scope of this paper.

In Appendix B an elaboration can be found for the case that the degradation process can be accurately described by a linear stochastic state space model as considered in [20].

Note that at the degradation forecasting stage, we just predict the values of all degradation measures according to all degradation modes. When computing the system reliability (see Section V), the information from the different degradation measures will be combined. The information from the different fault-specific models will be merged only in the maintenance optimization (see Section VI).

V. System reliability

A. *Multivariate definitions*

Two prognostic measures are (future) system reliability and remaining useful life [4], [16], [21], [38]. Although the remaining useful life is most commonly used, in this paper we focus on the system reliability. We made this choice because this measure best fits the subsequent maintenance optimization process (see Section III). Before elaborating on the system reliability, we define system failure in the multivariate case.

In the univariate case, system failure is usually defined as the degradation measure $\mathbf{X}(\cdot)$ being larger than or equal to a predefined threshold λ, i.e.:

$$\mathbf{X}(\tau) \begin{cases} < \lambda & \Longrightarrow \text{system is functional at } \tau \\ \geq \lambda & \Longrightarrow \text{system fails at } \tau \end{cases} \tag{8}$$

Failure definition (8) can be extended to the multivariate case by defining a failure in mode c_i as $g_i(\mathbf{X}(\cdot))$ being larger than or equal to a predefined threshold λ_i, i.e.:

$$g_i\big(\mathbf{X}(\tau)\big) \begin{cases} < \lambda_i & \Longrightarrow \text{no failure in mode } c_i \text{ at } \tau \\ \geq \lambda_i & \Longrightarrow \text{system fails in mode } c_i \text{ at } \tau \end{cases} \tag{9}$$

with $g_i(\cdot)$, $i = 1, \ldots, p$, application-specific functions, which we will elaborated on in Section V-B. Accordingly, system failure is defined as:

$$\begin{array}{lll} g_1\big(\mathbf{X}(\tau)\big) < \lambda_1 \text{ and} \ldots \text{and } g_p\big(\mathbf{X}(\tau)\big) < \lambda_p & \Longrightarrow & \text{functional at } \tau \\ g_1\big(\mathbf{X}(\tau)\big) \geq \lambda_1 \text{ or} \ldots \text{or } g_p\big(\mathbf{X}(\tau)\big) \geq \lambda_p & \Longrightarrow & \text{failure at } \tau \end{array} \tag{10}$$

The *system reliability* (P_{func}) is defined as the probability that the system is functional, i.e. does not fail [21]. Based on (9), the system reliability with respect to failure mode c_i at time τ is the probability that $g_i(\mathbf{X}(\tau))$ is smaller than λ_i, i.e.

$$P_{\text{func},i}(\tau) = p\Big(g_i\big(\mathbf{X}(\tau)\big) < \lambda_i\Big) \tag{11}$$

The overall system reliability at time τ is defined as the probability that the system is functional, i.e. is not in any of the failure modes c_1 till c_p:

$$P_{\text{func}}(\tau) = p\Big(g_1\big(\mathbf{X}(\tau)\big) < \lambda_1, \dots, g_p\big(\mathbf{X}(\tau)\big) < \lambda_p\Big) \tag{12}$$

From these definitions, we conclude that the functions $P^j_{\text{fail},i}$ as defined in (4) are related to predictions of the system reliability with respect to failure mode i conditional to degradation mode j. So, in accordance with the above definitions, (4) can be written as:

$$P^j_{\text{fail},i}(\tau) = 1 - P^j_{\text{func},i}(\tau) \tag{13}$$

with

$$P^j_{\text{func},i}(\tau) = p\Big(g_i\big(\mathbf{X}(\tau)\big) < \lambda_i \Big| H(\tau) = f_j\Big)$$

B. Determination of failure definition and system reliability

The functions $g_i(\cdot)$, $i = 1, \dots, p$, used to define system failure (9) are application-specific and depend on the relationships between the degradation measures X_ξ, $\xi = 1, \dots, z$. Here, we focus on three common types of relationships (see Figure 5 for 2-D example relationships):

1) complementary measures:
   a) independently assessable;
   b) not independently assessable;
2) redundant measures.

Measures X_ξ, $\xi = 1, \dots, z$, are *complementary* and *independently assessable* if it holds that the system is functional in mode c_i if and only if each measure X_ξ is below an individual threshold $\lambda_{i,\xi}$. For the 2-D example in Figure 5(a) this means that both $X_1 < \lambda_1$ and $X_2 < \lambda_2$ must hold for the system to be functional. In the case of *redundant measures*, only k out of z, $k < z$, of components X_ξ need to be below their threshold $\lambda_{i,\xi}$ for the system to be functioning in mode c_i (see Figure 5(b) for a 2-D example). For *complementary, not independently-assessable measures*, no relevant individual thresholds exist. For the system to be functioning, all functions $g_i(\cdot)$ of $\mathbf{X}$ should then just be below an overall threshold value λ_i. For the 2-D example in Figure 5(c), this means that

$$X_2 + \frac{\mathrm{k}_2}{\mathrm{k}_1} X_1 - \mathrm{k}_2 < 0$$

must hold for the system to be functional.

For brevity, in the sequel we omit the subscript i whenever the explicit reference to a particular failure mode is not necessary. For the same reason, we omit the time argument τ whenever possible.

1) Independently-assessable complementary measures: For measures that are complementary and independently assessable, the functions $g_i(\cdot)$, $i = 1, \dots, p$, can be chosen arbitrarily, as long as they satisfy:

$$g_i(\mathbf{X}) \begin{cases} \geq \lambda_i & \text{if } \max\left(X_1 - \lambda_{i,1}, \dots, X_z - \lambda_{i,z}\right) \geq 0 \\ < \lambda_i & \text{otherwise} \end{cases} \tag{14}$$

The system reliability with respect to failure mode c_i is computed as:

$$P_{\text{func},i} = \int_{-\infty}^{\lambda_{i,1}} \int_{-\infty}^{\lambda_{i,2}} \dots \int_{-\infty}^{\lambda_{i,z}} p(X_1, X_2, \dots, X_z) dX_z \dots dX_2 dX_1 \tag{15}$$

with $p(\cdot)$ the distribution function of the degradation measure $\mathbf{X}$, which follows from the degradation modeling and forecasting (see Section IV).

2) Redundant measures: Safety-critical systems are often equipped with redundancy, e.g. airplanes having more engines than necessarily for take-off. Systems can be redundant to varying degrees. The redundancy is lowest when $z-1$ out of z components need to be functioning for the whole system to be functioning, and highest when only 1 out of z components needs to be functioning for the whole system to be functioning. If the functioning of each redundant component is reflected by one degradation measure X_ξ, then for a k-out-of-z:G system[5] at least k out of z measures X_ξ, $\xi = 1, \dots, z$, need to be below their threshold $\lambda_{i,\xi}$ for the system to be functioning in mode c_i. So, $g_i(\cdot)$ should be chosen such that:

$$g_i^{(k)}(\mathbf{X}) \begin{cases} < \lambda_i & \text{if } \sum_{\xi=1}^{z} \alpha_{i,\xi} \geq k \\ \geq \lambda_i & \text{otherwise} \end{cases} \tag{16}$$

with:

$$\alpha_{i,\xi} = \begin{cases} 1 & \text{if } X_\xi < \lambda_{i,\xi} \\ 0 & \text{otherwise} \end{cases}$$

and the superscript (k) indicating that we consider a k-out-of-z:G system. The system reliability with respect to failure mode c_i is computed as:

$$\begin{aligned} P^{(k)}_{\text{func},i} &= \int\int \dots \int_{\Omega_i^{(k)}} p(X_1, X_2, \dots, X_z) dX_z \dots dX_2 dX_1 \\ &= \underbrace{\sum_{\iota=1}^{\frac{z!}{k!(z-k)!}} \int\int \dots \int_{\Omega_{i,\iota}^{(k)}} p(X_1, X_2, \dots, X_z) dX_z \dots dX_2 dX_1}_{R_i^{(k)}} \\ &\quad - kR_i^{(k+1)} \end{aligned} \tag{17}$$

with $\Omega_i^{(k)} = \Omega_{i,1}^{(k)} \cup \dots \cup \Omega_{i,\frac{z!}{k!(z-k)!}}^{(k)}$ the integration surface representing the subset of $\mathbf{X} \in \mathbb{R}^z$ for which the system is not in failure mode c_i and $\iota = 1, \dots, \frac{z!}{k!(z-k)!}$ the different configurations for which k degradation measures are in their desired region, with $\Omega_{i,\iota}^{(k)}$ the corresponding surfaces. The last term $kR_i^{(k+1)}$ corrects for the overlap between the integration surfaces associated with the different configurations $\iota = 1, \dots, \frac{z!}{k!(z-k)!}$. To illustrate, Figure 6 gives the integration

[5]A k-out-of-z:G system is a system that works well if at least k-out-of-z components work well.

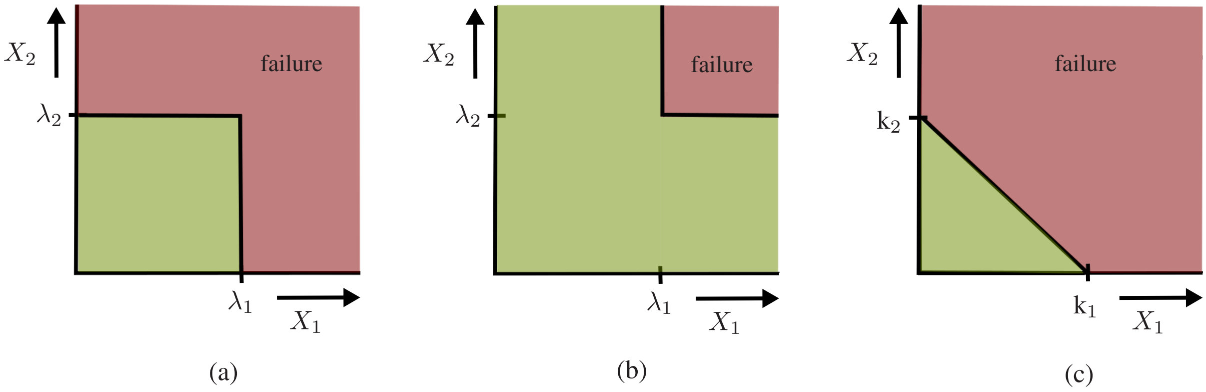


Fig. 5: 2-D illustration of three types of failure definitions. (a) independently-assessable complementary measures, (b) redundant measures, (c) not independently-assessable complementary measures, with k_1 and k_2 two constants.

surfaces $\Omega^{(3)}$, $\Omega^{(2)}$, and $\Omega^{(1)}$ for a three-dimensional case, which are defined as:

$$\begin{aligned}\Omega^{(3)} =& \{\mathbf{X} \in \mathbb{R}^3 : X_1 < \lambda_1 \text{ and } X_2 < \lambda_2 \text{ and } X_3 < \lambda_3\} \\ \Omega^{(2)} =& \{\mathbf{X} \in \mathbb{R}^3 : (X_1 < \lambda_1 \text{ and } X_2 < \lambda_2) \text{ or } \\ & (X_1 < \lambda_1 \text{ and } X_3 < \lambda_3) \text{ or } (X_2 < \lambda_2 \text{ and } X_3 < \lambda_3)\} \\ \Omega^{(1)} =& \{\mathbf{X} \in \mathbb{R}^3 : X_1 < \lambda_1 \text{ or } X_2 < \lambda_2 \text{ or } X_3 < \lambda_3\}\end{aligned}$$

3) Not individually-assessable complementary measures: In practice, it is common that the functioning of a system is defined as a combination of the degradation measures satisfying a certain criterion, e.g. the sum or product of measures X_ξ, $\xi = 1, \ldots, z$, should be below a threshold. In such situations, system reliability cannot be assessed based on individual threshold values. However, in such cases, the critical surface is generally smooth and can be written in the form:

$$s_{\text{cr}}(X_1, X_2, \ldots, X_z) = 0 \tag{18}$$

with $s_{\text{cr}}(\cdot)$ a continuous function (see Figure 7 for some two-dimensional example surfaces and the associated functions $s_{\text{cr}}(\cdot)$).

In this case, the integration bounds directly follow from $s_{\text{cr}}(\cdot)$.

4) Concluding remark: In the multivariate case, the failure definition and the associated computation of the system reliability depend on the relationships among the degradation measures. Three common relationships have been discussed. In general, the dependencies among degradation measures do not always fall within one category. Consider for example a system with four degradation measures X_1 till X_4 and failure defined as:

$$g(X_1, X_2) > \lambda_1 \text{ and } g'(X_3, X_4) > \lambda_2 \implies \text{system failure} \tag{19}$$

For this system, we have to deal with both redundant and not individually-assessable complementary measures. In such cases the strategies discussed before can be combined.

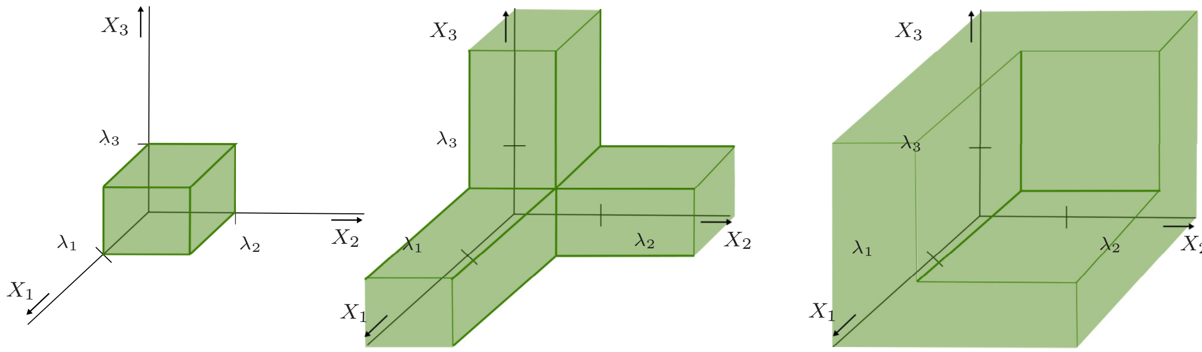
VI. Evaluation and Discussion

A realistic and thorough evaluation of the proposed approach is only possible for a particular application and in combination with a fault diagnosis and maintenance optimization approach. Such an evaluation goes beyond the scope of this paper. In this section, we will reflect on two main attributes of the proposed approach, namely:

1) its position within the condition-based maintenance process;
2) the added value of using multiple models over using a single model on the prediction quality, and its dependence on the diagnostic result.

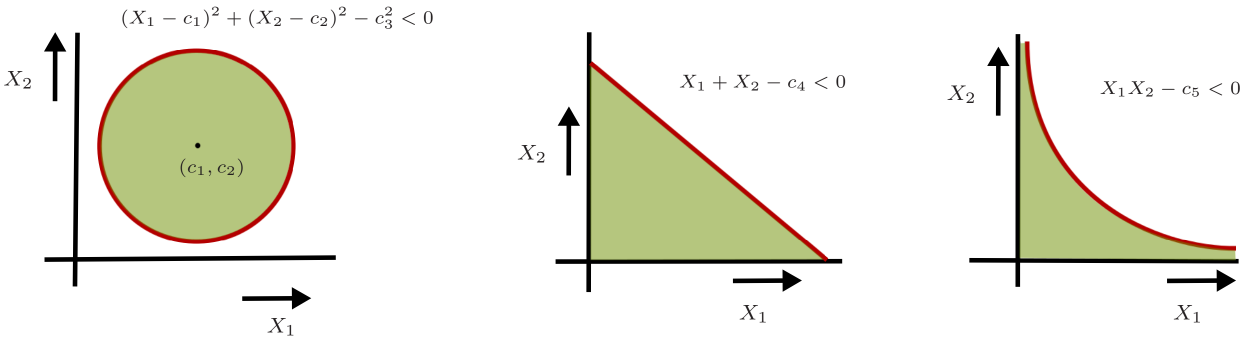
A. Position within the condition-based maintenance process

Procedure 1 summarizes the proposed prognosis strategy within a condition-based maintenance process. Although we ensure that the different processes are compatible with each other in the sense that the diagnostic and prognostic results support maintenance optimization, we do not impose further requirements on the diagnosis and maintenance optimization process. In particular, we only assume that the diagnosis process outputs a probability distribution over the system health state, and that decision making is done based on expected utilities. Even when the diagnostic result is specified using another uncertainty formalism (e.g. possibilities, fuzzy measures, mass functions) the proposed strategy is of use. In this case the alternative uncertainty distribution first has to be transformed into a probability distribution. For this task, transformation rules are available in literature [39], [40]. Moreover, if desired, another multivariate multiple-model forecasting algorithm can be used instead of the forecasting strategy outlined in Procedure 2. For example a multiple-model method based on gamma processes [12], [13] in the case that degradation behavior is best described by a monotonic process. We regard the freedom to independently select an optimal diagnosis and forecasting algorithm as a practical advantage. Indeed, problem characteristics vary widely among applications, and so the best combination of diagnosis and prognosis approach is highly application-specific. As another advantage, we regard the fact that the maintenance optimization model we rely on is based on cost functions that are easily assessed by practitioners



(a) (b) (c)

Fig. 6: 3-D visualization of the integration surfaces indicating the subsets of $\mathbf{X} \in \mathbb{R}^3$ for which the system is functional: (a) 3-out-of-3: G system; (b) 2-out-of-3: G system, (c) 1-out-of-3: G system.



(a) (b) (c)

Fig. 7: 2-D example surfaces indicating the subset of instances of $\mathbf{X} = [X_1 X_2]^\top$ for which the system is functional.

(e.g., costs of maintenance, costs associated with a failure, costs associated with downtime).

Procedure 1 Prognosis within condition-based maintenance at time τ_k.

Input: Failure functions $g_i(\cdot)$ and thresholds λ_i for $i = 1, \ldots, p$, set T of possible maintenance time instants

Fault diagnosis

generates $P\big(H(\tau_k)\big)$

Prognosis

for $t \in T, t > \tau_k$ **do**
  for $j = 1, \ldots, \ell$ **do**
    Determine $\mathbf{X}^j(t)$ using Procedure 2
    for $i = 1, \ldots, p$ **do**
      Determine $P^j_{\text{func},i}(t)$:

$$P^j_{\text{func},i}(t) = \int \int \cdots \int_{\Omega_i} p\big(\mathbf{X}^j(t)\big) dX_z \ldots dX_2 dX_1$$

      with $\Omega_i \in \mathbf{X}$ the surface for which $g_i(\mathbf{X}) < \lambda_i$
    end for
  end for
end for

Maintenance optimization

Based on $P(H(\tau_k))$ and $P^j_{\text{func},i}(t)$ $\forall t \in T, t > \tau_k$, $\forall j \in \{1, \ldots, \ell\}$, $\forall i \in \{1, \ldots, p\}$, determine the optimal maintenance decision, e.g. according to (3)-(4)

Output: Maintenance decision for τ_k

B. Multiple models

We motivated our choice for a multiple-model approach by the fact that system degradation may evolve differently over time for different types of faults. For example, for the railway case (see Figure 3) the ability to detect a free track decreases approximately linearly over time in the case of an electrical insulation problem, while the temporal degradation behavior is best described by an exponential model when a rail surface defect is present. Therefore, a natural choice is to use a linear model to forecast degradation resulting from an insulation problem, while using an exponential model to forecast degradation behavior as a consequence of a rail surface defect.

We conclude that a multiple-model approach has the potential to outperform a single-model approach with respect to prediction accuracy. We say "has the potential to" because the actual prediction performance of a multiple-model approach heavily relies on knowledge of the underlying degradation mode. In practice, we do not know with certainty which fault is present, and so which model best describes degradation behavior. This means that for online degradation forecasting the potential improvement in prediction accuracy cannot be fully utilized. Whether and to what extent a multiple-model approach will outperform a single-model approach with respect to prediction accuracy depends, among other things, on the accuracy of the diagnostic result. Although promising fault diagnosis methods have been proposed in the literature (see e.g. [41]–[43]), the achievable diagnostic accuracy is rather application-specific. Moreover, in general, diagnostic quality improves with the severity of the fault; so for incipient faults, diagnostic quality may be low.

Figure 8 shows two typical temporal behaviors of the diagnostic result for gradually evolving faults taken from [44]. These behaviors relate to a track circuit diagnosis task for which a Kalman filter approach has been used. In both Figures 8(a) and (b), the system is healthy till τ_{d}, i.e. $H(\tau) = h$ for $\tau < \tau_{\mathrm{d}}$. Afterwards, the system suffers from fault f_2, i.e. $H(\tau) = f_2$ for $\tau \geq \tau_{\mathrm{d}}$. From the diagnostic results in Figure 8, we conclude that in both cases the presence of a fault is almost instantly detected, i.e. $P(H(\tau) = h) \approx 0$ for $\tau > \tau_{\mathrm{d}}$. However, only from $\tau = \tau_{\mathrm{i}}$ on the system behavior is adequately diagnosed. In Figure 8(a), the fault is initially misdiagnosed, i.e. we conclude with a probability of approximately 80% that the system suffers from fault f_1. Slightly later, when more data have been collected, fault f_2 is correctly identified. In Figure 8(b) for τ between τ_{d} and τ_{i} there is (much) uncertainty about the cause of the faulty behavior. Initially all faults are plausible, i.e. both $P(H(\tau) = f_1)$, $P(H(\tau) = f_2)$, and $P(H(\tau) = f_3)$ are significantly larger than zero. Afterwards, doubt remains between faults f_2 and f_3 only. From time τ_{i}, the actual fault cause is identified with high accuracy, i.e. $P(H(\tau) = f_2) \approx 1$.

In general, the longer the fault is present, the more data of the faulty behavior are available, and the more accurate and reliable the diagnostic result is. How long it takes before adequate diagnostic results are obtained is however rather application-specific. Since in general both diagnostic and degradation forecasting performance improve over time, it is important to account for this time behavior in the subsequent decision making process.

VII. Conclusions

We have proposed a multiple-model approach to degradation modeling and forecasting for systems with multiple degradation and failure modes. A stochastic filtering approach is considered to handle the different sources of uncertainty inherent to degradation forecasting. Moreover, the links with the other tasks of the condition-based maintenance process, i.e. diagnosis and maintenance planning, have been established. We conclude that conditional predictions of future system reliability best support the subsequent decision making process. Accordingly, a framework has been proposed to determine the (future) system reliability in the multivariate case for different types of relationships among degradation measures.

We conclude that by using multiple models to forecast degradation behavior, the modeling error can be reduced. However, since the applicable model is selected based on the diagnostic result, the benefit of using multiple models over using a single model highly depends on the accuracy of the diagnostic result. Given the current quality of diagnosis methods, we do not expect this to be a serious drawback. However, caution is needed when faults are in their incipient phase. In this phase, the diagnostic results are often less accurate. A thorough analysis of the accuracy of diagnosis and prognosis results over time, and its implications on the subsequent maintenance optimization process is therefore an interesting topic for further research. As another topic for further research, we propose the thorough evaluation of the proposed approach within a condition-based maintenance process.

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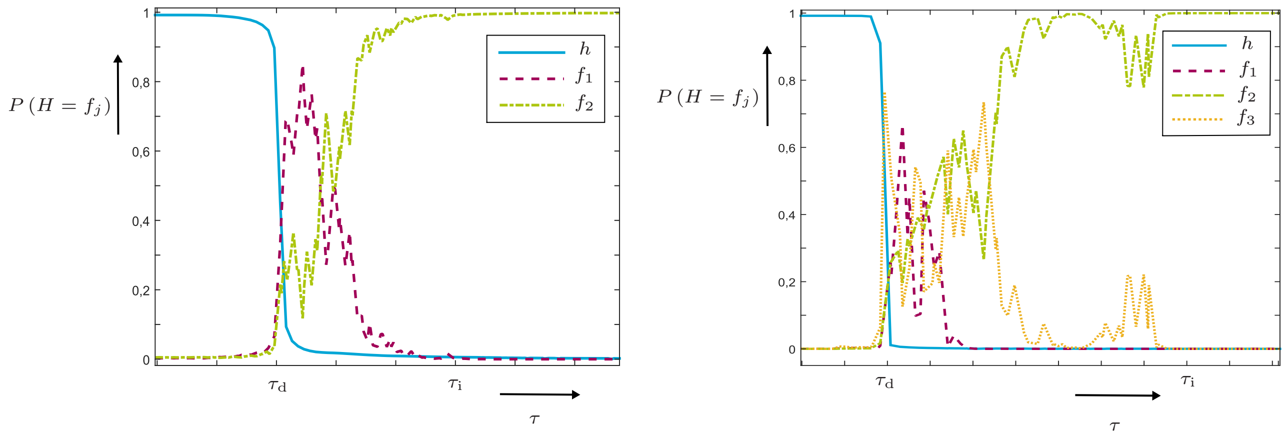

(a) (b)

Fig. 8: Examples of commonly observed time behaviors of the diagnostic result [44]. The monitored system is healthy till $\tau = \tau_\mathrm{d}$, and suffers from fault f_2 afterwards. From the diagnostic results, we conclude that: (a) fault f_2 is incorrectly diagnosed in its incipient phase; (b) in the incipient phase, there is doubt between the different faults.

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APPENDIX A
EXPECTED UTILITY THEORY

Expected-utility theory [28] provides a framework for determining the optimal action given probabilistic information regarding the situation you are in. Its two main ingredients are:

1) Utilities, which indicate the desirability of a particular action in a particular situation, i.e. utilities express preferences among the available choices.
2) Probabilities, which indicate how likely a particular situation is.

The expected utility $\mathbb{E}(u|d)$ of a decision $d \in \Theta_D$ is computed as:

$$\mathbb{E}(u|d) = \sum_{v \in \Theta_V} P(v) u(d, v) \tag{20}$$

with Θ_D the discrete set of possible decisions, Θ_V the set of possible situations, $u(d, v)$ the utility of decision d given situation v, and $P(v)$ the probability of v. Then, an optimal decision d^* is:

$$d^* = \arg \max_{d \in \Theta_D} \mathbb{E}(u|d) \tag{21}$$

APPENDIX B
MULTIPLE-MODEL KALMAN PREDICTION

Consider the case that the degradation process can be accurately described by a linear stochastic state space model as considered in [20], i.e. for all j, model m_j is of the form:

$$m_j(\tau_k, \theta_{j,k}) = \beta_j(\tau_k, \phi_j)\theta_{j,k} \tag{22}$$

with $\phi_j \in \mathbb{R}^{n_{\phi_j}}$ a vector of deterministic parameters, $\theta_{j,k} \in \mathbb{R}^{n_{\theta_j}} \sim N(\mu_{\theta_j}, \Sigma_{\theta_j})$ a vector of stochastic parameters, and β_j a $z \times n_{\theta_j}$ matrix. In this case, model (7) can be written in a linear form:

$$\mathbf{S}_k^j = \begin{pmatrix} \mathbf{X}_k^j \\ \theta_{j,k} \end{pmatrix} = A_{j,k}\mathbf{S}_{k-1}^j + \eta_k^j \tag{23a}$$

$$\mathbf{Y}_k = C\mathbf{S}_k^j + \epsilon_k \tag{23b}$$

with:

$$\begin{aligned}
A_{j,k} &= \begin{bmatrix} I & \beta_j(\tau_k, \phi_j) - \beta_j(\tau_{k-1}, \phi_j) \\ 0 & I \end{bmatrix} \\
\eta_k^j &= \begin{bmatrix} v_k^j \\ 0 \end{bmatrix} \sim N(0, Q_{j,k}) \\
Q_{j,k} &= \begin{bmatrix} \mathrm{diag}\big(\sigma_{j,1}^2(\tau_k - \tau_{k-1}), \ldots, \sigma_{j,z}^2(\tau_k - \tau_{k-1})\big) & 0 \\ 0 & 0 \end{bmatrix} \\
C &= \begin{bmatrix} I & 0 \end{bmatrix} \\
\epsilon_k &\sim N(0, R) \\
R &= \mathrm{diag}(\gamma_1^2, \ldots, \gamma_z^2)
\end{aligned}$$

Procedure 2 outlines the degradation estimation and forecasting based on the Kalman filter.

Procedure 2 Multiple-model degradation estimation and prediction at time τ_k.

Input: Previous states $\mathbf{S}^j(k-1|k-1)$, previous covariance matrices $P^j(k-1|k-1)$, and matrices $A_{j,k}$ and $Q_{j,k}$, for $j = 1, \ldots, \ell$; matrices C and R; failure criteria

Measure $\mathbf{Y}_k$

for $j = 1, \ldots, \ell$ **do**

Estimation of current degradation

Prediction step:

$$\mathbf{S}^j(k|k-1) = A_{j,k}\mathbf{S}^j(k-1|k-1)$$
$$P^j(k|k-1) = A_{j,k}P^j(k-1|k-1)A_{j,k}^\top + Q_{j,k}$$

Correction step:

$$K^j(k) = P^j(k|k-1)C^\top \left(CP^j(k|k-1)C^\top + R\right)^{-1}$$
$$\mathbf{S}^j(k|k) = \mathbf{S}^j(k|k-1) + K^j(k)\left(\mathbf{Y}_k - C\mathbf{S}^j(k|k-1)\right)$$
$$P^j(k|k) = \left(I - K^j(k)C\right)P^j(k|k-1)$$

Prediction of future degradation

$n \leftarrow 0$

while failure criteria are not met **do**

$n \leftarrow n + 1$

n-step ahead prediction:

$$\mathbf{S}^j(k+n|k) = (A_{j,k})^n\, \mathbf{S}^j(k|k)$$
$$P^j(k+n|k) = (A_{j,k})^n\, P(k|k)(A_{j,k}^\top)^n + \sum_{l=0}^{n-1}(A_{j,k})^l Q_{j,k}\left(A_{j,k}^\top\right)^l$$

end while

end for

Output: predictions of the degradation measure $\mathbf{X}^j(\tau_q)$ according to all degradation modes d_j for $q = k, k+1, \ldots, k+n$