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Technical report 18-030

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S. Yuan, B. De Schutter, and S. Baldi, "Adaptive tracking control of switched linear systems using mode-dependent average dwell time," *Proceedings of the 2018 Eighth International Conference on Information Science and Technology (ICIST)*, Cordoba, Granada, and Seville, Spain, 7 pp., June–July 2018. doi:10.1109/ICIST.2018.8426176

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*This report can also be downloaded via https://pub.deschutter.info/abs/18_030.html

Adaptive tracking control of switched linear systems using mode-dependent average dwell time

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***Abstract*—This paper studies model reference adaptive control for switched linear systems with large parametric uncertainties. An aggregate leakage approach is proposed to develop a novel adaptive law, which overcomes the state-of-the-art assumption of knowing the upper and lower bounds of the parameter uncertainty. In addition, a switching law is developed based on mode-dependent average dwell time scheme, which exploits the information of the known reference model for every subsystem, i.e., average dwell time is realized in a subsystem sense. Based on the proposed time-constraint scheme, switching signals that are less conservative than those based on dwell time and average dwell time can be designed. Global uniform ultimate boundedness of the closed-loop adaptive switched system is guaranteed. Furthermore, the tracking error is shown to be upper bounded and also an ultimate bound is presented. Simulations using NASA GTM aircraft illustrate the proposed method.**

***Index Terms*—Adaptive control; switched linear systems; average dwell time;an aggregate leakage approach**

I. Introduction

Switched systems are a special class of hybrid systems, which consists of subsystems and a rule to regulate the switching behavior between them. The subsystems are also referred to as *modes*, while the rule as *switching law*. Switched systems are widely used to model many complex physical systems with hybrid dynamics, such as power converters [1], networked systems [2], smart buildings [3], and electro-hydraulic systems [4], and can be also used to approximate nonlinear systems around certain operating regions [5], [6]. In the past few decades the research on switched systems has attracted a lot of investigations, one of the main topics being how to design a family of switching signals that guarantee certain properties. Problems that have been studied cover stabilizing switching signals [7], [8], [9], switching signals which are robust against uncertainties or disturbances [10], [11], [4], [12], [13], and switching signals in combination with adaptive laws to cope with parameter uncertainties [5], [14], [15], [16], [17].

This paper is connected to this last line of research, i.e. adaptive control of switched systems with unknown parameters. It is well known that model reference adaptive control (MRAC) schemes can be used to cope with uncertainties in classical (non-switched) systems [18], [19], [20], [21], [22]. The main purpose of MRAC is to guarantee that the behavior of the system to be controlled matches the behavior of a desired model. Being the system dynamics unknown, appropriate adaptation mechanisms must be implemented in order to accomplish asymptotically the matching task. For uncertain switched systems, achieving the matching task involves a twofold design: together with the adaptive law, a switching law also should be carefully designed to guarantee stability of the uncertain closed-loop switched system. Model reference adaptive control of switched linear systems is a relatively recent research field: the authors in [5] proposed an adaptive law with a parameter projection to study switched linear system; in [14] a tracking adaptive control scheme for uncertain switched systems based on average dwell time is studied; an adaptive control with a parameter projection and a switching law based on ADT was proposed to study switched linear parameter-varying systems in [15]. However, the adaptive laws [5], [14], [15] for switched linear systems are designed based on a parameter projection method, which means that the bounds of the actual parameter must be *a priori* known for average dwell time. This can potentially limit the scope for the implementation of adaptive control strategies. Moreover, the design of switching laws in uncertain switched systems involves in general quite restricted families of switching laws: in fact, the state of the art has shown that, for switched systems without uncertainty, the design of switching signals based on mode-dependent average dwell time (MDADT) [23], [24] can lead to less conservative results than average dwell time (ADT) and dwell time (DT). Conservativeness is intended as the time interval required to switch from one mode to another (which should be as short as possible as to approach arbitrarily fast switching). The open question is whether conservativeness can be reduced also in an adaptive framework without requiring any knowledge of the bounds of actual parameters.

The main contribution of this work is twofold: on the one hand, we develop a novel adaptive law using an aggregate leakage approach that does not require any knowledge of the bounds of the parametric uncertainties. On the other hand, we enlarge, in an adaptive setting, the family of stabilizing switching laws. By combining a multiple Lyapunov function with MDADT, we show that the closed-loop switched system is globally uniformly ultimately bounded. In addition, we study the transient and steady-state performance of the tracking error, which is a relevant problem in adaptive control systems: an upper bound and an ultimate bound of the tracking error are derived.

The paper is organized as follows: Section 2 introduces the control objective of this work. Section 3 proposes a family of adaptive laws and a switching law to design the adaptive controller. Section 4 gives the main stability results of the adaptive closed-loop switched system. Section 5 uses NASA GTM models to illustrate the proposed control methods. Some conclusive remarks are given in Section 6.

Notations: The notations used in this paper are standard. $\mathbb{N}^+$ and $\mathbb{R}$ denote the set of positive natural numbers and real numbers, respectively. The notation $P = P^T > 0$ represents a symmetric positive definite matrix, where the superscript "T" denotes the transpose of P. The notations $\lambda_{\max}(*)$ and $\lambda_{\min}(*)$ represent the maximum and minimum eigenvalue of a square matrix $*$, respectively. The operator $\mathrm{tr}(\cdot)$ is the trace of a matrix. The notation $\|\cdot\|$ is the Euclidean norm. The identity matrix is denoted by I.

II. Preliminaries and Problem Formulation

The paper is focused on uncertain switched linear systems of form of the following differential equations:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + b_{\sigma(t)}u(t), \quad \sigma(t) \in \mathcal{M} = \{1,\cdots,M\} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u : [0,\infty) \to \mathbb{R}$ is a piecewise continuous input, $\sigma(\cdot)$ is the switching signal, and the capital letter M represents the number of subsystems (or modes). Uncertainty arises due to the fact that the entries of $A_p \in \mathbb{R}^{n\times n}$ and $b_p \in \mathbb{R}^{n\times 1}$ are unknown for any $p \in \mathcal{M}$. In order to have a well-posed control problem, the pair (A_p, b_p) is controllable for all $p \in \mathcal{M}$.

A family of reference models representing the desired behavior of each subsystem is given as follows:

$$\dot{x}_{\mathrm{m}}(t) = A_{\mathrm{m}\sigma(t)}x_{\mathrm{m}}(t) + b_{\mathrm{m}\sigma(t)}r(t), \quad \sigma(t) \in \mathcal{M} \tag{2}$$

where $x_{\mathrm{m}} \in \mathbb{R}^n$ is the desired state, and $r \in \mathbb{R}$ is a bounded reference input. The matrices $A_{\mathrm{m}p} \in \mathbb{R}^{n\times n}$ and $b_{\mathrm{m}p} \in \mathbb{R}^{n\times 1}$ are known and $A_{\mathrm{m}p}$, $p \in \mathcal{M}$, are Hurwitz matrices (to have a stable desired behavior). The nominal state feedback controller that makes the switched system (1) behave like the reference models (2) is $u(t) = k^{*T}_{\sigma(t)}(t)x(t) + l^*_{\sigma(t)}(t)r(t)$, where $k^*_p \in \mathbb{R}^{n\times 1}$ and $l^*_p \in \mathbb{R}$, $p \in \mathcal{M}$, are nominal parameters, which can be calculated by the so-called matching condition [25]:

$$A_p + b_p k^{*T}_p = A_{\mathrm{m}p}, \; b_p l^*_p = b_{\mathrm{m}p}, \quad p \in \mathcal{M}. \tag{3}$$

Since the parameters A_p and b_p are unknown, we cannot calculate k^*_p and l^*_p using (3), but we must estimate them. Hence, the state feedback controller is designed as:

$$u(t) = k^T_{\sigma(t)}(t)x(t) + l_{\sigma(t)}(t)r(t) \tag{4}$$

where k_p and l_p are the estimates of k^*_p and l^*_p, $p \in \mathcal{M}$, respectively. In addition, the following tracking error is defined as $e(t) = x(t) - x_{\mathrm{m}}(t)$. By substituting (4) into (1), and by subtracting (2), we have the following dynamics of the tracking error

$$\dot{e}(t) = A_{\mathrm{m}\sigma(t)}e(t) + b_{\sigma(t)}(\tilde{k}^T_{\sigma(t)}(t)x(t) + \tilde{l}_{\sigma(t)}(t)r(t)) \tag{5}$$

where $\tilde{k}_p = k_p - k^*_p$ and $\tilde{l}_p = l_p - l^*_p$, $p \in \mathcal{M}$. $\tilde{k}_p$ and $\tilde{l}_p$ are the parameter estimation errors.

For later analysis, the following definitions are given:

Definition 1: [23] For a switching signal $\sigma(\cdot)$, let $N_{\sigma p}(t_1,t_2)$, $t_2 \geq t_1 \geq 0$, $p \in \mathcal{M}$, be the number of times that subsystem p is activated over interval $[t_1, t_2)$ and $T_p(t_1,t_2)$ denotes the total active time of subsystem p over the interval $[t_1,t_2]$, $p \in \mathcal{M}$. We say that $\sigma(\cdot)$ has a *mode-dependent average dwell time* (MDADT) $\tau_{\mathrm{a}p}$ if there exist positive numbers N_{0p} and $\tau_{\mathrm{a}p}$ such that the condition

$$N_{\sigma p}(t_1,t_2) \leq N_{0p} + \frac{T_p(t_1,t_2)}{\tau_{\mathrm{a}p}}, \quad \forall t_2 \geq t_1 \geq 0. \tag{6}$$

where N_{0p} are called mode-dependent chatter bounds, and $\tau_{\mathrm{a}p}$ mode-dependent average dwell time bounds.

Definition 2: [17] The switched system with parametric uncertainties (1) under switching signal $\sigma(\cdot)$ is *globally uniformly ultimately bounded* (GUUB) if there exists a finite positive number b_{T} such that for every initial function x_{t_0}, there exists a finite positive number T such that $\|x(t)\| \leq b_{\mathrm{T}}$ for all $t \geq t_0 + T$. Any positive number b_{T} for which this condition holds is called *ultimate bound*.

We are now ready to present the control objective for the switched system (1) as follows:

Problem 1: Given the uncertain switched system (1) and the reference models (2), design an adaptive mechanism for k_p and l_p in (4) without any knowledge of the bounds of k_p and l_p, and a MDADT switching law for $\sigma(\cdot)$ that guarantee boundedness of all signals of the closed-loop adaptive switched system, and GUUB of the tracking error (5). Furthermore, derive transient and steady-state bounds for the tracking error (5) under such designed adaptive and switching laws.

III. Methodology

In this section, to solve *Problem 1*, an adaptive law and a switching law are proposed. Since $A_{\mathrm{m}p}$ is stable, there exists a real symmetric matrix $P_p > 0$ for a given positive number κ_p of subsystem $p \in \mathcal{M}$ such that

$$A^T_{\mathrm{m}p}P_p + P_pA_{\mathrm{m}p} + \kappa_p P_p \leq 0. \tag{7}$$

We define $\alpha = \max_{p\in\mathcal{M}} \lambda_{\max}(P_p)$ and $\beta = \min_{p\in\mathcal{M}} \lambda_{\min}(P_p)$.

A. Adaptive Law

Before introducing the adaptive law, one assumption needs to be made: the sign of $l^*_{\sigma(t)}$ is known, which is widely used in adaptive law of linear systems to guarantee the boundedness of signals in closed-loop adaptive systems [25]. Let p, $p \in \mathcal{M}$, denote the index of the active subsystem for $t \in [t_l, \, t_{l+1})$. Therefore, the adaptive law with an aggregate leakage approach is given as follows, for $t \in [t_l, \, t_{l+1})$,

$$\dot{k}_p = -\mathrm{sign}[l^*_p]\Gamma_p x e^T P_p b_{\mathrm{m}p} - \delta^k_p \Gamma_p (k_p - k_{p,0}) \tag{8a}$$
$$\dot{l}_p = -\mathrm{sign}[l^*_p]\gamma_p r e^T(t) P_p b_{\mathrm{m}p} - \delta^l_p \gamma_p (l_p - l_{p,0}) \tag{8b}$$
$$\dot{k}_q = -\delta^k_q \Gamma_q (k_q - k_{q,0}) \tag{8c}$$
$$\dot{l}_q = -\delta^l_q \gamma_q (l_q - l_{q,0}) \tag{8d}$$

for $p,q \in \mathcal{M}$ with $p \neq q$, where the positive definite matrix $\Gamma_p \in \mathbb{R}^{n\times n}$ and positive constant $\gamma_p \in \mathbb{R}$ are the adaptive gains, $k_{p,0}$ and $l_{p,0}$, $p \in \mathcal{M}$, represent the initial guesses of the parameter estimates of the current active subsystem, and the positive constants $\delta_p^k \in \mathbb{R}$ and $\delta_p^l \in \mathbb{R}$ are the leakage rates, which are designed to satisfy

$$\begin{aligned} \delta_p^k - \max_{p\in\mathcal{M}}\left\{\kappa_p\right\}\lambda_{\max}\left(\Gamma_p^{-1}\right) \geq 0 \\ \delta_p^l - \max_{p\in\mathcal{M}}\left\{\kappa_p\right\}\gamma_p^{-1} \geq 0. \end{aligned} \tag{9}$$

Remark 1: To prevent the parameter estimates from diverging far away from the actual parameters due to the switching between different subsystems, a parameter projection has been commonly adopted in [5], [14], [15]. This requires the knowledge of the bounds of the actual parameters. In this work, a leakage approach from robust adaptive control [25] is exploited and extended to adaptive tracking control of switched linear systems. As a consequence, in contrast with the adaptive laws in [5], [14], [15], in (8) the assumption about the knowledge of upper and lower bounds of the parameter estimates is removed.

Remark 2: Some guidelines are given for the selections of the parameters in (8). The leakage rates δ_p^k and δ_p^l are designed to guarantee the stability of the switched systems, which are expected to be small for better tracking performance [25]. This requires κ_p to be small, and Γ_p, γ_p to be large enough. In addition, the initial guesses $k_{p,0}$ and $l_{p,0}$ can be reset at the beginning of each active interval of subsystem p according to the tracking performance, e.g. using a supervisory architecture [19].

B. Switching Law

A switching law is proposed based on the MDADT strategy as follows,

$$\tau_{\mathrm{a}p} > \tau_{\mathrm{a}p}^* = \frac{1}{\xi\kappa_p}\ln\mu_p, \quad \forall p \in \mathcal{M} \tag{10}$$

where $\mu_p = \alpha/\lambda_{\min}(P_p)$ and $\xi \in (0,\ 1)$ is a user-defined positive constant which will be clarified in the next section.

IV. Main Results

The proposed adaptive and switching laws lead to the following stability results.

Theorem 1: With the control law (4), adaptive law (8) and switching law (10) based on MDADT, the GUUB stability of the uncertain switched system (1) can be guaranteed. The norm of the tracking error is upper bounded by, $\forall t \geq t_0$,

$$\|e\|^2 \leq \frac{1}{\beta}\left(\sum_{p=1}^{N} N_{0p}\ln\mu_p\right)\max\left\{c_1,\ \frac{\alpha c_2}{\beta\min_{p\in\mathcal{M}}\left\{\kappa_p\right\}(1-\xi)}\right\}$$

where the positive constants c_1 and c_2 depend on the initial estimates and the real values of the controller parameters. In addition, the ultimate bound for the tracking error lies in the following interval,

$$b \in \left[0,\ \frac{1}{\beta}\sqrt{\exp\left(\sum_{k=1}^{M} N_{0k}\ln\mu_k\right)\frac{\alpha\Xi}{\min_{p\in\mathcal{M}}\left\{\kappa_p\right\}(1-\xi)}}\right]. \tag{11}$$

Proof: Let us consider the following quadratic Lyapunov function,

$$V(t) = e^T(t)P_{\sigma(t)}e(t) + \sum_{p=1}^{M}\frac{1}{|l_p^*|}\tilde{k}_p^T(t)\Gamma_p^{-1}\tilde{k}_p(t) + \sum_{p=1}^{M}\frac{1}{|l_p^*|}\tilde{l}_p^2(t)\gamma_p^{-1}. \tag{12}$$

Generally, P_p are different for different subsystems, which indicates that $V(t)$ might be continuous w.r.t. time only in the intervals between two consecutive switches. To study the behavior of (12) between two consecutive switching instants, we consider an active interval $t \in [t_l,\ t_{l+1})$. Then, according to (5), (7), and (8), the derivative of $V(t)$ w.r.t. time is, for $t \in [t_l,\ t_{l+1})$,

$$\begin{aligned} \dot{V}(t) = & -\kappa_{\sigma(t_l)}e^T(t)P_{\sigma(t_l)}e(t) - 2\sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^k\tilde{k}_p^T(t)\left(k_p(t)-k_{p,0}\right) \\ & -2\sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^l\tilde{l}_p(t)\left(l_p(t)-l_{p,0}\right) \\ = & -\kappa_{\sigma(t_l)}e^T(t)P_{\sigma(t_l)}e(t) - \sum_{p=1}^{M}\frac{2}{|l_p^*|}\delta_p^k\tilde{k}_p^T(t)\left(\tilde{k}_p(t)+k_p^*-k_{p,0}\right) \\ & -2\sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^l\tilde{l}_p(t)\left(\tilde{l}_p(t)+l_p^*-l_{p,0}\right) \\ \leq & -\kappa_{\sigma(t_l)}e^T(t)P_{\sigma(t_l)}e(t) - \sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^k\tilde{k}_p^T(t)\tilde{k}_p(t) \\ & +\sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^k\|k_p^*-k_{p,0}\|^2 - \sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^l\tilde{l}_p^2(t) \\ & +\sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^l\|l_p^*-l_{p,0}\|^2 \end{aligned} \tag{13}$$

where the last inequality holds according to Young's inequality. Hence, according to (12), the following holds

$$\begin{aligned} \dot{V}(t) \leq & -\kappa_{\sigma(t_l)}V(t) + \kappa_{\sigma(t_l)}\sum_{p=1}^{M}\frac{1}{|l_p^*|}\tilde{k}_p^T(t)\Gamma_p^{-1}\tilde{k}_p(t) \\ & +\kappa_{\sigma(t_l)}\sum_{p=1}^{M}\frac{1}{|l_p^*|}\tilde{l}_p^2(t)\gamma_p^{-1} - \sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^k\tilde{k}_p^T(t)\tilde{k}_p(t) \\ & +\sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^k\|k_p^*-k_{p,0}\|^2 - \sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^l\tilde{l}_p^2(t) \\ & +\sum_{p=1}^{M}\frac{1}{|l_p^*|}\delta_p^l\|l_p^*-l_{p,0}\|^2 \\ \leq & -\kappa_{\sigma(t_l)}V(t) + \sum_{p=1}^{M}\frac{1}{|l_p^*|}\left[\kappa_{\sigma(t_l)}\lambda_{\max}\left(\Gamma_p^{-1}\right)-\delta_p^k\right]\|\tilde{k}_p(t)\|^2 \\ & +\sum_{p=1}^{M}\frac{1}{|l_p^*|}\left[\kappa_{\sigma(t_l)}\gamma_p^{-1}-\delta_p^l\right]\tilde{l}_p^2(t) + \Xi \end{aligned} \tag{14}$$

where a finite constant Ξ is defined as

$$\Xi = \sum_{p=1}^{M}\frac{1}{|l_p^*|}\left(\delta_p^k\|k_p^*-k_{p,0}\|^2 + \delta_p^l\|l_p^*-l_{p,0}\|^2\right).$$

Due to (9), it follows from (14) that, for $\xi \in (0,\ 1)$

$$\begin{aligned}\dot{V}(t) \leq & -\kappa_{\sigma(t_l)}V(t) + \Xi \\ \leq & -\kappa_{\sigma(t_l)}\xi V(t) - \kappa_{\sigma(t_l)}(1-\xi)V(t) + \Xi.\end{aligned} \tag{15}$$

We define a positive finite number

$$\overline{B} = \frac{\Xi}{\min_{p\in\mathcal{M}}\{\kappa_p\}(1-\xi)}. \tag{16}$$

Now we get two possible behaviors of the Lyapunov function during two consecutive switching instants according to (15)–(16):

- For $V(t) \geq \overline{B}$, we have $\dot{V}(t) \leq -\kappa_{\sigma(t_l)}\xi V(t)$ i.e., $V(t)$ is decreasing exponentially with a countable rate.
- For $V(t) < \overline{B}$, it follows that $V(t)$ might be increasing.

Next, we study $V(t)$ at the switching instants. Let us consider the Lyapunov function at the switching instant t_{l+1}, $\forall l \in \mathbb{N}^+$. Subsystem $\sigma(t_{l+1}^-)$ is active when $t \in [t_l,\ t_{l+1})$ and subsystem $\sigma(t_{l+1})$ is active when $t \in [t_{l+1},\ t_{l+2})$. At the switching instant t_{l+1}, we have

$$\begin{aligned}V(t_{l+1}^-) = & \ e^T(t_{l+1}^-)P_{\sigma(t_{l+1}^-)}e(t_{l+1}^-) + \sum_{p=1}^{M}\frac{1}{|l_p^*|}\tilde{k}_p^T(t_{l+1}^-)\Gamma_p^{-1}\tilde{k}_p(t_{l+1}^-) \\ & + \sum_{p=1}^{M}\frac{1}{|l_p^*|}\tilde{l}_p^2(t_{l+1}^-)\gamma_p^{-1}\end{aligned}$$

and

$$\begin{aligned}V(t_{l+1}) = & \ e^T(t_{l+1})P_{t_{l+1}}e(t_{l+1}) + \sum_{p=1}^{M}\frac{1}{|l_p^*|}\tilde{k}_p^T(t_{l+1})\Gamma_p^{-1}\tilde{k}_p(t_{l+1}) \\ & + \sum_{p=1}^{M}\frac{1}{|l_p^*|}\tilde{l}_p^2(t_{l+1})\gamma_p^{-1}.\end{aligned}$$

Due to the continuity of the tracking error $e(t)$ and the parameter estimates, we have $e(t_{l+1}) = e(t_{l+1}^-)$, $\tilde{k}_p(t_{l+1}) = \tilde{k}_p(t_{l+1}^-)$, and $\tilde{l}_p(t_{l+1}) = \tilde{l}_p(t_{l+1}^-)$. Then, considering the fact that $e^T(t)P_{\sigma(t)}e(t) \leq \alpha e^T(t)e(t)$ and $e^T(t)P_{\sigma(t)}e(t) \geq \lambda_{\min}(P_{\sigma(t)})e^T(t)e(t)$, it follows at the switching instant t_{l+1}as follows,

$$\begin{aligned}V(t_{l+1}) - V(t_{l+1}^-) = & \ e^T(t_{l+1})\left(P_{\sigma(t_{l+1})} - P_{\sigma(t_{l+1}^-)}\right)e(t_{l+1}) \\ \leq & \frac{\alpha - \lambda_{\max}(P_{\sigma(t_{l+1}^-)})}{\lambda_{\min}(P_{\sigma(t_{l+1}^-)})}e^T(t_{l+1})P_{\sigma(t_{l+1}^-)}e(t_{l+1}).\end{aligned}$$

i.e.,

$$V(t_{l+1}) \leq \mu_{\sigma(t_{l+1}^-)}V(t_{l+1}^-) \tag{17}$$

with $\mu_{\sigma(t_{l+1}^-)} = \alpha/\lambda_{\min}(P_{\sigma(t_{l+1}^-)})$.

We are now ready to analyze the overall behavior of $V(t)$ by (15)–(17). Starting from the initial condition $V(t_0)$, there are two scenarios: *a)* $V(t_0) \geq \overline{B}$; *b)* $V(t_0) < \overline{B}$.

Scenario a) $V(t_0) \geq \overline{B}$. We assume that the Lyapunov function is located outside the bound $\overline{B}$ for $t \in [t_0,\ t_0+T_1)$, where $V(t)$ is decreasing exponentially at a countable rate between two consecutive switching instants. Denote the number of intervals that subsystem p, $p \in \mathcal{M}$, is active by N_{1p}. In light of this, it follows from (15) and (17) that, for $t \in [t_0,\ t_0+T_1)$,

$$\begin{aligned}V(t) \leq & \prod_{p=1}^{N}\mu_p^{N_{1p}}\exp\left\{-\sum_{p=1}^{N}\sum_{i=1}^{N_{1p}}(t_{p_i+1}-t_{p_i})\kappa_p\xi\right\}V(t_0) \\ = & \ \exp\left(\sum_{p=1}^{M}N_{1p}\ln\mu_p\right)\exp\left(-\sum_{p=1}^{M}T_p\kappa_p\xi\right)V(t_0) \\ \leq & \ \exp\left\{\sum_{p=1}^{N}\left[\left(N_{0p}+\frac{T_p}{\tau_{\mathrm{a}p}}\right)\ln\mu_p - T_p\kappa_p\xi\right]\right\}V(t_0) \\ \leq & \ \exp\left(\sum_{p=1}^{N}N_{0p}\ln\mu_p\right)\exp\left[\sum_{p=1}^{N}\left(\frac{\ln\mu_p}{\tau_{ap}} - \kappa_p\xi\right)T_p\right]V(t_0)\end{aligned} \tag{18}$$

where T_p is the total time when subsystem p is active for $t \in [t_0,\ t_0+T_1)$. By substituting MDADT $\tau_{\mathrm{a}p} > \ln\mu_p/\xi\kappa_p$ to (18), $V(t)$ is attracted into the finite interval $[0, \overline{B}]$ with sufficiently large positive number T_1. To calculate the value of $V(t_0+T_1)$, we focus on the worst case: when $t = t_0+T_1$, a switching behavior is activated. Then, the interval $[0, \overline{B}]$ increases to be $[0, \overline{B}\alpha/\beta]$, where the coefficient α/β is given by (17). As time elapses, $V(t)$ may possibly diverge and become far more than $\overline{B}\alpha/\beta$ due to fast switches for $t > t_0+t_1$. With performing this analysis recursively, it can be noticed that fast switches possibly take place intermittently throughout the whole time horizon, which can only guarantee that the Lyapunov function $V(\cdot)$ enters and then exceeds the bound $\overline{B}\alpha/\beta$ intermittently (because of fast switches) over the whole time horizon. The worse case for the ultimate bound of the Lyapunov function is that fast switches denoted by N_{0k} take place when the Lyapunov function becomes larger than the bound $\overline{B}\alpha/\beta$. This indicates that we can only guarantee the following ultimate bound of the Lyapunov function:

$$b_V \leq \exp\left(\sum_{p=1}^{M}N_{0k}\ln\mu_k\right). \tag{19}$$

Scenario b) $V(t_0) < \overline{B}$. The Lyapunov function is assumed to be non-decreasing in the beginning, and it might become larger than the bound $\overline{B}$. In light of this, making use of similar analysis of Scenario *(a)*, the same ultimate bound b_V of the Lyapunov function $V(\cdot)$ is achieved as in (19). Hence, we can conclude that the switched system (1) is GUUB via (19) according to the definite of GUUB. In addition, making use of (18), an upper bound of $V(\cdot)$ can be easily obtained with a switching law based on MDADT (10), which is shown as follows

$$V(t) \leq \exp\left(\sum_{p=1}^{N}N_{0p}\ln\mu_p\right)\max\left\{V(t_0),\ \frac{\alpha}{\beta}\overline{B}\right\}. \tag{20}$$

for $\forall t \geq t_0$. Since $V(t) \geq \beta\|e(t)\|^2$, we have, $\forall t \geq t_0$,

$$\|e(t)\|^2 \leq \frac{1}{\beta}\left(\sum_{p=1}^{N}N_{0p}\ln\mu_p\right)\max\left\{V(t_0),\ \frac{\alpha}{\beta}\overline{B}\right\}. \tag{21}$$

Furthermore, using (19), an ultimate bound of the tracking error is achieved as in (11). Substituting $c_1 = V(t_0)$, and $c_2 = \Xi$ to (21) and (11) gives rise to the results in Theorem 1. ■

Remark 3: The positive number ξ can illustrate the trade off between the length of the MDADT according to (10) and the performance of the tracking error according to (21) and (11). Large ξ results in long MDADT, which means that the switching signal is more conservative and we have a smaller bound on the ultimate tracking error. On the contrary, small ξ leads to short MDADT and larger ultimate bound of the tracking error. Finally, when the reference signal is zero, the tracking error turns out to be a regulation error, and the adaptive laws in (8) can still guarantee the main results of Theorem 1.

V. Example

The NASA GTM example in [5], [26] is adopted to illustrate the proposed method. Let us select $M = 2$, and trim the GTM at steady-state, straight, wings-level flight condition at 80 knots and 90 knots at 800 ft., respectively, to obtain a switched linear model. The control input is the elevator deflection, and the unknown parameter matrices are as follows

$$A_1 = \begin{bmatrix} -0.0293 & 0.2460 & -0.0899 & -0.3210 \\ -0.2611 & -3.0403 & 1.2973 & -0.0222 \\ 1.7458 & -32.0173 & -3.8364 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -0.0380 & 0.2786 & -0.0750 & -0.3213 \\ -0.2440 & -3.4119 & 1.4623 & -0.0165 \\ 1.3633 & -35.8069 & -4.4019 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$
$$b_1 = \begin{bmatrix} 0.0031 & -0.6953 & -85.2589 & 0 \end{bmatrix}^T$$
$$b_2 = \begin{bmatrix} -0.0010 & -0.8703 & -108.6559 & 0 \end{bmatrix}^T.$$

(a) (**Design of the reference models**) Using a common practice in many applications, the reference models are based on LQR control [27]. Two LQR controllers $u_p = k_p^* x(t)$ with $Q = \mathrm{diag}([1\ 0\ 0\ 0])$, and $R = 1$ are used as reference models. Note that the resulting LQR feedback gains are unknown to the designer: $k_1^* = \begin{bmatrix} -0.9156 & -0.2260 & 0.0208 & 0.3076 \end{bmatrix}$, $k_2^* = \begin{bmatrix} -0.9176 & -0.2054 & 0.0173 & 0.3222 \end{bmatrix}$. Then, the designer knows only the reference models,

$$A_{\mathrm{m}1} = \begin{bmatrix} -0.0321 & 0.2453 & -0.0898 & -0.3200 \\ 0.3755 & -2.8831 & 1.2829 & -0.2361 \\ 79.8084 & -12.7449 & -5.6073 & -26.2261 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$
$$A_{\mathrm{m}2} = \begin{bmatrix} -0.0371 & 0.2788 & -0.0750 & -0.3216 \\ 0.5546 & -3.2331 & 1.4473 & -0.2969 \\ 101.0614 & -13.4886 & -6.2783 & -35.0082 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$

which satisfies the matching condition $A_{\mathrm{m}p} = A_p + b_p k_p^*$, and $b_{\mathrm{m}p} = b_p$ with $l_p^* = 1$, $p \in \{1, 2\}$.

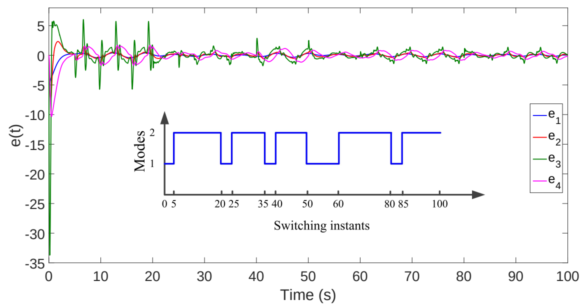

Fig. 1 The tracking error.

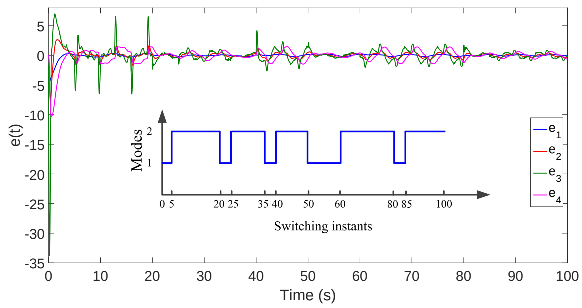

Fig. 2 The tracking error with unmodeled dynamics and disturbances.

(b) (**Model reference adaptive control**) Let $\kappa_1 = 1.2$, $\kappa_2 = 0.45$, solving (7) gives rise to the following positive definite matrices

$$P_1 = \begin{bmatrix} 5.3538 & -0.0335 & -0.0438 & -1.0257 \\ -0.0335 & 0.2765 & -0.0257 & -0.0189 \\ -0.0438 & -0.0257 & 0.0189 & 0.0257 \\ -1.0257 & -0.0189 & 0.0257 & 0.4225 \end{bmatrix}$$
$$P_2 = \begin{bmatrix} 4.6899 & -0.0568 & -0.0131 & -1.0084 \\ -0.0568 & 0.2570 & -0.0181 & -0.0342 \\ -0.0131 & -0.0181 & 0.0150 & 0.0171 \\ -1.0084 & -0.0342 & 0.0171 & 0.4986 \end{bmatrix}.$$

Let $\xi = 0.95$, and we obtain the MDADT $\tau_{\mathrm{a}1}^* = 5.3067$, $\tau_{\mathrm{a}2}^* = 14.1512$, by (10), and the ADT $\tau_{\mathrm{a}} = 14.1512$ by the method in [14], as shown in Table 1.

TABLE I: Comparison between two switching strategies.

Switching strategies	ADT	MDADT
Switching signal	$\tau_{\mathrm{a}}^* = 14.1512$ $(\mu = 423.96)$ $(\kappa = 0.45)$	$\tau_{\mathrm{a}1}^* = 5.3067$ $\tau_{\mathrm{a}2}^* = 14.1512$ $(\mu = 423.96)$ $(\kappa_1 = 1.2,\ \kappa_2 = 0.45)$

Note that the class of MDADT switching signals that stabilizes the switched system is larger than the class of ADT switching signals. Select $\tau_{\mathrm{a}1} = 6 > \tau_{\mathrm{a}1}^*$, $\tau_{\mathrm{a}2} = 15 > \tau_{\mathrm{a}2}^*$ for MDADT. We design a switching signal based on MDADT with $N_{01} = 4$, $N_{02} = 2$, as shown in Fig. 1. Let $\Gamma_1 = \Gamma_2 = I$, $\gamma_1 = \gamma_2 = 1$, $\delta_1^k = \delta_1^l = \delta_2^k = \delta_2^l = 1.2$, $\xi = 0.95$. With the initial condition $x_0 = \begin{bmatrix} 5 & 0 & 0 & 0 \end{bmatrix}^T$, $k_p(0) = 0.8k_p^*$, $l_p(0) = 0.8l_p^*$, $\forall p \in \mathcal{M}$. The performance of the tracking error is given in Fig. 1. It is observed that the tracking error is bounded via MDADT switching law without requiring any knowledge of bounds of the actual parameters, which illustrates the effectiveness of the proposed adaptive law (8) and switching law (10).

Then, we consider some unmodeled dynamics and external disturbances to show the robustness of the proposed adaptive law. Consider the unmodeled part of the system matrices and actuator vectors as follows

$$\Delta A_1 = \begin{bmatrix} -0.0150 & 0.0500 & 0.0100 & -0.0100 \\ -0.0200 & -0.1000 & 0.1000 & -0.0100 \\ 0.1200 & -0.5000 & -0.1000 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\Delta A_2 = \begin{bmatrix} -0.0100 & 0.1000 & -0.0100 & -0.0200 \\ -0.0200 & -0.1500 & 0.1000 & -0.0100 \\ 0.1300 & -0.3500 & -0.2000 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\Delta b_1 = \begin{bmatrix} 0.002 & -0.100 & -3.000 & 0 \end{bmatrix}^T$$
$$\Delta b_2 = \begin{bmatrix} -0.001 & -0.100 & -5.000 & 0 \end{bmatrix}^T$$

and the additive process disturbance $d(t) = [0.1\sin(5t) \quad 0.2\cos(10t) \quad 0.15\sin(\pi t) \quad 0]^T$. The same parameters and initial conditions are adopted. The tracking performance is given in Fig. 2. We see that the tracking error is bounded, which implies that the proposed adaptive control scheme shows robustness to some unmodeled dynamics and disturbances.

VI. Conclusion

In this work, a novel model reference adaptive control scheme has been developed for switched linear systems with parametric uncertainties. The adaptive law based on an aggregate leakage method has been introduced, which does not need any knowledge of the space the actual parameters reside. The switching law has been developed based on mode-dependent average dwell time, which guarantees less conservative switching signals in terms of the time interval required to switch from one mode to another (which should be as short as possible to as to approach arbitrarily fast switching). The analysis has shown that the adaptive switched closed-loop system is globally uniformly ultimately bounded under the proposed adaptive law and switching law. Moreover, an upper bound and an ultimate bound of the tracking error have been also derived. A practical example of NASA GTM has been exploited to show the effectiveness of the proposed method.

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