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Distributed Chance-Constrained Model Predictive Control for Condition-Based Maintenance Planning for Railway Infrastructures

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1 Introduction

Maintenance is essential for the reliability, availability, and safety of a railway network, which is composed of various infrastructures like tracks, tunnels, stations, switches, overhead wiring, signaling systems, and safety control systems. In this paper we focus on track maintenance, which in general takes up a large portion of the annual maintenance budget of a railway infrastructure network, e.g., 40% for the Dutch railway network [1]. As shown in Fig. 1, a railway track contains different assets, e.g., rails, ballasts, sleepers, fastenings, welds, etc., that are interconnected and work together. These assets suffer from quality degradation over time due to regular usage. For example, the contact between wheel and rail leads to squats, a typical rolling contact fatigue that first appears on the rail surface and might cause rail breakage if not treated properly [2]. Early-stage squats can be effectively treated by grinding, while late-stage squats can only be addressed by rail replacement [3].

Due to the high cost of railway track maintenance interventions (e.g., over EUR 10 000 for one grinding operation), and the limited resource for track maintenance (e.g., limited track possession time for maintenance), how to plan maintenance interventions in a cost-efficient way without sacrificing the safety and reliability of the whole network has become a primary concern for railway infrastructure managers. This explains why most European countries have started a shift from reactive maintenance to proactive maintenance in recent years [4,5]. Condition-based maintenance [6,7], where maintenance interventions are planned based on the "condition" of the asset, has been considered the most promising predictive maintenance strategy in various fields [8,9], as most system failures are preceded by one or more indicative signals [10].

We consider condition-based maintenance optimization [11, 12], where the optimal planning of maintenance interventions is based on an explicit mathematical model describing the deterioration dynamics of the asset. This deterioration model can be either deterministic or stochastic. Examples of deterministic models include the linear model used in [13] to describe track quality degradation over tonnage, and the exponential model proposed in [14] for track geometry deterioration over time. The main advantage of deterministic model is that the resulting optimization problem is easier to solve than in case a more complex stochastic model is used. However, as a deterministic model only captures the nominal deterioration behavior of an asset, the resulting maintenance plan might not be robust enough in the presence of various sources of randomness like model uncertainties and measurement errors. In this case stochastic models are preferred. A bi-variant Gamma process is used in [15] to describe the evolution of longitudinal and transverse levels for a French high-speed line. A grey-box model is proposed in [16] to identify the stochastic aging process of track geometry using Monte Carlo simulation. Dagum probabilities are used in [17] to characterize the reduction of the standard deviation of the longitudinal level over time. In [18], a fuzzy Takagi-Sugeno internal model is applied to capture the most representative dynamics of squat evolution.

To make the proposed approach applicable to a wide range of defects in general railway infrastructures, we use a piecewise-affine model with bounded uncertain parameters as the deterioration model. The main contribution of this chapter is the development of a model-based, optimization-based approach for

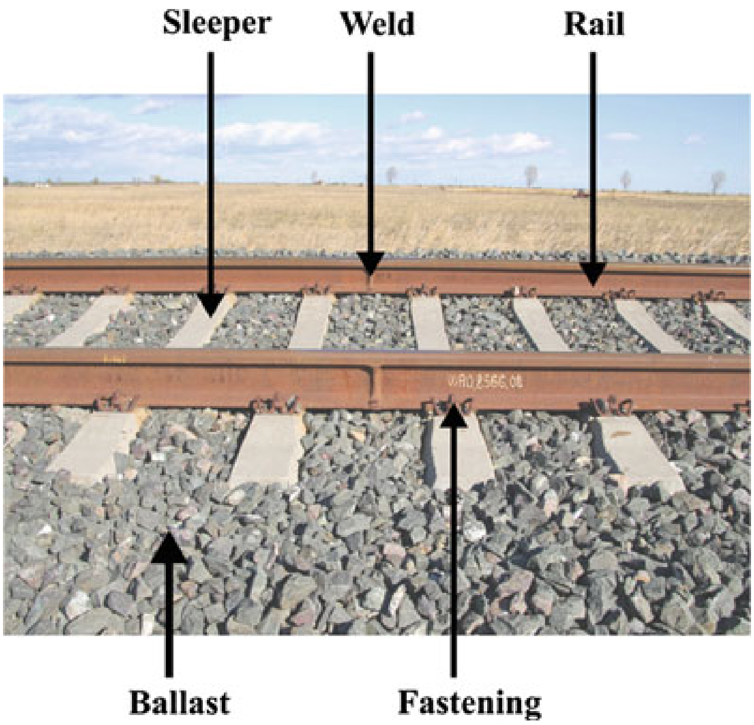

Figure 1: Components of railway track.

condition-based maintenance planning of railway infrastructures. The developed approach is robust but nonconservative, and the proposed distributed solution methods guarantee tractability even for large-scale infrastructure systems.

The paper is organized as follows: the theoretical background of the proposed approach is presented in Sect. 2, and the problem formulation is given in Sect. 3. Two distributed solution approaches are explained in Sect. 4. A numerical case study with computational experiments and comparison to other approaches is presented in Sect. 5. Finally, we conclude this work and provide future working directions in Sect. 6.

2 Preliminaries

We use Model Predictive Control (MPC) [19, 20] as the basic methodology for optimal condition-based maintenance planning for railway infrastructures. MPC follows a *receding horizon principle*. An optimization problem is solved at each sampling time step to predict the optimal sequence of maintenance actions for a given *prediction horizon*, based on the information (e.g., measurement data) available at the current time step. Only the first step of the maintenance action sequence is applied to the system, and a new optimization problem is solved at the next time step with new information. The prediction horizon is in general much shorter than the *planning horizon*, so the MPC optimization problem at each time step is much easier to solve than the correspondent optimization problem for the entire planning horizon. Although the MPC controller does not guarantee closed-loop optimality, in practice it usually gives a good control performance [21].

2.1 Hybrid and Distributed MPC

MPC has been applied to several real-world optimization problems like risk management [22] and supply chain management [23, 24]. If the system involved in these problems contains both continuous and discrete dynamics, we call it hybrid system. One way to address such a hybrid system is to transform it into a Mixed Logical Dynamical (MLD) system [25] and to solve a Mixed Integer Programming (MIP) problem at each time step. Another way is to adopt the concept of Time Instant Optimization (TIO) [26] and transform the MPC optimization containing both continuous and discrete decision variables into a non-smooth optimization problem with only continuous decision variables. Since both MIP problems and non-smooth optimization problems are NP-hard, hybrid MPC usually becomes computationally intractable for large-scale systems. In this case a distributed optimization scheme is usually adopted to improve the scalability of the MPC approach. In the control literature, most of the distributed optimization approaches are Lagrangian-based, e.g., Alternating Direction Method of Multipliers (ADMM) [27], and there is no guarantee of convergence to a global optimum for MIP problems. A continuous relaxation of binary variables is used in [28,29], yielding a bound on the objective function value to warm-start the MIP problem. A practical approach is proposed in [30] for a class of networked hybrid MPC. This heuristic first determines the binary decision variables in the local problems, and then transforms the Mixed Integer Quadratic Programming (MIQP) problem into a set of Quadratic Programming (QP) problems via distributed coordination. One non-Lagrangian-based distributed method for MIP problems is the Distributed Robust Safe But Knowledgeable (DRSBK) algorithm [31], which adopts a constraint tightening technique.

In the operations research literature, Benders decomposition [32] and Dantzig-Wolfe decomposition [33] are the most well-known decomposition methods for large-scale Linear Programming (LP) and Mixed Integer Linear Programming (MILP) problems. Benders decomposition is designed for problems coupled through common variables, while Dantzig-Wolfe decomposition is for problems coupled through common constraints. Benders decomposition can provide global optimal solution for MILP problems in which the integer decision variables are only in the coupling variables. An up-to-date review on Benders decomposition is provided in [34]. Dantzig-Wolfe decomposition only solves an LP relaxation for MILP problems. One example of applying Dantzig-Wolfe decomposition to hybrid MPC is [35], which provides a suboptimal solution of the MILP problem via column generation.

2.2 Chance-Constrained MPC

Real-world problems like maintenance planning are influenced by various sources of randomness like model uncertainties, measurement error, and missing data. Robust control [36,37], where control performance and constraint satisfaction are guaranteed when the uncertainties are within a specific range, might lead to a very conservative control strategy. In this case, the concept of chance-constrained optimization [38] can be adopted to achieve a balance between robustness and optimality. Chance-constrained MPC, where the probabilistic constraints are formulated as chance-constrained constraints and the objective is to optimize the expected value of the objective function, has been applied to various cases in industries like drinking water network management [39], hospital pharmacy stock management [40], and condition-based planning of railway infrastructures [41].

For chance-constrained optimization problems with known probability distributions of uncertainties, analytical approximation methods [42] are the most suitable solution approaches. When the probability distributions of uncertainties are unknown, scenario-based approaches [43] and sample average approximation methods [44] should be considered. Both approaches are based on randomization of uncertainties. The major difference is that scenario-based approaches have more restrictive assumptions on the convexity of the chance-constrained optimization problem, but require less randomized scenarios to obtain the same probabilistic guarantee as sample average approximation methods. On the other hand, sampling average approximation methods, which are based on Monte Carlo simulation, do not require convexity of the chance-constrained problem, but need a large number of scenarios to achieve an acceptable probabilistic guarantee.

Since most scenario-based approaches require the chance constraints to be convex with respect to the uncertain parameters, their applications to MILP chance-constrained problems are scarce. One notable example is [45]. However, the proposed bound in [45] on the number of scenarios is very conservative, and thus not suitable for large-scale chance-constrained problems. In this case, we choose a two-level approach [46] that lies between robust approach and scenario-based approach.

3 Problem Formulation

In this section, we first describe the deterioration model in Sect. 3.1. The local chance-constrained MPC problem is formulated in Sect. 3.2, and the two-stage robust scenario-based approach to approximate the chance-constrained MPC problem is explained in Sect. 3.3. Finally, the centralized MLD-MPC problem that have to be solved at each time step is formulated in Sect. 3.4. Some important symbols used in this section are presented in Table 1.

3.1 Deterioration Model

For the planning of track maintenance activities, we divide a piece of railway track into N nonoverlapping sections, as shown in Fig. 2. The following discrete-time state-space model is used to describe the independent

Table 1: Important symbols used in Sect. 3.

Symbol	Meaning
$x_{j,k}$	State of section j at time step k
$u_{j,k}$	Maintenance option applied to section j at time step k
$\theta_{j,k}$	Realizations of all the uncertain parameters for section j at time step k
$v_{j,k}$	New binary and continuous decision variables in the transformed MLD model
N_{P}	Prediction horizon
$\hat{x}_{j,k+l\|k}$	Estimated state of section j at time step $k+l$, based on the information available at time step k
$\tilde{x}_{j,k}$	Estimated state of section j from time step $k+1$ to time step $k+N_{\mathrm{P}}$
$\tilde{u}_{j,k}$	Maintenance option applied to section j from time step k to time step $k+N_{\mathrm{P}}-1$; same notation applies to $\tilde{\theta}_{j,k}, \tilde{v}_{j,k}$
$\tilde{x}_{j,k}^{(s)}$	Scenario s of $\tilde{x}_{j,k}$; similar notation applies to $\tilde{\theta}_{j,k}^{(s)}, \tilde{v}_{j,k}^{(s)}$

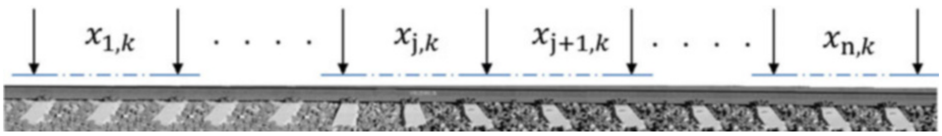

Figure 2: Illustration of track sections for a single railway line.

deterioration dynamics of each section $j \in \{1, \ldots, n\}$:

$$\begin{aligned} x_{j,k+1} &= f_j(x_{j,k}, u_{j,k}, \theta_{j,k}) \\ &= \begin{cases} f_j^1(x_{j,k}, \theta_{j,k}) & \text{if } u_{j,k} = 1 \text{ (no maintenance)} \\ f_j^q(x_{j,k}, \theta_{j,k}) & \text{if } u_{j,k} = q \quad \forall q \in \{2, \ldots, N-1\} \\ f_j^N(\theta_{j,k}) & \text{if } u_{j,k} = N \text{ (full renewal)}, \end{cases} \end{aligned} \tag{1}$$

where the vector $x_{j,k} = \left[x_{j,k}^{\mathrm{con}} x_{j,k}^{\mathrm{aux}} \right]^{\mathrm{T}} \in \mathscr{X}_j$ denotes the state of section j at time step k. In particular, $x_{j,k}^{\mathrm{con}}$ indicates the "condition" of the track section, while $x_{j,k}^{\mathrm{aux}}$ is an auxiliary state that can be viewed as the "memory" of the track section, e.g., the number of grindings that have been applied to this section since the last rail replacement. This auxiliary state is useful to capture the inefficiency of track maintenance activities. The discrete scalar $u_{j,k} \in \mathscr{U}_j = \{1, \ldots, N\}$ denotes the maintenance options, including maintenance activities and the "no maintenance" option, that is applied to section j. Finally, the vector $\theta_{j,k} \in \Theta_j$ contains the realizations of all the uncertain parameters for system j at time step k. Our only assumption on the uncertain parameters is that Θ_j is a bounded hyperbox.

We assume that for any $q \in \{1, \ldots, N\}$, the function f_j^q is either piecewise affine or linear with respect to $x_{j,k}$. This is not a very restrictive assumption, as piecewise-affine functions can approximate any nonlinear function with arbitrary accuracy.

3.2 Local Chance-Constrained MPC Problem

Let N_P denote the prediction horizon. Define:

$$\begin{aligned}\tilde{x}_{j,k} &= \left[\begin{array}{ccc}\hat{x}^\mathrm{T}_{j,k+1|k} & \dots & \hat{x}^\mathrm{T}_{j,k+N_\mathrm{P}|k}\end{array}\right]^\mathrm{T} \\ \tilde{u}_{j,k} &= \left[\begin{array}{ccc}u_{j,k} & \dots & u_{j,k+N_\mathrm{P}-1}\end{array}\right]^\mathrm{T} \\ \tilde{\theta}_{j,k} &= \left[\begin{array}{ccc}\theta^\mathrm{T}_{j,k} & \dots & \theta^\mathrm{T}_{j,k+N_\mathrm{P}-1}\end{array}\right]^\mathrm{T}\end{aligned} \tag{2}$$

where $\hat{x}^\mathrm{T}_{j,k+l|k}$ denotes the estimated state of section j at time step $k+l$, based on the information available at time step k. Define

$$J_j^\mathrm{Deg}(\tilde{x}_{j,k}) = \|\tilde{x}^\mathrm{con}_{j,k}\|_1, \tag{3}$$

where $\|\cdot\|_1$ denotes the 1 -norm, and P is a nonnegative weighting matrix. This term calculates the accumulated condition deterioration within the prediction window. Define the indicator function I_X, which takes value 1 if the statement X is true, and 0 otherwise. We then define

$$J_j^\mathrm{Maint}(\tilde{u}_{j,k}) = \sum_{l=0}^{N_\mathrm{P}-1}\sum_{q=1}^{N} c^\mathrm{Maint}_{q,j} I_{u_{j,k+l}=q}, \tag{4}$$

which computes the total maintenance costs for section j within the entire prediction window. The objective function for each local MPC controller can then be expressed as:

$$J_j(\tilde{x}_{j,k}, \tilde{u}_{j,k}) = J_j^\mathrm{Deg}(\tilde{x}_{j,k}) + \phi_j J_j^\mathrm{Maint}(\tilde{u}_{j,k}), \tag{5}$$

where the weighting parameter ϕ_j captures the trade-off between condition deterioration and maintenance costs. Finally, the chance-constrained MPC problem for section j can then be formulated as:

$$\min_{\tilde{u}_{j,k}} \mathbb{E}_{\tilde{\theta}_{j,k}}\left[J_j(\tilde{x}_{j,k}, \tilde{u}_{j,k})\right] \tag{6}$$

$$\text{subject to: } \tilde{x}_{j,k} = \tilde{f}_j(\tilde{u}_{j,k}, \tilde{\theta}_{j,k}; x_{j,k}) \tag{7}$$

$$\mathbb{P}_{\tilde{\theta}_{j,k}}\left[\max_{l=1,\dots,N_\mathrm{P}} \hat{x}^\mathrm{con}_{j,k+l|k}(\tilde{u}_{j,k}, \tilde{\theta}_{j,k}; x_{j,k}) \le x^\mathrm{con}_\mathrm{max}\right] \ge 1-\epsilon_j, \tag{8}$$

where the objective (6) is to minimize the expected condition deterioration and maintenance costs. The N_P-step prediction model (7) can be computed by recursive substitution of (1). Constraint (8) is the chance constraint, stating that the probability that the maximal degradation level within the prediction horizon is no more than the maintenance threshold $x^\mathrm{con}_\mathrm{max}$ is at least $1-\epsilon_j$, where the violation level ϵ_j is a small positive value, e.g., 0.05.

3.3 Two-Stage Robust Scenario-Based Approach

We apply the two-stage approach developed in [46] to approximate the chance-constrained problem (6)–(8) with a confidence level β_j indicating that the optimal solution of the resulting deterministic problem is also an ϵ-level solution of the originate chance-constrained problem with a probability at least $1-\beta_j$, where β_j is a small positive value.

First, we generate the scenario set $\mathscr{H}_j$ satisfying the following condition [47]:

$$|\mathscr{H}_j| \ge \left\lceil \frac{1}{\epsilon_j}\cdot\frac{e}{e-1}\left(2\dim\left(\tilde{\Theta}_j\right) - 1 + \ln\frac{1}{\beta_j}\right)\right\rceil \tag{9}$$

and solve the following convex scenario-based optimization problem:

$$\min_{\left\{(\underline{\tau}_i, \bar{\tau}_i)\right\}_{i=1}^{\dim(\tilde{\Theta}_j)}} \sum_{i=1}^{\dim(\tilde{\Theta}_j)} \bar{\tau}_i - \underline{\tau}_i \tag{10}$$

$$\text{subject to: } \left(\tilde{\theta}_{j,k}\right)_i^{(h)} \in [\underline{\tau}_i, \bar{\tau}_i] \quad \forall h \in \mathscr{H}, \forall i \in \left\{1, \dots, \dim(\tilde{\Theta}_j)\right\} \tag{11}$$

to obtain the smallest hyperbox $\mathscr{B}_j^*$ covering all scenarios in $\mathscr{H}_j$. The notation $\left(\tilde{\theta}_{j,k}\right)^{(h)}$ denotes the realization of $\tilde{\theta}_{j,k}$ for scenario h, and the symbol $(v)_i$ denotes the i-th entry of vector v.

Then we solve the robust optimization problem

$$\min_{\tilde{u}_{j,k}} \frac{1}{|\mathscr{H}_j|} \sum_{h \in \mathscr{H}_j} J_j\left(\tilde{x}_{j,k}^{(h)}, \tilde{u}_{j,k}\right) \tag{12}$$

$$\text{subject to: } \tilde{x}_{j,k}^{(h)} = \tilde{f}_j\left(\tilde{u}_{j,k}, \tilde{\theta}_{j,k}^{(h)}; x_{j,k}\right) \quad \forall h \in \mathscr{H}_j \tag{13}$$

$$\max_{\tilde{\theta}_{j,k} \in \mathscr{B}_j^* \cap \tilde{\Theta}_j} \max_{l=1,\ldots,N_\mathrm{P}} \hat{x}_{j,k+l|k}^{\mathrm{con}}\left(\tilde{u}_{j,k}, \tilde{\theta}_{j,k}; x_{j,k}\right) \leq x_{\max}^{\mathrm{con}}. \tag{14}$$

Furthermore, define the worst-case scenario w as

$$\tilde{\theta}_{j,k}^{(w)} \in \operatorname*{argmax}_{\tilde{\theta}_{j,k} \in \mathscr{B}_j^* \cap \tilde{\Theta}_j} \max_{l,\ldots,N_\mathrm{P}} \hat{x}_{j,k+l|k}^{\mathrm{con}}\left(\tilde{u}_{j,k}, \tilde{\theta}_{j,k}; x_{j,k}\right), \tag{15}$$

and replace the robust constraint (14) by the following linear constraint:

$$P_j \tilde{x}_{j,k}^{(w)}\left(\tilde{u}_{j,k}, \tilde{\theta}_{j,k}^{(w)}; x_{j,k}\right) \leq x_{\max}^{\mathrm{con}}, \tag{16}$$

where the matrix P_j satisfies $P_j \tilde{x}_{j,k} = \tilde{x}_{j,k}^{\mathrm{con}}$. The local chance-constrained MPC problem (6)–(8) can then be approximated by the deterministic optimization problem (12), (13), (16) with the local scenario set $\mathscr{S}_j = |\mathscr{H}_j| \cup \{w\}$.

3.4 MLD-MPC Problem

For each scenario $s \in \mathscr{S}_j$, we can transform the local deterioration model (1) into the following standard MLD model [25]:

$$x_{j,k+1}^{(s)} = A_j^{(s)} x_{j,k}^{(s)} + B_j^{(s)} v_{j,k}^{(s)} \tag{17}$$

$$E_{j,1}^{(s)} v_{j,k}^{(s)} \leq E_{j,2}^{(s)} x_{j,k}^{(s)} + E_{j,3}^{(s)}, \tag{18}$$

where the new decision variable $v_{j,k}^{(s)}$ contains all the binary and continuous decision variables in the transformed MLD model. An example of how to transform the deterioration dynamics of a generic railway asset can be found in [48].

Define $\tilde{v}_{j,k}$ similar to $\tilde{u}_{j,k}$ as in (2). Furthermore, define $\tilde{v}_k = \left[\left(\tilde{v}_{j,k}^{(1)}\right)^\mathrm{T} \ldots \left(\tilde{v}_{j,k}^{(|\mathscr{S}_j|)}\right)^\mathrm{T}\right]^\mathrm{T} \in \tilde{\mathscr{V}}_j$. Let $\tilde{v}_k = \left[\tilde{v}_{1,k}^\mathrm{T} \ldots \tilde{v}_{n,k}^\mathrm{T}\right]^\mathrm{T}$. The MPC optimization problem for the whole systems can then be expressed in the following compact MILP formulation:

$$\min_{\tilde{v}_k} \sum_{j=1}^{n} c_j \tilde{v}_{j,k} \tag{19}$$

$$\text{subject to: } \sum_{j=1}^{n} R_j \tilde{v}_{j,k} \leq r \tag{20}$$

$$F_j \tilde{v}_{j,k} \leq l_j \quad \forall j \in \{1, \ldots, n\}. \tag{21}$$

The objective function (19) is obtained by substituting (17) into the local objective function (12) for every section j. The linear constraint (20) is the global coupling constraint on resources, e.g., available track possession time for maintenance. Constraints (21) are the local constraints for each track section, including the deterministic approximation of the local chance constraint, and all the linear constraints from the transformation of the hybrid dynamics into an MLD model.

4 Distributed Optimization

The centralized MPC problem (19)–(21) is an NP-hard MILP problem, where the number of binary decision variables is proportional to the number of sections and the dimension of uncertain parameters. It becomes intractable for a railway infrastructure divided into a large number of sections, or for high-dimensional uncertainties. To improve the scalability of the proposed approach, we investigate two distributed optimization schemes. We call the first one the DWD algorithm, as it is based on Dantzig-Wolfe decomposition [49]. The second one is a modified version of the DRSBK algorithm [31] that uses a constraint tightening technique [50].

4.1 Dantzig-Wolfe Decomposition

Define the polyhedron $\mathscr{P}_{j,k} = \left\{ \tilde{v}_{j,k} \in \tilde{\mathscr{V}}_j : F_j \tilde{v}_{j,k} \leq l_j \right\}$, which is the feasible region of the j-th local MPC problem. The set $\mathscr{G}_{j,k}$ that contains all the extreme points, i.e., columns, of the convex hull of $\mathscr{P}_{j,k}$, is called the *generating set* of the j-th subproblem. According to Minkowski's theorem [51], every point in a compact polyhedron can be represented by a convex combination of the extreme points. For each column $g \in \mathscr{G}_{j,k}$, let $\tilde{v}_{j,k}^{[g]}$ denote the value of $\tilde{v}_{j,k}$ at column g, and let $\mu_{j,g}$ denote the weight assigned to column g. Furthermore, define $\mu_j = \left[\mu_{j,1} \ \dots \ \mu_{j,|\mathscr{G}_{j,k}|}\right]^{\mathrm{T}}$ and $\mu = \left[\mu_1^{\mathrm{T}} \ \dots \ \mu_n^{\mathrm{T}}\right]^{\mathrm{T}}$. The *master problem* can then be defined as:

$$\min_{\mu} \sum_{j=1}^{n} \sum_{g \in \mathscr{G}_{j,k}} c_j \tilde{v}_{j,k}^{[g]} \mu_{j,g} \tag{22}$$

$$\text{subject to: } \sum_{j=1}^{n} \sum_{g \in \mathscr{G}_j} \left(R_j \tilde{v}_{j,k}^{[g]} \right) \mu_{j,g} \leq r \tag{23}$$

$$\sum_{g \in \mathscr{G}_j} \mu_{j,g} = 1 \quad \forall j \in \{1, \dots, n\} \tag{24}$$

$$\mu_{j,g} \geq 0 \quad \forall g \in \mathscr{G}_{j,k}, \ \forall j \in \{1, \dots, n\}. \tag{25}$$

This master problem is a reformulation of the LP-relaxation of the centralized MPC problem (19)–(21).

As the size of the generating set $\mathscr{G}_{j,k}$ is usually large, column generation [52], which starts with an initial partial generating set $\mathscr{G}_{j,k}^{\mathrm{s}} \subset \mathscr{G}_{j,k}$, is usually used to improve computational efficiency. Instead of solving the master problem, a *restricted master problem* that can be obtained by simply replacing $\mathscr{G}_{j,k}$ by $\mathscr{G}_{j,k}^{\mathrm{s}}$ in (22)–(25) is solved. The dual of this restricted master problem can be written as:

$$\max_{\lambda,\pi} -r\lambda + \sum_{j=1}^{n} \pi_j \tag{26}$$

$$\text{subject to: } \lambda \left(-R_j \tilde{v}_{j,k}^{[g]} \right) + \pi_j \leq c_j \tilde{v}_{j,k}^{[g]} \quad \forall g \in \mathscr{G}_{j,k}^{\mathrm{s}}, \forall j \in \{1, \dots, n\} \tag{27}$$

$$\lambda \geq 0 \tag{28}$$

$$\pi \in \mathbb{R}^n. \tag{29}$$

Let μ^* and (λ^*, π^*) denote the optimal solutions of the restricted master problem and its dual, respectively. The *reduced cost* of section j can then be obtained by solving the following pricing subproblem:

$$\begin{aligned} \rho_j &= \min_{g \in \mathscr{G}_{j,k}} c_j \tilde{v}_{j,k}^{[g]} + \lambda^* \left(R_j \tilde{v}_{j,k}^{[g]} \right) - \pi_j^* \\ &= \min_{\tilde{v}_{j,k} \in \mathscr{P}_{j,k}} c_j \tilde{v}_{j,k} + \lambda^* \left(R_j \tilde{v}_{j,k} \right) - \pi_j^* \end{aligned} \tag{30}$$

which is an MILP. We only add the new column, i.e., the optimal solution of (30), into the partial generating set $\mathscr{G}_{j,k}^{\mathrm{S}}$, when the reduced cost ρ_j is negative. Furthermore, an upper bound on the objective function value of the centralized MPC problem is obtained whenever μ^* is binary, and a lower bound is given by:

$$\begin{aligned} q(\lambda^*) &= \inf_{\tilde{v}_k \in \times_{j=1}^{n} \mathscr{P}_{j,k}} \sum_{j=1}^{n} c_j \tilde{v}_{j,k} + \lambda^* \left(\sum_{i=1}^{n} R_j \tilde{v}_{j,k} - r \right) \\ &= -\lambda^* r + \sum_{j=1}^{n} \left(\rho_j + \pi_j^* \right) \end{aligned} \tag{31}$$

which is the Lagrangian dual of the centralized MPC problem.

The column generation procedure terminates when all the reduced costs are 0, or when the upper bound meets the lower bound. In particular, if the procedure ends with a binary μ^*, then we have also found

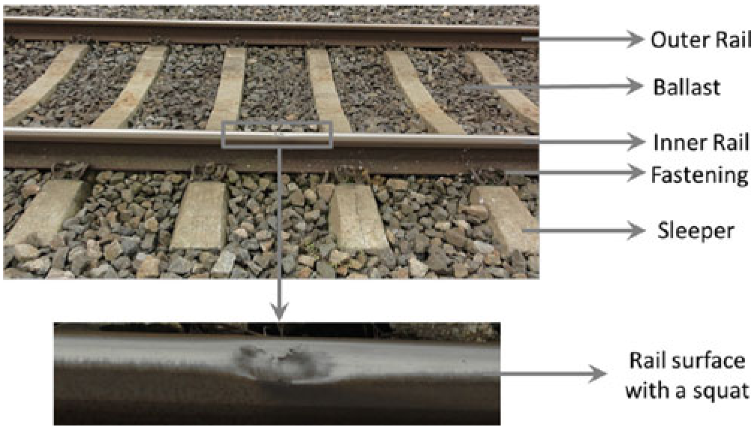

Figure 3: A severe squat on the rail surface.

the global optimal solution for the centralized MPC problem. If not, then a suboptimal solution of the centralized MPC problem can be found by solving the restricted master problem using the partial generating sets obtained at the end of the column generation procedure [35].

4.2 Constraint Tightening

We modify the DRSBK algorithm [31], which is based on a constraint tightening technique. First, we generate a random sequence s that is a permutation of the set $\{1, \ldots, n\}$. This sequence specifies the order of solving the subproblems. Then for each section j, we define the following subproblem:

$$\min_{\tilde{v}_{j,k} \in \mathscr{P}_{j,k}} c_j \tilde{v}_{j,k} \tag{32}$$

$$\text{subject to: } R_j \tilde{v}_{j,k} \leq r - \sum_{i=1, i \neq j}^{n} R_i \tilde{v}_{i,k}^{\dagger}, \tag{33}$$

where the local feasible region $\mathscr{P}_{j,k}$ is defined the same way as in Sect. 4.1. The left-hand side of constraint (33) is the resource allocated to section j, while the right-hand side represents the global resource reduced by the resource allocated to all the other sections. If the i-th subproblem is already solved before the j-th problem, then $\tilde{v}_{i,k}^{\dagger}$ denotes its optimum at time step k, otherwise $\tilde{v}_{i,k}^{\dagger}$ denotes the optimal solution of the i-th problem at time step $k - 1$.

If the subproblem (32)–(33) is infeasible for any section j, a new sequence s is generated, and the subproblems are solved in a new order. The iteration terminates when all the subproblems are feasible, and the difference of global objective function values between the current iteration and the previous iteration is less than the optimality tolerance. Unlike column generation, where the solution improves over each iteration, this random algorithm might need a large number of iterations for convergence. However, in practice this random algorithm works surprisingly well for MILP problems with a relatively small number of coupling constraints.

5 Case Studies

5.1 Settings

A numerical case study is performed on the optimal treatment of squats, a type of rolling contact fatigue. The evolution of a squat depends on the dynamic wheel-rail contact. A severe squat is shown in Fig. 3. The severity of a squat is determined by its visual length, which can be measured by techniques like axle box acceleration [53, 54], eddy current testing [55], or ultrasonic surface waves [56]. The degradation level, i.e., condition, of each section can be computed by aggregating the individual squat measurements within the section, as in [41]. For convenience we normalize the degradation level to $[0, 1]$.

We consider three maintenance options, no maintenance, grinding, and replacing, to be applied to each track section. The deterioration model of section j can then be expressed as:

$$\begin{aligned} x_{j,k+1}^{\text{con}} &= f_j^{\text{con}}(x_{j,k}^{\text{con}}, u_{j,k}, \theta_{j,k}) \\ &= \begin{cases} f_j^{\text{Deg}}(x_{j,k}^{\text{con}}, \theta_{j,k}) & \text{if } u_{j,k} = 1 \text{ (no maintenance)} \\ f_j^{\text{Gr}}(x_{j,k}^{\text{con}}, \theta_{j,k}) & \text{if } u_{j,k} = 2 \text{ (grinding)} \\ 0 & \text{if } u_{j,k} = 3 \text{ (replacing)} \end{cases} \end{aligned} \tag{34}$$

$$\begin{aligned} x_{j,k+1}^{\text{aux}} &= f_j^{\text{aux}}(x_{j,k}^{\text{aux}}, u_{j,k}) \\ &= \begin{cases} x_{j,k}^{\text{aux}} & \text{if } u_{j,k} = 1 \text{ (no maintenance)} \\ x_{j,k}^{\text{aux}} + 1 & \text{if } u_{j,k} = 2 \text{ (grinding)} \\ 0 & \text{if } u_{j,k} = 3 \text{ (replacing).} \end{cases} \end{aligned} \tag{35}$$

The auxiliary state $x_{j,k}^{\text{aux}}$ counts the number of grindings on section j since the last renewal. The functions f_j^{Deg} and f_j^{Gr} in (34) are both piecewise-affine in the current condition $x_{j,k}^{\text{con}}$, i.e.

$$f_j^{\text{Deg}}(x_{j,k}^{\text{con}}) = \begin{cases} y_{j,1}^{\text{int}} + \frac{y_{j,2}^{\text{int}} - y_{j,1}^{\text{int}}}{x_{j,1}^{\text{swi}}} x_{j,k}^{\text{con}} & \text{if } x_{j,k}^{\text{con}} \in \left[0, x_{j,1}^{\text{swi}}\right) \\ y_{j,2}^{\text{int}} + \frac{y_{j,3}^{\text{int}} - y_{j,2}^{\text{int}}}{x_{j,2}^{\text{swi}} - x_{j,1}^{\text{swi}}} \left(x_{j,k}^{\text{con}} - x_{j,1}^{\text{swi}}\right) & \text{if } x_{j,k}^{\text{con}} \in \left[x_{j,1}^{\text{swi}}, x_{j,2}^{\text{swi}}\right) \\ y_{j,3}^{\text{int}} + \frac{y_{j,4}^{\text{int}} - y_{j,3}^{\text{int}}}{1 - x_{j,2}^{\text{swi}}} \left(x_{j,k}^{\text{con}} - x_{j,2}^{\text{swi}}\right) & \text{if } x_{j,k}^{\text{con}} \in \left[x_{j,2}^{\text{swi}}, 1\right], \end{cases} \tag{36}$$

$$f_j^{\text{Gr}}(x_{j,k}^{\text{con}}) = \begin{cases} 0 & \text{if } x_{j,k}^{\text{con}} \le x_j^{\text{eff}} \\ \frac{y_j^{\text{sev}}}{x_j^{\text{sev}} - x_j^{\text{eff}}} \left(x_{j,k}^{\text{con}} - x_j^{\text{eff}}\right) & \text{if } x_j^{\text{eff}} < x_{j,k}^{\text{con}} \le x_j^{\text{sev}} \\ y_j^{\text{sev}} + \frac{y_j^{\max} - y_j^{\text{sev}}}{1 - x_j^{\text{sev}}} \left(x_{j,k}^{\text{con}} - x_j^{\text{sev}}\right) & \text{if } x_{j,k}^{\text{con}} > x_j^{\text{sev}}. \end{cases} \tag{37}$$

Five different deterioration models are used, and the model parameters are given in Table 3 in Appendix A. The maintenance threshold $x_{\max}^{\text{con}}$ is 0.95, and the following deterministic constraints are imposed on the auxiliary state:

$$x_{j,k+l}^{\text{aux}} \le x_{\max}^{\text{aux}} \quad \forall j \in \{1, \dots, n\},\ \forall l \in \{1, \dots, N_{\text{P}}\}, \tag{38}$$

to bound the maximal number of consecutive grindings on one track section. We set $x_{\max}^{\text{aux}} = 10$ in the case study.

Finally, we have the following global constraint:

$$\sum_{j=1}^{n} I_{u_{j,k}=1} \le n_{\max}^{\text{Gr}} \quad \forall l \in \{1, \dots, N_{\text{P}}\} \tag{39}$$

to bound the maximal number of sections that can be ground at each time step.

The proposed approach is implemented in Matlab R2016b, on a desktop computer with an Intel Xeon E5-1620 eight-core CPU and 64 GB of RAM, running a 64-bit version of SUSE Linux Enterprise Desktop 12. All the MILP and LP problems are solved by CPLEX V12.7.0.

5.2 Representative Run

A representative run with 53 track sections is performed to illustrate the proposed MPC approach. The length of each track section can range from 200 m to 5 km . Note that the size of the MPC optimization problem depends on the number of track sections in the network, rather than the length of each track section. For the same physical network, a finer partition captures the condition of a section more accurately, at the cost of heavier computational demand. The sampling time is 3 months, and the planning horizon is 20 steps, i.e., 5 years. The prediction horizon $N_{\text{P}} = 3$, and the maximal number of sections that can be ground is 15 . The maximum number of section that can be ground at each time step is determined by multiple practical factors like the sampling time step (the larger the sampling time step, the more available track possession time for maintenance) and section length (longer section indicates more maintenance time to treat each section, thus less sections that can be ground). The realizations of the uncertain parameters within the planning window are randomly generated by Gaussian distribution. The simulation results of one of the 53 sections are shown in Fig. 4. From Fig. 4a we can see that the degradation level of this track section is kept below the maintenance threshold for the entire planning horizon. Due to the high maintenance cost,

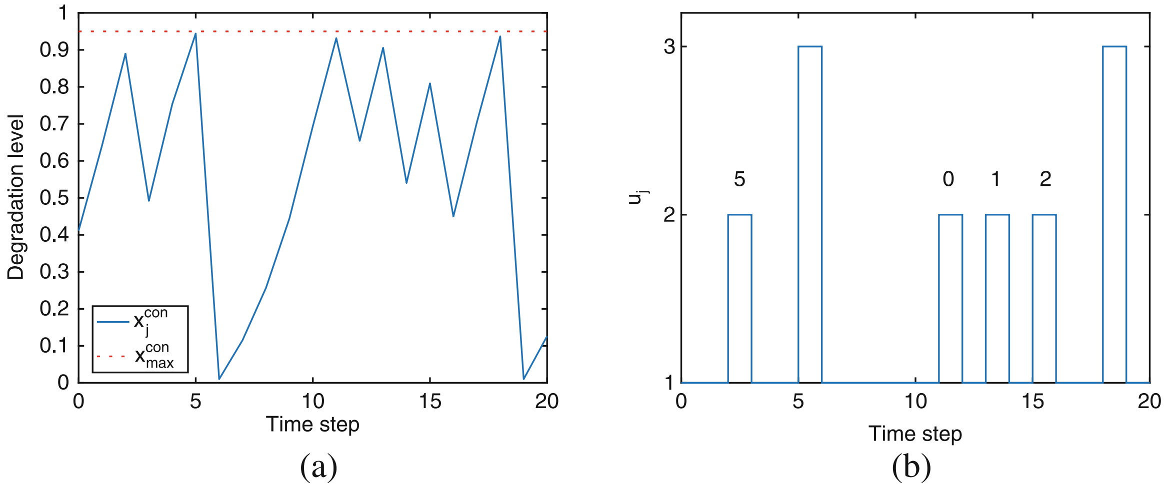

Figure 4: Simulation results for section 24 by the chance-constrained MPC based on column generation. The number above each grinding action is the number of previous grindings on section 24 since the last replacement. (a) Simulated degradation levels within the planning horizon. (b) Interventions suggested by the MPC controller.

maintenance interventions, including grinding and replacing, are suggested when the degradation level is relatively high (above 0.8). Replacing is suggested when the degradation level almost hits the threshold, and there is a long interval (7 time steps) of no maintenance after rail replacement.

An overview of the simulation results of the whole network at one time step is shown in Fig. 5. In total 11 grindings and 2 replacements are suggested at the current time step, keeping the degradation levels of the whole network under the maintenance threshold at the next time step.

5.3 Computational Comparisons

We test the performance of the two distributed optimization algorithms on 12 randomly generated chance-constrained MPC optimization problems with the number of sections ranging from 10 to 120. The centralized approach becomes intractable (out of memory) when the number of sections reaches 130. The comparison of the 12 test problems is shown in Fig. 6. The DWD algorithm is the fastest one in all the 12 test problems. Moreover, the CPU time increases almost linearly as the size of the problem grows. The DRSBK algorithm does not show much advantage over the centralized method for small problems with no more than 30 sections. However, as the computation time of the DRSBK algorithm also grows linearly, the reduction in CPU time becomes more obvious for larger problems, especially those with more than 100 sections. The centralized approach is the slowest one in most of the test problems.

Neither of the two distributed algorithms provides theoretical guarantee on convergence to global optimum. However, the DWD algorithm is able to obtain global optimum in all the test problems. DRSBK algorithm converges to the global optimum in all the test problems except the one with 80 sections. It converges to a local optimum 70% away from the global optimum.

In summary, the DWD algorithm performs the best among the three solution methods. The centralized approach always provides global optimal solution, but its scalability is poor. The DRSBK algorithm is faster and more scalable than centralized approach. However, it might converge to a local optimum very far away from the global optimum. The DWD algorithm is the fastest among the three algorithms, and it converges to the global optimum in all the test cases. Moreover, due to its distributed nature, it is suitable for large-scale railway networks divided into many sections, as tractability of the DWD algorithm mainly depends on the tractability of the local pricing problem (30), which is an MILP of the same size as the centralized MPC problem for one single section.

5.4 Comparison with Alternative Approaches

We compare the results of the proposed chance-constrained MPC (solved by the DWD algorithm) with two alternative maintenance planning approaches. The first one is the nominal MPC approach, which uses a deterministic deterioration model that considers only the mean values of the uncertain parameters. The

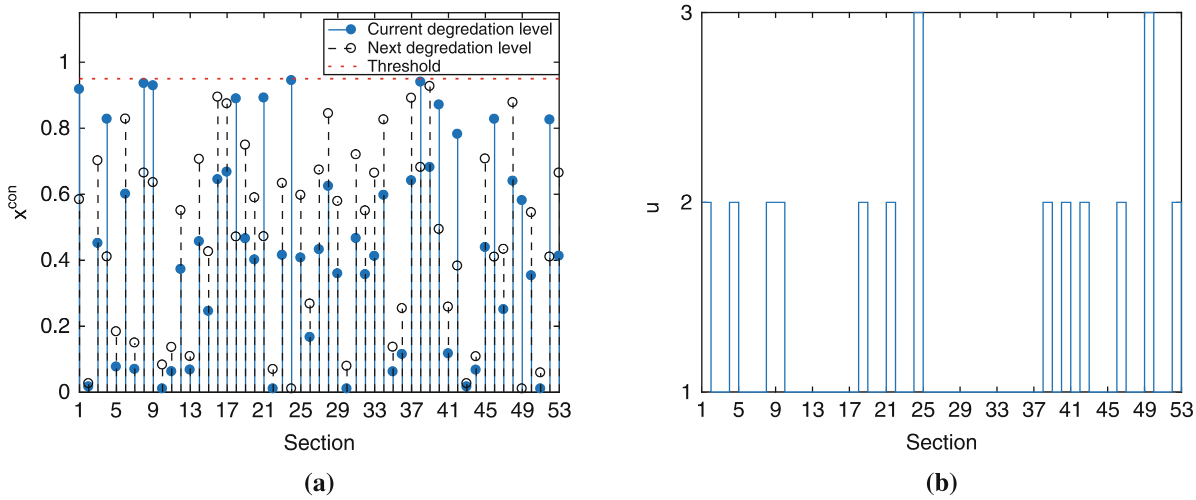

Figure 5: Simulation results for the whole railway network at representative time step ($k = 6$). (a) Degradation levels of the whole railway network at time step 6 (current time step) and time step 7 (next time step). (b) Interventions suggested by the high-level MPC controller at time step 6 for the whole railway network.

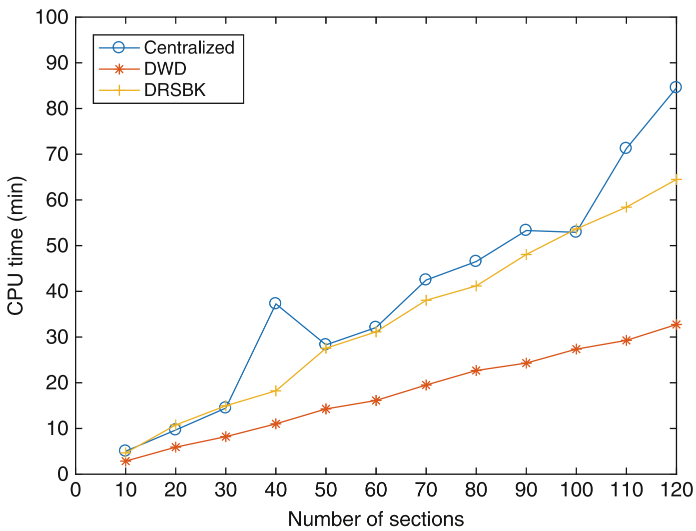

Figure 6: CPU time of the centralized approach and two distributed approaches.

Table 2: Comparison between the proposed chance-constrained MPC approach (with subscript "CC") solved by the DWD algorithm, the nominal approach (with subscript "Nom"), and the cyclic approach (with subscript "Cyc").

Run	Constraint violation			Closed-loop performance			CPU time (h)	
	$v_{\mathrm{CC}}(\%)$	$v_{\mathrm{Nom}}(\%)$	$v_{\mathrm{Cyc}}(\%)$	$\frac{J_{\mathrm{CC}}}{J_{\mathrm{Cyc}}}(\%)$	$\frac{J_{\mathrm{Nom}}}{J_{\mathrm{Cyc}}}(\%)$	J_{Cyc}	T_{CC}	T_{Nom}
1	0	0.063	0	39.335	34.148	670,502	5.671	0.003
2	0	0.006	0	38.127	36.577	670,504	5.075	0.003
3	0	0.353	0	37.635	35.043	670,503	5.062	0.003
4	0	0.129	0	37.606	33.344	670,502	5.703	0.003
5	0	0	0	36.354	34.536	670,502	5.141	0.003
6	0	0.082	0	36.413	35.803	670,502	5.802	0.003
7	0	0.021	0	39.425	36.250	670,503	5.134	0.003
8	0	0.053	0	38.440	35.028	670,500	5.126	0.003
9	0	0.0344	0	40.244	33.359	670,503	5.088	0.003
10	0	0.172	0	38.902	34.656	670,503	5.082	0.003

other one is the cyclic approach following a time-based maintenance strategy, and performing grinding and replacing at a predetermined optimal interval. The formulation of the cyclic approach is given in Appendix B.

We compare robustness, optimality, and computational efficiency of the three maintenance planning approaches. Robustness is measured by maximal constraint violation v defined as:

$$v = \max\left(\frac{x_{\mathrm{worst}}^{\mathrm{con}} - x_{\max}^{\mathrm{con}}}{x_{\max}^{\mathrm{con}}}, 0\right) \tag{40}$$

where $x_{\mathrm{worst}}^{\mathrm{con}}$ is the highest degradation level of all sections within the entire planning horizon. Optimality is measured by the closed-loop objective function value, which is obtained by evaluating all the local objective function values (5) for the entire planning horizon and summing them up. Computational efficiency is measured by the CPU time needed for solving all the MPC optimization problems for all the 20 time steps. Since the cyclic approach is an offline optimization approach, i.e., it solves only one optimization problem for the entire planning horizon, we only compare the computational efficiency of the two MPC approaches.

We create 10 test runs where the realizations of the uncertain parameters within the planning horizon are randomly generated by a Gaussian distribution. The comparison of the three approaches for the 10 test runs is shown in Table 2. Both the chance-constrained MPC approach and the cyclic maintenance approach are robust, as neither of them has constraint violations for the 10 test runs. However, the cyclic approach shows much worse closed-loop performance. It is very conservative and tends to plan more maintenance than necessary. The nominal MPC approach has a slightly lower closed-loop objective function value than the chance-constrained MPC approach, and a much shorter CPU time. However, it is not robust, as it has constraint violations in 9 out of the 10 test runs. So in comparison, the proposed chance-constrained MPC provides an excellent balance between robustness and optimality, despite its high computational demand.

6 Conclusions and Future Work

In this paper we have developed a chance-constrained MPC approach for optimal condition-based maintenance planning for railway infrastructures. Two distributed optimization algorithms, the DWD algorithm based on Dantzig-Wolfe decomposition, and the modified Distributed Robust Safe But Knowledgeable (DRSBK) algorithm [31], have been investigated to improve the scalability of the proposed MPC approach. Computational experiments have shown that column generation is able to obtain the global optimum with

Table 3: Parameters of the functions f_j^{Deg} and f_j^{Gr} for five different models. Both the nominal values and the 95% nonsimultaneous confidence bounds (given in the square brackets) are provided for all uncertain parameters.

	Model				
Parameter	1	2	3	4	5
$x_{j,1}^{\mathrm{swi}}$	0.512	0.526	0.543	0.363	0.563
$x_{j,2}^{\mathrm{swi}}$	0.683	0.784	0.781	0.621	0.798
$y_{j,1}^{\mathrm{int}}$	0.107 [0.086,0.128]	0 [0,0]	0.051 [0.040,0.063]	0.076 [0.036,0.115]	0.058 [0.049,0.068]
$y_{j,2}^{\mathrm{int}}$	0.783 [0.776,0.790]	0.849 [0.845,0.853]	0.815 [0.809,0.821]	0.624 [0.615,0.633]	0.805 [0.800,0.809]
$y_{j,3}^{\mathrm{int}}$	0.929 [0.924,0.934]	0.975 [0.967,0.983]	0.972 [0.966,0.977]	0.859 [0.853,0.865],	0.963 [0.958,0.968]
$y_{j,4}^{\mathrm{int}}$	1 [0.997,1.003]	1 [0.997,1.004]	1 [0.998,1.002]	1 [0.994,1.006]	1 [0.998,1.002]
x_j^{eff}	0.156	0.177	0.172	0.141	0.106
x_j^{sev}	0.899	0.810	0.880	0.938	0.882
y_j^{sev}	0.506 [0.494,0.518]	0.516 [0.505,0.527]	0.502 [0.490,0.514]	0.506 [0.490,0.521]	0.443 [0.432,0.455]
$y_j^{\max}$	0.957 [0.944,0.970]	0.991 [0.981,1]	0.977 [0.965,0.990]	0.922 [0.905,0.939]	0.944 [0.931,0.956]

a much shorter CPU time. Comparison with two alternative maintenance planning approaches has shown that the proposed chance-constrained MPC approach is robust and cost-effective.

In the future, it is interesting to consider heterogeneous components, e.g., rail and switches, in maintenance planning. Another interesting extension would be joint condition-based maintenance planning and train scheduling. Furthermore, a business case study with historical measurement data and actual maintenance costs can be performed to demonstrate the applicability of the proposed MPC approach for real-world railway track maintenance planning problems. For this purpose, a suitable key performance indicator should be chosen to evaluate the condition of each track section, and sufficient data should be used to identify the deterioration model.

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A Parameters for Case Study

See Table 3.

B Cyclic Approach

Let $t_{0,j}$ denote the time instant of the first replacement on section j. Grinding is performed every $T_{\mathrm{Gr},j}$ after the first replacement for section j. Furthermore, we assume that replacement is performed after r consecutive grindings since the last replacement on section j. Let k_{end} denote the planning horizon. Then the offline optimization problem of the cyclic maintenance approach can be formulated as:

$$\min_{t_0, T_{\mathrm{Gr}}, r} \sum_{k=1}^{k_{\mathrm{end}}} \sum_{j=1}^{n} x_{j,k}^{\mathrm{con}} + \lambda \sum_{q=2}^{3} c_{q,j}^{\mathrm{Maint}} I_{u_{j,k}=q} \tag{41}$$

subject to

$$x_{j,k+1} = f_j\big(x_{j,k}, u_{j,k}; \mathbb{E}(\theta_{j,k})\big) \quad \forall j \in \{1,\dots,n\},\ \forall k \in \{0,\dots,k_{\mathrm{end}}-1\} \tag{42}$$

$$x_{j,k}^{\mathrm{con}} \le x_{\max}^{\mathrm{con}}, \quad x_{j,k}^{\mathrm{aux}} \le x_{\max}^{\mathrm{aux}} \quad \forall j \in \{1,\dots,n\},\ \forall k \in \{1,\dots,k_{\mathrm{end}}\} \tag{43}$$

$$u_{j,k} = \begin{cases} 2, & \text{if } (k - t_{0,j}) \bmod \mathrm{round}\,(T_{\mathrm{Gr},j}) = 0 \\ 3, & \text{if } k = t_{0,j} \text{ or } (k - t_{0,j}) \bmod \mathrm{round}\,(r T_{\mathrm{Gr},j}) = 0 \\ 1, & \text{otherwise} \end{cases} \tag{44}$$

$$\forall j \in \{1,\dots,n\},\ \forall k \in \{1,\dots,k_{\mathrm{end}}\}$$

$$1 \le t_{0,j} \le T_{\max} \quad \forall j \in \{1,\dots,n\} \tag{45}$$

$$1 \le T_{j,\mathrm{Gr}} \le T_{\max} \quad \forall j \in \{1,\dots,n\} \tag{46}$$

$$1 \le \mu_j \le \mu_{\max} \quad \forall j \in \{1,\dots,n\}. \tag{47}$$

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