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A Markov Traffic Model for Signalized Traffic Networks Based on Bayesian Estimation

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Abstract: In order to better understand the stochastic dynamic features of signalized traffic networks, we propose a Markov traffic model to simulate the dynamics of traffic link flow density for signalized urban traffic networks with demand uncertainty. In this model, we have four different state modes for the link according to different congestion levels of the link. Each link can only be in one of the four link state modes at any time, and the transition probability from one state to the other state is estimated by Bayesian estimation based on the distributions of the dynamic traffic flow densities, and the posterior probabilities. Therefore, we use a first-order Markov Chain Model to describe the dynamics of the traffic flow evolution process. We illustrate our approach for a small traffic network. Compared with the data from the microscopic traffic simulator SUMO, the proposed model can estimate the link traffic densities accurately and can give a reliable estimation of the uncertainties in the dynamic process of signalized traffic networks.

Keywords: Markov traffic model, Traffic signals, Bayesian, Urban traffic network.

1. INTRODUCTION

Many uncertainties exist in traffic flow dynamics in real traffic networks. These uncertainties are usually expressed with link capacity and traffic demands, and rarely conveyed in traffic models, especially for signalized traffic networks. In order to better understand dynamic features and stochastic features of signalized stochastic traffic networks, this research investigates the randomness in link capacity and traffic demands.

Assuming that both the link capacity (supply) and traffic demand are known, deterministic methods have been used by researchers to describe the performance of traffic networks. However, in real traffic networks, link capacity and traffic demand are subject to stochastic fluctuations. Uncertainty in traffic supply is caused by various disturbances on the road, such as accidents, road affairs or weather (Chen and Zhou (2010)). Traffic control measures, such as traffic signal and ramp metering, can also cause changes in link capacity. On the other hand, there are many causes for the uncertainty of traffic demand. Travel demand fluctuations may be caused by time factors (time of day, day of the week or seasonality), special events, weather conditions, etc.

The dynamic traffic flow model is an important component in dynamic traffic distribution as well as in real-time traffic control and management. In order to simulate dynamic random traffic, a lot of researches have been done on the establishment and verification of different models. For instance, Baras et al. (2005) developed a point queue model with assumption of Poisson arrivals and departures that adds measurement noise to the vehicle's communication. Lan and Davis (1997) proposed a Markov Chain model to describe the traffic flow between signalized intersections by dividing one link into several sections. Inspired by this, Yu

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and Recker (2006) developed a Markov adaptive control model with coarse state division and a large time step to describe the arriving traffic flow. The numerical iterative algorithm is used to determine the optimal decision of the controlled Markov process. On this basis, Tordeux et al. (2014) proposed a Markov jump model with discrete traffic states and constant transition probabilities derived from historical real traffic data or simulations. Another type of stochastic dynamic traffic flow model (Jabari and Liu (2012)) is derived from a deterministic traffic flow model by adding random noise to the equation or by using a stochastic distribution instead of deterministic traffic variables. Based on the CTM model (Daganzo (1995)), a lot of models have been proposed to consider the randomness in urban traffic networks. Sumalee et al. (2011) put forward the stochastic CTM model (SCTM) for freeways by integrating the random link Fundamental Diagrams (FD) into the CTM dynamics to generate the stochastic traffic state evolution. Besides, based on the link transmission model (LTM) (Yperman et al. (2006)), Flötteröd and Osorio (2017) proposed a Stochastic Link Transmission Model (SLTM) that captures random traffic dynamics within a link through a system of four finite-space capacity queues with lagged flows. These models combined the stochastic models with existing dynamic traffic models to formulate the stochastic features in the traffic supply and the traffic flow propagation.

In this paper, we propose a Markov traffic model for signalized traffic networks with uncertainty in demand. This model is a stochastic state transition model considering demand uncertainty and other exogenous sources of uncertainties. The dynamic evolution of the model is achieved by iteratively updating the link density of four link state modes. According to the two boundary states of the link, the state mode of the link can be determined. The transition probability is estimated by Bayesian estimation based on the distribution of dynamic traffic flow densities derived at each time step, and the posterior probabilities obtained through historical traffic data. The traffic density at the next time step can be obtained by the calculation formula of the traffic flow density of the transferred mode, which is initially discussed in Lin et al. (2018). The model can describe changes in link density and flow under the influence of traffic signal settings, and it provides the possibility to better analyze the uncertainty in the dynamic iteration of urban traffic flow network.

2. MODEL DESCRIPTION

An important feature of traffic flow density is that the traffic volume predicted for the next time period has a strong but not massively deterministic relationship with the current and recent state. The Markov chain model can express this feature well and it is very suitable for describing the traffic flow density. In the Markov chain model, the state to be predicted obeys the probability distribution, and the probability of the next state depends on the current and previous states. In this section, we adopt this idea and use a Markov Chain model to describe the traffic flow density with demand uncertainty. Firstly, we briefly introduce the Markov Chain model and divide the link state into 4 different modes based on the traffic density of the upstream and downstream of the link, and introduce the dynamic process of the link in 4 state modes. Then, we propose a method which is based on the distribution of the traffic flow density to calculate the transition probability. Finally, we explain some traffic volumes (leaving and receiving flow, entering and accepted flow, etc) in the dynamic process of the traffic network and introduce the dynamic process of state transferring.

2.1 Markov Chain Model for Traffic Flow Density

A Markov chain is a general model that can explain the natural change with a mathematical method. It was proposed by the famous Russian mathematician Markov around 1910. Markov processes are an important aspect of stochastic process theory in probability theory. After a hundred years of development, Markov processes have penetrated into various fields and played an important role. People will find many phenomena with the continuous development of time in the research of practical problems. There are also some phenomena or processes that can be expressed as follows: when the present is known, the future and the past of this process of change are irrelevant. In other words, the future situation of this process does not depend on the past development and change. We call the process with the above properties a Markov process. When the time and state of Markov process are discrete, such Markov process is called Markov chain. The mathematical expression of Markov chain is as follows: Define a random sequence $\{X(t), t \in T\}$, where $T = \{0, 1, 2 \cdots\}$, and the state space is $S = \{s_0, s_1, s_2, \cdots\}$. If at any time t and any state $s_0, s_1, \cdots, s_{t-1}, s_i, s_j$, the random sequence always satisfies with

$$\begin{aligned} &P\{X_{t+1}=s_j|X_t=s_i, X_{t-1}=s_{n-1}, \cdots, X_1=s_1, X_0=s_0\} \\ &= P\{X_{t+1}=s_j|X_t=s_i\}. \end{aligned} \tag{1}$$

then we call this random sequence Markov chain. In Eq. (1), $P\{X_{t+1} = s_j|X_t = s_i\}$ is the transition probability from time step t to time step $t+1$. The above formula defines the Markov property at the same time as the definition of Markov chain, which is also called "Memorylessness", i.e, the random variable of step $t+1$ is conditionally independent of the rest of the random variables after the random variable of step t is given.

2.2 Link State Modes and Transition Probabilities

In an optimal situation, the shape of the FD (Fundamental Diagram) on each link is supposed to be triangular. Considering the congestion of the link in the real-life signalized traffic networks, the traffic capacity of each link will be reduced to a certain extent (Wu et al. (2011) Lo (1999)), as shown in Fig. 1. Due to the presence of a traffic signal, the shape of the FD at the upstream entrance of the link is triangular, and the shape of the FD at the downstream exit of the link is trapezoidal (Lin et al. (2018)).

As it is shown in Fig. 1, ρ_c is the critical density, ρ_{cl} and ρ_{cu} are the lower and upper critical traffic flow density when the traffic capacity is limited; ρ_J is the junction traffic flow density, v_f is the free-flow speed, and w_c is the spillback speed. The traffic capacity is limited by the fraction of the green time over the cycle time on the link:

$$Q'_M = \frac{Q_M \cdot g}{c} \tag{2}$$

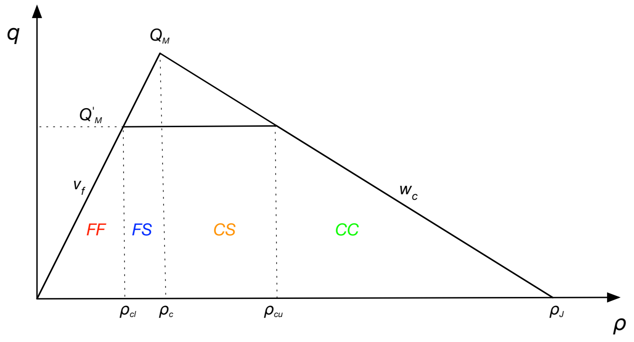

Fig. 1. Illustration for the link state modes: Free flow-Free flow(FF), Free flow-Saturation(FS), Congestion-Congestion(CC), Congestion-Saturation(CS)

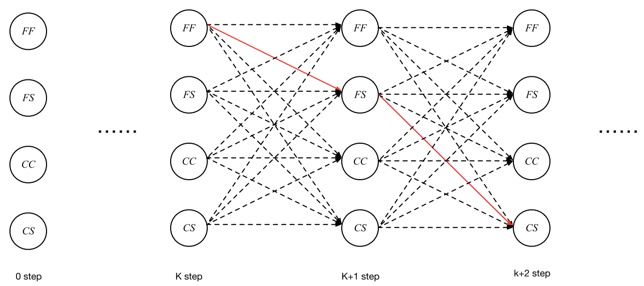

Fig. 2. Iterative Process of Markov Traffic Model

where Q_M and Q'_M are the capacity and the limited capacity of the FD, g is the green time on the link, and c is the cycle time of the traffic lights.

According to the different degree of congestion at the upper and lower boundary. the link state mode of each link can be classified into four different modes Lin et al. (2018). As shown in Fig. 1, the four state modes can be expressed as:

(1) Free flow-Free flow(FF): the upstream boundary of the link is in free-flow state and the downstream boundary of the link is also in free-flow state.
(2) Free flow-Saturation(FS): the upstream boundary of the link is in free-flow state and the downstream boundary of the link is in saturation state.
(3) Congestion-Congestion(CC): the upstream boundary of the link is in congestion state and the downstream boundary of the link is also in congestion state.
(4) Congestion-Saturation(CS): the upstream boundary of the link is in congestion state and the downstream boundary of the link is in saturation state.

As Fig. 2 shows, the state mode set of the Markov traffic model is $M = \{FF, FS, CC, CS\}$, and it is possible for each state mode to switch to any other state mode in the state mode set. The transition probability transferring from mode n to m at time step k is defined as $P_{n,m}(k)(n, m \in M)$. So, according to Bayesian estimation, we have

$$P_{n,m}(k) = P\left[\rho_m(k+1)|\rho_n(k)\right] = \frac{P\left[\rho_m(k+1)\right]P\left[\rho_n(k)|\rho_m(k+1)\right]}{\sum_{m\in M} P\left[\rho_m(k+1)\right]P\left[\rho_n(k)|\rho_{m(k+1)}\right]}. \quad (3)$$

We assume that the transition probability $P_{n,m}(k)$ is equal to the conditional probability of mode m at time step $k+1$ under the condition of having mode n at time step k, where $P[\rho_n(k)|\rho_m(k+1)]$ is the posteriori probability of mode n at time step k under the condition of having mode m at time step $k+1$, which can be statistically obtained through history data. Thus, Eq. (3) can be written as

$$P_{n,m}(k) = \frac{P[\rho_m(k+1)]P[\rho_n|\rho_m]}{\sum_{m\in M} P[(\rho_m(k+1)]P[\rho_n|\rho_m]}. \quad (4)$$

where $P[\rho_n|\rho_m]$ is the posteriori probability statistically estimated through historical data. We suppose that the traffic flow density on the link follows the Normal distribution as $\rho_n(k) \sim N(\mu_n(k), \sigma_n(k))$; then the transition probabilities of four different state modes can be formulated as

$$P[\rho_n(k)] = \begin{cases} Pr\{0 \le \rho_n(k) < \rho_{cl}\} & n = FF \\ Pr\{\rho_{cl} \le \rho_n(k) \le \rho_c\} & n = FS \\ Pr\{\rho_{cu} < \rho_n(k) \le \rho_J\} & n = CC \\ Pr\{\rho_c < \rho_n(k) \le \rho_{cu}\} & n = CS \end{cases} \quad (5)$$

Where $P[\rho_n(k)]$ is the probability that the link is in the mode n at time step k. Then the transition probability transferring from mode n to mode m can be calculated by the distribution of traffic flow density in Eq. (4). After that, the link state will jump to the state mode with the highest transition probability at time step $k+1$.

As long as the mean $\mu(k)$ and the variance $\sigma(k)$ are obtained, the transfer probability of Markov model can be calculated based on formula Eq. (4). In the next section, we will make a model of traffic flow density in four state modes and explain some traffic volumes in the state transferring.

2.3 Link Models for Different Link State Modes

Considering the different modes, entering and leaving flows and traffic signals, the dynamic evolution of the traffic flow density at time step k on a link can be formulated as

$$\rho(k+1) = A\rho(k) + B_0\rho(k)U(k) + B_1U(k) + Dd(k) + C, \quad (6)$$
$$U(k) = \beta(k)\gamma(k), \quad (7)$$

where A, B_0, B_1, D and C are constant parameters, $U(k)$ is the scalar control input at time step k, which is assumed to be a deterministic value; it is composed of the turning rate vector $\beta(k)$ and the vector of green time splits $\gamma(k)$ of the traffic signal at the intersection at time step k. More over $d(k)$ includes the stochastic disturbances from outside the link:

$$d(k) = [q_E(k)d_i(k)d_0(k)q_A(k)], \quad (8)$$

where q_E is the input flow of the link, which composed of all the leaving flows of the upstream links, q_A is the output flow of the link, which is determined by all available receive flow of the downstream links, and d_i and d_o are the disturbance flows that get in and out of the link.

The control input $U(k)$ can be defined as

$$U(k) = \beta_{th}(k)\gamma_{th}(k) + \beta_l(k)\gamma_l(k), \quad (9)$$

where $\beta_{th}(k)$ and $\beta_l(k)$ are the ratios of going straight and turning left at the intersection at time step k, and $\gamma_{th}(k)$ and $\gamma_l(k)$ are the green signal splits of going straight and turning left at intersections at time step k.

In the stochastic link flow model, we can further write four different dynamic models according to the different probability of the link mode. For the link state mode FF:

$$\rho(k+1) = A\rho(k) + B_0\rho(k)U(k) + Dd(k), \tag{10}$$

where $A = 1 + \beta_r\beta_0$, $B_0 = -\frac{T_s}{l}v_f$, $D = [\frac{T_s}{l}000]$, and l is the length of the link, T_s is the simulation time step.

For the link state mode FS:

$$\rho(k+1) = A\rho(k) + B_1U(k) + Dd(k) + C, \tag{11}$$

where $A = 1$, $B_1 = -\frac{T_s}{l}Q_M$, $D = [\frac{T_s}{l}000]$, and $C = \beta_r\beta_l$.

For the link state mode CC:

$$\rho(k+1) = A\rho(k) + Dd(k) + C, \tag{12}$$

where $A = 1 + \frac{T_s}{l}w_c$, $D = [000 - \frac{T_s}{l}]$, and $C = -\frac{T_s}{l}w_c\rho_J$.

For the link state mode CS:

$$\rho(k+1) = A\rho(k) + B_1U(k) + Dd(k) + C, \tag{13}$$

where $A = 1 + \frac{T_s}{l}w_c$, $B_1 = -\frac{T_s}{l}Q_M$, $C = -\frac{T_s}{l}w_c + \beta_r\beta_l$.

2.4 Link Mean Density and Auto-Correlation

According to Eq. (6), the link density can be calculated as

$$\mu(k+1) = (A + B_0U(K))\mu(K) + B_1U(K) + DE(d(k)) + C. \tag{14}$$

Let $\Omega(k) = \mathbb{E}\left(\rho^2(k)\right)$. The auto-correlation of the link density can be calculated as

$$\begin{aligned}\Omega(k+1) = F_1(k)\Omega(k) + F_0(k)\mu(k) + G(k)E(d(k)) \\ + E(d^T(k)D^T(k)D(k)d(k)) + H(k)\end{aligned} \tag{15}$$

where

$$F_1(k) = A^2 + B_0^2U^2(k) + 2AB_0U(k), \tag{16}$$

$$\begin{aligned}F_0(k) = 2B_0B_1U^2(k) + 2(B_0D(k)E(d(K)) + AB_1 \\ + B_0C)U(k) + 2ADE(d(k)) + 2AC,\end{aligned} \tag{17}$$

$$G(k) = 2B_1D(k)U(k) + 2CD, \tag{18}$$

$$H(k) = B_1^2U^2(k) + 2B_1CU(k) + C^2. \tag{19}$$

Then, the mean and variance of the link density at time step k can be written as

$$\mu(k) = \rho(k), \tag{20}$$

$$\sigma(k) = \sqrt{\Omega(k) - \mu^2(k)}. \tag{21}$$

2.5 Leaving and Receiving Flows of the Link

In the previous section, we defined four different state modes for the link based on the different congestion levels of the link. The leaving flow and the receiving flow of the link are exactly related to the congestion levels of the link. So when the link is in different traffic modes, the leaving flow and the receiving flow are also different. According to the Fundamental Diagram in Fig. 1, the leaving flow of the link can be written as

$$q_{L,FF}(k) = \rho(k)V_fU(k), \tag{22}$$

$$q_{L,FS}(k) = q_{L,CS}(k) = Q_MU(k), \tag{23}$$

$$q_{L,CC}(k) = q_A(k), \tag{24}$$

which means that the leaving flow of the link totally depends on the accepting flow of the downstream links and congestion levels.

Similarly, the receiving flow of the link can be defined as

$$q_{R,FF}(k) = q_{R,FS}(k) = Q_M, \tag{25}$$

$$q_{R,CC}(k) = q_{R,CS}(k) = w_c(\rho(k) - \rho_J), \tag{26}$$

which shows that the receiving flow of the link depends on the capacity and the congestion flow of the link.

2.6 Entering and Accepted Link Flows

In the FF and FS mode, the traffic flow upstream of the link is in free-flow state, the entering link flow is always provided by the upstream link; in the CC mode, the upstream link is in congestion mode, and it cannot provide space for the entering link flow, thus the accepted link flow only depends on the available space of the downstream link.

The entering flow of link i can be expressed as the sum of all the flows from upstream links as

$$q_{E,i}(k) = \sum_{u\in I_i}\beta_{u,i}q_u(k), \tag{27}$$

where $\beta_{u,i}$ is the turning ratio of the flow turning from link u to link i, $q_u(k)$ is the leaving flow of link u at time step k, and $q_{E,i}(k)$ is the entering flow of link i at time step k.

In the CC mode, the accepted traffic flow can be separated into two situations for each downstream. The probabilities can be written as

$$P_{1,d}(k) = Pr\{q_{R,d}(k) \leq Q_{M,i}\beta_{i,d}\gamma_{i,d}(k)\}, \tag{28}$$

$$P_{1,d}(k) = Pr\{q_{R,d}(k) > Q_{M,i}\beta_{i,d}\gamma_{i,d}(k)\}, \tag{29}$$

where $Q_{M,i}$ is the capacity flow of link i, $q_{R,d}(k)$ is the receiving flow of link d at time step k , and $\gamma_{i,d}(k)$ is the green time split for the flow turning from link i to link d at time step k.

Thus, the accepted flow of link i can be written as

$$q_{A,i}(k) = \sum_{d\in O_i}[P_{1,d}(k)q_{R,d}(k) + P_{2,d}(k)Q_{M,i}\beta_{i,d}\gamma_{i,d}(k)]. \tag{30}$$

2.7 State Transferring

According to the model described above, we could derive the mean and the variance of link density in any mode and at any time step. Then, we can obtain the distribution of traffic flow density throughout the evolution of the traffic states Thus we could calculate the probability of every mode at all time steps based on Eq. (5). Therefore, the transition probability can be estimated by Bayesian estimation, based on these probabilities predicted by the dynamic model, and the posteriori probability obtained from the historical traffic data, as in Eq. (4). Hence we obtain the transition probability of traffic flow density of Markov traffic model. During the iteration of the model, we suppose that the link state will jump to the state mode with highest transition probability at each iteration. As is shown in Fig. 2, we assume the link is in the FF mode at time step k. With the increase of traffic demand, the traffic density in the road network will gradually accumulate and the probability of transferring to the FS mode will become the highest. Then the link state mode will switch from the FF mode to the FS mode at time step $k+1$. If the demand keeps on increasing, the probability of transferring will be changed and the probability of transferring to the CS mode will become the highest. The link state mode will switch from the FS mode to the CS mode at time step $k+2$, in which the traffic congestion will be propagated to the upstream link.

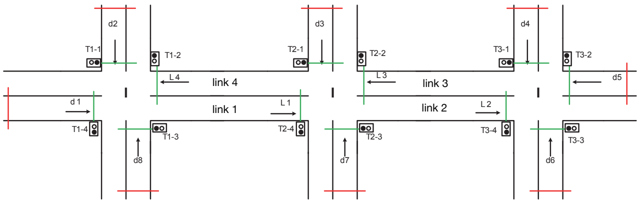

Fig. 3. A small test network

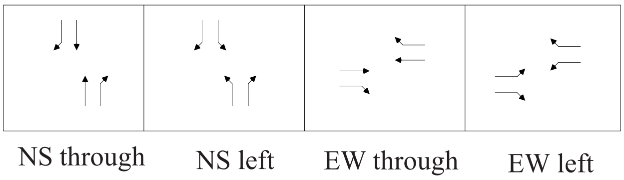

Fig. 4. The phases of traffic signal

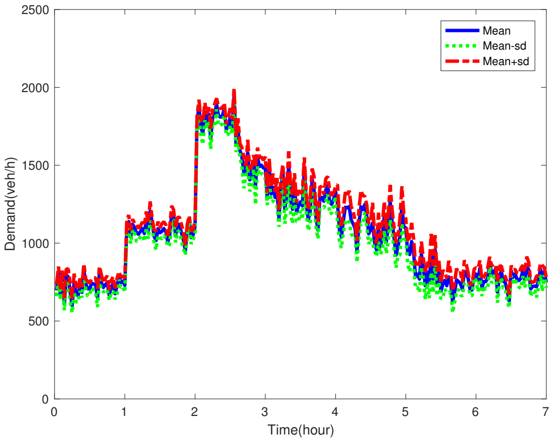

Fig. 5. The stochastic traffic demand

3. SIMULATION

To estimate the effectiveness of the Markov traffic model, we use the micro traffic simulator SUMO to build a traffic network that provides stochastic traffic demand, as well as real values of traffic density and traffic flow. The traffic demand generated by SUMO is given to the Markov traffic model, and the link state will jump to the state mode with the highest transition probability at each iteration. Finally, we compare the estimated link density in Markov model with the data obtained from SUMO.

3.1 Network and Signal Setup

We build a simple traffic network, which has 3 signalized intersections, 12 links in the network, and 12 sets of green time splits. As shown in Fig. 3. The stochastic traffic demand is provided to the network through 8 external access links. As shown in Fig. 4, there are four traffic signal stages: North South through, North South left, East West through, and East West left. Stochastic traffic demands are generated for all the entry links of the network for 7 hours,The stochastic traffic demand is shown in Fig. 5.

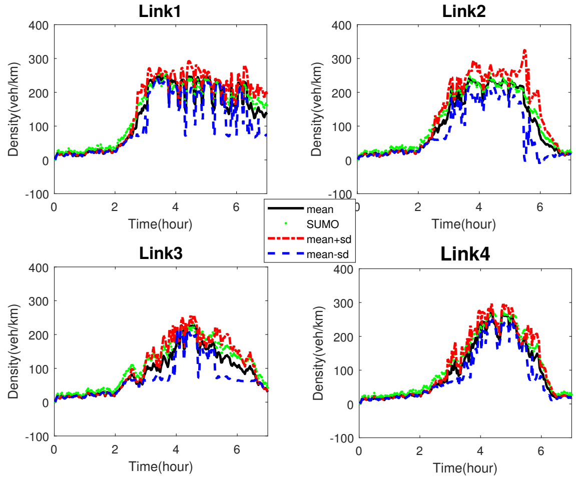

Fig. 6. The estimated link densities vs. the SUMO real link densities

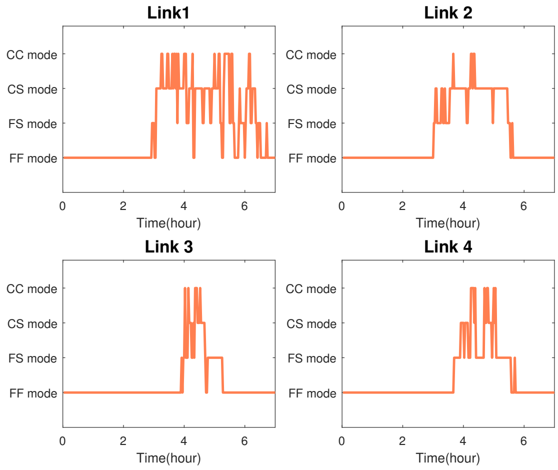

Fig. 7. The transition of 4 state modes

3.2 Results

Compared with the actual values of Markov traffic model and SUMO in Fig. 6, the results show that the average link density simulated by Markov traffic model is in good agreement with link densities from SUMO. As we can see in Fig .7, four links are all in FF mode in first three hours, and the estimated data match well with SUMO data. After that, link become more congested than link, and the state modes of the link are switched into CS mode and CC mode. At the same time, when the link gets congested, the estimation standard deviation of the link density from the stochastic model increases, which proves that the higher the average link density is, the more uncertainties of the link density will have.

If the link is more congested, based on the previous derivation of Markov traffic model, the standard deviation of link density is more affected by the high average and automatic-correlation of links in the previous time steps. This means that congestion will increase the uncertainty of link density estimation. In other words, congestion is the main cause of the random disturbance in the link traffic flow, which leads to more uncertainty. As Fig. 7 shows, links 1, 2 are more in CC mode, but link 3, 4 are more in FF mode. This means that traffic congestion under high traffic demands will make the Markov traffic model highly random in determining the link state mode.

In general, the results of the simulation are basically in agreement with the real data from SUMO, and can reasonably explain the relationship between the congestion condition and uncertainties. In addition, the proposed model is computational fast compared to SUMO. Therefore, the Markov traffic model can be used to study the propagation of urban traffic demand and its uncertainty, and can also be used to optimize the traffic signal of urban traffic network with demand uncertainty.

4. CONCLUSIONS

In this paper, we have proposed a Markov traffic model to describe the traffic link flow in an urban traffic network with signalized intersections. This model is a stochastic state transition model considering demand uncertainty. In the proposed model, the link state is formulated as four different modes based on different congestion levels of the link. The dynamic process of traffic network is described by the state transferring, and the transition probability from one state to another state is approximated by Bayesian estimation based on the distribution of traffic flow density at neighboring time steps. The Markov traffic model is a stochastic state transition model considering demand uncertainty and other exogenous sources of uncertainties.

For the simulation of a small traffic networks, we compared the results with the real values from SUMO, and verified the reliability of the Markov traffic model. According to the state transferring of different links at different time steps, we can reasonably explain the relationship between traffic congestion and uncertainties: when the link is in a congested state, the standard deviation of its traffic flow density will become larger which means that congestion is the main cause of random disturbance of link traffic, and it leads to more uncertainties. The results show that the Markov traffic model can be used to study the propagation of urban traffic dynamic process and its uncertainty, and can also be used to optimize the traffic signal of urban traffic network with demand uncertainty.

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