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Logic-based traffic flow control for ramp metering and variable speed limits – Part 1: Controller

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Abstract—This paper proposes a Logic-Based Traffic Flow Control algorithm (LB-TFC) for integrated control of Ramp Metering (RM) installations and Variable Speed Limits (VSLs) in order to reduce traffic jams created at bottlenecks. LB-TFC estimates, for each control time step, the number of vehicles that should be held back or released by the control measures (i.e. the VSLs and the RM rates) in order to avoid the capacity drop (maximizing the outflow of the bottleneck). Afterwards, based on the resulting estimated number of vehicles, the VSLs and/or the RM rates are increased or decreased in a pre-specified order.

In order to avoid or reduce traffic breakdowns, the proposed controller (LB-TFC) anticipates the future evolution of the bottleneck density by using a feed-forward structure. As a result, the performance of the controller is very efficient and similar to the one obtained with an optimal controller while the implementation of the controller (with an almost instantaneous computation time) and the tuning of the parameters are easy.

In the second part of this work, published in a separate paper ('Part 2: Simulation and Comparison'), LB-TFC is simulated, analyzed and compared for two freeways (one synthetic network and one stretch of the ring-road freeway SE-30 in Seville, Spain).

Index Terms—Ramp Metering, Variable Speed Limits, Feed-forward control, Freeway traffic control.

I. INTRODUCTION

Freeway traffic congestion causes many social and economic problems in daily life (waste of fuel and time, increased pollution, greater accident risk, ...). Since constructing new infrastructure is not always a viable option, much research has been focused on solving these problems by using dynamic traffic control.

Nowadays, ramp metering is the most successfully and widely used freeway traffic control measure. In fact, ramp metering has already been profitably implemented in practice in France, United States, Germany, Australia, and several other countries [1], [2]. On the other hand, the use of Variable Speed Limits (VSLs) is a promising measure for reducing or avoiding traffic jams on freeways (see [3] for a review). Although traditionally VSLs have been mostly implemented in order to reduce the risk of accidents (safety-oriented VSL control [4]), VSLs have emerged during the last years as a potential traffic management measure for increasing freeway efficiency (congestion-oriented VSL control [5]–[10]). This

paper focuses on congestion-oriented VSL algorithms since the goal of this work is to propose a control algorithm that reduces or removes traffic jams created at bottlenecks.

Previously proposed control algorithms for either RM, VSLs, or the integrated control of both kinds of control measures can be classified in two main groups:

- **Optimization-based controllers:** Many optimization-based control algorithms (mainly using Model Predictive Control [11]) have been theoretically proposed for RM [12], [13], for VSLs [14], [15], and for the integrated control of both RM and VSLs [5], [8], [9]. However, these algorithms are usually too complicated to be implemented in real time, mainly because their computation time rapidly increases with the size of the traffic network and because, moreover, optimization-based algorithms are not usually robust in case of communication or measurement errors. For these reasons, to the best of our knowledge, there are no implementations in practice using this kind of controllers.
- **Easy-to-implement controllers:** On the other hand, several easy-to-implement control algorithms have been proposed in the literature for RM, for VSLs, and for the integrated control of both RM and VSLs.
 - For RM, ALINEA [16] is one of the most widely deployed ramp metering strategy. ALINEA is based on a feedback structure and is derived by use of classical automatic control methods [17]. However, as stated in some references such as [18]–[20], if the bottleneck is far away from the on-ramp (more than just a few hundred meters), then ALINEA is not always effective (even having negative effects in some cases if the control parameter is not properly tuned for the considered scenario or if the critical density is not properly estimated). In order to deal with this problem, PI-ALINEA was proposed in [21], improving the performance given by ALINEA for a distant bottleneck. However, the obtained behavior can still be substantially improved, especially in terms of robustness. This improvement is achieved in [20], where a feed-forward controller (FF-ALINEA) is proposed that approaches the optimal behavior while being very robust in cases where the demands differ from the ones used for calibration.
 - For VSLs, the most well known easy-to-implement strategies are SPECIALIST [7] and MTFC [6]. SPECIALIST, a control algorithm based on shock wave theory, is able to reduce or avoid isolated shock

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waves that do not necessarily always occur at the same time of the day or that do not have similar magnitudes. However, SPECIALIST is not designed to deal with the traffic congestion created at freeway bottlenecks. On the other hand, MTFC, a local VSL controller with feedback of the flow downstream of the VSL application area and of the density at the bottleneck area, is able to successfully reduce or avoid the capacity drop and the onset of congestion at bottlenecks [22]. Nevertheless, its behavior strongly depends on the scenario that has been used for the calibration of the control parameters. As a result, although MTFC usually performs close to the optimal solution for the scenario used for calibration, the performance is highly decreased if these control parameters are applied for a scenario with substantially different traffic conditions [10]. Other approaches are QL-VSL [23] and SPERT [24]. These algorithms allow an easy implementation while approaching the performance of an optimal controller. However, unfortunately, the off-line effort needed for the training of the network (in the case of QL-VSL), or for the computation of the optimal solution (in the case of SPERT) may cause that, for medium and large scale networks, these algorithms are also very difficult (if not impossible) to implement in practice. Recently, a logic-based controller (LB-VSL) that is able to robustly approach the performance of an VSL optimal controller, while implementation and tuning are fast and easy, was proposed in [10].

- The most well-known easy-to-implement control algorithm for the integrated control of RM and VSLs is, to the best of our knowledge, the one proposed in [25], which integrates the use of MTFC for VSLs and PI-ALINEA for RM. However, this algorithm has the same limitations (in terms of robustness) as PI-ALINEA and MTFC considered individually. In [26], an extension of MTFC is presented including the additional feature of managing queue lengths and handling downstream distant bottlenecks. Another interesting easy-to-implement control algorithm for integrated control of RM and VSLs is SPECIALIST-RM [27], which is based on the field-tested VSL strategy SPECIALIST and deals with the particular case of moving limited-length jams. However, as for SPECIALIST, the algorithm is designed to reduce isolated shock waves, but it can in general not always properly reduce congestion created at freeway bottlenecks.

The main contribution of this work is the proposal of an easy-to-implement integrated control algorithm (Logic-Based Traffic Flow Control or LB-TFC for shorty), for VSLs and RM installations, aimed at avoiding or reducing congestion at bottlenecks, that is able to overtake the limitations of previously proposed integrated controllers. In fact, LB-TFC is, to the best of our knowledge, the first integrated controller for RM and VSLs that is able to robustly approach the performance of

an optimal controller when dealing with congestion created at a bottleneck for different traffic demand profiles while the tuning of the control parameters is simple and intuitive and the computation of the control inputs is almost instantaneous.

This robust and quasi-optimal performance is achieved through the use of a feed-forward structure that anticipates the activation and deactivation of the RM installations and of the VSLs before reaching the critical density. As a result, LB-TFC naturally activates and deactivates the control measures at the right time without relying on parameters that depend on the scenario (and on how fast the bottleneck density is decreasing or increasing). Therefore, since the performances of both VSLs and RM control algorithms are highly related with a proper activation time, the proposed controller is able to locally approach the Total Time Spent (TTS) reduction of an optimal controller.

This work is divided in two parts, each one presented in a separate paper. Firstly, in ‘Part 1: Controller’, the control structure and the equations of LB-TFC are presented and derived. Subsequently, in ‘Part 2: Simulation and Comparison’ [28], LB-TFC is simulated, analyzed and compared for two freeways (one synthetic network and one stretch of the ring-road freeway SE-30 in Seville, Spain).

The structure and equations of the proposed controller (LB-TFC) are based on the ones used for FF-ALINEA [20] and for LB-VSL [10].

This paper (‘Part 1: Controller’) is structured as follows: Section II derives and justifies the equations used by the proposed controller. Subsequently, in Section III, the control logic that is needed for the implementation of LB-TFC is presented. In Section IV, LB-TFC is simulated for synthetic case study in order to show an application of the control method. Finally, the main conclusions are drawn in Section V.

II. DERIVATION OF THE CONTROLLER

In order to define the equations and structure of the proposed controller, the network shown in Fig. 1, which includes a bottleneck due to a reduction in the number of lanes, will be used as an illustrative example. The freeway is discretized in time, with time step T and time step counter k , and in space, in consecutive segments indicated by the index i , which are dynamically characterized by the space-mean speed $v_i(k)$ and the space-mean density $\rho_i(k)$. The network includes a bottleneck with a length of L_B and λ_B lanes, which can create a traffic jam on the upstream segments. The mean outflow of each segment can be easily computed for each time step k using $q_i(k) = \lambda_i \rho_i(k) v_i(k)$, where λ_i is the number of lanes of segment i .

In order to avoid congestion, there is a set of control measures (RM installations and VSLs) located at a certain distance upstream of the bottleneck (with L_A the distance between the most upstream considered control measure and the bottleneck). The index j is running over the entire set of control measures upstream of the bottleneck (starting with the most upstream one which is indicated by $j = 1$).

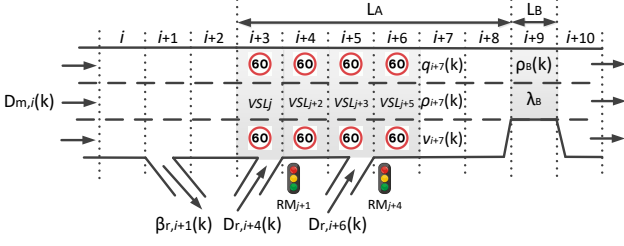


Fig. 1. Example of traffic network upstream of a bottleneck with density $\rho_B(k)$ and λ_B lanes, where j is the index of the VSLs and RM installations located upstream of the bottleneck, i is the index for the segments, $D_{m,i}(k)$ and $D_{r,i}(k)$ are the mainline and on-ramp demands, and $\beta_{r,i}(k)$ are off-ramp split ratios.

A. Simplified modeling

In order to achieve a control algorithm that is able to anticipate the future evolution of the bottleneck density in order to reduce (or avoid) traffic breakdowns, this paper proposes the use of a feed-forward control structure that makes use of the measurements of the density measured at the bottleneck ($\rho_B(k)$) and of the available measurements of the speeds and flows from detectors located upstream of the bottleneck ($q_i(k)$, $v_i(k)$). Using these measurements, the proposed controller computes, for each time step k , the number of vehicles that should be held back (or released) in order to avoid capacity drop after a certain period of time $T_{ff}(k)$.

In order to find an appropriate equation for the computation of this number of vehicles, this paper proposes the use of a simplified model in order to predict the evolution of bottleneck density (since the goal is to obtain an easy-to-implement controller). Firstly, the value of the density of the bottleneck after a given period of time $T_{ff}(k)$ is predicted based on the conservation of vehicles:

$$\rho_B\left(k + \frac{T_{ff}(k)}{T}\right) = \rho_B(k) + \frac{T_{ff}(k)}{\lambda_B L_B} (Q_{iB}(k) - Q_{oB}(k)) \quad (1)$$

where T is the controller time step, $Q_{iB}(k)$ is the average flow over all lanes (veh/h) entering the bottleneck during $[kT, kT + T_{ff}(k))$, $Q_{oB}(k)$ is the outflow of the bottleneck during the same period, and $T_{ff}(k)$ is a certain period of time determined such that all the vehicles located at time step k within a certain distance L_A upstream of the bottleneck are able to reach the bottleneck. This period of time $T_{ff}(k)$ is computed using the mean speed $\hat{v}_A(k)$ within the distance L_A upstream of the bottleneck: $T_{ff}(k) = L_A / \hat{v}_A(k)$. The determination of L_A and the estimation method used for the flow entering the bottleneck ($Q_{iB}(k)$) and for the mean speed upstream of the bottleneck ($\hat{v}_A(k)$) are explained in Section II-C.

Subsequently, the equation used for the prediction of the bottleneck density ((1)) is simplified based on following assumptions:

- The period of time $T_{ff}(k)$ is chosen such that all the vehicles located at time step k within the distance L_A are able to enter the bottleneck.
- It is assumed that the vehicles reaching the bottleneck during $[kT, kT + T_{ff}(k))$ have been traveling during this

period at a mean speed $\hat{v}_A(k)$ before they enter the bottleneck.

- The outflow of the bottleneck ($Q_{oB}(k)$) is considered to be equal to the capacity of the bottleneck (\bar{C}_B) for the calculation of the number of vehicles to hold back, and equal to the congested outflow (capacity minus capacity drop) of the bottleneck (C_B) for the calculation of the number of vehicles to release. Although this assumption is only true if the bottleneck is being controlled around the critical density, it is quite useful for control purposes (as it can be seen in the simulation results in the second part of the paper).

These two different values are used for $Q_{oB}(k)$ since, in order to avoid oscillations in the response of the controller while still activating the control measures at the right time, it is desirable to slightly underestimate the number of vehicles to release (i.e. to only release vehicles when it is clear that they do not have to be metered any more). Moreover, the activation of the control measures (for which the number of vehicles to hold back is computed) will be generally carried out before the bottleneck has reached congestion. On the other hand, the deactivation of the control measures (for which the number of vehicles to release is computed) will be generally carried out when the bottleneck is still congested (with the corresponding decrease in the capacity due to the capacity drop).

Moreover, it should be noted that the capacity of a bottleneck can be subject to day-to-day stochastic variations. However, the proposed controller will rely on both the critical density and the capacity, therefore increasing the robustness in the case of capacity variations.

With these assumptions, (1) can be rewritten as:

$$\rho_B\left(k + \frac{T_{ff}(k)}{T}\right) = \rho_B(k) + \frac{L_A(Q_{iB}(k) - Q_{oB}(k))}{\lambda_B L_B \hat{v}_A(k)} \quad (2)$$

with $Q_{oB}(k) = \bar{C}_B$ for holding vehicles and $Q_{oB}(k) = C_B$ for releasing vehicles.

In practice, \bar{C}_B and C_B are tuning parameters that have to be set between the congested outflow (capacity minus capacity drop) and the capacity of the bottleneck. The tuning of \bar{C}_B and C_B is discussed in the second part of this work [28].

B. Number of vehicles to hold back/release

As previously mentioned, the proposed controller has to calculate the number of vehicles that have to be held back ($V_{hold,1}(k)$) or released ($V_{rel,1}(k)$) in order to avoid the appearance of capacity drop. This computation is done by imposing that the density after $T_{ff}(k)$ has to be equal to the critical density (i.e. imposing that the flow reaches the capacity of the bottleneck).

Firstly, considering that we are computing the number of vehicles to hold back ($Q_{oB}(k) = \bar{C}_B$), (2) can be rewritten as:

$$\rho_B\left(k + \frac{T_{ff}(k)}{T}\right) = \rho_B(k) + \frac{L_A(Q_{iB}(k) - \bar{C}_B)}{\lambda_B L_B \hat{v}_A(k)} \quad (3)$$

Consequently, in order to keep $\rho_B(k + T_{ff}(k)/T)$ around the critical density and taking into account that $V_{hold,1}(k) \geq 0$, the

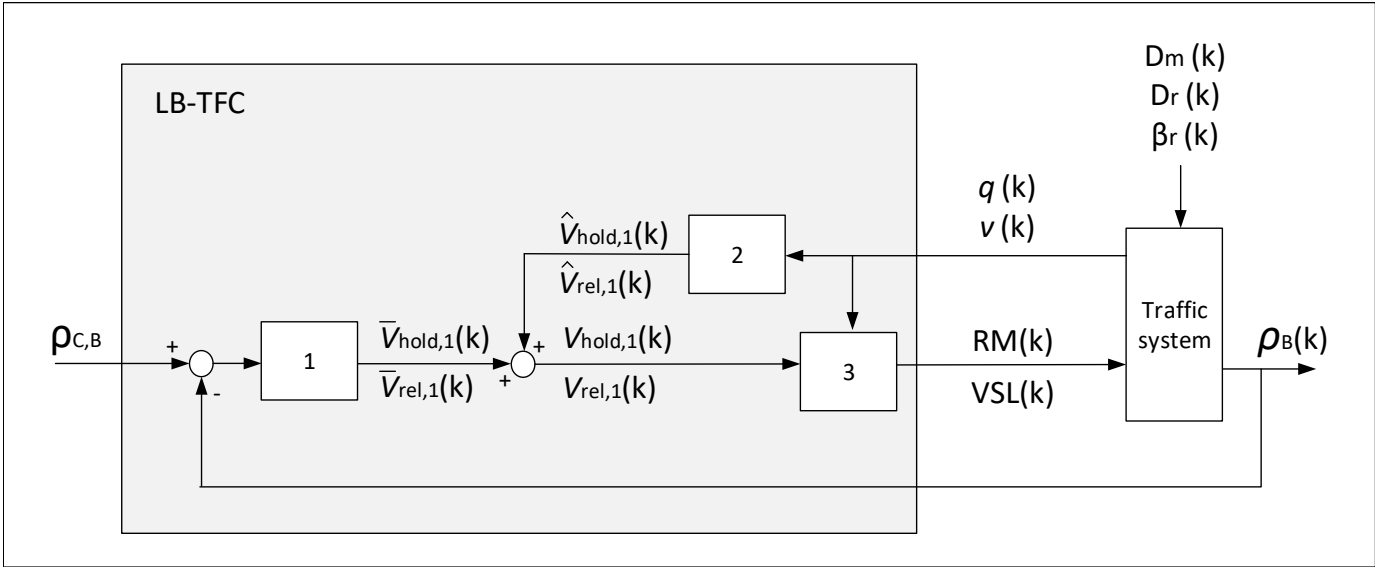


Fig. 2. Control structure for LB-TFC, where $\bar{V}_{hold,1}(k)$ and $\bar{V}_{rel,1}(k)$ are the number of vehicles that have to be held back or released, respectively, because of the current value of the bottleneck density and $\hat{V}_{hold,1}(k)$ and $\hat{V}_{rel,1}(k)$ are the number of vehicles that have to be held back or released, respectively, because of the flow reaching the bottleneck. Moreover, $D_m(k)$, $D_r(k)$, $\beta_r(k)$, $q(k)$, $v(k)$, $VSL(k)$, and $RM(k)$ are vectors containing the values of the corresponding variables at time step k for the entire set of considered segments or control measures, respectively.

final equation for the number of vehicles to hold back by the most upstream control measure (which is indicated by $j = 1$) can be obtained from (3):

$$V_{hold,1}(k) = \max \left(0, \lambda_B L_B \left(\rho_B(k) + \frac{T_{ff}(k)}{T} \right) - \rho_{c,B} \right) \quad (4)$$

$$= \max \left(0, \frac{L_A}{\hat{v}_A(k)} \left(Q_{iB}(k) - \bar{C}_B \right) - \lambda_B L_B \left(\rho_{c,B} - \rho_B(k) \right) \right)$$

where $\rho_{c,B}$ is the critical density of the bottleneck.

Equation (4) is composed of two terms:

- The first term ($\hat{V}_{hold,1}(k) = \frac{L_A}{\hat{v}_A(k)} (Q_{iB}(k) - \bar{C}_B)$) computes the number of vehicles that have to be held back if the flow reaching the bottleneck during period of time $[kT, kT + T_{ff}(k))$ is larger than the capacity.
- The second term ($\bar{V}_{hold,1}(k) = -\lambda_B L_B (\rho_{c,B} - \rho_B(k))$) computes the number of vehicles that have to be held back if the current density of the bottleneck is larger than the critical one.

The addition of both terms in (4) implies the adoption of a feedback structure ($\bar{V}_{hold,1}(k)$ and $\bar{V}_{rel,1}(k)$) combined with a feed-forward structure ($\hat{V}_{hold,1}(k)$ and $\hat{V}_{rel,1}(k)$) as shown in Fig. 2. In the figure, blocks 1 and 2 estimate the number of vehicles that have to be held back or released due to, respectively, the current density of the bottleneck and the flow reaching the bottleneck. Block 3 uses the estimated number of vehicles to hold back/release in order to compute the value of the ramp metering rates and speed limits that will be implemented.

It has to be taken into account that in the cases that the controller is not going to be able to actually avoid congestion, the real number of vehicles that has to be held back will be larger than $\hat{V}_{hold,1}(k) + \bar{V}_{hold,1}(k)$. However, since the control inputs will be saturated in these circumstances, the

implemented value of the control inputs will be the same as using a larger, and more realistic, value of $V_{hold,1}(k)$.

An equivalent procedure is undertaken for the number of vehicles to release ($V_{rel}(k)$) resulting in:

$$V_{rel,1}(k) = \max \left(0, -\frac{L_A}{\hat{v}_A(k)} (Q_{iB}(k) - \bar{C}_B) + \lambda_B L_B (\rho_{c,B} - \rho_B(k)) \right) \quad (5)$$

It has to be taken into account that, if $\bar{C}_B > \underline{C}_B$, either $V_{hold,1}(k)$, $V_{rel,1}(k)$, or both are equal to 0 for any time step ($V_{hold,1}(k)V_{rel,1}(k) = 0 \forall k$).

After computing the number of vehicles to hold back/release, the proposed controller increases/decreases the most upstream control measure (which is indicated by $j = 1$) by using the equations included in Section II-D (for the case or a RM installation) or in Section II-F (for the case of a VSL).

For the following control measures ($j > 1$), the number of vehicles to hold back/release is updated taking into account the number of vehicles that have been actually held back/released by the control measure indicated by $j - 1$. This update is done by using the equations included in Section II-E, for a RM installation, and Section II-G, for a VSL.

C. Estimation of entering flow and mean speed

Equations (4) and (5) require the online estimation of $Q_{iB}(k)$ and $\hat{v}_A(k)$ and the selection of a distance L_A . The procedure used for the estimation of these variables can be adapted based on the topology of the network and the number of detectors available.

In this work, we assume that L_A is the distance between the bottleneck and the segment where the first (upstream) RM installation or VSL is located (see Fig. 1). This distance should be short enough to only include the traffic jam caused by the

bottleneck (and no other source of congestion) but long enough to allow the controller to anticipate the future evolution of the bottleneck density. This anticipation allows the controller to compensate the delay between the application of the control inputs and its effects on the bottleneck density. In future work, the effect of selecting longer and/or shorter values for L_A will be studied.

If there are detectors in the entire set of segments upstream the bottleneck, an accurate and easy method to estimate $\hat{v}_A(k)$ and $Q_{iB}(k)$ is to compute the weighted summation of all the speeds and flows:

$$\hat{v}_A(k) = \frac{\sum_{i \in D_A} v_i(k) \hat{L}_i}{L_A} \quad Q_{iB}(k) = \frac{\sum_{i \in D_A} q_i(k) \hat{L}_i}{L_A} \quad (6)$$

where A is the freeway stretch between the bottleneck and the first detector, D_A is the set of detectors located on freeway stretch A , $v_i(k)$ and $q_i(k)$ are the speed and flow measured at the detector located on segment i , and \hat{L}_i is the distance between detector i and detector $i + 1$.

In the case of having less measurements available, other estimation methods can be used without a substantial reduction in the close-loop performance as shown in [20].

- If there is only one detector available upstream the bottleneck located on segment m , $Q_{iB}(k)$ and $\hat{v}_A(k)$ can be estimated by taking the temporal mean of the measurements during the last period of length T_A (using the current value of the speed measurements in order to predict T_A):

$$T_A = L_A / v_m(k) \quad \hat{v}_A(k) = \sum_{l=0}^{[T_A/T]} \frac{v_m(k-l)}{T_A/T} \quad (7)$$

$$Q_{iB}(k) = \sum_{l=0}^{[T_A/T]} \frac{q_m(k-l)}{T_A/T}$$

where $q_m(k)$ and $v_m(k)$ are flow and speed measurements on time step k for segment m , L_A is the distance between this detector and the bottleneck.

- If there are no speed measurements available, the free-flow speed can be used as an estimator of the mean speed: $\hat{v}_A(k) = v_f$.
- If there are on-ramps or off-ramps located between the bottleneck and the first considered detector, the on-ramps and off-ramps flows have to be taken into account during the estimation of $Q_{iB}(k)$.

For example, if there was one on-ramp located upstream of the bottleneck, the estimated flow would have to be adapted using the following equation:

$$Q_{iB}(k) = \frac{\sum_{i \in D_{A,before}^{On}} \left((q_i(k) + q_r(k)) \hat{L}_i \right)}{L_A} \quad (8)$$

$$+ \frac{\sum_{i \in D_{A,after}^{On}} \left(q_i(k) \hat{L}_i \right)}{L_A}$$

where $q_r(k)$ is the flow of the on-ramp at time step k , $D_{A,before}^{On}$ is the set of detectors in freeway stretch A located before (i.e. upstream of) the on-ramp, and

$D_{A,after}^{On}$ is the set of detectors in freeway stretch A located after (i.e. downstream of) the on-ramp.

Equivalently, if there were one off-ramp located upstream of the bottleneck, the estimated flow would have to be adapted using the following equation:

$$Q_{iB}(k) = \frac{\sum_{i \in D_{A,before}^{Off}} \left(q_i(k) \hat{L}_i \right)}{L_A} \quad (9)$$

$$+ (1 - \beta(k)) \frac{\sum_{i \in D_{A,after}^{Off}} \left(q_i(k) \hat{L}_i \right)}{L_A}$$

where $\beta(k)$ is the split ratio of the off-ramp at time step k , $D_{A,before}^{Off}$ is the set of detectors in freeway stretch A located before (upstream) of the off-ramp, and $D_{A,after}^{Off}$ is the set of detectors in freeway stretch A located after (downstream) of the off-ramp.

- The use of, at least, one flow measurement is necessary for the application of the controller.

D. Decreasing/Increasing an RM rate

This section derives the equations used for the computation of the RM rate. Firstly, it has to be pointed out that the number of vehicles to hold back $V_{hold,j}$ (and, equivalently, $V_{rel,j}$) reflects the number of additional vehicles that have been held back (or released) taking account of the traffic states of the current time step. That means that if an RM installation was already active in time step $k-1$ (allowing a flow $q_{r_j}(k)$ to enter the freeway), the RM installation has to hold $V_{RM,j}(k)$ additional vehicles with respect to the rate applied in time step $k-1$ and, therefore, to reduce the on-ramp flow with a quantity $\nabla q_{r_j}(k)$ vehicles per hour with respect to $q_{r_j}(k)$. Therefore, for the case of holding back vehicles ($V_{hold,j}(k) > 0$):

$$\nabla q_{r_j}(k) = V_{hold,j}(k) / T \quad (10)$$

where r_j is the index of the on-ramp where RM installation j is located, and $q_{r_j}(k)$ is the flow at time step k on the on-ramp r_j .

On the other hand, assuming that there is enough space to hold back all the requested vehicles, the reduction in the on-ramp flow due to a change of the RM rate ($\nabla q_{r_j}(k)$) can be also computed according to the following equation:

$$\nabla q_{r_j}(k) = q_{r_j}(k) - RM_j^{all}(k) C_j \quad (11)$$

where $RM_j^{all}(k)$ is the ramp metering rate (i.e. the percentage of on-ramp flow that the RM installation allows to enter the freeway) at time step k that would have to be applied in order to hold back all the vehicles, and C_j is the capacity of the on-ramp.

As a result, using (10) and (11), $RM_j^{all}(k)$ is obtained as follows:

$$RM_j^{all}(k) = q_{r_j}(k) / C_j - V_{hold,j}(k) / (T C_j) \quad (12)$$

$$= \frac{T q_{r_j}(k) - V_{hold,j}(k)}{T C_j}$$

Subsequently, the maximum number of vehicles that can be waiting on the on-ramp queue has to be taken into account

since it is a limitation that is imperative in real applications. Thus, if the previously computed RM rate would result in an on-ramp queue length that is larger than its maximum value, the rate has to be increased. In order to find this minimum rate, the equation for the dynamics of an on-ramp queue, used in many macroscopic models such as METANET [29] or CTM [30], is used:

$$w_j(k+1) = w_j(k) + T(D_j(k) - C_j \text{RM}_j^{\bar{w}}(k)) \quad (13)$$

where $w_j(k)$ is the number of vehicles waiting at time step k in the queue of the on-ramp corresponding to RM installation j , $D_j(k)$ is the demand of vehicles in this on-ramp (which can be estimated by the ramp flow upstream of the queue of vehicles), and $\text{RM}_j^{\bar{w}}(k)$ is the minimum ramp metering rate that has to be applied at time step k in order to keep the queue of vehicles waiting on the corresponding queue under a predefined value \bar{w}_j .

Taking into account (13) and that, in this case, the on-ramp queue of the following time step has to be equal to its maximum value ($w_j(k+1) = \bar{w}_j$), the ramp metering rate that keeps the on-ramp queue within the queue constraint is obtained:

$$\text{RM}_j^{\bar{w}}(k) = \frac{D_j(k)}{C_j} + \frac{(w_j(k) - \bar{w}_j)}{C_j T} \quad (14)$$

Nevertheless, it has to be pointed that other choices for the computation of the ramp metering rate that respect the maximum on-ramp queue (such as the one used in [25]) can be used without a substantial change in the formulation of the controller.

Lastly, taking into account that the RM rate should not be incremented if $V_{\text{hold},j}(k) > 0$, the final equation for the ramp metering rate in the case of holding vehicles ($\text{RM}_j^{\text{hold}}(k)$) is obtained:

$$\begin{aligned} \text{RM}_j^{\text{hold}}(k) & \quad (15) \\ &= \min \left(\text{RM}_j(k-1), \max \left(\text{RM}_j^{\text{all}}(k), \text{RM}_j^{\bar{w}}(k) \right) \right) \\ &= \min \left(\text{RM}_j(k-1), \right. \\ & \quad \left. \max \left(\frac{Tq_{r_j}(k) - V_{\text{hold},j}(k)}{TC_j}, \frac{D_j(k)}{C_j} + \frac{(w_j(k) - \bar{w}_j)}{C_j T} \right) \right) \end{aligned}$$

Equivalently, the equation for the ramp metering rate in the case of releasing vehicles ($\text{RM}_j^{\text{rel}}(k)$) can be obtained following an analogous procedure for the number of vehicles to release:

$$\begin{aligned} \text{RM}_j^{\text{rel}}(k) & \quad (16) \\ &= \max \left(\text{RM}_j^{\bar{w}}(k), \text{RM}_j(k-1), \frac{Tq_{r_j}(k) + V_{\text{rel},j}(k)}{TC_j} \right) \end{aligned}$$

$$V_{\text{RM},j}(k) = \begin{cases} \max \left(T(q_{r_j}(k) - C_j \text{RM}_j(k)), -w_j(k) \right) & \text{if } \text{RM}_j(k) \neq \text{RM}_j(k-1) \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

If other implementation constraints, apart from the maximum queue on the on-ramps, are considered (such as the maximum and minimum rates of change of the RM values, maximum and minimum increment of the RM rate in one time step,...), they have to be included in (15) and (16).

Finally, the RM rate that has to be implemented ($\text{RM}_j(k)$) is defined based on the value of $V_{\text{hold},j}(k)$ and $V_{\text{rel},j}(k)$:

$$\text{RM}_j(k) = \begin{cases} \text{RM}_j^{\text{hold}}(k) & \text{if } V_{\text{hold},j}(k) > 0 \\ \text{RM}_j^{\text{rel}}(k) & \text{if } V_{\text{rel},j}(k) > 0 \\ \text{RM}_j(k-1) & \text{otherwise} \end{cases} \quad (17)$$

The number of vehicles that are going to be actually held back ($V_{\text{RM},j}(k) > 0$) or released ($V_{\text{RM},j}(k) < 0$) by the RM installation j ($V_{\text{RM},j}(k)$) is computed using (18). This equation is obtained using (12) and taking into account that the maximum number of vehicles that an on-ramp can release is the current number of vehicles waiting on the queue and that $V_{\text{RM},j}$ should not be updated if the ramp metering has not been actually increased or decreased.

E. Updating $V_{\text{hold},j}$ and $V_{\text{rel},j}$ in the case of RM

After the new value for the RM rate for RM installation j has been found, the proposed controller computes the remaining number of vehicles that have to be still held back ($V_{\text{hold},j+1}(k)$) or released ($V_{\text{rel},j+1}(k)$) by the following control measures (if any). This update is computed taking into account that the remaining number of vehicles has to be positive and by subtracting the number vehicles that have been already held back (19) or released (20) by the RM:

$$V_{\text{hold},j+1}(k) = \max \left(0, V_{\text{hold},j}(k) - V_{\text{RM},j}(k) \right) \quad (19)$$

$$V_{\text{rel},j+1}(k) = \max \left(0, V_{\text{rel},j}(k) + V_{\text{RM},j}(k) \right) \quad (20)$$

F. Decreasing/Increasing a VSL

The value of the VSL ($\text{VSL}_j(k)$) that has to be implemented in order to hold back or release a certain number of vehicles is computed taking into account the following assumptions:

- The segment in which VSL_j is located is uncongested. If the VSLs are located upstream of the bottleneck and there is no other source of congestion, this assumption is true before the congestion is created on the bottleneck and while the bottleneck density is controlled around the critical density. When the congestion is already reaching the segments where the VSLs are located, this assumption is not true anymore. However, in the case that the congestion is already propagating upstream, the potential reduction of the congestion that can be achieved by the VSLs is minimal.

- The flow of the segment in which VSL_j is located after a certain and relatively short period of time is equal to the flow of the same segment before the change of the speed limit value. This assumption will be true if the first assumption holds (the segment is uncongested).
- The mean speed after the change on the corresponding speed limit is the posted limit multiplied by a compliance factor $(1 + \alpha_{i_j})$. If other assumptions about the VSL-induced speed were considered (such as, a VSL-induced speed that decreases with the density as proposed in [31]), they could be included in the formulation without substantial changes.

Taking these assumptions into account, it can be stated that:

$$v_{\text{before},i_j}(k)\rho_{\text{before},i_j}(k) = v_{\text{after},i_j}(k)\rho_{\text{after},i_j}(k) \quad (21)$$

$$v_{\text{after},i_j}(k) = (1 + \alpha_{i_j})VSL_j^{\text{all}}(k) \quad (22)$$

where $VSL_j^{\text{all}}(k)$ is the speed limit at time step k that would have to be applied in order to hold back all the vehicles, and $\rho_{\text{before},i_j}(k)$, $v_{\text{before},i_j}(k)$, $\rho_{\text{after},i_j}(k)$, and $v_{\text{after},i_j}(k)$ are, respectively, the densities and speeds of segment i_j , where the VSL denoted by j is located, before and after the change in the value of the speed limit.

Moreover, the number of vehicles that are going to be held back by the VSL after a certain (relatively short) period of time is equal to:

$$V_{\text{hold},j}(k) = \lambda_{i_j} L_{i_j} (\rho_{\text{after},i_j}(k) - \rho_{\text{before},i_j}(k)) \quad (23)$$

Therefore, using (21), (22), and (23) and taking into account that $\rho_{\text{before},i_j}(k) = \rho_{i_j}(k)$ and $v_{\text{before},i_j}(k) = v_{i_j}(k)$, the value of the speed limit that should be applied in order to store $V_{\text{hold},j}(k)$ vehicles is obtained as:

$$VSL_j^{\text{all}}(k) = \frac{L_{i_j} \lambda_{i_j} v_{i_j}(k) \rho_{i_j}(k)}{(1 + \alpha_{i_j})(L_{i_j} \lambda_{i_j} \rho_{i_j}(k) + V_{\text{hold},j}(k))} \quad (24)$$

Finally, considering that the value of the VSL should not be increased if $V_{\text{hold},j}(k) > 0$, the final equation for the speed limit ($VSL_j^{\text{hold}}(k)$) in the case of holding vehicles is obtained:

$$\begin{aligned} VSL_j^{\text{hold}}(k) &= \min \left(VSL_j(k-1), VSL_j^{\text{all}}(k) \right) \\ &= \min \left(VSL_j(k-1), \frac{L_{i_j} \lambda_{i_j} v_{i_j}(k) \rho_{i_j}(k)}{(1 + \alpha_{i_j})(L_{i_j} \lambda_{i_j} \rho_{i_j}(k) + V_{\text{hold},j}(k))} \right) \end{aligned} \quad (25)$$

Equivalently, the equation for the variable speed limit in the case of releasing vehicles can be obtained following an analogous procedure for the number of vehicles to release. However, in this case it has to be considered that if $V_{\text{rel},j}(k) \geq L_{i,j} \lambda_{i,j} \rho_{i,j}(k)$, the VSL denoted by j has to be increased as much as possible; however, the value of the VSL computed using (24) would be negative. As a consequence, $VSL_j^{\text{rel}}(k)$ is defined as a piece-wise function in order to avoid this undesirable effect in such a way that, if the number of vehicles

in the current segment (i.e. $L_{i,j} \lambda_{i,j} \rho_{i,j}(k)$) is lower than the number of vehicles to release (i.e. $V_{\text{rel},j}(k)$), VSL j is increased as much as possible:

$$\begin{aligned} VSL_j^{\text{rel}}(k) &= \max(VSL_j(k-1), Y_j(k)) \\ Y_j(k) &= \begin{cases} \overline{VSL}_j & \text{if } \rho_{i_j}(k) \leq \frac{V_{\text{rel},j}(k)}{L_{i_j} \lambda_{i_j}} \\ \frac{L_{i_j} \lambda_{i_j} v_{i_j}(k) \rho_{i_j}(k)}{(1 + \alpha_{i_j})(L_{i_j} \lambda_{i_j} \rho_{i_j}(k) - V_{\text{rel},j}(k))} & \text{otherwise} \end{cases} \end{aligned} \quad (26)$$

where $VSL_j^{\text{rel}}(k)$ is the VSL that has to be applied if $V_{\text{rel},j}(k) > 0$ and \overline{VSL}_j is the maximum speed limit that can be applied on segment j .

If other implementation constraints are considered, they have to be included in (25) and (26).

Finally, the speed limit that has to be implemented ($VSL_j(k)$) is defined based on the value of $V_{\text{hold},j}(k)$ and $V_{\text{rel},j}(k)$:

$$VSL_j(k) = \begin{cases} VSL_j^{\text{hold}}(k) & \text{if } V_{\text{hold},j}(k) > 0 \\ VSL_j^{\text{rel}}(k) & \text{if } V_{\text{rel},j}(k) > 0 \\ VSL_j(k-1) & \text{otherwise} \end{cases} \quad (27)$$

The number of vehicles that are going to be held back/released by the VSL ($V_{\text{VSL},j}(k)$) is computed by (28). This equation is obtained using (24) and taking into account that $V_{\text{VSL},j}$ should not be updated if the VSL denoted by index j has not been actually increased or decreased.

G. Updating $V_{\text{hold},j}$ and $V_{\text{rel},j}$ in the case of VSL

Similarly as was done in Section II-E for a RM installation, the following equations update the value of $V_{\text{hold},j}(k)$ and $V_{\text{rel},j}(k)$ if the value of the VSL j has been increased or decreased:

$$V_{\text{hold},j+1}(k) = \max \left(0, V_{\text{hold},j}(k) - V_{\text{VSL},j}(k) \right) \quad (29)$$

$$V_{\text{rel},j+1}(k) = \max \left(0, V_{\text{rel},j}(k) + V_{\text{VSL},j}(k) \right) \quad (30)$$

III. LOGIC-BASED TRAFFIC FLOW CONTROL (LB-TFC) FOR FREEWAY BOTTLENECKS

This section presents the control logic that is used for the implementation of LB-TFC for each time step k (see the flow chart shown in Fig. 3). The equations needed for the implementation of LB-TFC are referred to in the flow chart.

As can be seen in Fig. 3, after receiving the new traffic measurements for time step k , the control algorithm first computes the number of vehicles that have to be held back ($V_{\text{hold},1}(k)$) or released ($V_{\text{rel},1}(k)$) so as to keep the bottleneck flow around its capacity. This number of vehicles is calculated based on the bottleneck density and on the mean speed and flow upstream of the bottleneck (i.e. the mean speed and

$$V_{\text{VSL},j}(k) = \begin{cases} \lambda_{i_j} L_{i_j} \left(\frac{v_{i_j}(k) \rho_{i_j}(k)}{(1 + \alpha_{i_j}) VSL_j(k)} - \rho_{i_j}(k) \right) & \text{if } VSL_j(k) \neq VSL_j(k-1) \\ 0 & \text{otherwise} \end{cases} \quad (28)$$

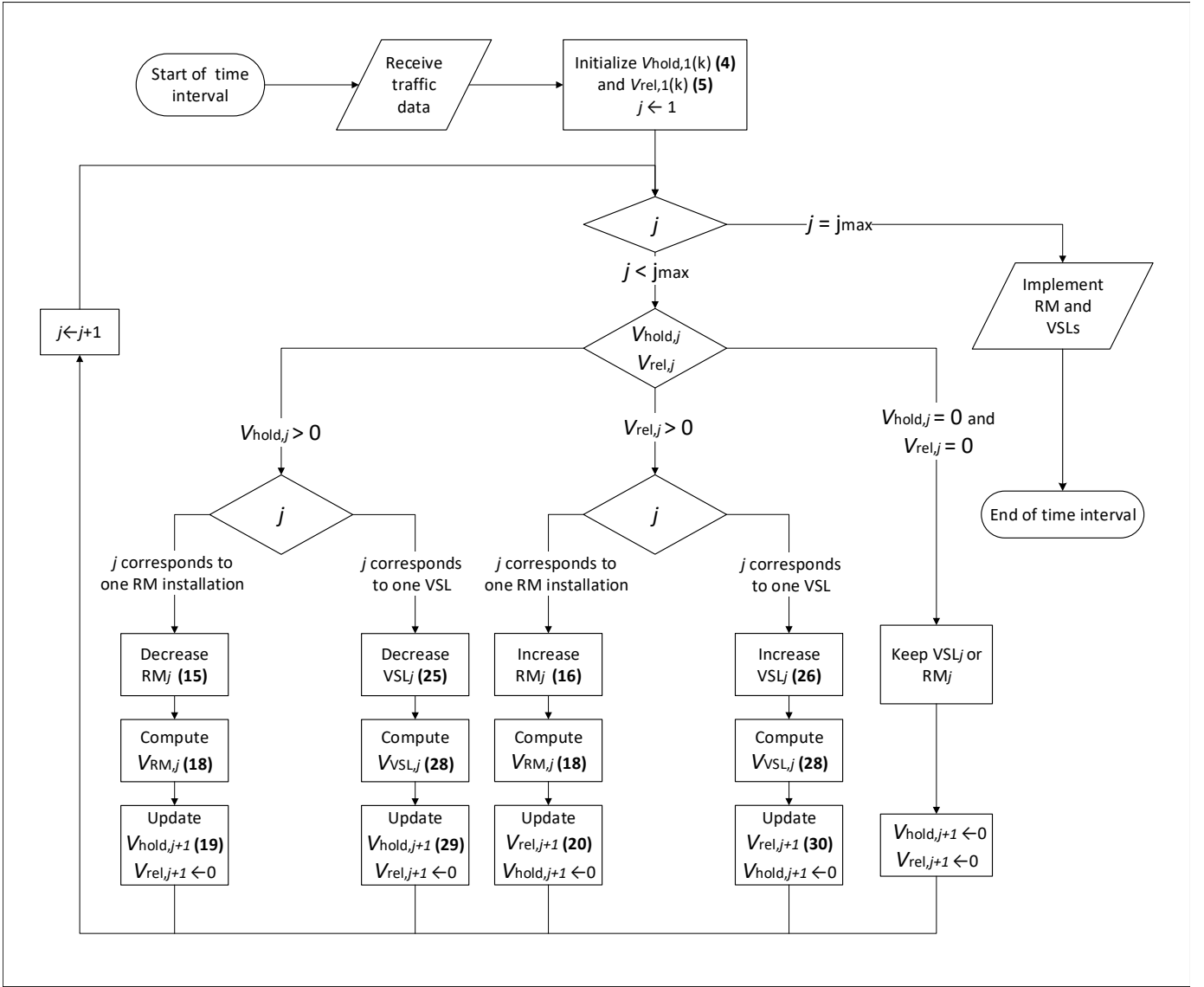


Fig. 3. Control logic for the implementation of LB-TFC, where j is an index running over the control measures upstream of the bottleneck.

flow for the segments within a distance L_A upstream of the bottleneck) using (4) and (5).

Subsequently, the RM rates and the VSLs are decreased or increased sequentially, using (15), (16), (25), and (26), respectively. The order in which the VSLs and the RM installations are activated/deactivated is defined by the index j . In this work, the index j is running from the most upstream control measure to the one closer to the bottleneck. However, in future works the use of other orders (and their optimality) to specify the control measures that will be activated sooner and later could be investigated.

Lastly, the remaining number of vehicles that have to be still held back or released (and, therefore, considered for control measure $j + 1$) after decreasing the RM or VSL denoted by index j is computed by (19), (20), (29) and (30).

The iterative process continues until all the VSLs and RM rates have been decreased/increased/kept ($j = j_{\max}$).

It should be noted that LB-TFC is designed for the control

of one single bottleneck. Therefore, in the case of multiple interacting bottlenecks, a high-level controller should set, for each controller time step, which bottleneck is managed by each control measure.

IV. CASE STUDY I

A. Synthetic freeway

The case study considered in this paper uses a synthetic 12 km long freeway stretch in order to simulate the response given by LB-TFC. In the second part of the paper ('Part 2: Simulation and Comparison') [28], the performance and responses achieved with LB-TFC will be compared with the ones obtained with MTFC + PI-ALINEA and with the optimal solution. The used freeway stretch, shown in Fig. 4, has been already used for the simulation of the FF-ALINEA algorithm in [20] and for the simulation of the LB-VSL algorithm in [10].

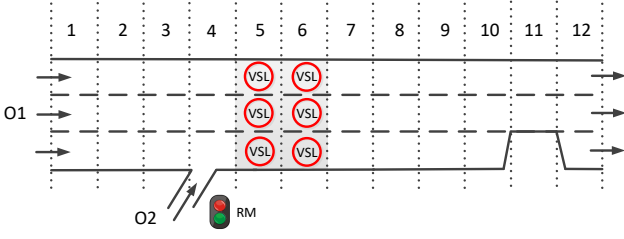


Fig. 4. Freeway stretch considered for the first case study.

The freeway has $N = 12$ segments with $\lambda_i = 3$ lanes and with a length of $L_i = 1$ km for each segment, one controlled on-ramp at the beginning of segment 4, and one lane drop in segment 11 (i.e. segment 11 has only 2 lanes). Because of the lane drop, segment 11 is a bottleneck that will create congestion if the demands are high enough. There is an RM installation on the on-ramp located on segment 4 and there are two segments equipped with VSLs (segment 5 and 6). The VSLs are only allowed to take a limited number of discrete values in the set $\{40, 50, 60, 70, 80, 90, 100\}$ km/h and they can only be increased or decreased with 10 km/h for each controller time step. These implementation constraints have been included in equations (25) and (26).

In order to simulate the considered freeway stretch, the METANET model [29] has been used. The effects of the VSLs have been included using the VSL model proposed in [5] with a compliance parameter of $\alpha = 0.1$. The simulation time chosen is three hours, corresponding to 180 controller sample steps (because of the length of the controller time step is selected as $T_c = 60$ s) and 1080 simulation steps (because of the length of the simulation time step is selected as $T = 10$ s). All the METANET parameters are considered to be the same for all the segments. The on-ramp has a capacity of $C_{r,4} = 2000$ veh/h, the free-flow speed v_f is 110 km/h, the critical density ρ_c is 32 veh/(km·lane), the maximum density ρ_m is 180 veh/(km·lane), and the time constant τ is 18 s. The rest of the model parameters can be seen in Table I and they have the same value as in [20] and [10]. As proposed in [5], the model takes different values for μ (μ_H and μ_L) depending on the downstream density.

TABLE I
METANET PARAMETERS USED IN THE SIMULATION

a	μ_H	μ_L	ϕ	K	δ
2	20 km ² /h	80 km ² /h	0.1	40	0.01

For this case study, five different mainline demands and one on-ramp demand (shown in Fig. 5) are considered. Moreover, two values for \bar{w} (the maximum number of vehicles waiting on the on-ramp queue) are considered (50 veh and 200 veh). As a result, 10 scenarios, with different levels of congestion, are used combining the demands and the queue constraints as listed in Table II.

For the implementation of LB-TFC, it has been considered that there are flow and speed detectors available for all the segments between the on-ramp and the bottleneck (segments

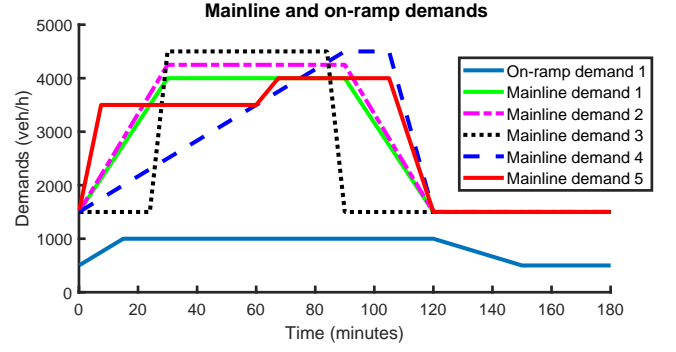


Fig. 5. Mainline and on-ramp demands

TABLE II
CONSIDERED SCENARIOS

Scenario	1	2	3	4	5	6	7	8	9	10
Demand	D_1	D_1	D_2	D_2	D_3	D_3	D_4	D_4	D_5	D_5
\bar{w}	200	50	200	50	200	50	200	50	200	50

4 up to and including 11) and that the first control input that is activated in case of congestion ($j = 1$) is the ramp metering installation located on segment 4, followed ($j = 2$) by the VSL located on segment 5, and finishing ($j = 3$) with the VSL located on segment 6.

The critical density of the bottleneck has been estimated using the flows and densities obtained by simulating METANET for Scenario 1 without any activation of the speed limits. More concretely, the fundamental diagram of the no-control case has been constructed and the value of the density corresponding to maximum outflows of the bottleneck has been taken as the critical density. As in previous references [19], [20], the obtained critical density ($\rho_{c,b} = 36.78$ veh/(km·lane)) is larger than the one given by the METANET fundamental diagram (32 veh/(km·lane)).

B. Results

In this section, the proposed controller is tested for the 10 scenarios considered and the results are compared with the ones obtained with the no-control case.

The values of the controller parameters (shown in Table III) used for the simulation of LB-TFC for each scenario i (\bar{C}_B^i , \bar{C}_B^i) have been found by minimizing the Total Time Spent (TTS) for the given scenario i (equation (1) in the second part of the work). For the estimation of $Q_{ib}(k)$ and $\hat{v}_A(k)$, needed for LB-TFC, the flows and speeds from segments 4 to 11 have been taken as measurements for equation (6).

The obtained numerical results are shown in Table IV. Analyzing the results, it can be seen that the LB-TFC is able to substantially reduce the TTS for the entire set of considered scenarios.

Subsequently, the speeds contour plots obtained for Scenario 1, which uses the Mainline Demand 1 (shown in Fig. 5) and has a maximum on-ramp queue of 200 vehicles, for the uncontrolled case and for LB-TFC are shown in Fig. 6.

In the figure, it can be seen that, for the no-control case, a traffic jam is created due to congestion appearing at the

TABLE III
OPTIMAL CONTROL PARAMETERS FOR LB-TFC OPTIMIZED FOR SCENARIO i

Scenario	1	2	3	4	5	6	7	8	9	10	All
$\overline{C_B^i}$ (veh/h)	4784.0	4827.0	4835.4	4871.4	4880.0	4830.9	4801.6	4810.3	4794.2	4796.3	4817.2
$\underline{C_B^i}$ (veh/h)	4729.6	3035.2	4694.4	3003.1	3023.7	2995.2	3180.5	2622.8	4689.7	3455.0	4706.2

TABLE IV
TTS (VEH·H) FOR DIFFERENT CONTROLLERS AND SCENARIOS

Scen.	1	2	3	4	5	6	7	8	9	10
Uncontrolled	2860.7	2860.7	3820.1	3820.1	3007.2	3007.2	2464.8	2464.8	2490.4	2490.4
LB-TFC	1455.7	1710.0	2458.6	3001.4	2175.7	2498.8	1825.9	2049.4	1594.8	1948.3
Red(%)	-49.1	-40.2	-35.6	-21.4	-27.6	-16.9	-25.9	-17.0	-36.0	-21.8

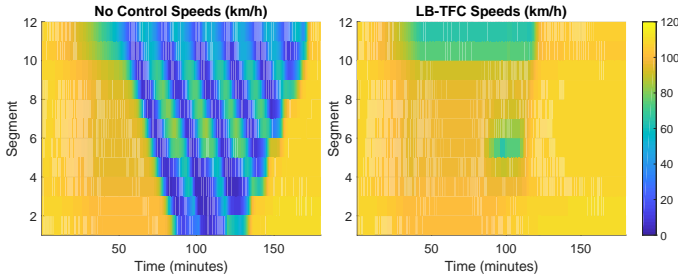


Fig. 6. Speed contour plots for Scenario 1

bottleneck. The congestion creates shock waves that propagate upstream, causing a considerable decrease of the TTS as can be seen in Table IV. On the other hand, LB-TFC is able to totally resolve, for this scenario, the congestion created for the no-control case. This removal of the traffic jam is achieved by the use of the ramp metering rates and the speed limits as shown in Fig. 7.

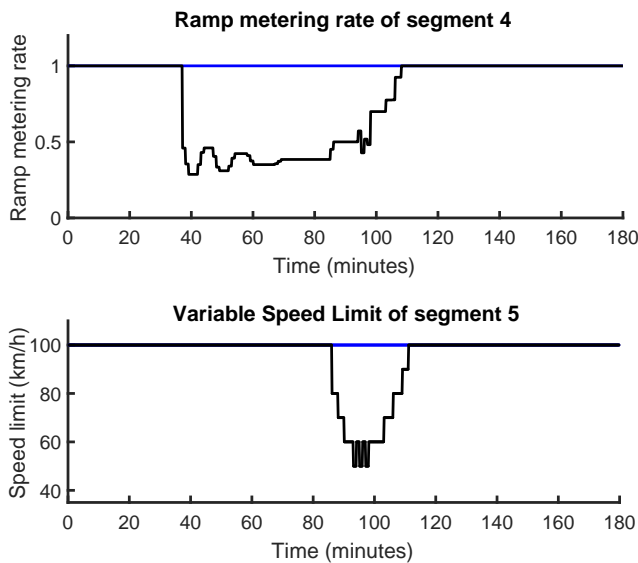


Fig. 7. RM rates, VSLs, and ramp queues for Scenario 1

In this figure, it can be seen that LB-TFC shows light oscillations for the VSLs. In the case that these oscillations are undesirable due to implementation constraints or other reasons, they can be avoided by changing the parameters of the controllers (reducing $\underline{C_B^i}$, or increasing $\overline{C_B^i}$) entailing slight reductions in performance in terms of TTS.

V. CONCLUSIONS

This paper proposes a new Logic-Based Traffic Flow Control (LB-TFC) algorithm for integrated control of variable speed limits and ramp metering installations in order to reduce or avoid traffic congestion at bottlenecks. The proposed controller is based on the estimation of the number of vehicles that have to be held back or released in order to keep the outflow of the bottleneck around the capacity. Based on this number, the ramp metering rates and the variable speed limits are accordingly increased or decreased.

In the second part of the paper ('Part 2: Simulation and Comparison') [28], LB-TFC will be simulated, analyzed, and compared for two freeways (one synthetic network and one stretch of the ring-road freeway SE-30 in Seville, Spain).

The main advantage of the LB-TFC, compared with previously proposed controllers for the integrated control of ramp metering installations and variable speed limits, is that it is easy-to-implement, because the tuning of the control parameters is very intuitive and simple and the computation of the control inputs is almost instantaneous, while its performance is robust and effective.

In future work, the integration of LB-TFC with a controller for reversible lanes, such as the one proposed in [32], and with other control measures, such as lane closure or route guidance, will be investigated. Moreover, the use of different orders for the application of the control inputs and the effect of selecting longer and/or shorter values for L_A will be also studied in future works.

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