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Decomposition and distributed optimization of real-time traffic management for large-scale railway networks – Supplementary material

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Abstract

This document contains supplementary material for the paper "Decomposition and distributed optimization of real-time traffic management for large-scale railway networks" by X. Luan, B. De Schutter, L. Meng, and F. Corman, *Transportation Research Part B*, vol. 141, pp. 72-97, Nov. 2020.

1. Instance details

Railway network

The layout of the Dutch railway network under consideration is presented in Figure 1, as well as the station abbreviations used by the Dutch railways. As shown, there are 4 major stations, including Ut (Utrecht Central), Ah (Arnhem), Nm (Nijmegen), and Ht ('sHertogenbosch), plus other 40 minor stations. The network consists of 891 nodes and 968 links. There are 46 links (i.e., block sections) that trains can use in both directions, and the other links are practically used in a single direction by the current timetable.

Train information

Table 1 gives the information on the train routes and the number of trains on each route. There are total 154 trains, including 70 intercity trains and 84 local trains (more specifically, 4 ICE3 trains, 10 ICM10 trains, 14 ICM12 trains, 16 ICM3 trains, 18 IRM12 trains, 8 IRM4 trains, 68 Mat64 trains, and 16 SGM3 trains). The trains can change their local routes, i.e., the tracks and platforms used in station areas, while their global routes are considered to be fixed.

Primary delay

In the experiments, we consider 10 randomly generated delay scenarios. For each delay scenario, each train is given a randomly generated primary delay time at its origin by following a 3-parameter Weibull distribution. The parameters used are as follows:

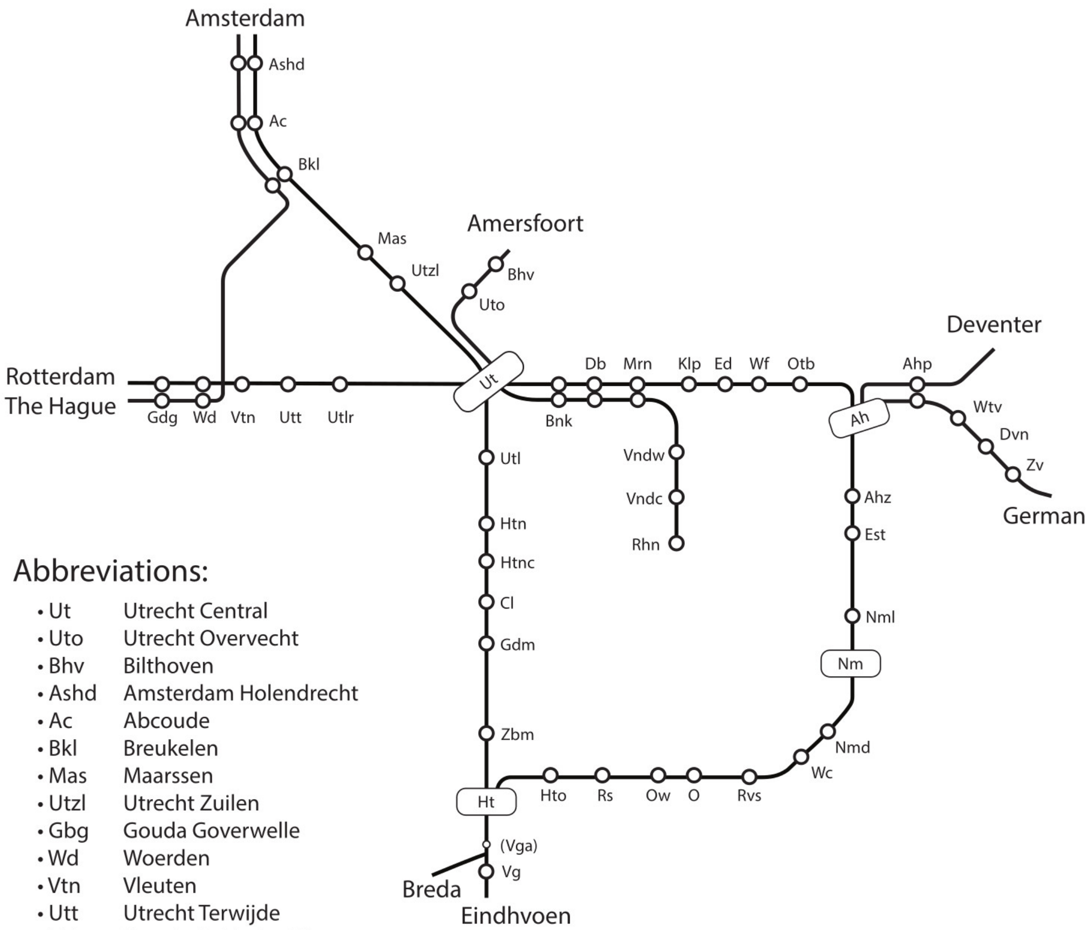
- for intercity trains, scale = 394, shape = 2.27, shift = 315;
- for local trains, scale = 235, shape = 3.00, shift = 186.

The parameters have been obtained by fitting to real life data, as explained in (Corman et al., 2011). Figure 2 illustrates the primary delays (on the y-axis) followed by trains (on the x-axis) in each delay scenario.

2. Experiment details

Decomposition couplings and lower bound (LB)

When decomposing a large problem into several subproblems, some constraints have to be relaxed, and those relaxed constraints actually result in couplings among subproblems. As we have explained in the paper, when implementing the GEO decomposition, the train transition (TTC) constraint becomes the coupling constraint, ensuring that the train departure time from a region must equal the train arrival time at the consecutive region. When using the TRA decomposition, the capacity (CAP) constraint becomes the coupling constraint, forcing that each block section can only be occupied by one train at any time. The coupling constraints of the



Abbreviations:

- Ut Utrecht Central
- Uto Utrecht Overvecht
- Bhv Bilthoven
- Ashd Amsterdam Holendrecht
- Ac Abcoude
- Bkl Breukelen
- Mas Maarssen
- Utzl Utrecht Zuilen
- Gbg Gouda Goverwelle
- Wd Woerden
- Vtn Vleuten
- Utt Utrecht Terwijde
- Utlr Utrecht Leidsche Rijn
- Bnk Bunnik
- Db Driebergen Zeist
- Mrn Maarn
- Klp Veenendaal De Klomp
- Ed Ede-Wageningen
- Wf Wolfheze
- Otb Oosterbeek
- Vndw Veenendaal West
- Vndc Veenendaal Centrum
- Rhn Rhenen
- Ah Arnhem Centraal
- Ahp Arnhem Velperpoort
- Wtv Westervoort
- Dvn Duiven
- Zv Zevenaar
- Ahz Arnhem Zuid
- Est Elst
- Nml Nijmegen Lent
- Nm Nijmegen
- Nmd Nijmegen Dukenburg
- Wc Wijchen
- Rvs Ravenstein
- O Oss
- Ow Oss West
- Rs Rosmalen
- Hto s-Hertogenbosch Oost
- Utl Utrecht Lunetten
- Htn Houten
- Htnc Houten Castellum
- Cl Culemborg
- Gmd Geldermalsen
- Zbm Zaltbommel
- Ht 's-Hertogenbosch

Figure 1: The railway network layout

Table 1: Train routes

Route index	Route (non-directional)	Number of trains
1	Ashd - Ac - Bkl - Mas - Utzl - Ut - Utl - Htn - Htnc - Cl - Gdm - Zbm - Ht - Vg(a)	6
2	Ashd - Ac - Bkl - Mas - Utzl - Ut - Bnk - Db - Mrn - Klp - Ed - Wf - Otb - Ah	3
3	Ah - Ahz - Est - Nml - Nm - Nmd - Wc - Rvs - O - Ow - Rs - Hto - Ht - Vg(a)	3
4	Ut - Bnk - Db - Mrn - Klp - Ed - Wf - Otb - Ah - Ahp - Wtv - Dvn - Zv	2
5	Ut - Bnk - Db - Mrn - Klp - Ed - Wf - Otb - Ah - Ahz - Est - Nml - Nm	4
6	Gdg - Wd - Vtn - Utt - Utlr - Ut - Bnk - Db - Mrn - Klp - Ed	2
7	Ut - Bnk - Db - Mrn - Klp - Ed - WF - Otb - Ah	2
8	Ht - Hto - Rs - Ow - O - Rvs - Wc - Nmd - Nm	5
9	Rvs - Wc - Nmd - Nm - Nml - Est - Ahz - Ah	6
10	Ashd - Ac - Bkl - Mas - Utzl - Ut - Utl - Htn	2
11	Gdg - Wd - Vtn - Utt - Utlr - Ut - Uto - Bhv	10
12	Ut - Utl - Htn - Htnc - Cl - Gdm - Zbm - Ht	4
13	Ut - Bnk - Db - Mrn - Vndw - Vndc - Rhn	8
14	Ashd - Ac - Bkl - Mas - Utzl - Ut	7
15	Nm - Nml - Est - Ahz - Ah - Ahp	10
16	Gdg - Wd - Vtn - Utt - Utlr - Ut	10
17	Ut - Utl - Htn - Htnc - Cl - Gdm	6
18	Ashd - Ac - Bkl - Wd - Gdg	7
19	Ah - Ahp - Wtv - Dvn - Zv	9
20	Ashd - Ac - Bkl - Mas	2
21	Ut - Utlr - Utt - Vtn	2
22	Ed - Wf - Otb - Ah	4
23	Gdm - Zbm - Ht	4
24	Gdg - Wd - Vtn	4
25	Vg(a) - Ht - Hto	7
26	Bhv - Uto - Ut	21
27	Ut - Bnk - Db	4

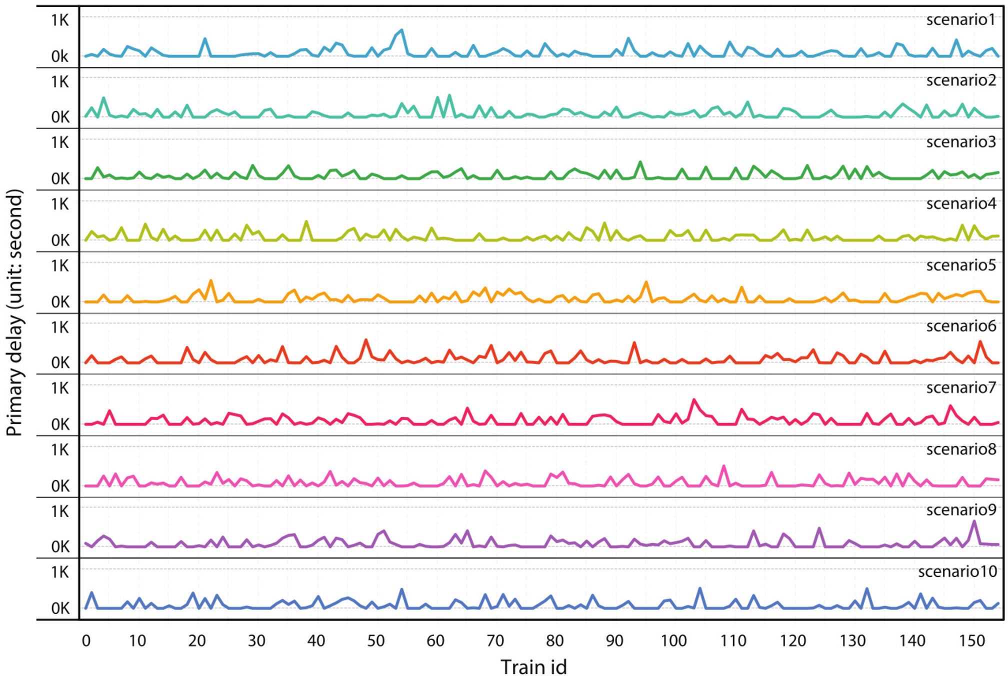


Figure 2: Primary delays of trains for 10 delay scenarios

Table 2: Decomposition couplings and lower bounds

Decom-position	Number of re-gions/Time interval size	Number of subproblems	Total number of TTC constraints	Total number of CAP constraints	Number of couplings	Number of couplings violated in LB	Feasibility of LB	Objective value of LB i.e., delays (unit: second)
GEO	3	3	4318	-	38	38	99.12%	7125.00
	5	5	4318	-	140	140	96.76%	6313.60
	7	7	4318	-	220	220	94.91%	5886.35
	9	9	4318	-	258	258	94.03%	5338.00
TRA	-	154	-	22498	22498	784	96.52%	5873.50
TIN	5	7	4318	22498	23008	1624	93.94%	1654.00
	10	14	4318	22498	22787	1070	96.01%	2797.80

TIN decomposition include both the TTC and CAP constraints. Lower bounds are generated by solving the resulting subproblems separately and independently, i.e., by aggregating the solutions of individual subproblems and ignoring all couplings among the subproblems.

Table 2 gives some preliminary information about the decomposition and the lower bound (LB) solution, presenting the average of the 10 delay scenarios. The results of Table 2 corresponds to the discussion in Sections 6.2.2 and 6.3.3. For the GEO decomposition, the network shown in Figure 1 is partitioned into 3, 5, 7, and 9 regions, and for the TIN decomposition, we set the time interval length to be 5 and 10 min . Let us give examples for each decomposition method to interpret the information presented in the table.

In the original optimization problem, the total number of the TTC constraints is 4318, and 38 of them become the coupling constraints in the 3-region case of the GEO decomposition. If solving the subproblems separately, all these 38 TTC coupling constraints are violated in the LB solution. We use the number of the violated TTC constraints divided by the total number of TTC constraints to measure the feasibility of the LB solutions. Thus, the feasibility rate of the LB solution is $1 - 38/4318 = 99.12\%$. The average objective value of the LB solutions is 7125.00.

The total number of the CAP constraints is 22498 in the original problem, and all of them are coupling constraints in the TRA decomposition. When independently scheduling trains, there are 784 CAP constraints violated in the LB solution. Similarly, the feasibility rate of the LB solution is $1 - 748/22498 = 96.52\%$, and the average objective value of the LB solutions is 5873.50 .

In the 5-min time interval case of the TIN decomposition, there are 23008 constraints recognized as coupling constraints, including both the TTC and CAP constraints. In the LB solutions, 1624 of them are violated, resulting in a LB feasibility rate of 93.94%, calculated by $1 - 1624/(4318 + 22498) = 93.94\%$. The average object value of the LB solutions is very loose, only 1654.00 .

Train delay location

Table 3 summarizes the number of train conflicts that occur at every station and segment in the LB solutions of the TRA decomposition. The conflicts mean that trains request the same infrastructure at the same time, and we count conflict for each pair of trains on each block section. This means that the situation where 4 trains are using the same block section at the same time is counted as 6 conflicts.

Table 3: Location and number of conflicts in the LB solutions

Index	Location	Number of conflicts									
		scen. 1	scen. 2	scen. 3	scen. 4	scen. 5	scen. 6	scen. 7	scen. 8	scen. 9	scen. 10
1	Ut	21	17	20	10	15	22	15	21	26	27
2	Ut-Utlr	11	14	11	9	9	12	5	11	14	13
3	Utlr	5	7	5	3	5	7	3	6	7	6
4	Utlr-Utt	5	7	5	4	5	7	4	6	7	6
5	Utt	8	9	8	6	11	9	7	10	10	9
6	Utt-Vtn	5	6	5	5	7	4	4	7	6	5
7	Vtn	8	16	14	7	14	8	8	13	13	13
8	Vtn-Wd	24	36	35	29	30	23	21	32	34	33
9	Wd	6	8	5	9	8	9	7	6	8	9
10	Wd-Gdg	12	12	11	13	12	14	20	16	16	14
11	Gdg	0	0	0	0	0	0	0	0	0	0

(continued on next page)

Table 3, continued

Index	Location	Number of conflicts									
		scen. 1	scen. 2	scen. 3	scen. 4	scen. 5	scen. 6	scen. 7	scen. 8	scen. 9	scen. 10
12	Ahz	2	1	0	0	1	0	1	1	1	0
13	Ahz-Ah	12	7	14	12	17	18	18	14	17	9
14	Ah	4	2	2	5	3	4	4	2	3	3
15	Est	1	1	1	2	2	0	0	0	1	2
16	Est-Nml	9	5	4	10	14	0	0	4	1	14
17	Nml	0	0	0	0	0	0	0	0	0	0
18	Nml-Nm	5	2	3	2	2	2	3	2	4	3
19	Nm	1	0	1	0	0	0	0	0	0	0
20	Mas	28	19	8	19	13	10	13	13	6	15
21	Mas-Bkl	13	15	3	10	9	6	9	9	0	9
22	Bkl	30	21	11	18	11	10	15	12	14	17
23	Bkl-Ac	6	16	1	8	8	6	8	8	1	8
24	Ac	2	3	2	3	3	2	2	3	2	4
25	Ac-Ashd	2	4	2	3	3	2	2	3	2	4
26	Ashd	1	4	0	2	2	1	2	2	0	2
27	Ah-Otb	4	5	5	4	4	6	3	5	4	3
28	Otb-Wf	4	6	6	4	4	6	2	6	4	8
29	Wf	2	7	7	4	3	5	6	5	5	6
30	Wf-Ed	0	7	8	5	5	7	12	9	13	10
31	Ed	0	1	1	1	1	1	2	2	1	1
32	Ed-Klp	0	0	0	0	0	0	5	5	0	0
33	Klp-Mrn	13	9	22	9	16	21	29	36	0	15
34	Mrn-Db	3	3	8	7	15	27	17	28	7	14
35	Db	2	2	3	6	6	8	9	6	3	7
36	Db-Bnk	4	6	10	11	5	6	18	15	7	10
37	Bnk-Ut	12	12	12	15	9	2	27	9	30	20
38	Ut-Utzl	8	5	3	5	5	3	4	6	0	4
39	Utzl	1	7	1	4	4	1	4	4	0	4
40	Utzl-Mas	3	3	2	3	2	2	2	2	1	2
41	Vg	0	0	0	0	0	0	0	0	0	0
42	Vg-Ht	0	0	0	0	0	0	0	0	0	0
43	Ht	2	6	4	5	4	7	4	5	6	4
44	Ht-Hto	0	0	0	1	0	0	0	0	0	0
45	Hto-Rs	0	0	2	0	0	0	0	0	0	0
46	Rs-Ow	0	0	7	0	0	0	0	0	0	0
47	Ow-O	0	0	0	0	0	0	0	0	0	0
48	O	0	0	3	1	1	0	1	0	1	1
49	O-Rvs	0	0	6	0	0	0	0	0	0	0
50	Rvs	0	0	1	0	0	0	0	0	0	0
51	Rvs-Wc	1	0	8	1	2	1	0	0	0	2
52	Wc-Nmd	0	0	0	1	1	0	0	0	0	1
53	Nmd-Nm	2	1	3	2	2	1	1	1	2	2
54	Est-Ahz	9	12	9	8	13	5	6	12	8	6
55	Uto-Ut	4	4	4	8	4	12	4	8	4	15
56	Gdm	0	3	0	0	2	2	1	2	1	1
57	Gdm-Cl	10	21	8	9	10	18	7	11	7	16
58	Cl-Htnc	4	3	5	6	1	1	6	5	0	2
59	Htnc-Htn	3	3	3	2	0	0	2	2	0	2
60	Htn	1	3	2	2	0	0	4	4	0	2
61	Htn-Utl	6	4	5	4	6	1	7	5	1	3
62	Utl-Ut	9	2	5	2	4	5	4	3	4	4
63	Otb	0	0	0	0	0	0	0	0	0	0
64	Ah-Ahp	5	6	3	3	2	8	4	4	6	4
65	Ahp-Wtv	2	4	0	0	1	7	3	3	7	2

(continued on next page)

Table 3, continued

Index	Location	Number of conflicts									
		scen. 1	scen. 2	scen. 3	scen. 4	scen. 5	scen. 6	scen. 7	scen. 8	scen. 9	scen. 10
66	Dvn-Zv	2	2	2	2	2	2	2	2	2	1
67	Zv	0	0	0	0	0	0	0	0	0	0
68	Zv-Zvo	0	0	0	0	0	0	0	0	0	0
69	Zbm-Ht	13	9	12	4	9	9	12	7	9	13
70	Uto	6	2	3	3	4	5	2	4	4	8
71	Uto-Bhv	9	4	7	7	13	10	9	4	9	14
72	Cl	0	1	0	0	1	1	0	0	0	1
73	Gdm-Zbm	9	12	11	1	2	4	8	3	10	9
74	Mrn-Vndw	0	0	0	0	0	0	0	0	0	0
75	Vndw	0	0	0	0	0	0	0	0	0	0
76	Vndw-Vndc	0	0	0	0	0	0	0	0	0	0
77	Vndc	0	0	0	0	0	0	0	0	0	0
78	Wd-Bkl	0	1	0	4	1	1	1	1	1	1
79	Wtv	0	1	0	0	1	2	1	1	2	0
80	Wtv-Dvn	5	4	0	2	8	11	9	6	10	6
Total number of conflict		379	413	372	345	377	384	409	432	362	449

Performance of the algorithms Table 4 reports the detailed results regarding the performance evaluation discussed in Section 6.3.1.

Columns 5-6 give the number of iterations and the cumulative CPU time consumed for finding the first feasible solution. Column 7 gives the objective value of the first feasible solution found, i.e., the total train delays. Similarly, columns 9–10 present the information on the best feasible solution found. In columns 11–12, the optimality gap of the first and best feasible solution is given.

In Table 5, we present the optimality gap between the CEN solution and the best feasible solution found, in a form that fully corresponds to Figure 6(a) of the paper.

In Table 6, we present the cumulative computation (CPU) time for finding the best feasible solution, in a form that fully corresponds to Figure 6(b) of the paper.

References

Corman, F., D'Ariano, A., Pranzo, M., & Hansen, I. A. (2011). Effectiveness of dynamic reordering and rerouting of trains in a complicated and densely occupied station area. *Transportation Planning and Technology*, 34, 341–362.

Table 4: Results of the three decompositions and the three algorithms, regarding the CPU computation time, objective value, and optimality gap

1	2	3	4	5	6	7	8	9	10	11	12
Decomposition Algorithm	Cost to go	Setting	# of regions / Time interval size	# of iteration of first solution	CPU time of first solution (second)	Objective of first solution (second)	# of iteration of best solution	CPU time of best solution (second)	Objective of best solution (second)	Optimality gap of first solution	Optimality gap of best solution
CEN	-	-	-	-	791.49	7586.55	-	791.49	7586.55	0.00%	0.00%
GEO - ADMM	No	ϱ^{small}	3	7	229.93	14184.40	7	229.93	14184.40	42.76%	42.76%
GEO - ADMM	No	ϱ^{small}	5	59	937.11	21932.41	59	937.11	21932.41	61.99%	61.99%
GEO - ADMM	No	ϱ^{small}	7	72	537.54	25482.64	72	537.54	25482.64	68.28%	68.28%
GEO - ADMM	No	ϱ^{small}	9	88	903.97	37821.68	88	903.97	37821.68	76.60%	76.60%
GEO - ADMM	No	ϱ^{large}	3	1	26.40	16345.50	1	26.40	16345.50	50.25%	50.25%
GEO - ADMM	No	ϱ^{large}	5	12	161.49	26866.61	12	161.49	26866.61	69.36%	69.36%
GEO - ADMM	No	ϱ^{large}	7	14	140.59	27695.54	14	140.59	27695.54	71.55%	71.55%
GEO - ADMM	No	ϱ^{large}	9	17	208.76	39945.93	17	208.76	39945.93	79.71%	79.71%
GEO - ADMM	Yes	ϱ^{small}	3	3	42.88	11230.98	3	42.88	11230.98	31.68%	31.68%
GEO - ADMM	Yes	ϱ^{small}	5	15	126.27	13436.19	15	126.27	13436.19	42.97%	42.97%
GEO - ADMM	Yes	ϱ^{small}	7	27	116.52	12211.61	27	116.52	12211.61	37.99%	37.99%
GEO - ADMM	Yes	ϱ^{small}	9	64	338.78	14581.62	64	338.78	14581.62	47.78%	47.78%
GEO - ADMM	Yes	ϱ^{large}	3	2	30.66	11108.75	2	30.66	11108.75	31.22%	31.22%
GEO - ADMM	Yes	ϱ^{large}	5	6	64.21	14374.22	6	64.21	14374.22	46.94%	46.94%
GEO - ADMM	Yes	ϱ^{large}	7	6	39.74	12994.94	6	39.74	12994.94	41.65%	41.65%
GEO - ADMM	Yes	ϱ^{large}	9	12	91.11	15461.11	12	91.11	15461.11	51.01%	51.01%
GEO - ADMM	Yes	ϱ^{linear}	3	4	63.65	10523.98	4	63.65	10523.98	27.46%	27.46%
GEO - ADMM	Yes	ϱ^{linear}	5	19	214.22	13101.58	19	214.22	13101.58	40.73%	40.73%
GEO - ADMM	Yes	ϱ^{linear}	7	28	148.02	12570.64	28	148.02	12570.64	39.78%	39.78%
GEO - ADMM	Yes	ϱ^{linear}	9	32	208.93	14177.58	32	208.93	14177.58	46.44%	46.44%
GEO - ADMM	Yes	ϱ^{exp}	3		54.53	10294.82	4	54.53	10294.82	25.44%	25.44%
GEO - ADMM	Yes	ϱ^{exp}	5	26	219.34	12154.25	26	219.34	12154.25	36.72%	36.72%
GEO - ADMM	Yes	ϱ^{exp}	7	24	129.90	12184.20	24	129.90	12184.20	37.86%	37.86%
GEO - ADMM	Yes	ϱ^{exp}	9	39	224.50	14563.61	39	224.50	14563.61	47.73%	47.73%
GEO - PR	-	-	3	2	17.71	10251.42	7	51.96	9444.00	26.90%	20.12%
GEO - PR	-	-	5	-	-	-	-	-	-	-	-
GEO - PR	-	-	7	-	-	-	-	-	-	-	-
GEO - PR	-	-	9	-	-	-	-	-	-	-	-
GEO - CDRSBK	-	-	3	1	48.75	7586.55	1	48.75	7586.55	0.00%	0.00%
GEO - CDRSBK	-	-	5	1	39.65	8078.40	3	116.35	7586.55	3.73%	0.00%
GEO - CDRSBK	-	-	7	1	33.49	15383.48	4	75.10	7586.55	40.04%	0.00%
GEO - CDRSBK	-	-	9	1	33.98	15539.73	4	97.00	7586.55	25.49%	0.00%
TRA - ADMM	Yes		-	5	144.65	15004.87	5	144.65	15004.87	47.53%	47.53%
TRA - ADMM	Yes	ϱ^{large}	-	3	78.31	21118.94	3	78.31	21118.94	62.52%	62.52%
TRA - ADMM	Yes	ϱ^{linear}	-	5	115.65	16206.45	5	115.65	16206.45	52.34%	52.34%
TRA - ADMM	Yes	ϱ^{exp}	-	7	162.56	14879.89	7	162.56	14879.89	47.23%	47.23%
TRA - PR	-	-	-	1	1.74	33036.41	29	49.39	17796.63	76.35%	57.21%
TRA - CDRSBK	-	-	-	4	9.24	7619.35	4	9.24	7619.35	0.48%	0.48%
TIN - ADMM	Yes	ϱ^{small}	5	157	1408.36	23973.69	157	1408.36	23973.69	67.80%	67.80%
TIN - ADMM	Yes	ϱ^{small}	10	105	1505.17	22207.54	105	1505.17	22207.54	64.79%	64.79%
TIN - ADMM	Yes	ϱ^{large}	5	68	682.57	30323.62	68	682.57	30323.62	74.88%	74.88%
TIN - ADMM	Yes	ϱ^{large}	10	54	815.08	28497.90	54	815.08	28497.90	73.24%	73.24%
TIN - ADMM	Yes	ϱ^{linear}	5	80	813.15	24090.47	80	813.15	24090.47	68.33%	68.33%
TIN - ADMM	Yes	ϱ^{linear}	10	70	934.44	23149.34	70	934.44	23149.34	66.08%	66.08%
TIN - ADMM	Yes	ϱ^{exp}	5	93	820.96	26110.91	93	820.96	26110.91	70.75%	70.75%
TIN - ADMM	Yes	ϱ^{exp}	10	79	980.95	23803.52	79	980.95	23803.52	67.46%	67.46%
TIN - PR	-	-	5	-	-	-	-	-	-	-	-
TIN - PR	-	-	10	-	-	-	-	-	-	-	-
TIN - CDRSBK	-	-	5	1	2.48	12991.36	10	21.10	10881.93	41.44%	30.47%
TIN - CDRSBK	-	-	10	1	10.53	8376.80	2	14.76	8299.20	9.64%	8.80%

* No feasible solution is found by GEO-PR in the cases of 5, 7, and 9 regions and by TIN-PR. Therefore, no result is provided.

Table 5: Optimality gap between the CEN solution and the best feasible solution

Algorithm	Setting	CEN	GEO				TRA	TIN	
		-	3	5	7	9	-	5	10
CEN	-	0.00%							
ADMM	ϱ^{small}		31.68%	42.97%	37.99%	47.78%	47.53%	67.80%	64.79%
	ϱ^{large}		31.22%	46.94%	41.65%	51.01%	62.52%	74.88%	73.24%
	ϱ^{linear}		27.46%	40.73%	39.78%	46.44%	52.34%	68.33%	66.08%
	ϱ^{exp}		25.44%	36.72%	37.86%	47.73%	47.23%	70.75%	67.46%
PR	-		20.12%	-	-	-	57.21%	-	-
CDRSBK	-		0.00%	0.00%	0.00%	0.00%	0.48%	30.47%	8.80%

Table 6: Computation time of the best feasible solution (unit: second)

Algorithm	Setting	CEN	GEO				TRA	TIN	
		-	3	5	7	9	-	5	10
CEN	-	791.49							
ADMM	ϱ^{small}		42.88	126.27	116.52	338.78	144.65	1408.36	1505.17
	ϱ^{large}		30.66	64.21	39.74	91.11	78.31	682.57	815.08
	ϱ^{linear}		63.65	214.22	148.02	208.93	115.65	813.15	934.44
	ϱ^{exp}		54.53	219.34	129.90	224.50	162.56	820.96	980.95
PR	-		51.96	-	-	-	49.39	-	-
CDRSBK	-		48.75	116.35	75.10	97.00	9.24	21.10	14.76