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An Efficient Dispersion Model for Control of Emission Levels in the Vicinity of Freeways

Michiel Damoiseaux¹ and Bart De Schutter²

Abstract—Greenhouse gasses emitted by vehicles on a freeway displace to the area downwind of the freeway with potential health risks as a result [1]. In order to predict and control the distribution of pollutants towards target areas near highways, such as hospitals, schools, and residences, a model could be used. Multiple authors have proposed so-called emission dispersion models, but none is yet suited for on-line control of freeway networks. This is due to high computational costs, inapplicability in constantly changing traffic networks, or deviating purposes of the models. This work proposes a new dispersion model that is able to model the distribution of pollutant gasses in the vicinity of a freeway and that is applicable in real-time traffic control. The proposed model, the Line Source Gaussian Puff (LSGP) model, is a modification of the existing Gaussian puff model to make it suitable for dispersion modeling for freeway sections. This is done by implementing line sources instead of point sources. The results of the case study show that the new model is able to minimize the amount of pollutant gasses nearby a freeway with a low computational complexity in a model predictive control scheme.

I. INTRODUCTION

One of the main sources of greenhouse gasses is the vast and still expanding road transport sector. The number of vehicles utilizing freeways increases every day, with more and more traffic jams and pollution as a result. Lately, the development of smart traffic management systems seems to be a sustainable solution to address these issues, and extensive research is being done on these systems [2], [3].

Traffic management systems, using variable speed limits and ramp metering, can be used to reduce congestion, and therewith the total time spent by the vehicles in a freeway network can be minimized [4]. Besides improving the traffic flow in a freeway network, these traffic management systems are also capable of reducing the total amount of emitted pollutants by the freeway system [5] and of minimizing the amount of emissions dispersed to a target area [6], [7]. In order to perform simulations with traffic management systems or to use them in a model-based predictive control scheme, prediction models are required to predict the future evolution of the system. The METANET model [8] and the VT-macro [9] model are renowned models to simulate the traffic flow and the emitted emissions by the vehicles on the road. Due to a good trade-off between model accuracy and computational complexity these models are very suitable for on-line traffic control. However, for modeling the dispersion of emitted pollutants by vehicles on a freeway and for use in on-line traffic control, there are not yet models that provide a similar good trade-off between model accuracy and computational complexity.

Dispersion models proposed in literature [10]–[12] range from very accurate time-dependent models to coarse and fast models. Generally, dispersion models depend on variables such as wind speed and direction, air temperature, rainfall, the topography of the area, and the presence of obstacles.

A type of dispersion models that are increasingly used are the Computational Fluid Dynamic (CFD) models [10]. These models use numerical methods to compute the dispersion of emissions based on the laws of fluid dynamics, mass conservation, momentum conservation, and pollutant transport. This is done by solving the Navier-Stokes equation in three dimensions with finite-difference and finite-volume methods. CFD models use a fine grid to capture all the relevant turbulence scales, leading to a high complexity. The high complexity ensures a realistic representation of reality. However, it also invokes high computational demands. Large eddy simulation [13] and Reynold-average Navier-Stokes [14] are methods that decrease the computational burden, but the remaining computational cost is still high, making real-time applications impossible.

Another type of models used to simulate the dispersion of emissions are Lagrangian models [11], also known as particle-tracking models. This type of model describes fluid elements that follow the instantaneous flow. The paths followed by each fluid element, e.g., a pollutant, is computed individually by the Lagrangian models. These trajectories are driven by wind fields, buoyancy, and turbulence. The accumulation of all the trajectories of the single fluid elements results in a concentration distribution of pollutant gasses in the atmosphere close to the source. The simulation of all the trajectories of the individual elements results in high computation times.

Lastly, the Gaussian models are commonly used [12]. By the use of diffusion equations, a concentration distribution in the vicinity of a source is computed. These equations are functions of the distance from the release point or of the time since release. The models are relatively simple, which results in a low computation time. On the downside, the Gaussian models are steady-state, and therefore, not applicable in situations where the wind or the emission rate is changing constantly. The Gaussian puff model is a non-stationary and non-homogeneous variant of the Gaussian model. A concentration field is then described by a series of independent puffs of emissions. The puffs are released every time step and displaced by the wind. This makes them dependent on temporal and spatial variations in the wind and on alternating emission rates. The Gaussian puff models are time-dependent and space-dependent dispersion models

with a low computation time, which are requirements for the model to be used in on-line traffic control. The main drawback of this model is the release of puffs from a point source. A freeway is a stretched source, and therefore, using a point source is an inconvenient and inaccurate way of modeling the dispersion of emissions from a freeway.

Due to high computational costs, the inapplicability in constantly changing traffic networks, or deviating purposes of models such as stack emission modeling, the models proposed in literature are not suitable for on-line freeway traffic control, and therefore, a new dispersion model is required.

The main contribution of this paper is the proposal of a new freeway emission dispersion model that provides a good trade-off between model accuracy and computational complexity. This paper proposes the Line Source Gaussian Puff (LSGP) model, which is a modified version of the Gaussian puff model, in order to make it suitable for the modeling of emissions dispersed from freeways. The LSGP model is capable of accurately simulating the concentration distribution of emissions near a freeway within a reasonable computation time, which makes it applicable for on-line traffic control. In the remainder of this paper, the formulation of the LSGP model will be described, and the model will be validated and compared with existing models.

II. DISPERSION MODELING

This section proposes a new dispersion model, the Line Source Gaussian Puff (LSGP) model, which is able to model the concentration distribution in the vicinity of a freeway in a realistic manner and with a low computation time. In this model, the displacement of pollutant gasses is treated as a superposition of various puffs of emission that are displaced by the wind, influenced both by the speed and the direction of the wind, in the surroundings of the source.

A. Puff displacement

The puffs, created at every time step on the freeway segments, disperse in time over the surrounding area near the freeway. The trajectory of the center of a puff is wind-driven. The following equations describe how the puff center displaces in the x-direction and the y-direction by the wind:

$$x_{\text{puff}}(k+1) = x_{\text{puff}}(k) - TV_w(k) \sin(\phi_w(k)) \quad (1)$$

$$y_{\text{puff}}(k+1) = y_{\text{puff}}(k) - TV_w(k) \cos(\phi_w(k)) \quad (2)$$

where $x_{\text{puff}}(k)$ and $y_{\text{puff}}(k)$ are, respectively, the x and y coordinates of the puff at time step k , T is the sampling time, $V_w(k)$ represents the speed of the wind, and $\phi_w(k)$ the direction of the wind.

B. Concentration distribution

In a Gaussian puff model every puff has a concentration distribution around the center of the puff in order to create a smooth concentration field after accumulating the separate puffs. In the existing Gaussian puff model, with a point source, the distribution is Gaussian in the x - and y -direction.

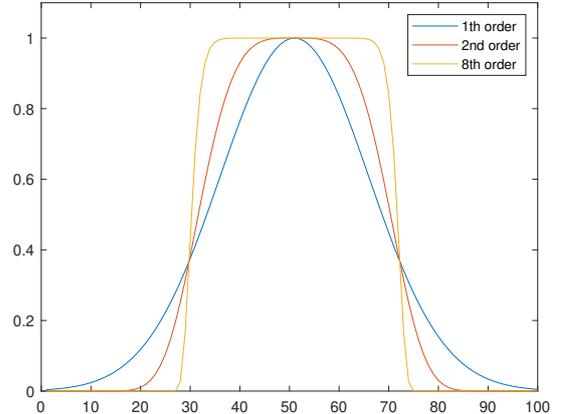


Fig. 1: Super-Gaussian distribution for different orders.

This distribution can be seen in Figure 2a and is represented by the following equation:

$$C(x, y) = A \exp \left(- \left(\frac{(x - x_0)^2}{2\sigma_X^2} \right) - \left(\frac{(y - y_0)^2}{2\sigma_Y^2} \right) \right) \quad (3)$$

where C is the concentration of emissions, A is the amplitude, σ_X and σ_Y are respectively the standard deviations in the x - and y -direction, and x_0 and y_0 represent the coordinates of the center of the puff.

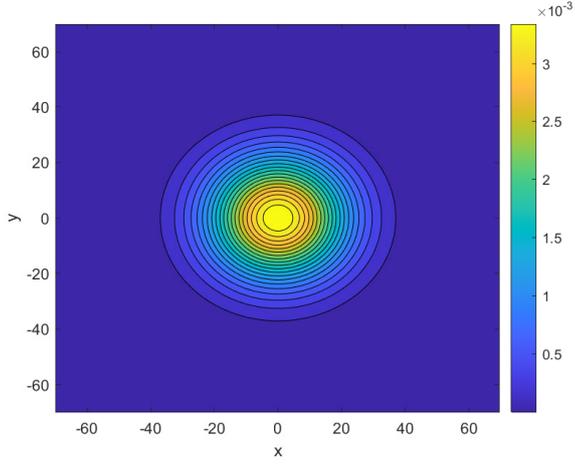
The objective of the new model is to represent the dispersion of emissions of a freeway. When a puff is released at a freeway segment, the concentration in the length of the freeway section should be uniform. With a Gaussian distribution, the concentration in the center of the freeway section will be high, and the further away from the center, the lower the concentration. In order to create a uniform distribution in the longitudinal direction of the freeway, the model should be formed in such a way that it involves a line source instead of a point source.

In this work, a super-Gaussian distribution is used to modify the regular Gaussian distribution of the Gaussian puff model and to create a line source model. This super-Gaussian distribution is a Gaussian distribution with the term in the exponent raised to the power P :

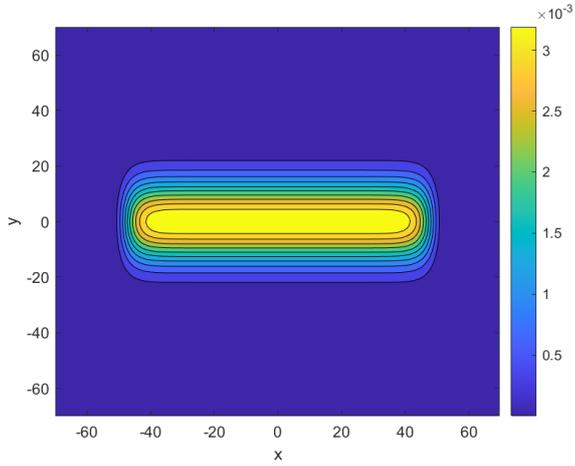
$$C(x) = A \exp \left(- \left(\frac{(x - x_0)^2}{2\sigma_X^2} \right)^P \right). \quad (4)$$

In Figure 1, the different shapes of two-dimensional super-Gaussian distributions are shown. A regular Gaussian distribution, with $P = 1$, is depicted by the blue line. This has a recognizable cone-like shape. For $P = 2$, the orange line appears. Clearly, the top of the curve is becoming wider and flatter. When $P = 8$, the yellow line, the distribution results in a flat-topped curve. The higher the order P , the flatter the top, with a square distribution when P goes to infinity.

In order to model the dispersion of a freeway, a line source is created by using a super-Gaussian distribution in the longitudinal direction of the freeway. A regular Gaussian



(a) Concentration distribution of a point source



(b) Concentration distribution of a line source

Fig. 2: Emission distributions of a single puff.

distribution is used in this work in the direction perpendicular to the freeway to create a three-dimensional concentration field. The resulting distribution of a single puff can be seen in Figure 2b. In this contour plot, a uniform distribution, shaped like a freeway section, can be identified in the yellow area. In this way, a line-shaped distribution of emissions is created and can be used in the modeling of the dispersion of freeway emissions.

C. Dynamics of the LSGP model

The LSGP model is formulated by an equation that describes the concentration field of individual puffs. This equation is:

$$C_{x,y}(i, j, k) = \frac{\xi \bar{J}_{\bar{y},i}(k)}{j} \cdot \exp\left(-\frac{\left(\frac{(x-x_0)^2}{2D_x}\right)^8 - \left(\frac{(y-y_0)^2}{2D_y}\right)}{j}\right) \quad (5)$$

where $C_{x,y}(i, j, k)$ is the concentration field at time step k of a puff released j time steps ago from section i , and $\bar{J}_{\bar{y},i}(k)$ is the amount of pollution \bar{y} emitted by freeway section i at time step k . Note that j represents the degradation of concentration in the puff for every time step since its release on the freeway. In this work, a P_x value of 8 is used. This is done because it represents a flat-topped line source and still models the transition region in the concentration of pollutants on the edges of the freeway section before dropping to zero. Furthermore, three unknown parameters occur: D_x , D_y , and ξ , where D_x and D_y represent the spread of a puff in the direction of the freeway and in the direction perpendicular to the freeway, respectively, and ξ is used as a correction factor on the amplitude of the puff. This factor separates the pollutants emitted by the source into a part at ground level, which is used in this work, and a part flowing into the higher atmosphere.

The total concentration distribution in the vicinity of a pollutant source at a certain time step can be evaluated by the sum of the concentration fields of the individual puffs:

$$C_{\text{total}}(k) = \sum_{i=1}^{N_{\text{seg}}} \sum_{j=1}^{j_{\text{max}}} C_{x,y}(i, j, k). \quad (6)$$

D. Parameter estimation

To establish a relevant model for real-world applications out of the formulation proposed above, the unknown parameters have to be estimated based on real-world data. In this work, it is chosen to use data gathered by a CFD model built in COMSOL, which is a finite-element solver. CFD models are increasingly realistic and a reasonable alternative to field measurements.

A freeway section with a length of 100 meters is modeled in COMSOL. This freeway section is constantly emitting an amount of pollutant gasses, which are dispersed by the wind into the vicinity of the freeway. The wind direction is perpendicular to the freeway and the wind speed is constant for every simulation in COMSOL. The COMSOL model generates a concentration distribution of pollutant gasses downwind of the freeway section. In this concentration distribution a grid of sampling points is placed, where the average concentration is measured for square boxes with sides of 2 meters. Within these boxes the average ground level of emissions is measured.

In order to approach a real-world situation, the concentration distribution of the LSGP model should be equal to the concentration distribution of the COMSOL model. To accomplish this, a grid of s_x by s_y sampling points is placed on the LSGP model at the same location as the grid for the COMSOL model. Subsequently, the parameters are estimated by solving the following minimization problem for a fixed time instant:

$$\min_{D_x, D_y, \xi} \left[\sum_{i=1}^{s_x} \sum_{j=1}^{s_y} (C_{\text{COMSOL}}(i, j) - C(i, j))^2 \right] \quad (7)$$

where the total squared error between the concentration distribution generated by COMSOL and the concentration

Parameter	Error
D_x	0.27%
D_y	1.26%
ξ	2.40%

TABLE I: Numerical error parameters.

distribution determined by the LSGP model is minimized. The optimal parameters can be found with a nonlinear least-squares optimization method. A multi-start approach should be used in order to approach the global minimum of the optimization problem.

The set of optimal parameters differ for different wind speeds. To avoid determining the optimal set of parameters for every wind speed, it is useful to find a correlation between the parameters for different wind speeds. In this study, it was chosen to linearly interpolate between the values of the parameters for a set of wind speeds. To validate the correctness of the resulting continuous function, parameters of various wind speeds outside the basic set were evaluated and plotted on top of the continuous function.

In this particular case, the parameters were estimated for the odd wind speeds from 1 m/s until 21 m/s. The interpolated functions are shown in Figures 3a, 3b, and 3c as the blue lines, for the parameters D_x , D_y , and ξ respectively. The functions for D_x and D_y show some irregularities for higher wind speeds. This can be explained by differences in data for the wind speeds. If the data for those wind speed would be collected several times and the average would be taken, the irregularities would cease to exist. In the same figures, the parameters for the even wind speeds from 2 m/s until 20 m/s are plotted as stars. These parameters are estimated in the same way as above. It can be observed that the stars are on or close to the interpolated line for all three of the parameters.

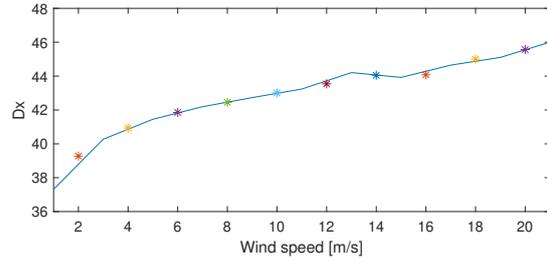
Moreover, a numerical experiment is done to analyze the deviation between the real parameters and the interpolated ones. In Table I, the average percentage error is shown. The errors are small and therefore, it can be concluded that the interpolated line can be used as a continuous function for the parameters of different wind speeds.

III. MODEL VALIDATION

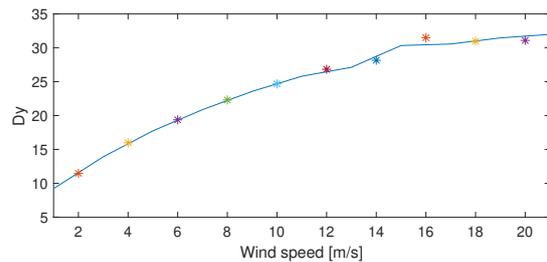
In this section, the LSGP model proposed above will be validated. This is done by comparing the results with a validation data set build in COMSOL and by comparing the new model with existing dispersion models.

A. Validation method

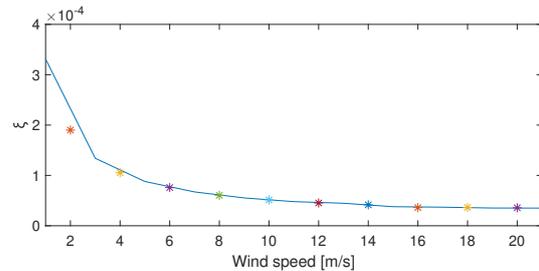
Numerical validation can be done by evaluating the difference between the validation data and the data created by the new model. This then involves the absolute error. However, as the absolute error is a poor way to validate the new dispersion model, a normalized-error method is necessary to properly validate the new dispersion model. In this study,



(a) Validation of the continuous function for D_x .



(b) Validation of the continuous function for D_y .



(c) Validation of the continuous function for ξ .

Fig. 3: Continuous functions for the estimated parameters with the validation points.

the NRMSE is used, which is defined as follows:

$$\text{NRMSE} = \frac{\sqrt{\frac{\sum_{i=1}^{s_x} \sum_{j=1}^{s_y} (C_{\text{COMSOL}}(i,j) - C_{\text{total}}(i,j))^2}{s_x \cdot s_y}}}{C_{\text{max}} - C_{\text{min}}} \quad (8)$$

This method applies the quadratic mean of the difference between the validation data and the new dispersion model. Thereafter, it is normalized by the range of the data, the difference between the minimum and maximum value of the data set. If the NRMSE value is 0, this implies a perfect fit between the data and the model. For a value of 1, there is no correlation between the data and the model.

B. Validation

In order to validate a data-based model, a new set of data, the validation data, is needed. For this study, the validation data is gained by a new evaluation in COMSOL. In order to prevent that the model is fitted only on the sampling points used to estimate the parameters and not on the entire pollutant cloud, for this evaluation the sampling points are placed in different locations. In Table II, the NRMSE is shown for several wind speeds. The new dispersion model has an error of about 0.05, which can be seen as a deviation

Wind speed [m/s]	Absolute error	NRMSE
8	0.000354	0.0526
9	0.000296	0.0541
10	0.000254	0.0554
11	0.000219	0.0564
12	0.000197	0.0582

TABLE II: Numerical validation results.

Wind speed [m/s]	NRMSE		
	LSGP	point source Gaussian	Zegeye
10	0.055	0.138	0.192

TABLE III: Numerical error of three dispersion models to the validation data.

of 5%. The normalized error is slightly increasing for higher wind speeds. This can be explained by the fact that then there are fewer puffs, perpendicular to the freeway, to model the same distance. For example, with a wind speed of 5 m/s, the model consists 200 puffs to cover 1 km, while with a wind speed of 20 m/s the model only consists 50 puffs in 1 km. This can be solved by a finer discretization in time for higher wind speeds, although that will create a higher computational burden.

C. Comparison with other models

The objective of the new model was to create a time-dependent dispersion model with a low computation time and with a line source. To accomplish this, the point source Gaussian puff model was modified. In this section, it will be tested whether the new modified line source model has a reduced error with the validation data compared to the point source Gaussian puff model. The expanding grid-based dispersion model proposed by Zegeye et al. [6] is also included in the comparison.

The numerical error of the various models can be found in Table III. The line source model has the smallest normalized error of about 5% compared to a 14% and a 19% error for the point source Gaussian puff model and Zegeye's model respectively. Therefore, the LSGP model performs the best of the three models. Moreover, our simulations have shown that the computational complexity of the new model is comparable to that of Zegeye's model and the Gaussian puff model, and significant lower than the computational complexity of a CFD model or a Lagrangian model.

IV. CASE STUDY: FREEWAY CONTROL

In order to find out whether the new dispersion model can decrease the amount of pollutant gasses in target areas, a case study will be performed in this section. This case study involves a freeway network that will be controlled with a model predictive control scheme.

The freeway network used by Hegyi et al. in [15] is used to evaluate the performance of the new model. This particular freeway network is chosen because is both simple and proven to be realistic. The particular layout of this case study can be controlled sufficiently and is therefore useful to determine

Parameter	Value	unit	Parameter	Value	unit
T	10	[s]	v_{free}	102	[km/h]
τ	18	[s]	θ	60	[km ² /h]
δ	0.0122	[-]	κ	40	[veh/lane/km]
a_m	1.867	[-]	ρ_{max}	180	[veh/lane/km]
α	0.1	[-]	ρ_{crit}	33.5	[veh/lane/km]

TABLE IV: Parameters used in the METANET model.

the performance of the new dispersion model. A schematic overview of the case study is given in Figure 4. This freeway network consists of two main lanes ($\lambda=2$) that are 6 km long and that are divided into 6 segments with a length (L) of 1 km. The main lanes have an origin in segment 1 where the mainstream demand is coming in. A second origin is located at the 5th segment, where an on-ramp is located. The flow on this freeway network can be influenced by two variable speed limits (VSLs) and a metered on-ramp. The VSLs are located at the 3th and 4th segment. The target area, where the dispersed emission levels will be minimized in this case study, is a 0.04 km² square 500 m away from the freeway. The traffic on the freeway network is simulated with the METANET model and the amount of emitted gasses by the vehicles in the network is simulated with the VT-macro model; for a detailed description on these models the reader is referred to [8], [9]. Lastly, the dispersion of the emissions produced by the vehicles on the freeway network is modeled with the new LSGP model.

A. System parameters

The traffic network described above will be simulated for 2.5 hours with a sampling time (T) of 10 [s]. The system parameters used for the METANET model are the same to those used by Hegyi et al. in [15]. These parameters can be seen in Table IV. The demand profile used for the origins can be seen in Figure 5. Furthermore, the LSGP model uses the total amount of emitted gasses for every segment and for every time step produced by the VT-macro model. In this case study the width of the puffs is chosen to be the same length as the length of a freeway segment: 1 km; in this way every freeway segment is represented by one puff. To reduce the computational complexity, the puffs are released every 10 seconds. For these puffs, with a new discretization compared to Section III, new parameter values for D_x , D_y , and ξ are estimated. In order to calculate where the puff center is located at every time step, the model uses the wind speed and wind direction. In real-world scenarios, the wind is constantly changing. In order to analyze whether the new model is compatible with these changes, the wind speed and wind direction are changing over time in this case study and their profiles can be seen in Figure 6.

B. Objective function

Model Predictive Control (MPC) optimizes an objective function to find the optimal control input sequence. The objective function to be minimized for this case study is

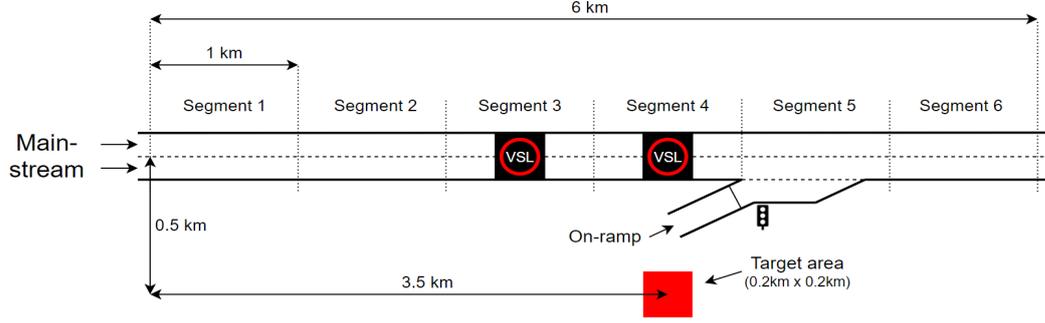


Fig. 4: Schematic overview of the case study.

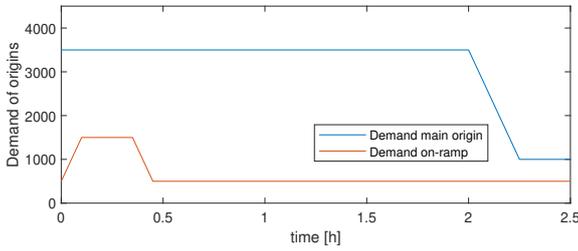


Fig. 5: Demand profile used in the case study.

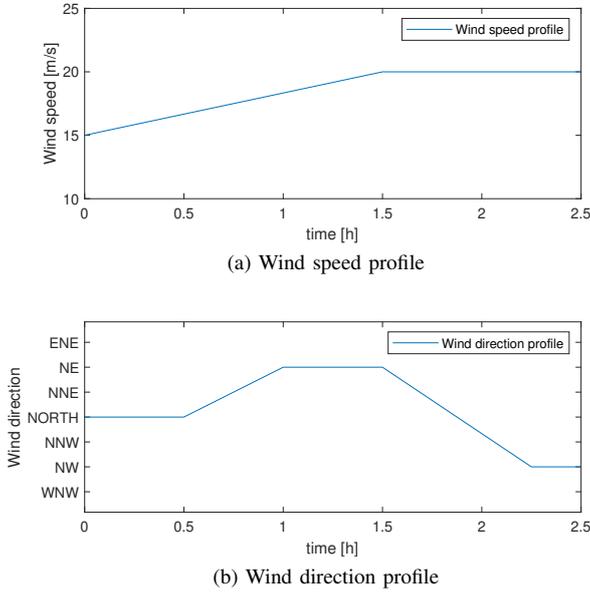


Fig. 6: Wind profile used in the case study.

defined as

$$J(k_c) = \frac{\zeta_1}{\text{TTS}_n} \left[T_c \sum_{k=k_c}^{k+N_p} \left(\sum_i \rho_i(k) \lambda L + \sum_o w_o(k) \right) \right] + \frac{\zeta_2}{\text{TEG}_n} \left[T_c \sum_{k=k_c}^{k+N_p} \bar{J}_{\text{CO}_2, i}(k) \right]$$

$$+ \frac{\zeta_3}{\text{TDE}_n} \left[T_c \sum_{k=k_c}^{k+N_p} C_{\text{target}}(k) \right] + \zeta_4 \left[(u_r(k) - u_r(k-1))^2 \right] + \sum_{i \in I_{\text{VSL}}} \left[\left(\frac{u_{\text{VSL}}(k) - u_{\text{VSL}}(k-1)}{v_{\text{free}}} \right)^2 \right] + \zeta_5 \left[\max(w_r(k) - w_{\text{max}}, 0) \right]. \quad (9)$$

The first three terms in this objective function are the model performance criteria. The first term will minimize the Total Time Spent (TTS) of all the vehicles in the traffic network, the second term will minimize the Total amount of Emitted Gasses (TEG) by all the vehicles in the network, and the third term will ensure that the Total amount of Dispersed Emissions (TDE) is minimized in the target area. These three model performance criteria lead to a multi-criteria optimization. In this case, the TTS partial objective function is conflicting with the TEG and TDE partial objective functions, and therefore, trade-offs have to be made to find an optimal control input sequence. This is done by the weighted-sum strategy. In order to put more weight on one of the three performance criteria, ζ_1 , ζ_2 , and ζ_3 are used as weighting factors. For example, if $\zeta_1 = 1$ and $\zeta_2 = \zeta_3 = 0$ only the total time spent on the traffic network is minimized and the sustainable objectives are neglected. Moreover, TTS_n , TEG_n , and TDE_n are normalization factors used to create an objective function where all the separate objective functions are weighted identically. In this case study, the no-control values of the TTS, TEG, and TDE are used as the normalization factor. The last two terms in the objective function are penalty terms. Large changes in control measures can result in an unsafe or uncomfortable driving experience. Therefore, the first of the two penalty terms will ensure that the fluctuation over time of the traffic control inputs will be small. The second penalty term is used to prevent the queue in front of the on-ramp from becoming too long. Long queues can result in spill-back to street intersections outside of the freeway network, which is undesirable. In this case study the maximum queue length (w_{max}) is set to 150 vehicles. For the penalty terms, also a

weighting factor is used to prevent that the penalty terms are too strict or too loose. For both of the penalty terms three weighting factor: 10%, 1%, and 0.1%, have been tested to find the most appropriate one. With a 10% weighting factor, the control inputs stayed on one value for the entire simulation and therefore that value was too strict, while with a 0.1% weighting factor, there was no penalty on the fluctuation at all. The best results followed from a weighting factor of 1%, and therefore, in the rest of the case study we use $\zeta_4 = \zeta_5 = 0.01$.

C. Control parameters

An MPC scheme aims to find the optimal input control sequence for every controller time step, where for this case study we select a value of 60 [s] for the controller sampling time T_c . The control horizon N_c and the prediction horizon N_p are also same as those used by Hegyi et al. in [15]: $N_c = 5$ and $N_p = 7$. The control input for the VSLs can vary between 20 km/h and 120 km/h and for the on-ramp between 0 and 1. The optimal control input sequence is determined by solving the constrained optimization problem defined by (9), the model equations, and the bounds on the VSLs and the ramp metering rates, in Matlab with an active-set algorithm for nonlinear, non-convex constrained optimization. The active-set algorithm was chosen after a comparison with the SQP and the interior-point algorithm. The active-set algorithm resulted in a better result within a lower computation time. For every controller time step, a multi-start approach with 40 runs has been applied to prevent getting stuck in a local minimum.

D. CPU specifications

All the simulations are done on an HP ZBook Studio G5. This device contains an Intel Core i7-8750H processor with 16 GB of installed RAM. The R2020b version of MATLAB is used for the simulations, and all optimization problems are solved with the Matlab Optimization Toolbox.

E. Case study results

In the case study a freeway traffic network is simulated and controlled. The aim of the simulation is to minimize a multi-criteria function with conflicting performance criteria. In order to find an optimal situation, five scenarios are simulated. In Table V, the scenarios are indicated schematically. The first case is a no-control scenario to create a benchmark, the following three scenarios aim to minimize the separate partial objectives individually, and the last scenario has the objective to find a minimum for all of the objectives at once. The simulation of the distinctive scenarios led to various results; these are shown in Table VI. From the results, it can be seen that when a partial objective function is minimized individually, the performance for this partial objective is clearly improved compared to the no-control case. Therefore, the three models and the MPC scheme are working as desired. On the downside, an improvement in the performance for a partial objective also results in the decreased performance for a different partial objective.

Scenario	Objective	Weighting factors		
		ζ_1	ζ_2	ζ_3
S ₁	No control	0	0	0
S ₂	TTS	1	0	0
S ₃	TEG	0	1	0
S ₄	TDE	0	0	1
S ₅	TTS, TEG & TDE	1/3	1/3	1/3

TABLE V: Different scenarios simulated in this case study.

Scenario	Performance criteria		
	TTS [h]	TEG [kg]	TDE [kg]
S ₁	1457	92.5	0.0233
S ₂	1220 (-16.7%)	93.5 (+1.01%)	0.0239 (+2.58%)
S ₃	2978 (+104.4%)	80.6 (-12.85%)	0.0203 (-12.88%)
S ₄	2756 (+89.1%)	80.9 (-12.56%)	0.0199 (-14.59%)
S ₅	2685 (+84.3%)	81.2 (-12.20%)	0.0199 (-14.59%)

TABLE VI: Results of the different scenarios of the case study.

Especially in scenarios 3 and 4, where the sustainability objectives are minimized, the conflicting objective, the total time spent, increases substantially. In the case where all the objectives are weighted with the same amount, the results show that the sustainability objectives are minimized and the TTS objective is neglected.

In some cases, it can be useful to have an optimal TTS in the freeway traffic network, e.g., in rush hours. In this case, the TTS objective can be weighted stronger. On the other hand, when the traffic is less crowded and there is smog, it can be useful to protect certain target areas from high concentrations of pollutant gasses. In that case, the new dispersion model can be used to minimize the pollution in on-line traffic control. In this way, the model is useful and the weighting factors should be selected in a suitable way for specific situations.

V. CONCLUSIONS

We have developed a new freeway emission dispersion model with an accurate representation of reality and a low computational complexity. This was done by extending the Gaussian puff model with a line source, forming the new LSGP model. The model has been validated on a validation data set generated in COMSOL. From the results, it followed that the model has a normalized error of about 5%. This can be seen as a very accurate representation of reality, and therefore, the model is performing as we aimed for. Moreover, the computational complexity is kept within reasonable limits, making it applicable in on-line traffic control.

In order to show that the new model is a contribution to the research already done in the field of dispersion modeling, the new model has been compared with other dispersion models. The model has been compared with the Gaussian puff model, of which the new model is an extension, and with a dispersion model Zegeye et al. [6] proposed especially for the dispersion of freeway emissions. From the results, it can be concluded that the new dispersion model has a smaller error, of 5%, compared to a 14% and a 19% error of the point source Gaussian puff model and Zegeye's

model respectively, with a comparable computation time. However, if the computational burden of the new model is compared with CFD models or Lagrangian models, where the computation time can reach up to several hours, the new model has a significantly lower computation time.

In order to validate whether the model is applicable in on-line traffic control, the LSGP model has been used in a case study to minimize the total dispersed emissions toward a target area. In this simulation, the model performed very well and was able to significantly decrease the amount of emissions in the target area compared to a no-control case.

Performing a field study to collect dispersion data of a real-life scenario and fitting the LSGP model on that data set is a topic for future research. Furthermore, it would be interesting to model the dispersion of traffic emissions in an urban canyon instead of an open freeway environment. Lastly, as the effects of turbulence created by the traffic flow is not taken into account in this model, in future research those effects could be considered.

REFERENCES

- [1] D. Brugge, J. L. Durant, and C. Rioux. Near-highway pollutants in motor vehicle exhaust: A review of epidemiologic evidence of cardiac and pulmonary health risks. *Environmental Health*, 6, 2007.
- [2] M. Papageorgiou, C. Diakaki, V. Dinopoulou, A. Kotsialos, and Y. Wang. Review of road traffic control strategies. *Proceedings of the IEEE*, 91(12):2043–2067, 2003.
- [3] C. Pasquale, S. Sacone, S. Siri, A. Ferrara. Traffic control for freeway networks with sustainability-related objectives: Review and future challenges. *Annual Reviews in Control*, Volume 48:312-324, 2019.
- [4] A. Hegyi, B. De Schutter, and H. Hellendoorn. Model predictive control for optimal coordination of ramp metering and variable speed limits. *Transportation Research Part C: Emerging Technologies*, 13(3):185–209, 2005.
- [5] S. K. Zegeye, B. De Schutter, H. Hellendoorn, and E. Breunese. Reduction of travel times and traffic emissions using model predictive control. In *2009 American Control Conference*, pages 5392–5397, 2009.
- [6] S. K. Zegeye, B. De Schutter, J. Hellendoorn, and E. Breunese. Reduction of areawide emissions using an efficient model-based traffic control strategy. *2011 IEEE Forum on Integrated and Sustainable Transportation Systems, FISTS 2011*, 2011.
- [7] A. Csikós, I. Varga, and K. Hangos. Modeling of the dispersion of motorway traffic emission for control purposes. *Transportation Research Part C: Emerging Technologies*, 58:598-616, 2015.
- [8] A. Messmer and M. Papageorgiou. METANET: a macroscopic simulation program for motorway networks. *Traffic Engineering & Control*, 31:466–470, 1990.
- [9] S.K. Zegeye, B. De Schutter, J. Hellendoorn, E.A. Breunese, and A. Hegyi. Integrated macroscopic traffic flow, emission, and fuel consumption model for control purposes. *Transportation Research Part C: Emerging Technologies*, 31:158–171, 2013.
- [10] Y. Tominaga and T. Stathopoulos. CFD simulation of near-field pollutant dispersion in the urban environment: A review of current modeling techniques. *Atmospheric Environment*, 79:716–730, 2013.
- [11] M. Rotach, W. Mathias. Simulation of urban-scale dispersion using a Lagrangian stochastic dispersion model. *Boundary-Layer Meteorology*, 3:379-410, 2001.
- [12] D. Heist, V. Isakov, S. Perry, M. Snyder, A. Venkatram, C. Hood, J. Stocker, D. Carruthers, S. Arunachalam, and R. C. Owen. Estimating near-road pollutant dispersion: A model inter-comparison. *Transportation Research Part D: Transport and Environment*, 25:93–105, 2013.
- [13] U. Piomelli. Large-eddy simulation: achievements and challenges. *Progress in Aerospace Sciences*, 35:335-362, 1999.
- [14] Y. Tominaga and T. Stathopoulos. CFD modeling of pollution dispersion in a street canyon: Comparison between LES and RANS. *Journal of Wind Engineering and Industrial Aerodynamics*, 4:340-348, 2011.
- [15] A. Hegyi, B. De Schutter, and H. Hellendoorn. Model predictive control for optimal coordination of ramp metering and variable speed limits. *Transportation Research Part C: Emerging Technologies*, 13(3):185–209, 2005.