Delft University of Technology

Delft Center for Systems and Control

Technical report 22-011

Timetable scheduling for passenger-centric urban rail networks: Model predictive control based on a novel absorption model*

X. Liu, A. Dabiri, and B. De Schutter

If you want to cite this report, please use the following reference instead:
X. Liu, A. Dabiri, and B. De Schutter, "Timetable scheduling for passenger-centric urban rail networks: Model predictive control based on a novel absorption model," *Proceedings of the 2022 IEEE Conference on Control Technology and Applications (CCTA)*, Trieste, Italy, pp. 1147–1152, Aug. 2022. doi:10.1109/CCTA49430.2022.9966045

Delft Center for Systems and Control
Delft University of Technology
Mekelweg 2, 2628 CD Delft
The Netherlands
phone: +31-15-278.24.73 (secretary)
URL: https://www.dcsc.tudelft.nl

*This report can also be downloaded via https://pub.deschutter.info/abs/22_011.html

Timetable Scheduling for Passenger-Centric Urban Rail Networks: Model Predictive Control based on a Novel Absorption Model

Xiaoyu Liu, Azita Dabiri, and Bart De Schutter, *Fellow, IEEE*

***Abstract*— Timetable scheduling plays a key role in daily operations of urban rail transit systems, as it determines the quality of service provided to passengers. In order to develop efficient timetable scheduling methods, it is necessary to develop a proper model to integrate timetable-related and passenger-related factors in urban rail network efficiently. In this paper, a novel passenger absorption model for passenger-centric urban rail networks is established. The model explicitly integrates time-varying passenger origin-destination demands and the departure frequency of each line for real-time timetable scheduling. Then, a model predictive control (MPC) method for the timetable scheduling problem is proposed based on the developed model. The resulting MPC optimization problem can be formulated as a mixed-integer programming (MILP) problem, which can be solved efficiently by using the existing MILP solvers. The effectiveness of the absorption model and the corresponding MILP-based MPC approach is illustrated through the case study based on two Beijing subway lines.**

I. INTRODUCTION

Urban rail transit has become one of the most important public transportation modes in big cities (e.g., Beijing, London, New York) because of its safety, stability, high efficiency, and sustainability. The main goal of the urban rail transit systems is to provide satisfactory service to passengers. Timetable scheduling is regarded as an effective way to improve the quality of service and to reduce the operation costs under the infrastructure limitations. With the rapidly growing passenger demands in recent years, it has become increasingly challenging to generate a high-quality passenger-centric timetable where both passenger-related factors and operator-related factors in urban rail networks are jointly taken into account.

Many studies in railway timetable scheduling focus on optimizing arrival and departure times of trains at each platform in the railway network, with the aim of minimizing objectives such as passenger travel times [1], passenger waiting times [2], station crowdedness [3], deviation from the planned timetable [4], or a combination of them. In the railway network, passenger demands are often represented as several time-varying origin-destination (OD) pairs, which can be obtained according to entering and exiting flows of stations or historical data of automatic fare collection systems. Passenger OD demands would largely influence the performance of a timetable. An efficient passenger-centric timetable should properly take passenger OD demands into account [5], [3], [6].

The departure frequency (i.e., how many trains depart from a platform during a certain period) significantly influences the quality of service, as it determines an upper bound on the transport capacity. As passenger demands are time-varying, the departure frequency in railway networks varies throughout a day, e.g., the departure frequency of metro networks in peak hours is usually higher than in off-peak hours so as to transport more passengers. Passengers prefer high-frequency lines so that they have a better chance of boarding trains without large waiting times. On the other hand, higher departure frequencies will lead to higher operational costs. In this context, optimizing departure frequencies for each platform is more important than determining specific departure and arrival times for attending passenger demands and improving the quality of service of railway transportation systems [7]. Once the departure frequency has been determined, the detailed departure and arrival times can be determined at the lower level with a more detailed train operation model and/or passenger flow model, which is however not in the scope of this paper.

Most research related to optimizing transit frequencies is conducted in the context of urban transit networks, e.g., bus networks [8], [9]. However, the urban rail transit system has its own characteristics, i.e., train braking distances are relatively long, and trains operate with strict constraints of signaling systems. An efficient departure frequency control method is required for urban rail networks to meet the time-varying passenger demands while considering operation costs and infrastructure constraints. De-Los-Santos et al. [10] used an exact algorithm and a heuristic approach to design line frequencies and train capacities to maximize the profit of metro networks. Canca et al. [6] developed a mixed-integer nonlinear programming approach to optimize line frequencies and train capacities in dense railway rapid transit networks. However, these studies do not consider the detailed number of passengers accurately, leaving an open gap of further improving the passenger satisfaction.

Formulating the timetable scheduling problem generally leads to a constrained control problem. Model predictive control (MPC) is a well recognized effective real-time method to control constrained systems [11], [12]. In [13], a real-time timetable scheduling approach was developed based on the switching max-plus-linear models to minimize the operational costs and train delays. An MPC method was designed to deal with train rescheduling problems in the complex station areas in [14]. In [15], a state space model

*This work is supported by the China Scholarship Council under Grant 202007090003.

The authors are with the Delft Center for Systems and Control, Delft University of Technology, 2628CD Delft, The Netherlands x.liu-20@tudelft.nl, a.dabiri@tudelft.nl, b.deschutter@tudelft.nl

was developed to describe the passenger load of trains and the evolution of departure time, and an MPC approach is then proposed. By adjusting timetables and passenger loads, the headway and timetable deviations of a metro line is optimized. In [16], MPC was also used for railway disruptions, and the MPC optimization problem was transformed into an MILP problem to reduce the computational complexity. The successful application of MPC in the above studies has inspired us to develop an efficient MPC approach for timetable scheduling of urban rail transit networks. This also implies the development of a novel model is crucial for the application of MPC approach in real-time timetable scheduling.

The main contributions of this paper are as follows:

1) A novel passenger absorption model that can explicitly handle time-varying passenger origin-destination demands in urban rail networks is proposed.
2) In contrast to most of the existing models, the absorption model deals with passenger flows by involving departure frequencies. So the transport capacity of the network can be optimized while keeping a balanced trade-off between model accuracy and computational efficiency.
3) An MPC-based approach is developed for the timetable scheduling problem based on the proposed model where the MPC optimization problem can be transformed into a mixed-integer linear programming (MILP) problem, which can be solved efficiently by existing MILP solvers.

The remainder of the paper is organized as follows. In Section II, a novel passenger absorption model is proposed for urban rail network. In Section III, a model predictive control scheme is used for determining the number of trains departing at each platform, and the MPC optimization problem is transformed into MILP problem. Section IV provides a case study based on two Beijing subway lines. Finally, Section V summarizes the paper and provides recommendations for future research.

II. Passenger Absorption Model

Passenger demands are generally represented by time-varying origin-destination matrices. Incorporating time-varying passenger demands is a challenging task in timetable scheduling problems, as it would greatly increase the computational burden. Passenger demands usually change gradually throughout the day without sudden changes. Therefore, we discretize the planning time span into several periods of length T, where passenger demands in each period are assumed to be constant. The number of trains departing at each platform during each period is the decision variable in this model. Fig. 1 provides a typical time-varying passenger arrival rate and the approximate profile of the arrival rate for the passenger absorption model. In real life, the passenger flow data are typically collected periodically, e.g., the total flows over each half hour are recorded and stored in Beijing Subway. Therefore, the piecewise constant approximation is consistent with the utilization of practical passenger flow data.

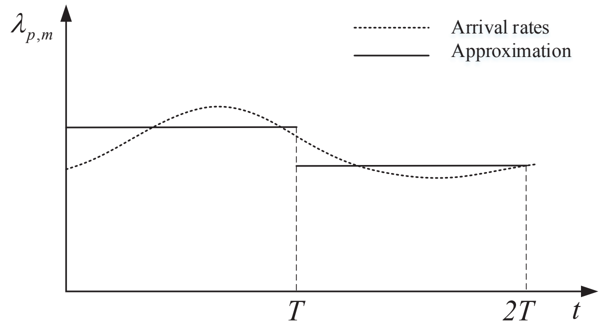

Fig. 1. Illustration of piecewise constant approximation of passenger arrival rates.

In urban rail transit systems, the line indicates the route of one specific type of train services with the same stopping platforms/stations. Generally, different lines use different tracks and platforms in urban rail transit systems.

In the passenger absorption model, the number of passengers remaining at a platform at each period can be updated as follows:

$$\begin{aligned} n_{p,m}(k+1) = & n_{p,m}(k) + \lambda_{p,m}(k)T \\ & + n_{p,m}^{\text{arrive,trans}}(k) - n_{p,m}^{\text{absorb}}(k), \end{aligned} \quad (1)$$

where $n_{p,m}(k)$ is the number of passengers waiting at platform p with station m as their destination at the beginning of period k; $\lambda_{p,m}(k)$ denotes the passenger arrival rate at platform p with station m as their destination during period k; $n_{p,m}^{\text{absorb}}(k)$ represents the number of passengers at platform p with station m as their destination absorbed by trains during period k, and $n_{p,m}^{\text{arrive,trans}}(k)$ is the number of transferring passengers arriving at platform p during period k with station m as their destination.

The number of passengers $n_p^{\text{wait}}(k)$ waiting at platform p for boarding trains during period k is

$$n_p^{\text{wait}}(k) = n_p(k) + \lambda_p(k)T + n_p^{\text{arrive,trans}}(k), \quad (2)$$

where $n_p(k) = \sum_{m \in S} n_{p,m}(k)$, $\lambda_p(k) = \sum_{m \in S} \lambda_{p,m}(k)$, and $n_p^{\text{arrive,trans}}(k) = \sum_{m \in S} n_{p,m}^{\text{arrive,trans}}(k)$, with S the set of all stations in the urban rail network.

Then, the total number of passengers $n_p^{\text{absorb}}(k)$ absorbed by trains at platform p during period k can be calculated by

$$n_p^{\text{absorb}}(k) = \min\left(n_p^{\text{wait}}(k), C_p(k)\right), \quad (3)$$

where $C_p(k)$ is the total remaining capacity of trains visiting platform p during period k.

The number of passengers $n_{p,m}^{\text{absorb}}(k)$ absorbed by trains at platform p with station m as their destination during period k can be approximated by

$$n_{p,m}^{\text{absorb}}(k) = \frac{\lambda_{p,m}(k)}{\lambda_p(k)} n_p^{\text{absorb}}(k), \quad (4)$$

which means the proportion of absorbed passengers with different destinations is assumed to be equal to the proportion of passengers arriving in the current period. As $\lambda_{p,m}(k)$ and $\lambda_p(k)$ are known constants that can calculated based on historical data, (4) corresponds to a linear relation between $n_{p,m}^{\text{absorb}}(k)$ and $n_p^{\text{absorb}}(k)$.

The total capacity provided by trains visiting platform p during period k can be computed by

$$C_p(k) = f_p(k) \cdot C_{\max} - \sum_{m \in S} n_{p,m}^{\text{train}}(k) + \sum_{m \in S} n_{p,m}^{\text{alight}}(k), \tag{5}$$

where $f_p(k)$ represents the number of trains visiting platform p during period k, $C_{\max}$ is the maximum capacity of a train, $n_{p,m}^{\text{train}}(k)$ is the number of passengers on board of trains arriving at platform p with station m as their destination during period k, and $n_{p,m}^{\text{alight}}(k)$ denotes the number of passengers alighting from platform p with destination m during period k.

In this paper, all trains are assumed to depart from the starting platform of a line, visit every platform of the line, and finally arrive at the terminal platform, i.e., short-turn, shunting, and stop-skipping are not considered. In this context, the number of trains departing at each platform along a line is connected to the number of trains departing at the first platform of the line. As there are several trains arriving at a platform during period k, we can define ψ_p as the average time for a train departing from the first platform of a line to arrive at the current platform[1]. Then, we define

$$\delta_p = \text{floor}\left\{\frac{\psi_p}{T}\right\}, \tag{6}$$

$$\gamma_p = \text{rem}\{\psi_p, T\}, \tag{7}$$

where floor$\{x\}$ denotes the largest integer smaller than or equal to x, and rem$\{\psi_p, T\}$ refers to the remainder of the division of ψ_p by T. Hence, we have

$$\psi_p = \delta_p T + \gamma_p, \quad 0 \le \gamma_p < T. \tag{8}$$

Therefore, $f_p(k)$, which is the number of trains visiting platform p during period k can be approximated by

$$f_p(k) = \frac{T - \gamma_p}{T} f_{\text{line}(p)}(k - \delta_p) + \frac{\gamma_p}{T} f_{\text{line}(p)}(k - \delta_p - 1), \tag{9}$$

where line(p) defines the starting platform of the line corresponding to platform p.

Due to safety requirements, the number of trains arriving at platform p during period k should also satisfy

$$f_p(k)\left(h_p^{\min} + \tau_p^{\min}\right) \le T, \tag{10}$$

where $h_p^{\min}$ represents the minimum headway between two trains at platform p, and $\tau_p^{\min}$ is the minimum dwell time at platform p.

[1] ψ_p can be determined based on the historical data of the timetable.

The number of passengers $n_{p,m}^{\text{train}}(k)$ on board of trains when trains arrive at platform p during period k can be calculated by

$$n_{p,m}^{\text{train}}(k) = \frac{T - \bar{r}_{\text{p}^{\text{pla}}(p)}}{T} n_{\text{p}^{\text{pla}}(p),m}^{\text{depart}}(k) + \frac{\bar{r}_{\text{p}^{\text{pla}}(p)}}{T} n_{\text{p}^{\text{pla}}(p),m}^{\text{depart}}(k-1), \tag{11}$$

where $n_{\text{p}^{\text{pla}}(p),m}^{\text{depart}}(k)$ denotes the number of passengers departing from the predecessor platform of platform p with station m as their destination during period k. As several trains arrive at platform p during period k, $\bar{r}_{\text{p}^{\text{pla}}(p)}$ represents the average running time of trains from the predecessor platform, for the line to which platform p belongs, to platform p. Since we aim at determining the number of trains over a relatively long time window, we assume that $T \gg \bar{r}_{\text{p}^{\text{pla}}(p)}$.

The number of passengers $n_{p,q,m}^{\text{trans}}(k)$, transferring from platform p to platform q with station m as their destination during period k, can be computed by

$$n_{p,q,m}^{\text{trans}}(k) = \beta_{p,q,m}^{\text{train}}(k)\, n_{p,m}^{\text{train}}(k), \forall q \in \text{plat}(p)/\{p\}, \tag{12}$$

where plat(p) denotes the set of platforms at the same station as platform p, $\beta_{p,q,m}^{\text{train}}(k)$ is the splitting rate of passengers with station m as their destination transferring from platform p to $q \in \text{plat}(p)$ during period k, with

$$\sum_{q \in \text{plat}(p)} \beta_{p,q,m}^{\text{train}}(k) = 1. \tag{13}$$

Then, the number of passengers $n_{p,m}^{\text{alight}}(k)$ alighting from trains at platform p with destination m during period k can be calculated by

$$n_{p,m}^{\text{alight}}(k) = \begin{cases} \sum\limits_{q \in \text{plat}(p)/\{p\}} n_{p,q,m}^{\text{trans}}(k), & \text{if } m \in S/\{\text{sta}(p)\}, \\ n_{p,m}^{\text{train}}(k), & \text{if } m = \text{sta}(p), \end{cases} \tag{14}$$

where sta$(\cdot)$ defines a mapping between a platform and its corresponding station.

Therefore, the number of passengers $n_{p,m}^{\text{depart}}(k)$ departing from platform p with station m as their destination during period k can be calculated by

$$n_{p,m}^{\text{depart}}(k) = n_{p,m}^{\text{train}}(k) - n_{p,m}^{\text{alight}}(k) + n_{p,m}^{\text{absorb}}(k). \tag{15}$$

The arrival rate of passengers transferring from the other platforms of the station and arriving at platform p during period k is

$$n_{p,m}^{\text{arrive,trans}}(k) = \sum_{q \in \text{plat}(p)/\{p\}} \left(\frac{T - \theta_{q,p}^{\text{trans}}}{T} n_{q,p,m}^{\text{trans}}(k) + \frac{\theta_{q,p}^{\text{trans}}}{T} n_{q,p,m}^{\text{trans}}(k-1) \right), \tag{16}$$

where $\theta_{q,p}^{\text{trans}}$ represents the average passenger walking time for passengers from platform q to platform p.

III. Model Predictive Control for Real-time Timetable Scheduling

Based on the passenger absorption model proposed in Section II, the total number of passenger $w_p(k)$ at platform p who cannot board the train during period k is

$$w_p(k) = n_p^{\text{wait}}(k) - n_p^{\text{absorb}}(k), \tag{17}$$

where P is the set of all platforms in the considered urban rail network. Then, the total passenger travel time in the network during period k can be described by

$$J^{\text{pass}}(k) = \sum_{p\in P} \left(w_p(k)T + n_p^{\text{depart}}(k)\bar{r}_p + n_p^{\text{arrive,trans}}(k)\theta_{q,p}^{\text{trans}} \right). \tag{18}$$

It is obvious that $J^{\text{pass}}(k)$ can be minimized by using as many trains as available running with the minimum headway; however, the operational cost of using too many trains is usually very high. In real life, adding trains would lead to additional cost, e.g., energy consumption, crew scheduling costs, and maintenance costs. Hence, a penalty term is included to make a trade-off between passenger satisfaction and operational costs. Therefore, the objective function of the timetable scheduling problem is

$$J = \sum_{k=k_0}^{k_0+N-1} \left(J^{\text{pass}}(k) + \xi \sum_{p\in P} f_p(k)E \right), \tag{19}$$

where ξ is a weight balancing the two objectives, E represents the average operational cost for the departure of one train at a platform, and N is the number of periods in the planning window.

Therefore, the optimization problem for real-time timetable scheduling based on the proposed passenger absorption model at period k_0 is

$$\begin{cases} \min\limits_{\mathbf{f}(k_0)} \ J = \sum\limits_{k=k_0}^{k_0+N-1} \left(J^{\text{pass}}(k) + \xi \sum\limits_{p\in P} f_p(k)E \right) \\ \text{s.t.} \quad (1)-(5),\ (9)-(12),\ (14)-(17), \end{cases} \tag{20}$$

where $\mathbf{f}(k_0)$ collects the variables $f_p(k_0+k)$ of all the platforms for $k = 0, 1, \cdots, N-1$, i.e.,

$$\mathbf{f}(k_0) = [f_p(k_0+1), \cdots, f_p(k_0+k), \cdots, f_p(k_0+N-1)]^{\text{T}}. \tag{21}$$

Solving optimization problem (20) leads to a sequence of decision variables for period k_0 to k_0+N-1, and in MPC only the first decision variable for $k = k_0$ is implemented, and at the next period the prediction window is shifted for one period, resulting in a new optimization problem.

The MPC optimization problem (20) is a nonlinear nonconvex optimization problem. Sequential quadratic programming (SQP) algorithm is typically used in many fields to solve nonlinear nonconvex optimization problems [17], [1]. However, SQP might result in a local optimal solution for problem (20), and should be implemented with multi-start SQP to improve the solution quality.

Another approach is to transform the nonlinear optimization problem (20) into a mixed-integer linear programming (MILP) problem by using the method in [18], [19]. Define a auxiliary binary variable $\rho_{k,p}$ and an auxiliary variable $\mu_{k,p}$ and let

$$\mu_{k,p} = n_p^{\text{wait}}(k) - C_p(k). \tag{22}$$

Then, the statement $\mu_{k,p} \leq 0 \Leftrightarrow \rho_{k,p} = 1$ is true if and only if

$$\begin{cases} \mu_{k,p} \leq M_{k,p}(1-\rho_{k,p}), \\ \mu_{k,p} \geq \varepsilon + (m_{k,p} - \varepsilon)\rho_{k,p}, \end{cases} \tag{23}$$

with $m_{k,p}$ and $M_{k,p}$ are the minimum value and the maximum value of $f_{k,p}$, respectively, and ε is a small positive number. Therefore, (3) can be rewritten as

$$n_p^{\text{absorb}}(k) = \rho_{k,p} n_p^{\text{wait}}(k) + (1-\rho_{k,p}) C_p(k). \tag{24}$$

It is worth noting that there still some nonlinear terms in (24), i.e., $\rho_{k,p} \cdot n_p^{\text{wait}}(k)$ and $\rho_{k,p} \cdot C_p(k)$. The products of binary variables and real valued variables can also be transformed to linear terms by using the method in [18], [19]. Define an auxiliary real-valued variable $z_{k,p} = \rho_{k,p} \cdot n_p^{\text{wait}}(k)$. Therefore, $z_{k,p} = \rho_{k,p} \cdot n_p^{\text{wait}}(k)$ is equivalent to

$$\begin{cases} z_{k,p} \leq M_n\ \rho_{k,p}, \\ z_{k,p} \geq m_n\ \rho_{k,p}, \\ z_{k,p} \leq n_p^{\text{wait}}(k) - m_n(1-\rho_{k,p}), \\ z_{k,p} \geq n_p^{\text{wait}}(k) - M_n(1-\rho_{k,p}), \end{cases} \tag{25}$$

where m_n and M_n are the minimum and maximum values of $n_p^{\text{wait}}(k)$, respectively. Similarly, $\rho_{k,p} \cdot C_p(k)$ can also be transformed into linear inequalities [18], [19].

Based on the transformation, the MPC optimization problem can be written as an MILP problem of the following form:

$$\begin{cases} \min\limits_{\mathbf{f}(k_0)} \ J = c^{\text{T}}(k_0)\mathbf{f}(k_0) + d^{\text{T}}(k_0)z(k_0) \\ \text{s.t.} \quad A(k_0)\mathbf{f}(k_0) + B(k_0)z(k_0) \leq b(k_0), \end{cases} \tag{26}$$

where $\mathbf{f}(k_0)$ contains the decision variables of all platforms in the urban rail network of the planning window, $z(k_0)$ represents the auxiliary binary variable, $c(k_0)$ and $d(k_0)$ denote the constant vectors of the problem in the current planning window. The constraint $A(k_0)\mathbf{f}(k_0) + B(k_0)z(k_0) \leq b(k_0)$ represents all the mixed-integer constraints in a matrix form.

As we can start with a feasible solution of the overall system, i.e., the departure frequency of the basic timetable, and we can always use the basic departure frequency, so that a feasible solution can always be found. Therefore, the recursive feasibility of MPC can be satisfied.

IV. Case Study

In this section, we perform the case study to illustrate the proposed passenger absorption model and the corresponding MPC approach based on a small part of subway network from Beijing subway network.

As shown in Fig. 2, we select two lines from Beijing subway network. The network includes two subway lines and each line has two directions. The network contains 19 stations and 40 platforms.

We use MATLAB (R2019b) at a computer with an Intel Xeon W-2223 CPU and 8GB RAM for simulation. Passenger demands of the network are generated based on real-life passenger flow data of the Beijing subway network. In the collected information, the passenger flow data of the automatic fare collection system are varying every half hour. Therefore, we set $T = 1800$ s in the case study. The main parameters related to the simulation are listed in TABLE I.

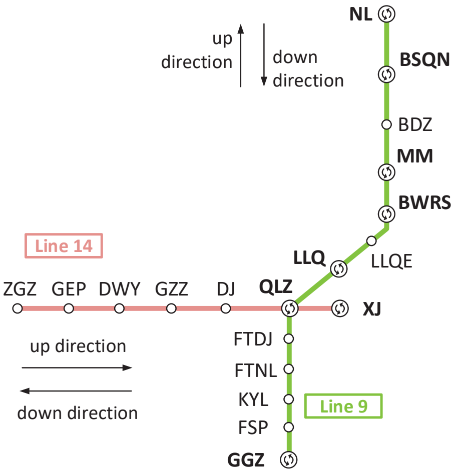

Fig. 2. Real-life network of 2 lines from Beijing subway network.

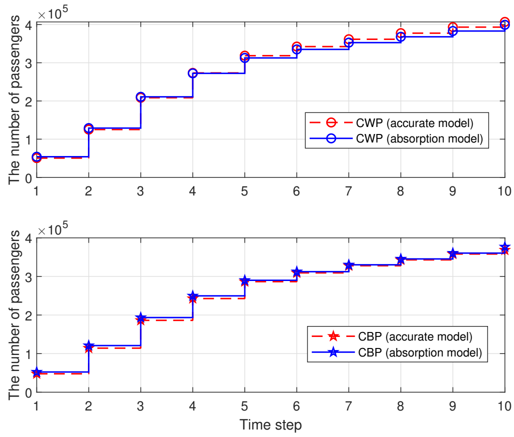

Fig. 3. Comparison of the absorption model and the accurate model.

TABLE I
PARAMETERS FOR THE METRO NETWORK

Parameters	Line 9	Line 14
Regular dwell time	60 s	60 s
Minimum dwell time	30 s	30 s
Regular headway	180 s	270 s
Minimum headway	108 s	108 s
Train capacity	2400 persons	2400 persons
Average transfer time	60 s	60 s
Period time	1800 s	1800 s

A. Assessment of the Absorption Model

The most accurate passenger-centric timetable scheduling model we found in the literature is the model of [5], [20]. For compactness, we regard the model of [5], [20] as "accurate model" in the remaining part of the section. Then, we compare the accuracy and efficiency of the passenger absorption model with the accurate model. The passenger absorption model focuses on the departure frequency at each period and does not involve detailed arrival and departure times of trains. Therefore, instead of comparing the numbers of passengers as a function of time, we compare the numbers at the end of each period. In the undersaturated or saturated situation, the capacity provided by trains when trains arrive at platforms, are larger than or equal to the required capacity; thus, all passengers are able to board the trains in time. Hence, we select the oversaturated situation for simulation, where efficient optimization approaches are important to facilitate more passenger to board the trains in time.

The cumulative number of waiting passengers (CBP) and the cumulative number of boarding passengers are two crucial factors related to passenger satisfaction. The simulation is conducted based on the passenger flow data from 7:00.

As Fig. 3 shows, the values of CWP and CBP are close to that of accurate model. The simulation time of the accurate model for the considered time window is 9.03s while the simulation time of the absorption model is 0.08s. The simulation time of the absorption model is reduced significantly, which implies that the absorption model can deal with the timetable scheduling problem more efficiently. The simulation results thus indicate that the absorption model can make a balanced trade-off between model accuracy and computation burden.

B. MPC for Timetable Scheduling of Urban Rail Networks

In this section, we perform case study to show the effectiveness of the developed MILP-based MPC approach. The prediction time window is 1.5 hours, i.e., the prediction horizon is 3, and the case study is conducted for 10 periods.

The MILP problem is solved by using `gurobi` solver implemented in MATLAB (R2019b). For the SQP algorithm, we apply the `fmincon` function in the MATLAB Optimization Toolbox. The basic timetable is generated according to the regular dwell time and regular headway in TABLE I. The performance, including the solution quality and the solution time, is compared with that of the basic timetable.

TABLE II
COMPARISON OF PERFORMANCE AND COMPUTATION TIME CORRESPONDING TO DIFFERENT APPROACHES

Method	Performance	CPU time (s)	
		t_{avrg}	$t_{\max}$
Basic timetable	$1.0298 \cdot 10^5$	-	-
SQP-based MPC	$8.9841 \cdot 10^4$	60.1	77.6
MILP-based MPC	$8.5447 \cdot 10^4$	23.5	27.6

The resulting MILP problem has 23584 continuous variables and 156 binary variables. The simulation results are shown in TABLE II. We can find that, in this case study, both SQP-based MPC and MILP-based MPC can largely improve the performance compared with the basic timetable, with the improvement of 12.76% and 17.03%, respectively. Instead of keeping constant departure frequency during the whole time window, SQP-based MPC and MILP-based MPC can adjust the departure frequency based on the real-time passenger demands, thus the performance is improved. SQP can fall into sub-optimal solutions of the nonlinear non-convex optimization problem, which will influence the performance of SQP-based MPC.

To illustrate the performance of each approach, the performance of each period are shown in Fig. 4. We can found from Fig. 4 that the performance obtained from MILP-based MPC is better than that of SQP-based MPC. Furthermore, the average CPU time of MILP-based MPC is reduced with a factor about 3 compared with SQP-based MPC. MILP-based MPC performs best with respect to the solution time and solution quality. The simulation results indicate that the MILP-based approach can be used in real time to determine departure frequencies in urban rail networks.

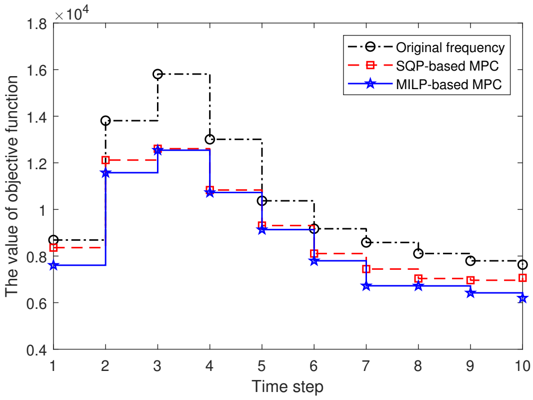

Fig. 4. The value of objective function at each time step.

V. Conclusions

In this paper, a novel passenger absorption model has been proposed for urban rail network timetable scheduling considering time-varying passenger origin-destination demands. The model divides the planning time window into several periods. By optimizing the number of trains visiting a platform during each period, an upper bound of the transport capacity per period is determined, so that passengers can be absorbed by trains at the platform. An MPC approach is developed for the real-time timetable scheduling problem. The passenger absorption model allows us to transform the MPC optimization problem into a mixed-integer linear programming problem, which can be solved efficiently by the existing solvers. Simulation results indicate that the absorption model can make a balanced trade-off between model accuracy and computational burden. Furthermore, we also shown that MILP-based MPC can help to greatly reduce the computation time while ensuring good control performance.

Our future work will involve developing two-level control methods to incorporate a more detailed train operation model and/or a more detailed passenger flow model into the passenger absorption model, so that the practically implementable timetable can be generated while keeping a balanced trade-off between computation time and control performance. To solve the timetable scheduling problem for large-scale networks, we will investigate efficient distributed control approaches. Future work will also investigate the potential of extending the absorption model to more general case of railway networks, including across-line operation, short-turning, shunting, and stop-skipping.

References

[1] Y. Wang, B. Ning, T. Tang, T. J. van den Boom, and B. De Schutter, "Efficient real-time train scheduling for urban rail transit systems using iterative convex programming," *IEEE Transactions on Intelligent Transportation Systems*, vol. 16, no. 6, pp. 3337–3352, 2015.

[2] H. Niu, X. Zhou, and R. Gao, "Train scheduling for minimizing passenger waiting time with time-dependent demand and skip-stop patterns: Nonlinear integer programming models with linear constraints," *Transportation Research Part B: Methodological*, vol. 76, pp. 117–135, 2015.

[3] J. Yin, A. D'Ariano, Y. Wang, L. Yang, and T. Tang, "Timetable coordination in a rail transit network with time-dependent passenger demand," *European Journal of Operational Research*, vol. 295, no. 1, pp. 183–202, 2021.

[4] X. Luan, Y. Wang, B. De Schutter, L. Meng, G. Lodewijks, and F. Corman, "Integration of real-time traffic management and train control for rail networks-part 1: Optimization problems and solution approaches," *Transportation Research Part B: Methodological*, vol. 115, pp. 41–71, 2018.

[5] Y. Wang, T. Tang, B. Ning, T. J. van den Boom, and B. De Schutter, "Passenger-demands-oriented train scheduling for an urban rail transit network," *Transportation Research Part C: Emerging Technologies*, vol. 60, pp. 1–23, 2015.

[6] D. Canca, E. Barrena, A. De-Los-Santos, and J. L. Andrade-Pineda, "Setting lines frequency and capacity in dense railway rapid transit networks with simultaneous passenger assignment," *Transportation Research Part B: Methodological*, vol. 93, pp. 251–267, 2016.

[7] A. Higgins and E. Kozan, "Modeling train delays in urban networks," *Transportation Science*, vol. 32, no. 4, pp. 346–357, 1998.

[8] Z. Gao, H. Sun, and L. L. Shan, "A continuous equilibrium network design model and algorithm for transit systems," *Transportation Research Part B: Methodological*, vol. 38, no. 3, pp. 235–250, 2004.

[9] F. Leurent, E. Chandakas, and A. Poulhès, "A traffic assignment model for passenger transit on a capacitated network: Bi-layer framework, line sub-models and large-scale application," *Transportation Research Part C: Emerging Technologies*, vol. 47, pp. 3–27, 2014.

[10] A. De-Los-Santos, G. Laporte, J. A. Mesa, and F. Perea, "Simultaneous frequency and capacity setting in uncapacitated metro lines in presence of a competing mode," *Transportation Research Procedia*, vol. 3, pp. 289–298, 2014.

[11] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. Scokaert, "Constrained model predictive control: Stability and optimality," *Automatica*, vol. 36, no. 6, pp. 789–814, 2000.

[12] D. Q. Mayne, "Model predictive control: Recent developments and future promise," *Automatica*, vol. 50, no. 12, pp. 2967–2986, 2014.

[13] T. J. van den Boom and B. De Schutter, "Modelling and control of discrete event systems using switching max-plus-linear systems," *Control Engineering Practice*, vol. 14, no. 10, pp. 1199–1211, 2006.

[14] G. Caimi, M. Fuchsberger, M. Laumanns, and M. Lüthi, "A model predictive control approach for discrete-time rescheduling in complex central railway station areas," *Computers & Operations Research*, vol. 39, no. 11, pp. 2578–2593, 2012.

[15] S. Li, M. M. Dessouky, L. Yang, and Z. Gao, "Joint optimal train regulation and passenger flow control strategy for high-frequency metro lines," *Transportation Research Part B: Methodological*, vol. 99, pp. 113–137, 2017.

[16] G. Cavone, T. van den Boom, L. Blenkers, M. Dotoli, C. Seatzu, and B. De Schutter, "An MPC-based rescheduling algorithm for disruptions and disturbances in large-scale railway networks," *IEEE Transactions on Automation Science and Engineering*, vol. 19, no. 1, pp. 99–112, 2022.

[17] P. T. Boggs and J. W. Tolle, "Sequential quadratic programming," *Acta Numerica*, vol. 4, pp. 1–51, 1995.

[18] A. Bemporad and M. Morari, "Control of systems integrating logic, dynamics, and constraints," *Automatica*, vol. 35, no. 3, pp. 407–427, 1999.

[19] H. P. Williams, *Model Building in Mathematical Programming*. John Wiley & Sons, 2013.

[20] N. Bešinović, Y. Wang, S. Zhu, E. Quaglietta, T. Tang, and R. M. Goverde, "A matheuristic for the integrated disruption management of traffic, passengers and stations in urban railway lines," *IEEE Transactions on Intelligent Transportation Systems*, 2021.