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Technical report 22-013

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M. Lv, B. De Schutter, C. Shi, and S. Baldi, "Logic-based distributed switching control for agents in power-chained form with multiple unknown control directions," *Automatica*, vol. 137, p. 110143, Mar. 2022. doi:10.1016/j.automatica.2021.110143

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Logic-based Distributed Switching Control for Agents in Power-Chained form with Multiple Unknown Control Directions

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Abstract

This work studies logic-based distributed switching control for nonlinear agents in power-chained form, where logic-based (switching) control arises from the online estimation of the control directions assumed to be unknown for all agents. Compared to the state-of-the-art logic-based mechanisms, the challenge of power-chained dynamics is that in general asymptotic tracking cannot be obtained, even for a single agent. To address this challenge, a new logic-based mechanism is proposed, which is orchestrated by a dynamic boundary function. The boundary function is decreasing in-between switching instants and monotonically increasing at the switching instants, depending on the jumps of an appropriately designed Lyapunov-like function. To remove chattering (i.e. two or more switching instants occurring consecutively with zero dwell time), a dynamic threshold is proposed, based on selecting the maximum values of the Lyapunov-like function before and after switching.

Key words: Logic-based switching, Distributed control, Dynamic boundary function, Unknown control directions

1 Introduction

Recent years have witnessed a tremendous progress in the field of distributed control of nonlinear multi-agent systems [1, 2, 3, 4, 5, 6]. Such results can be categorized according to two large families of nonlinear dynamics: strict-feedback [1, 2, 3, 6, 7, 8] and pure-feedback [4, 5, 9] dynamics. At the same time, another family of dynamics, namely *power-chained form*, has been attracting great attention. The reason is twofold: first, power-chained dynamics are a generalization of strict-feedback and pure-feedback dynamics since they include more general integrators (with positive odd-integer-powers) [10, 11, 12]; second, dynamics in power-chained form can describe relevant classes of practical systems such as dynamical boiler-turbine units [13], or hydraulic dynamics [14]. Besides, [10, 11, 12] have shown that some classes of under-actuated, weakly coupled mechanical systems with cubic force-deformation relations (nonlinear spring forces) can also be captured by power-chained form. It was shown that, even for a single agent, asymptotic tracking for this class of dynamics is structurally impossible, even locally, because the linearized dynamics contain uncontrollable modes whose eigenvalues are on the right half plane [10]. In fact, the results in the literature for power-chained dynamics achieve practical or semiglobal [10, 11, 12, 15, 16, 17, 18] stability, in place of asymptotic stability. This implies that distributed asymptotic tracking for power-chained dynamics, is also structurally impossible in general. Furthermore, state-of-the-art results [15, 18] for power-chained dynamics rely on the assumption that the agents' control directions (i.e. the signs of the control gain functions) are known *a priori*. When such *a priori* knowledge is not available [2], a popular approach to tackle this challenge is continuous parameter adaptation via Nussbaum functions [2, 19, 20, 21, 22, 23, 24], which has been used also for distributed control of strict-feedback or pure-feedback dynamics [2, 3, 4, 7, 8]. At the same time, because it is well-recognized that Nussbaum-based methods require additional complexity in the control design and continuous parameter adaptation may lead to large learning transients, several researchers have been engaged in the problem of overcoming continuous parameter adaptation by means of logic-based control [25, 26]. Notable settings where logic-based adaptation was employed include overcoming conventional continuous tuning of control parameters [27, 28, 29] and overcoming the conventional Nussbaum approach for strict-feedback dynamics [30, 31].

It is crucial to notice that the state-of-the-art logic-based mechanisms in [30, 31] for strict-feedback systems rely on monitor functions that monitor whether asymptotic tracking can be achieved (resulting in bounded energy of the tracking error) [30] or whether finite-time stabilization can be achieved (i.e. the tracking error converges to zero in finite time) [31]. Unfortunately, the same mechanism and monitor functions cannot be adopted for agents in power-chained form due to the aforementioned structural difficulty in achieving asymptotic tracking, see also [11, Examples 2.1 and 2.2]. Therefore, a different logic-based mechanism must be sought for distributed control of power-chained dynamics. This motivates the research question in this work: *is it possible to design a new logic-based mechanism for multi-agent systems in power-chained form with multiple unknown control directions even when asymptotic tracking cannot be*

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structurally obtained?

This paper provides a positive answer to this question with the following contributions:

i) To overcome the challenge that the exact value of the Lyapunov function is unavailable for logic-based adaptation, we propose a new Lyapunov-like function (cf. the discussion in Remark 3).

ii) We formally exclude any chattering phenomena by proposing a new dynamic threshold condition at the switching instants of the logic-based adaptation. It is worth noticing that state-of-the-art switching mechanisms cannot formally exclude chattering (cf. the discussion in Remark 4);

iii) To overcome the difficulty that no asymptotic tracking can be achieved for the power-chained form, we propose a new dynamic boundary function, which is decreasing in-between switching instants and possibly increasing at the switching instants of the logic-based adaptation (cf. Fig. 1 and the discussion in Remark 5);

Notations: The sets $\mathbb{R}$ and $\mathbb{R}^n$ stand for the set of real numbers and the n-dimensional Euclidean space, respectively; $\mathbb{N}_{\text{odd}}$ denotes the set of positive odd integers; $\|\cdot\|$ refers to either the Euclidean vector norm or the induced matrix 2-norm. Vectors are denoted in bold script, such as $\overline{\boldsymbol{\chi}}_{i,m}$ $\boldsymbol{W}_{i,m}$, $\boldsymbol{\varphi}_{i,m}$, $\boldsymbol{Z}_{i,m}$, $\boldsymbol{d}_{i,\sigma}$, and $\boldsymbol{h}_i(\cdot)$. For compactness and whenever unambiguous, some variable dependencies might be dropped throughout this paper, e.g. ι, $\psi_{i,m}$, $s_{i,m}$, $r_{i,m}$ can be used to represent $\iota(x_1,x_2)$, $\psi_{i,m}(\overline{\boldsymbol{\chi}}_{i,m})$, $s_{i,m}(\vartheta_{i,m},\alpha_{i,m-1})$, $r_{i,m}(\vartheta_{i,m},\alpha_{i,m-1})$, respectively.

2 Problem formulation and preliminaries

Let us first give some preliminaries on graph theory. The communication topology among agents is described by a directed graph $\mathscr{G} \triangleq (\mathscr{V},\mathscr{E})$, with $\mathscr{V} \triangleq \{0,1,\ldots,N\}$ the set of nodes (agents) and with $\mathscr{E} \subseteq \mathscr{V}\times\mathscr{V}$ the set of directed edges between two distinct agents. A directed edge $(j,i) \in \mathscr{E}$ represents that agent i can obtain information from agent j. The neighbor set of agent i is denoted by $\mathscr{N}_i = \{j|(j,i)\in\mathscr{E}\}$. Because agent 0 plays a special role (leader), let us consider the subgraph defined by $\overline{\mathscr{G}} \triangleq (\overline{\mathscr{V}},\overline{\mathscr{E}})$ with $\overline{\mathscr{V}} \triangleq \{1,2\ldots,N\}$ the set of follower agents and $\overline{\mathscr{E}}$ defined accordingly. For this subgraph, let us define the adjacency matrix $\overline{\mathscr{A}} = [a_{ij}] \in \mathbb{R}^{N\times N}$ as follows: if $(j,i)\in\overline{\mathscr{E}}$, then $a_{ij}=1$, otherwise $a_{ij}=0$. The Laplacian matrix $\mathscr{L}$ associated with $\mathscr{G}$ is defined as $\mathscr{L} = \begin{bmatrix} 0 & \mathbf{0}_{1\times N} \\ -b & \overline{\mathscr{L}}+\mathscr{B}\end{bmatrix}$ with $\mathscr{B} = \text{diag}[b_1,\ldots,b_N]$, where $b_i = 1$ if the leader $0\in\mathscr{N}_i$, and $b_i = 0$ otherwise. Moreover, $b = [b_1,\ldots,b_N]^{\mathrm{T}}$ and $\overline{\mathscr{L}} = \overline{\mathscr{D}} - \overline{\mathscr{A}}$ is the Laplacian matrix related to $\overline{\mathscr{G}}$ with $\overline{\mathscr{D}} = \text{diag}[d_1,\ldots,d_N]$, where $d_i = \sum_{j\in\mathscr{N}_i} a_{ij}$.

Consider a multi-agent system whose agents have the following nonlinear dynamics

$$\begin{cases} \dot{\chi}_{i,m} = \phi_{i,m}(\overline{\boldsymbol{\chi}}_{i,m}) + \psi_{i,m}(\overline{\boldsymbol{\chi}}_{i,m})\chi_{i,m+1}^{p_{i,m}}, \\ \dot{\chi}_{i,n_i} = \phi_{i,n_i}(\overline{\boldsymbol{\chi}}_{i,n_i}) + \psi_{i,n_i}(\overline{\boldsymbol{\chi}}_{i,n_i})u_i^{p_{i,n_i}}, \\ \quad y_i = \chi_{i,1}, \end{cases} \tag{1}$$

for $i = 1,\ldots,N$, $m = 1,\ldots,n_i-1$, where n_i is the dimension of system state $\overline{\boldsymbol{\chi}}_{i,n_i} = [\chi_{i,1},\ldots,\chi_{i,n_i}]^T \in \mathbb{R}^{n_i}$ and $\overline{\boldsymbol{\chi}}_{i,m} = [\chi_{i,1},\ldots,\chi_{i,m}]^T \in \mathbb{R}^m$. In (1), $p_{i,m}\in\mathbb{N}_{\text{odd}}$ are positive odd powers, and $u_i\in\mathbb{R}$ is the agent control input to be designed. The functions $\phi_{i,m}(\cdot)$ and $\psi_{i,m}(\cdot)$ are unknown locally Lipschitz continuous nonlinearities. The following assumptions are considered.

Assumption 1 For each follower i, the signs of $\psi_{i,m}(\cdot)$, called the *control directions*, are unknown and there exist known positive constants $\overline{\psi}_{i,m}$ and $\underline{\psi}_{i,m}$ such that

$$\underline{\psi}_{i,m} \leq |\psi_{i,m}(\cdot)| \leq \overline{\psi}_{i,m} \tag{2}$$

for $i = 1,\ldots,N$, $m = 1,\ldots,n_i$.

Assumption 2 [6] The leader agent 0 is represented by a leader output signal y_{r}, which is continuous, bounded and with bounded derivative; y_{r} is available only to the subset of follower agents i such that agent $0\in\mathscr{N}_i$, i.e. to those agents directly connected to the leader according to directed graph $\mathscr{G}$.

Assumption 3 [6] The directed graph $\mathscr{G}$ contains at least one directed spanning tree with the leader as the root. This implies that $\overline{\mathscr{L}}+\mathscr{B}$ is nonsingular.

Remark 1 The bounds in (2) are standardly assumed to ensure controllability of the system [11, 15, 16, 17]; Assumptions 2-3 are also standard in literature. The peculiar characteristic (and challenge) of (1) as compared to other multi-agent system models proposed in the literature, are the unknown multiple control directions in Assumption 1. Although some works have addressed multi-agent systems with unknown control directions [2, 3, 4, 7, 8], the dynamics therein are in the form of strict-feedback systems.

Define the consensus tracking error for the i-th follower as

$$\xi_{i,1} = \sum_{j\in\mathscr{N}_i} a_{ij}(y_i - y_j) + b_i(y_i - y_{\text{r}}), \tag{3}$$

for $i = 1,\ldots,N$. After collecting $\boldsymbol{\xi}_1 = [\xi_{1,1},\ldots,\xi_{N,1}]^T \in \mathbb{R}^N$, one has $\boldsymbol{\xi}_1 = (\overline{\mathscr{L}}+\mathscr{B})\boldsymbol{\omega}$, where $\boldsymbol{\omega} = \overline{\boldsymbol{y}} - \overline{\boldsymbol{y}}_{\text{r}}$ with $\overline{\boldsymbol{y}} = [y_1,\ldots,y_N]^T$ and $\overline{\boldsymbol{y}}_{\text{r}} = [y_{\text{r}},\ldots,y_{\text{r}}]^T$. Due to the nonsingularity of $\overline{\mathscr{L}}+\mathscr{B}$, it holds that $\|\boldsymbol{\omega}\| \leq \frac{\|\boldsymbol{\xi}_1\|}{\sigma_{\min}(\overline{\mathscr{L}}+\mathscr{B})}$, where $\sigma_{\min}(\overline{\mathscr{L}}+\mathscr{B})$ is the minimum singular value of $\overline{\mathscr{L}}+\mathscr{B}$. We impose a prescribed performance [32] on the consensus tracking error $\xi_{i,1}$ as $\underline{\xi}_{i,1}(t) \leq \xi_{i,1}(t) \leq \overline{\xi}_{i,1}(t)$ for $t\geq 0$, where $\overline{\xi}_{i,1}(t) = (\overline{\rho}_{i,1}-\rho_{i,\infty})\exp(-\overline{l}_{i,1}t)+\rho_{i,\infty}$ and $\underline{\xi}_{i,1}(t) = (\underline{\rho}_{i,1}+\rho_{i,\infty})\exp(-\underline{l}_{i,1}t)-\rho_{i,\infty}$ are the so-called performance functions [32], where $\overline{l}_{i,1}>0$ and $\underline{l}_{i,1}>0$ denote the minimum admissible convergence rates, $\rho_{i,\infty}>0$ is the maximum allowable tracking error at steady state, $\overline{\rho}_{i,1}>\rho_{i,\infty}>0$ and $\underline{\rho}_{i,1}<-\rho_{i,\infty}<0$ respectively represent the maximum and minimum bounds for $\xi_{i,1}(0)$. The following transformed consensus tracking error is then used for feedback:

$$\vartheta_{i,1}(t) = \ln\left(\frac{\xi_{i,1}(t)-\underline{\xi}_{i,1}(t)}{\overline{\xi}_{i,1}(t)-\xi_{i,1}(t)}\right). \tag{4}$$

Note that $\vartheta_{i,1}$ is monotonically increasing w.r.t. $\xi_{i,1}$ and that (4) implies that the consensus tracking error $\xi_{i,1}$ is within its imposed bounds provided $\vartheta_{i,1}$ is bounded [32].

Consensus tracking problem: Under dynamics (1) and Assumptions 1-3, the goal is to design u_i such that all closed-loop signals are semi-globally ultimately uniformly bounded, and the output of each follower agent i can follow the leader agent's signal y_{r} in spite of completely multiple unknown control directions.

Practical tracking [11, eq. (2.10)] (i.e. the tracking error converges to a residual set) will be sought, due to the fact that asymptotic tracking cannot be realized in general for dynamics (1) [11]. The following lemmas are instrumental in solving the practical tracking problem.

Lemma 1 [10] For any $x_1, x_2 \in \mathbb{R}$, given positive integers r_1, r_2 and any real-valued function $\iota(\cdot,\cdot) > 0$, it holds that

$$|x_1|^{r_1}|x_2|^{r_2} \leq \frac{r_2}{r_1+r_2}\iota(x_1,x_2)^{-\frac{r_1}{r_2}}|x_2|^{r_1+r_2} + \frac{r_1}{r_1+r_2}\iota(x_1,x_2)|x_1|^{r_1+r_2}. \quad (5)$$

Lemma 2 [18] For any x_1, $x_2 \in \mathbb{R}$, and positive odd integer $p \in \mathbb{N}_{\text{odd}}$, it holds that

$$(x_1+x_2)^p = r(x_1,x_2)x_1^p + s(x_1,x_2)x_2^p \quad (6)$$

where $r(x_1,x_2) \in [\underline{r}, \overline{r}]$ with $\underline{r} = 1-\delta$ and $\overline{r} = 1+\delta$, and $\delta = \sum_{k=1}^{p} \frac{p!}{k!(p-k)!}\frac{p-k}{p} l^{\frac{p}{p-k}}$ is a constant taking value in $(0,1)$ for some appropriately small constant l, and where $|s(x_1,x_2)| \leq \overline{s}(\delta)$ with $\overline{s}(\delta) = \sum_{k=1}^{p}\frac{p!}{k!(p-k)!}\frac{k}{p}l^{-\frac{p}{k}}$ a positive constant for a given l.

3 Adaptive Switching Consensus Protocol

The control design solving the consensus tracking problem comprises a *continuous* input (i.e. acting in-between two consecutive switching instants) and a *switching* mechanism (acting at the switching instants) to tune online some parameters of the continuous input. In this section, we focus on the continuous input, the design of which is well-established in literature under the assumption that the control directions are known [15].

After defining $\vartheta_{i,1}$ as in (4), and state errors

$$\vartheta_{i,m} = \chi_{i,m} - \alpha_{i,m-1}, \quad m = 2,\ldots,n_i, \quad (7)$$

the continuous control input comprises the so-called virtual laws $\alpha_{i,m}$ and the actual control u_i, designed as

$$\alpha_{i,1} = -h_{i,1}\Im_{i,1}^{\frac{1}{p_{i,1}}}\left(k_{i,1} + \epsilon_{i,1}^{\overline{p}_{i,1}}\widehat{\Theta}_{i,1}\Gamma_{i,1}^{\overline{p}_{i,1}} + \varrho_{i,1}^{\overline{p}_{i,1}}\right)^{\frac{1}{p_{i,1}}} \quad (8)$$

$$\Im_{i,1} = \vartheta_{i,1}^{p_{i,1}}\left[\ell_{i,1}\underline{\psi}_{i,1}(d_i+b_i)(1-\delta_{i,1})\right]^{-1},$$

$$\alpha_{i,m} = -h_{i,m}\Im_{i,m}^{\frac{1}{p_{i,m}}}\left(k_{i,m} + \epsilon_{i,m}^{\overline{p}_{i,m}}\widehat{\Theta}_{i,m}\Gamma_{i,m}^{\overline{p}_{i,m}} + \varrho_{i,m}^{\overline{p}_{i,m}}\right)^{\frac{1}{p_{i,m}}} \quad (9)$$

$$\Im_{i,m} = \vartheta_{i,m}^{p_{i,m}}\left[\underline{\psi}_{i,m}(1-\delta_{i,m})\right]^{-1}, (m = 1,\ldots,n_i)$$

$$u_i = \alpha_{i,n_i}, \ \delta_{i,n_i} = 0, \quad (10)$$

$$\dot{\widehat{\Theta}}_{i,m} = \gamma_{i,m}\left[\epsilon_{i,m}^{\overline{p}_{i,m}}\vartheta_{i,m}^{p_i+3}\Gamma_{i,m}^{\overline{p}_{i,m}} - \beta_{i,m}\widehat{\Theta}_{i,m}\right]. \quad (11)$$

with $\overline{p}_{i,m} = \frac{p_i+3}{p_i-p_{i,m}+3}$, $\underline{p}_{i,m} = \frac{p_i+3}{p_{i,m}}$, $p_i = \max_{m=1,\ldots,n_i}\{p_{i,m}\}$, and where $0 < \delta_{i,m} < 1$, $\varrho_{i,m} > 0$, $\epsilon_{i,m} > 0$, $\gamma_{i,m} > 0$ and $\beta_{i,m} > 0, (m = 1,\ldots,n_i)$ are design parameters. In (11), $\widehat{\Theta}_{i,m}$ is the estimate of $\Theta_{i,m} = \|\boldsymbol{W}_{i,m}^*\|^{\overline{p}_{i,m}}$ and $\Gamma_{i,m} = \|\boldsymbol{\varphi}_{i,m}\|$, which comes from appropriately designed function approximators (as detailed later on). Notice that the control design (8)-(11) is not complete, since the terms $k_{i,m}$ and $h_{i,m}$ are to be designed: these terms are necessary to tackle the multiple unknown control directions, and their design will be addressed in Sect. 4 via a switching mechanism. The rationale for the design (8)-(11) is given in the following steps.

Step $i,1$ $(i = 1,\ldots,N)$**:** The time derivative of $\vartheta_{i,1}$ along (1), (3), and (4) is

$$\dot{\vartheta}_{i,1} = l_{i,1}\dot{\xi}_{i,1} + H_{i,1} = l_{i,1}(d_i+b_i)\psi_{i,1}\chi_{i,2}^{p_{i,1}} + E_{i,1}, \quad (12)$$

where $l_{i,1} = \partial\vartheta_{i,1}/\partial\xi_{i,1} > 0$, $H_{i,1} = \left(\partial\vartheta_{i,1}/\partial\overline{\xi}_{i,1}\right)\dot{\overline{\xi}}_{i,1} + \left(\partial\vartheta_{i,1}/\partial\underline{\xi}_{i,1}\right)\dot{\underline{\xi}}_{i,1}$, and $E_{i,1} = l_{i,1}(d_i+b_i)\phi_{i,1} - l_{i,1}\sum_{j\in\mathcal{N}_i} a_{ij} \times(\phi_{i,1}+\psi_{i,1}\chi_{i,2}^{p_{i,1}}) - b_i\dot{y}_{\rm r} + H_{i,1}$. Along the same veins as [33], there exist some optimal weights $\boldsymbol{W}_{i,1}^*$, and a linear-in-the-parameter approximator $\boldsymbol{W}_{i,1}^*\boldsymbol{\varphi}_{i,1}(\boldsymbol{Z}_{i,1})$ for $|E_{i,1}|$ such that

$$\vartheta_{i,1}^{p_i-p_{i,1}+3}E_{i,1} \leq \left|\vartheta_{i,1}^{p_i-p_{i,1}+3}\right|\left[\boldsymbol{W}_{i,1}^*\boldsymbol{\varphi}_{i,1}(\boldsymbol{Z}_{i,1}) + \varepsilon_{i,1}(\boldsymbol{Z}_{i,1})\right] \leq \vartheta_{i,1}^{p_i+3}\left(\varrho_{i,1}^{\overline{p}_{i,1}} + \epsilon_{i,1}^{\overline{p}_{i,1}}\Theta_{i,1}\Gamma_{i,1}^{\overline{p}_{i,1}}\right) + \mu_{i,1},$$

where the last inequality uses Lemma 1. Furthermore, $\mu_{i,1} = \epsilon_{i,1}^{-\underline{p}_{i,1}} + \varrho_{i,1}^{-\underline{p}_{i,1}}\overline{\varepsilon}_{i,1}^{\underline{p}_{i,1}}$ with $\epsilon_{i,1} > 0$ and $\varrho_{i,1} > 0$ being design constants, $\varepsilon_{i,1}(\boldsymbol{Z}_{i,1})$ is the approximation error satisfying $\left|\varepsilon_{i,1}(\boldsymbol{Z}_{i,1})\right| \leq \overline{\varepsilon}_{i,1}$ on a compact set $\Omega_{i,1}$, $\boldsymbol{Z}_{i,1} = \left[\chi_{i,1}, \chi_{j,1,j\in\mathcal{N}_i}, \chi_{j,2,j\in\mathcal{N}_i}, b_iy_{\rm r}, b_i\dot{y}_r\right]^T \in \Omega_{i,1}$, and $\overline{\varepsilon}_{i,1} > 0$ a constant.

Remark 2 The continuous function $|E_{i,1}|$ in (12) embeds the effect of graph connectivity, since $|E_{i,1}|$ depends on the connectivity matrix a_{ij} and b_i. Note that, because the activation function $\boldsymbol{\varphi}_{i,1}(\cdot)$ of the linear-in-the-parameter approximation relies on the neighboring states, standard universal approximation results [34] of linear-in-the-parameter approximation still hold. Similar approximation ideas also can be found in [5, 34, 35]. Simulation results in this paper also validate this point (cf. Figs. 7 and 8).

Consider the Lyapunov function candidate

$$V_{i,1} = \frac{\vartheta_{i,1}^{p_i-p_{i,1}+4}}{p_i-p_{i,1}+4} + \frac{1}{2\gamma_{i,1}}\widetilde{\Theta}_{i,1}^2 \quad (13)$$

where $\widetilde{\Theta}_{i,1} = \Theta_{i,1} - \widehat{\Theta}_{i,1}$. According to Lemmas 1 and 2, it holds that

$$\vartheta_{i,1}^{p_i-p_{i,1}+3}\chi_{i,2}^{p_{i,1}} = s_{i,1}\vartheta_{i,1}^{p_i-p_{i,1}+3}\vartheta_{i,2}^{p_{i,1}} + r_{i,1}\vartheta_{i,1}^{p_i-p_{i,1}+3}\alpha_{i,1}^{p_{i,1}} < |s_{i,1}|\left(\vartheta_{i,1}^{p_i+3} + \vartheta_{i,2}^{p_i+3}\right) + r_{i,1}\vartheta_{i,1}^{p_i-p_{i,1}+3}\alpha_{i,1}^{p_{i,1}}. \quad (14)$$

Then, it follows from (12), (13), and (14) that the derivative of $V_{i,1}$ with respect to time is

$$\dot{V}_{i,1} < l_{i,1}(d_i+b_i)\vartheta_{i,1}^{p_i-p_{i,1}+3}\alpha_{i,1}^{p_{i,1}}h_{i,1}r_{i,1}|\psi_{i,1}| - \frac{\widetilde{\Theta}_{i,1}\dot{\widehat{\Theta}}_{i,1}}{\gamma_{i,1}} + \vartheta_{i,1}^{p_i+3}\left(\varrho_{i,1}^{\overline{p}_{i,1}} + \epsilon_{i,1}^{\overline{p}_{i,1}}\Theta_{i,1}\Gamma_{i,1}^{\overline{p}_{i,1}}\right) + \Delta_{i,1} + \mu_{i,1} + \varpi_{i,1}\left(\vartheta_{i,1}^{p_i+3} + \vartheta_{i,2}^{p_i+3}\right), \quad (15)$$

where $\Delta_{i,1} = l_{i,1}(d_i+b_i)r_{i,1}\vartheta_{i,1}^{p_i-p_{i,1}+3}\alpha_{i,1}^{p_{i,1}}(\text{sign}(\psi_{i,1}) - h_{i,1})|\psi_{i,1}|$, $\varpi_{i,1} = (d_i+b_i)l_{i,1}\overline{\psi}_{i,1}\overline{s}_{i,1}$, and we used the fact that $\psi_{i,1} = \text{sign}(\psi_{i,1})|\psi_{i,1}|$. Substituting the virtual control $\alpha_{i,1}$ (8) into (15) gives

$$\dot{V}_{i,1} < -(k_{i,1} - \varpi_{i,1})\vartheta_{i,1}^{p_i+3} + \vartheta_{i,1}^{p_i+3}\epsilon_{i,1}^{\overline{p}_{i,1}}\widetilde{\Theta}_{i,1}\Gamma_{i,1}^{\overline{p}_{i,1}} - \frac{\widetilde{\Theta}_{i,1}\dot{\widehat{\Theta}}_{i,1}}{\gamma_{i,1}} + \varpi_{i,1}\vartheta_{i,2}^{p_i+3} + \Delta_{i,1} + \mu_{i,1}. \quad (16)$$

Substituting the adaptive law $\dot{\widehat{\Theta}}_{i,1}$ (11) into (16) yields $\dot{V}_{i,1} < -c_{i,1}\vartheta_{i,1}^{p_i+3} + \varpi_{i,1}\vartheta_{i,2}^{p_i+3} - \beta_{i,1}\widetilde{\Theta}_{i,1}\widehat{\Theta}_{i,1} + \Delta_{i,1} + \mu_{i,1}$, where $c_{i,1} = k_{i,1} - \varpi_{i,1}$.

Step i,m $(i = 1,\ldots,N,\ m = 2,\ldots,n_i-1)$ **:** It follows from (1), (7), and (9) that the derivative of $\vartheta_{i,m}$ is

$$\dot{\vartheta}_{i,m} = \psi_{i,m}\chi_{i,m+1}^{p_{i,m}} + E_{i,m}, \quad (17)$$

where $E_{i,m} = \phi_{i,m} - \sum_{q=1}^{m-1}\frac{\partial\alpha_{i,m-1}}{\partial\chi_{i,q}}\left(\phi_{i,q} + \psi_{i,q}\chi_{i,q+1}^{p_{i,q}}\right) - \frac{\partial\alpha_{i,m-1}}{\partial y_{\rm r}}\dot{y}_{\rm r} - \sum_{q=1}^{m-1}\frac{\partial\alpha_{i,m-1}}{\partial\widehat{\Theta}_{i,q}}\dot{\widehat{\Theta}}_{i,q} - \sum_{j\in\mathcal{N}_i}a_{ij}\frac{\partial\alpha_{i,m-1}}{\partial\chi_{j,1}}\big(\phi_{j,1} +$

$\psi_{j,2}\chi_{j,2}^{p_{j,1}})$. Referring to Step $i,1$, there exist some optimal weights $\boldsymbol{W}_{i,m}^{*}$, and a linear-in-the-parameter approximator $\boldsymbol{W}_{i,m}^{*}\boldsymbol{\varphi}_{i,m}(\boldsymbol{Z}_{i,m})$ for $|E_{i,m}|$ such that

$$
\begin{aligned}
&\vartheta_{i,m}^{p_i-p_{i,m}+3}E_{i,m}\\
\leq&\left|\vartheta_{i,m}^{p_i-p_{i,m}+3}\right|\left[\boldsymbol{W}_{i,m}^{*}\boldsymbol{\varphi}_{i,m}(\boldsymbol{Z}_{i,m})+\varepsilon_{i,m}(\boldsymbol{Z}_{i,m})\right]\\
\leq&\vartheta_{i,m}^{p_i+3}\left(\varrho_{i,m}^{\overline{p}_{i,m}}+\epsilon_{i,m}^{\overline{p}_{i,m}}\Theta_{i,m}\Gamma_{i,m}^{\overline{p}_{i,m}}\right)+\mu_{i,m},
\end{aligned}
$$

where $\mu_{i,m}=\epsilon_{i,m}^{-\underline{p}_{i,m}}+\varrho_{i,m}^{-\underline{p}_{i,m}}\overline{\varepsilon}_{i,m}^{\underline{p}_{i,m}}$ with $\epsilon_{i,m}>0$ and $\varrho_{i,m}>0$ design constants, $\varepsilon_{i,m}(\boldsymbol{Z}_{i,m})$ is the approximation error satisfying $\left|\varepsilon_{i,m}(\boldsymbol{Z}_{i,m})\right|\leq\overline{\varepsilon}_{i,m}$ on a compact set $\Omega_{i,m}$, with $\boldsymbol{Z}_{i,m}=\Big[\overline{\boldsymbol{\chi}}_{i,m},\overline{\boldsymbol{\chi}}_{j,m},\frac{\partial\alpha_{i,m-1}}{\partial\chi_{j,1}},\frac{\partial\alpha_{i,m-1}}{\partial\chi_{i,1}},\ldots,\frac{\partial\alpha_{i,m-1}}{\partial\chi_{i,m-1}},\frac{\partial\alpha_{i,m-1}}{\partial\widehat{\Theta}_{i,1}},\ldots,\frac{\partial\alpha_{i,m-1}}{\partial\widehat{\Theta}_{i,m-1}},\widehat{\Theta}_{i,1},\ldots,\widehat{\Theta}_{i,m-1},\frac{\partial\alpha_{i,m-1}}{\partial y_{\mathrm{r}}},b_i y_{\mathrm{r}}\Big]_{j\in\mathcal{N}_i}^{T}\in\Omega_{i,m}$ and $\overline{\varepsilon}_{i,m}>0$ a constant. Consider the Lyapunov function candidate

$$
V_{i,m}=V_{i,m-1}+\frac{\vartheta_{i,m}^{p_i-p_{i,m}+4}}{p_i-p_{i,m}+4}+\frac{1}{2\gamma_{i,m}}\widetilde{\Theta}_{i,m}^{2}, \tag{18}
$$

where $\widetilde{\Theta}_{i,m}=\Theta_{i,m}-\widehat{\Theta}_{i,m}$. Following similar derivations as in Step $i,1$, the derivative of $V_{i,m}$ with respect to time is

$$
\begin{aligned}
\dot{V}_{i,m}<&-\sum\nolimits_{q=1}^{m}c_{i,q}\vartheta_{i,q}^{p_i+3}+\varpi_{i,m}\vartheta_{i,m+1}^{p_i+3}+\sum\nolimits_{q=1}^{m}\Delta_{i,q}\\
&+\sum\nolimits_{q=1}^{m}\left(\frac{\beta_{i,q}}{2}\left(\Theta_{i,q}^{2}-\widetilde{\Theta}_{i,q}^{2}\right)+\mu_{i,q}\right),
\end{aligned} \tag{19}
$$

where $c_{i,m}=k_{i,m}-\varpi_{i,m}-\varpi_{i,m-1}$, $\varpi_{i,m}=\overline{\psi}_{i,m}\overline{s}_{i,m}$, and $\Delta_{i,m}=\vartheta_{i,m}^{p_i-p_{i,m}+3}\alpha_{i,m}^{p_{i,m}}(\mathrm{sign}(\psi_{i,m})-h_{i,m})r_{i,m}|\psi_{i,m}|$, $(m=2,\ldots,n_i-1)$.

Step $i,n_i (i=1,\ldots,N)$ **:** For the last step, consider the Lyapunov function candidate

$$
V_{i,n_i}=V_{i,n_i-1}+\frac{\vartheta_{i,n_i}^{p_i-p_{i,n_i}+4}}{p_i-p_{i,n_i}+4}+\frac{1}{2\gamma_{i,n_i}}\widetilde{\Theta}_{i,n_i}^{2}, \tag{20}
$$

where $\widetilde{\Theta}_{i,n_i}=\Theta_{i,n_i}-\widehat{\Theta}_{i,n_i}$. Along similar lines as the previous steps, it is possible to conclude that

$$
\begin{aligned}
\dot{V}_{i,n_i}<&-\sum\nolimits_{q=1}^{n_i}c_{i,q}\vartheta_{i,q}^{p_i+3}+\sum\nolimits_{q=1}^{n_i}\left(\frac{\beta_{i,q}}{2}\left(\Theta_{i,q}^{2}-\widetilde{\Theta}_{i,q}^{2}\right)\right)\\
&+\sum\nolimits_{q=1}^{n_i}\mu_{i,q}+\sum\nolimits_{q=1}^{n_i}\Delta_{i,q},
\end{aligned} \tag{21}
$$

with $c_{i,n_i}=k_{i,n_i}-\varpi_{i,n_i-1}$ and $\Delta_{i,n_i}=\vartheta_{i,n_i}^{p_i-p_{i,n_i}+3}u_i^{p_{i,n_i}}\times(\mathrm{sign}(\psi_{i,n_i})-h_{i,n_i})|\psi_{i,n_i}|$. For any constant $\eta_i>0$, in light of Lemma 1, we have $\eta_i+\vartheta_{i,q}^{p_i+3}\geq\eta_i^{\frac{p_{i,q}-1}{p_i+3}}\vartheta_{i,q}^{p_i-p_{i,q}+4}$. Thus, (21) can be upper bounded as

$$
\dot{V}_{i,n_i}<-\varsigma_i V_{i,n_i}+\Xi_i+\sum\nolimits_{q=1}^{n_i}\Delta_{i,q}, \tag{22}
$$

where $\varsigma_i=\min\left\{\gamma_{i,q}\beta_{i,q},\ (p_i-p_{i,q}+4)c_{i,q}\eta_i^{\frac{p_{i,q}-1}{p_i+3}},i=1,\ldots,N,q=1,\ldots,n_i\right\}$, $\Xi_i=\sum_{q=1}^{n_i}(c_{i,q}\eta_i+\mu_{i,q})+\sum_{q=1}^{n_i}\frac{1}{2}\beta_{i,q}\Theta_{i,q}^{2}$.

The remaining problem is now the one of handling the term $\sum_{q=1}^{n_i}\Delta_{i,q}$ in (22) containing the signs of the control directions, which are unknown in view of Assumption 1. To tackle this term, a logic-based switching mechanism is proposed in the next section to adapt online the estimates $h_{i,m}$ of the multiple control directions.

4 Proposed Logic-based Design

Logic-based adaptation has been proposed in the literature for different classes of systems [30, 31, 36]. Because logic-based loops are switched systems [25, 37, 38], the concept of solution is intended in the sense of Carathéodory [25, Sect. 1.2.1]. Also, the subsequent switching mechanism is designed in such a way that chattering is avoided and the switching stops in finite time. Therefore, phenomena such as sliding mode or Zeno behavior, which are often a concern in switched systems, are avoided.

Algorithm 1 Logic-Based Distributed Switching Control Mechanism for the ith Follower Agent

```
Initialize: Set t_0 ← 0, σ ← 0, h_i(t_0) ← d_{i,0}, V̄_i(t_0) ≥ ℓ_{i,∞}(t_0) > 0 and V̄_i(t_0) ≥ V̂_{i,n_i}(t_0). Select positive design parameters ζ_{ℓ_{i,∞}} and ζ_{k_{i,m}}, i = 1,...,N, m = 1,...,n_i.
For every time t, for every agent i, calculate V̂_{i,n_i}(t), ℓ_i(t_σ,t), V̄_i(t), L_i(t_σ,t), and M_i(t_σ,t).
while (M_i(t_σ,t) ≥ 0), do
    Implement virtual control law (9), actual control
    law (10), and parameter adaptation law (11).
    h_i(t) ← d_{i,σ};
    ℓ_{i,∞}(t) ← ℓ_{i,∞}(t_σ);
    k_{i,m}(t) ← k_{i,m}(t_σ);
else
    σ ← σ + 1;
        if     σ is equal to 2^{n_i};
        then   σ ← 0;
        end if
    t_σ ← t;
    V̄_i(t_σ) ← max{ V̂_{i,n_i}(t_σ^-), V̂_{i,n_i}(t_σ) };
    h_i(t) ← d_{i,σ};
    ℓ_{i,∞}(t_σ) ← ℓ_{i,∞}(t_{σ-1}) + ζ_{ℓ_{i,∞}};
    ℓ_{i,∞}(t) ← ℓ_{i,∞}(t_σ);
    k_{i,m}(t_σ) ← k_{i,m}(t_{σ-1}) + ζ_{k_{i,m}};
    k_{i,m}(t) ← k_{i,m}(t_σ).
end while
```

4.1 Switching Mechanism

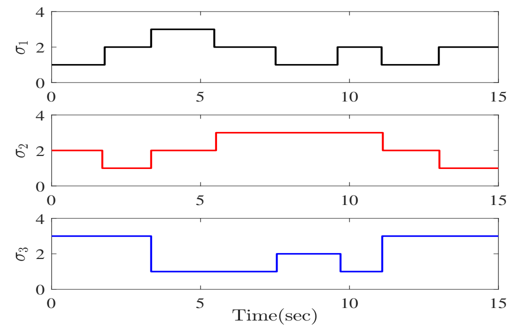


Figure 1. The sketch of the proposed switching mechanism.

We adopt a similar notation to [30], where the vectors $\boldsymbol{d}_{i,\sigma}\in\mathbb{R}^{n_i}$, whose elements are either 1 or -1, are used to represent all possible combinations of n_i control directions for each agent i. Accordingly, the switching sequence $\sigma(\cdot)$, taking values in $0,1,\ldots,2^{n_i}-1$, is a piecewise right-continuous function [25, Chap.1], and goes through all such possible combinations. For example, if $n_i=2$, we have four possible combinations: $\boldsymbol{d}_{i,0}=[-1,-1]^{T}$, $\boldsymbol{d}_{i,1}=[-1,1]^{T}$, $\boldsymbol{d}_{i,2}=[1,1]^{T}$, $\boldsymbol{d}_{i,3}=[1,-1]^{T}$. The order according to which the combinations are listed can be arbitrary, provided that all combinations are listed without repetitions. The reader can refer to [30] for more details on $\boldsymbol{d}_{i,\sigma}$. Please notice that each agent can exhibit its own switching sequence $\sigma_i(\cdot)$: however, in the following we will simply use $\sigma(\cdot)$ to avoid complicating the notation. Define $\boldsymbol{h}_i(t) = [h_{i,1}, \ldots, h_{i,n_i}]^T$ with $h_{i,m} \in \{-1, 1\}$, $m = 1, \ldots, n_i$. Let us now define

$$\overline{V}_i(t) = \max\left\{\ell_i(t_\sigma, t),\ \widehat{V}_{i,n_i}(t)\right\}, \tag{23}$$

$$\mathcal{L}_i(t_\sigma, t) = \ell_i(t_\sigma, t) - \overline{V}_i(t), \tag{24}$$

with

$$\widehat{V}_{i,n_i} = \sum_{m=1}^{n_i} \left\{ \frac{\vartheta_{i,m}^{p_i - p_{i,m} + 4}}{p_i - p_{i,m} + 4} + \frac{1}{2\gamma_{i,m}} \widehat{\Theta}_{i,m}^2 \right\} \tag{25}$$

and $\ell_i(t_\sigma, t)$ being a dynamic boundary function designed as $\ell_i(t_\sigma, t) = \big(\overline{V}_i(t_\sigma) - \ell_{i,\infty}(t_\sigma)\big) \exp\big(-\theta_i(t - t_\sigma)\big) + \ell_{i,\infty}(t_\sigma)$, where $\theta_i > 0$ is a design parameter. Let

$$\mathcal{M}_i(t_\sigma, t) = \mathcal{L}_i(t_\sigma, t) + \kappa_i \tag{26}$$

where $\kappa_i > 0$ is a preselected constant.

We are now in a position to present the logic-based mechanism for updating $\boldsymbol{h}_i(t)$, $\sigma(t)$, $k_{i,m}(t)$, $m = 1, \cdots, n_i$, and $\ell_{i,\infty}(t)$. After an initialization phase, the mechanism comprises a hold phase (i.e. σ is kept constant) and an update phase (i.e. σ is switched to a new value).

Initialization: $t_0 \leftarrow 0$, $\sigma \leftarrow 0$, $\boldsymbol{h}_i(t_0) \leftarrow \boldsymbol{d}_{i,0}$, $\overline{V}_i(t_0) \geq \ell_{i,\infty}(t_0) > 0$ and $\overline{V}_i(t_0) \geq \widehat{V}_{i,n_i}(t_0)$.

Hold phase: Phase in-between consecutive switching instants:

$$\begin{aligned}
&\text{while} && \mathcal{M}_i(t_\sigma, t) \geq 0, && (27)\\
&\text{do} && \boldsymbol{h}_i(t) \leftarrow \boldsymbol{d}_{i,\sigma}; && (28)\\
& && \ell_{i,\infty}(t) \leftarrow \ell_{i,\infty}(t_\sigma); && (29)\\
& && k_{i,m}(t) \leftarrow k_{i,m}(t_\sigma); && (30)\\
&\text{end while} && &&
\end{aligned}$$

at the same time, implement virtual control law (9), actual control law (10), and parameter adaptation law (11).

Update phase: Phase at the switching instant:

$$\begin{aligned}
&\text{if} && \mathcal{M}_i(t_\sigma, t) < 0, && (31)\\
&\text{then} && \sigma \leftarrow \sigma + 1; && (32)\\
&\quad\text{if} && \sigma \text{ is equal to } 2^{n_i}, && \\
&\quad\text{then} && \sigma \leftarrow 0; && (33)\\
&\quad\text{end if} && && \\
& && t_\sigma \leftarrow t; && (34)\\
& && \overline{V}_i(t_\sigma) \leftarrow \max\left\{\widehat{V}_{i,n_i}(t_\sigma^-), \widehat{V}_{i,n_i}(t_\sigma)\right\}; && (35)\\
& && \boldsymbol{h}_i(t) \leftarrow \boldsymbol{d}_{i,\sigma}; && (36)\\
& && \ell_{i,\infty}(t_\sigma) \leftarrow \ell_{i,\infty}(t_{\sigma-1}) + \zeta_{\ell_{i,\infty}}; && (37)\\
& && \ell_{i,\infty}(t) \leftarrow \ell_{i,\infty}(t_\sigma); && (38)\\
& && k_{i,m}(t_\sigma) \leftarrow k_{i,m}(t_{\sigma-1}) + \zeta_{k_{i,m}}; && (39)\\
& && k_{i,m}(t) \leftarrow k_{i,m}(t_\sigma); && (40)\\
&\text{end if} && &&
\end{aligned}$$

with $\zeta_{\ell_{i,\infty}} > 0$ and $\zeta_{k_{i,m}} > 0$ being design constants, $m = 1, \ldots, n_i$, $\sigma = 1, 2, \ldots$, and where t_σ^- denotes the value of t_σ when (31) is satisfied but $\boldsymbol{h}_i(t_\sigma)$, $\ell_{i,\infty}(t_\sigma)$, and $k_{i,m}(t_\sigma)$ have not been updated yet, and t_σ represents the time instant when (31) holds, and in the meantime, $\boldsymbol{h}_i(t_\sigma)$, $\ell_{i,\infty}(t_\sigma)$, and $k_{i,m}(t_\sigma)$ also have been updated according to (36)-(40).

The rationale for the proposed mechanism is as follows: the switching instants t_σ, $\sigma = 0, 1, \ldots$, occur whenever condition (31) is satisfied. The reset condition in (33) is necessary when all combinations in $\boldsymbol{d}_{i,\sigma}$ have been visited and thus it is necessary to start from the first one. The logic condition (35) circumvents the chattering phenomena at the switching instants t_σ, as elaborated in Remark 4.

The unique challenges of using logic-based mechanisms to handle multiple unknown control directions for power-chained form are elaborated in the following remarks:

Remark 3 A crucial challenge of the proposed logic-based switching is that the exact value of the Lyapunov function $V_{i,m}$ (18) is unavailable (as it contains the unknown constants $\Theta_{i,m}$ in $\widetilde{\Theta}_{i,m} = \Theta_{i,m} - \widehat{\Theta}_{i,m}$). Therefore, the unavailable Lyapunov function must be replaced by some estimate. To pursue this, the Lyapunov-like function $\widehat{V}_{i,n_i}$ (25) is proposed and designed in such a way as to establish the boundedness of the closed-loop signals (cf. appendix).

Remark 4 State-of-the-art logic-based mechanisms [30, 31, 39] cannot formally exclude chattering phenomena since they adopt $\ell_i(t_\sigma, t_\sigma) = \overline{V}_i(t_\sigma) = \widehat{V}_{i,n_i}(t_\sigma) = \widehat{V}_{i,n_i}(t_\sigma^-)$. More precisely, the update phase of [30, 31, 39] is designed as

$$\begin{aligned}
&\text{if} && \mathcal{M}_i(t_\sigma, t) < 0,\\
&\text{then} && t_\sigma \leftarrow t;\\
& && \overline{V}_i(t_\sigma) \leftarrow \widehat{V}_{i,n_i}(t_\sigma^-);\\
& && \boldsymbol{h}_i(t) \leftarrow \boldsymbol{d}_{i,\sigma};\\
& && k_{i,m}(t_\sigma) \leftarrow k_{i,m}(t_{\sigma-1}) + \zeta_{k_{i,m}};\\
&\text{end if} &&
\end{aligned}$$

In view of the discussions in [28, Remark 2 and the analysis after eq. (35)], it is theoretically possible for such mechanisms to yield an increase $\overline{V}_i(t_\sigma) = \widehat{V}_{i,n_i}(t_\sigma) > \widehat{V}_{i,n_i}(t_\sigma^-) + \kappa_i = \ell_i(t_\sigma, t_\sigma) + \kappa_i$, which indicates that $\mathcal{M}_i(t_\sigma, t) < 0$, leading to a new switching instant immediately after the previous one. This is because updating $\boldsymbol{h}_i(t)$ and $k_{i,m}$ may result in instantaneous changes in the tracking errors $\vartheta_{i,m}$, according to (7)-(10), which may lead to an increase of the value of the Lyapunov functions (13), (18), and (20). This could make the inequality (31) hold once more immediately after the previous time instant. To solve such issue, we exclude chattering phenomena by proposing a new dynamic threshold condition (35) at the switching instants, based on selecting the maximum values of the Lyapunov-like function before and after switching.

Remark 5 State-of-the-art logic-based designs for strict-feedback systems [30, 31, 39] rely on the fact that asymptotic tracking can be obtained for this class of systems: there exists at least one $\boldsymbol{d}_{i,\sigma}$, $\sigma \in \{0, 1, \ldots, 2^{n_i} - 1\}$, that leads to a vanishing tracking error. Unfortunately, it is well known in the literature that asymptotic tracking is impossible in general for the class of nonlinear systems (1) [11]. Therefore, the switching logic cannot rely on vanishing tracking errors. To overcome the above difficulty, we propose a new monitor function $\ell_i(\cdot)$, which is decreasing in-between switching instants and possibly increasing at switching instants. The role of $\ell_i(\cdot)$ is crucial to closed-loop stability through (23): $\ell_i(\cdot)$ is used to monitor the upper bound of the designed Lyapunov-like function $\widehat{V}_{i,n_i}$ as shown in Fig. 1. The distinguishing feature of $\ell_i(\cdot)$ is to allow $\widehat{V}_{i,n_i}$ to increase by a constant at every switching instant. Notice that the finite-switching mechanism guarantees that $\ell_i(\cdot)$ does not grow to infinity and thus closed-loop stability can be obtained.

4.2 Main stability result

To analyze the stability of the closed-loop system, we consider the global Lyapunov function

$$V=\sum_{i=1}^{N}V_{i,n_i},\ \text{and}\ \widehat{V}=\sum_{i=1}^{N}\widehat{V}_{i,n_i} \tag{41}$$

Theorem 1 *Under Assumptions 1-3, consider the closed-loop system consisting of the nonlinear multi-agent dynamics (1) in power-chained form and the logic-based switching control mechanism in Algorithm 1. Then, there exist positive design parameters $\varrho_{i,m}$, $\epsilon_{i,m}$, $\gamma_{i,m}$, $\beta_{i,m}$, $\eta_{i,m}$, and $k_{i,m}$ such that:*

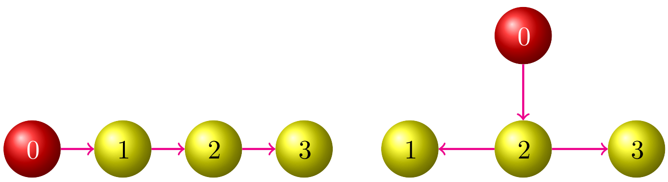
- *All closed-loop signals are semi-globally ultimately uniformly bounded and the prescribed performances of $\xi_{i,1}(t)$ are ensured, i.e., the inequality $\underline{\xi}_{i,1}(t)\le\xi_{i,1}(t)\le\overline{\xi}_{i,1}(t)$, $i=1,\ldots,N$, holds.*
- *Switching stops in finite time and $\omega(t)$ converges to the compact set*

$$\Omega^{\star}=\left\{\omega(t)\middle|\,\|\omega(t)\|_{t\to+\infty}\le\sqrt{\frac{(N^2+N-1)^2\sum_{i=1}^{N}\frac{\rho_{i,\infty}^2\left[\left(\exp(\overline{\vartheta}_{i,1})-1\right)\right]^2}{\left[1+\exp(\overline{\vartheta}_{i,1})\right]^2}}{N^{1-N}(N-1)^{N-1}}}\right\}$$

where $\overline{\vartheta}_{i,1}=\left[(p_i-p_{i,1}+4)(\ell_{i,\infty}(t_{\sigma_s})+\kappa_i)\right]^{\frac{1}{p_i-p_{i,1}+4}}$ with σ_s being a sufficiently large integer.

PROOF. See the appendix.

5 Simulation results



(*a*) (*b*)

Figure 2. Two different communication topologies

To validate the effectiveness of the proposed control method, two different communication topologies with one leader (labeled by 0) and three follower agents are considered as represented by the directed graph of Fig. 2. From Fig. 2-(a) and 2-(b), it can be seen that the signal of the leader is only accessible to follower 1 and follower 2, respectively. The following parameter settings are kept the same for both topologies. The leader output is $y_r=6\sin(0.5t)+6\sin(t)$ and the three follower agents are described by the following dynamics:

$$\text{Agent 1}\ \begin{cases}\dot{\chi}_{1,1}=1.5\cos(\chi_{1,1})\chi_{1,1}+0.8\chi_{1,2}^3,\\ \dot{\chi}_{1,2}=\chi_{1,1}\sin(\chi_{1,2})+\left(\tanh(\chi_{1,1})+1.2\right)u_1^5.\end{cases}$$

$$\text{Agent 2}\ \begin{cases}\dot{\chi}_{2,1}=1.25\chi_{2,1}+0.5\chi_{2,1}^3+1.5\chi_{2,2}^3,\\ \dot{\chi}_{2,2}=0.75\chi_{2,2}\chi_{2,1}^2+\left(\sin(\chi_{2,1})^2+0.75\right)u_2^5.\end{cases}$$

$$\text{Agent 3}\ \begin{cases}\dot{\chi}_{3,1}=0.5\left(\cos(\chi_{3,1})+\chi_{3,1}^2\right)+1.2\chi_{3,2}^3,\\ \dot{\chi}_{3,2}=\chi_{3,1}\sin(\chi_{3,2})+\left(|\cos(\chi_{3,1})|+0.2\right)u_3^5.\end{cases}$$

In our simulation, RBF NNs are used as linear-in-the-parameter approximators to approximate $|E_{i,j}(\boldsymbol{Z}_{i,j})|$, $i=1,2,3$, $j=1,2$, employing 64 nodes with centers evenly spaced in $[-1.5,1.5]\times[-1.5,1.5]\times[-1.5,1.5]\times[-1.5,1.5]\times[-1.5,1.5]\times[-1.5,1.5]\times[-1.5,1.5]\times[-1.5,1.5]\times[-1.5,1.5]$ and widths equal to 2. The initial conditions are selected as: $\chi_{1,1}(0)=0.75$, $\chi_{1,2}(0)=-1.75$, $\chi_{2,1}(0)=1.5$, $\chi_{2,2}(0)=-1.5$, $\chi_{3,1}(0)=1.75$, $\chi_{3,2}(0)=-1.2$, $\widehat{\Theta}_{1,1}(0)=6.5$, $\widehat{\Theta}_{1,2}(0)=7.5$, $\widehat{\Theta}_{2,1}(0)=4$, $\widehat{\Theta}_{2,2}(0)=3$, $\widehat{\Theta}_{3,1}(0)=6.5$, $\widehat{\Theta}_{3,2}(0)=4.75$, $\ell_{1,\infty}(0)=\ell_{2,\infty}(0)=\ell_{3,\infty}(0)=0.5$, $k_{1,1}(0)=k_{2,1}(0)=k_{3,1}(0)=6$, $k_{1,2}(0)=k_{2,2}(0)=k_{3,2}(0)=8$. The design parameters are chosen as: $\zeta_{k_{1,1}}=\zeta_{k_{1,2}}=1$, $\zeta_{k_{2,1}}=\zeta_{k_{2,2}}=\zeta_{k_{3,1}}=\zeta_{k_{3,2}}=1.5$, $\epsilon_{1,1}=\epsilon_{1,2}=\epsilon_{2,1}=\epsilon_{2,2}=\epsilon_{3,1}=\epsilon_{3,2}=1$, $\varrho_{1,1}=\varrho_{1,2}=\varrho_{2,1}=\varrho_{2,2}=\varrho_{3,1}=\varrho_{3,2}=1$, $\gamma_{1,1}=\gamma_{2,1}=\gamma_{3,1}=1$, $\gamma_{1,2}=\gamma_{2,2}=\gamma_{3,2}=0.4$, $\beta_{1,1}=\beta_{2,1}=\beta_{3,1}=0.8$, $\beta_{1,2}=\beta_{2,2}=\beta_{3,2}=6.25$, $\delta_{1,1}=\delta_{2,1}=\delta_{3,1}=0.25$, $\kappa_1=\kappa_2=\kappa_3=0.3$, $\theta_1=1.8$, $\theta_2=1.25$, $\theta_3=0.75$, $\zeta_{\ell_{1,\infty}}=\zeta_{\ell_{2,\infty}}=\zeta_{\ell_{3,\infty}}=0.5$, $\underline{\rho}_{1,1}=\underline{\rho}_{2,1}=\underline{\rho}_{3,1}=-6$, $\overline{\rho}_{1,1}=\overline{\rho}_{2,1}=\overline{\rho}_{3,1}=8$, $\underline{l}_{1,1}=\underline{l}_{2,1}=\underline{l}_{3,1}=3$, $\overline{l}_{1,1}=\overline{l}_{2,1}=\overline{l}_{3,1}=4$, and $\rho_{1,\infty}=\rho_{2,\infty}=\rho_{3,\infty}=0.95$.

The simulation results are shown in Figs. 3-8. Fig. 3-(a) and Fig. 5-(a) reveal that the tracking errors $\xi_{i,1}$, $i=1,2,3$, under the two topologies evolve within their respective bounds. Figs. 3-(b)-(c) and Figs. 5-(b)-(c) show that the functions $\widehat{V}_{i,n_i}$, $i=1,2,3$, under both topologies are upper bounded by ℓ_i, $i=1,2,3$, respectively. It can be seen from Figs. 4 and 6 that switching for both topologies stops in finite time and that the parameters $k_{i,j}$, $i=1,2,3$, $j=1,2$, are updated synchronously with the control directions $h_{i,j}$, $i=1,2,3$, $j=1,2$. Figs. 7 and 8 show that the NN approximators can achieve satisfactory approximation.

6 Conclusions

This work has proposed a logic-based switching mechanism for distributed switching tracking control of nonlinear multi-agent systems in power-chained form and with multiple unknown control directions. A novel dynamic boundary function that is decreasing in-between switching instants and possibly increasing at the switching instants has been devised to tackle the issue that asymptotic tracking cannot be achieved for such challenging nonlinear systems. An interesting problem to be investigated in the future is to combine logic-based update of the control directions with logic-based updates of the parameters.

Appendix

Proof of Theorem 1. We provide the proof through two stages. At stage 1, we show that the control goals of Theorem 1 are guaranteed on the interval $[0,+\infty)$ provided that the switching stops in finite time. At stage 2, we show by contradiction that indeed the switching stops in finite time.

Stage 1: Let $[0,t_s)$ be the maximum interval of the existence of the closed-loop solution, σ_s be the final switching index, and $t_{\sigma_s}<t_s$ be the time instant when the final switching occurs. Combining (23), (26), (27), and the fact that there is only a finite number of switchings, one can conclude

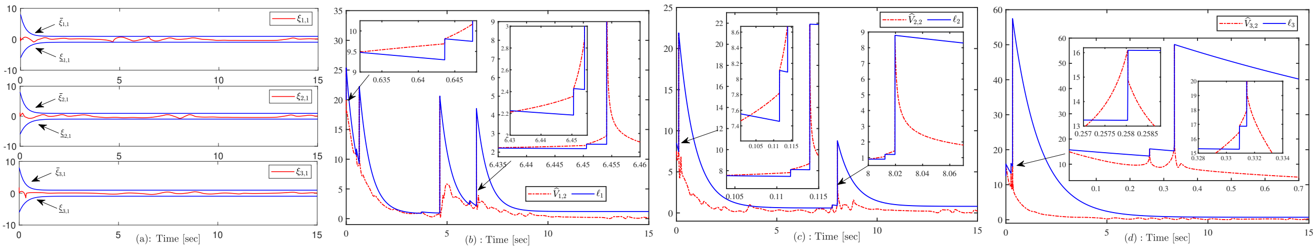


Figure 3. Topology in Fig. 2-(a): (a) trajectories of the consensus tracking errors $\xi_{1,1}$, $\xi_{2,1}$, and $\xi_{3,1}$; (b) trajectories of $\widehat{V}_{1,2}$ and ℓ_1; (c) trajectories of $\widehat{V}_{2,2}$ and ℓ_2; (d) trajectories of $\widehat{V}_{3,2}$ and ℓ_3.

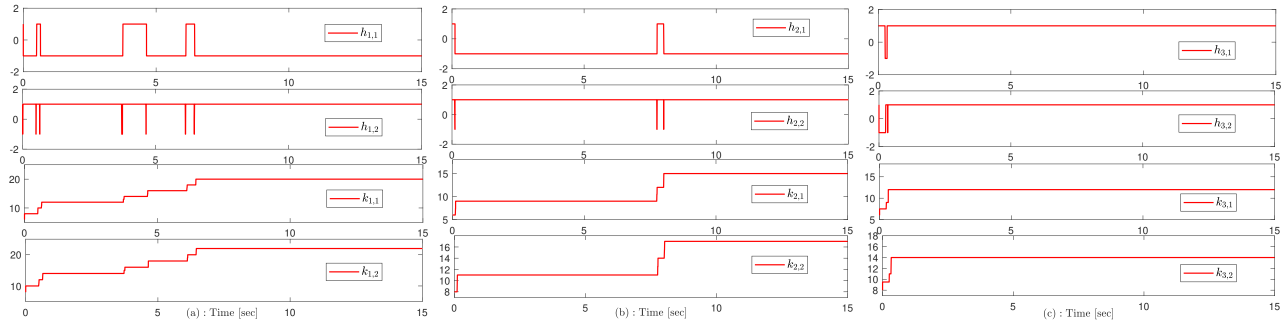


Figure 4. Topology in Fig. 2-(a): (a) evolution of $h_{1,1}$, $h_{1,2}$, $k_{1,1}$ and $k_{1,2}$; (b) evolution of $h_{2,1}$, $h_{2,2}$, $k_{2,1}$ and $k_{2,2}$; (c) evolution of $h_{3,1}$, $h_{3,2}$, $k_{3,1}$ and $k_{3,2}$.

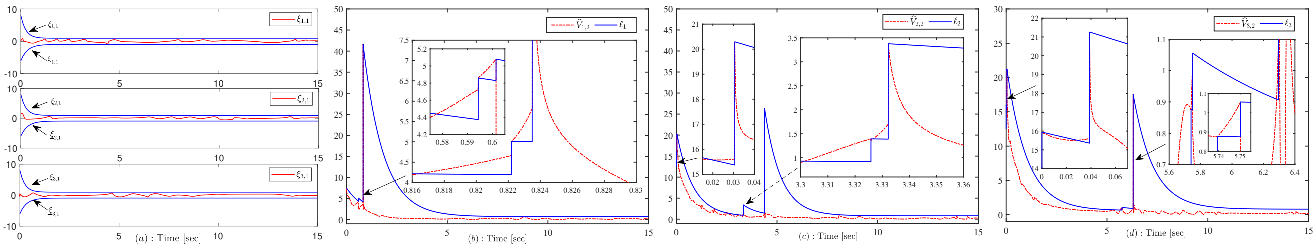


Figure 5. Topology in Fig. 2-(b): (a) trajectories of the consensus tracking errors $\xi_{1,1}$, $\xi_{2,1}$, and $\xi_{3,1}$; (b) trajectories of $\widehat{V}_{1,2}$ and ℓ_1; (c) trajectories of $\widehat{V}_{2,2}$ and ℓ_2; (d) trajectories of $\widehat{V}_{3,2}$ and ℓ_3.

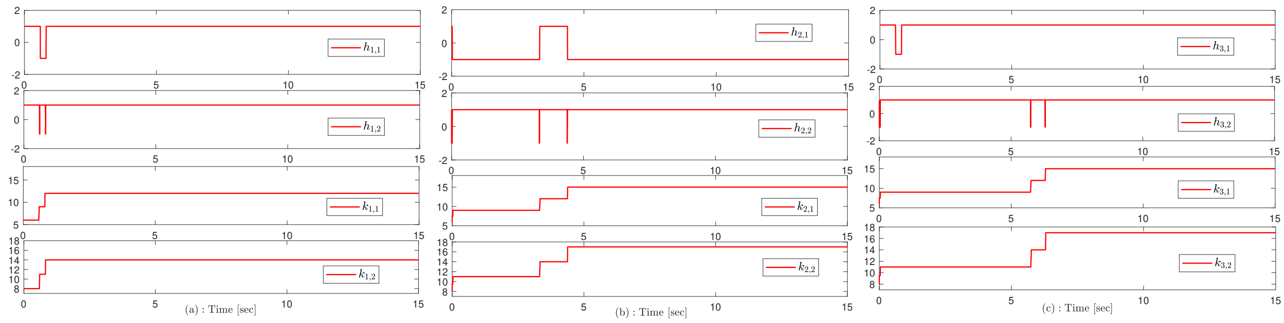


Figure 6. Topology in Fig. 2-(b): (a) evolution of $h_{1,1}$, $h_{1,2}$, $k_{1,1}$ and $k_{1,2}$ (a); (b) evolution of $h_{2,1}$, $h_{2,2}$, $k_{2,1}$ and $k_{2,2}$; (c) evolution of $h_{3,1}$, $h_{3,2}$, $k_{3,1}$ and $k_{3,2}$.

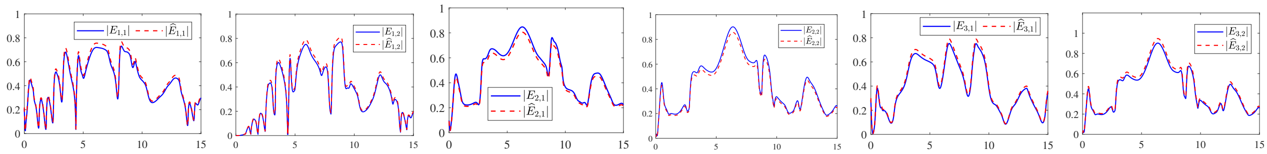


Figure 7. Topology in Fig. 2-(a): evolution of $|E_{1,1}|$, $|E_{1,2}|$, $|E_{2,1}|$, $|E_{2,2}|$, $|E_{3,1}|$, $|E_{3,2}|$, and their NN approximations $|\widehat{E}_{1,1}|$, $|\widehat{E}_{1,2}|$, $|\widehat{E}_{2,1}|$, $|\widehat{E}_{2,2}|$, $|\widehat{E}_{3,1}|$, $|\widehat{E}_{3,2}|$.

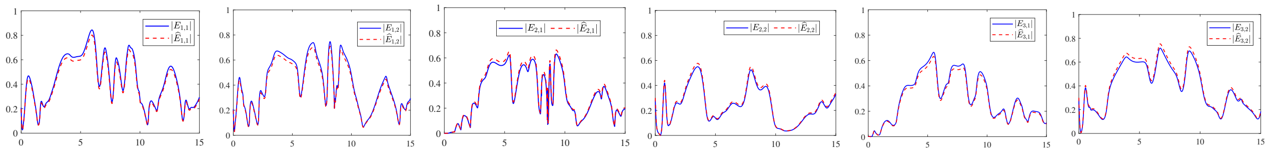


Figure 8. Topology in Fig. 2-(b): evolution of $|E_{1,1}|$, $|E_{1,2}|$, $|E_{2,1}|$, $|E_{2,2}|$, $|E_{3,1}|$, $|E_{3,2}|$, and their NN approximations $|\widehat{E}_{1,1}|$, $|\widehat{E}_{1,2}|$, $|\widehat{E}_{2,1}|$, $|\widehat{E}_{2,2}|$, $|\widehat{E}_{3,1}|$, $|\widehat{E}_{3,2}|$.

that after the final switching (i.e. for $t > t_{\sigma_s}$), it holds that $\mathcal{L}_i(t_{\sigma_s}, t) + \kappa_i \geq 0$, $t \in [t_{\sigma_s}, t_s)$, which indicates that

$$\widehat{V}_{i,n_i}(t) \leq \ell_i(t_{\sigma_s}, t) + \kappa_i, \ t \in [t_{\sigma_s}, t_s). \tag{42}$$

Thus, $\widehat{V}_{i,n_i}(\cdot)$, $\vartheta_{i,m}$ and $\widehat{\Theta}_{i,m}(\cdot)$, $i = 1, \ldots, N$, $m = 1, \ldots, n_i$, are bounded due to the boundedness of $\ell_i(\cdot)$ and κ_i on the interval $[t_{\sigma_s}, t_s)$. Furthermore, the virtual control laws $\alpha_{i,m}$, $m = 1, \ldots, n_i - 1$, $i = 1, \ldots, N$ and the actual control law u_i, $i = 1, \ldots, N$ are bounded on $[t_{\sigma_s}, t_s)$ according to (8)-(10). Thus, $\chi_{i,m}$, $\widetilde{\Theta}_{i,m}$, $m = 1, \ldots, n_i$, $i = 1, \ldots, N$, are bounded on $[t_{\sigma_s}, t_s)$ arising from the fact that $y_r(\cdot)$, $\Theta_{i,m}$, and $\widehat{\Theta}_{i,m}(\cdot)$ are bounded on $[t_{\sigma_s}, t_s)$. According to [40, Theorem 54, page. 476], no finite-time escape phenomena occurs, and thus $t_s = +\infty$. As a result, one concludes that all closed-loop signals are bounded on the entire time interval $[0, +\infty)$. Then, invoking (25) yields

$$\lim_{t\to+\infty} |\vartheta_{i,1}| \leq \left[(p_i - p_{i,1} + 4)(\ell_{i,\infty}(t_{\sigma_s}) + \kappa_i)\right]^{\frac{1}{p_i - p_{i,1} + 4}} \triangleq \overline{\vartheta}_{i,1}$$

which, in combination with the definition of $\vartheta_{i,1}$, gives

$$\lim_{t\to+\infty} \xi_{i,1}(t) \leq \frac{\rho_{i,\infty}\exp(\overline{\vartheta}_{i,1}) - \rho_{i,\infty}}{1 + \exp(\overline{\vartheta}_{i,1})}$$

After using a lower bound $\frac{\bar{N}}{N^2+N-1}$ [41] with $\bar{N} = \left(\frac{N-1}{N}\right)^{\frac{N-1}{2}}$ for $\sigma_{\min}\left(\mathscr{L} + \mathscr{B}\right)$, it follows that

$$\lim_{t\to+\infty} \|\omega(t)\| \leq \sqrt{\frac{(N^2+N-1)^2 \sum_{i=1}^{N} \frac{\rho_{i,\infty}^2\left[(\exp(\overline{\vartheta}_{i,1})-1)\right]^2}{\left[1+\exp(\overline{\vartheta}_{i,1})\right]^2}}{N^{1-N}(N-1)^{N-1}}}.$$

We are now in a position to discuss the existence of a compact set that makes the universal approximation ability valid, provided that the switching stops in finite time.

Consider the initial conditions $\overline{\boldsymbol{\chi}}_{i,m}(0)$ and $\widehat{\Theta}_{i,m}(0) \geq 0$, for $i = 1, \ldots, N, m = 1, \ldots, n_i$, satisfying $\widehat{V}\left(\overline{\boldsymbol{\chi}}_{i,m}(0), \widehat{\Theta}_{i,m}(0)\right) < \Upsilon_0$ with $\Upsilon_0 = \sum_{i=1}^{N} \hbar_i$ with $\hbar_i \triangleq \max_{\sigma\in\{0,1,\ldots,\sigma_s\}} \ell_i(t_\sigma, t_\sigma)$ and consider the compact set

$$\Omega_0 = \left\{\left(\overline{\boldsymbol{\chi}}_{i,m}(t), \widehat{\Theta}_{i,m}(t)\right)\middle|\widehat{V}\left(\overline{\boldsymbol{\chi}}_{i,m}, \widehat{\Theta}_{i,m}\right) \leq \Upsilon, \ t \geq 0\right\} \tag{43}$$

where $\Upsilon = \Upsilon_0 + \sum_{i=1}^{N} \kappa_i$. According to Algorithm 1, (41), and (42), $\hbar_i$ is bounded provided that the switching stops in finite time, and that the inequality

$$\widehat{V}\left(\overline{\boldsymbol{\chi}}_{i,m}(t), \widehat{\Theta}_{i,m}(t)\right) < \Upsilon, \tag{44}$$

holds true for all $t \geq 0$ provided that $\widehat{V}\left(\overline{\boldsymbol{\chi}}_{i,m}(0), \widehat{\Theta}_{i,m}(0)\right) < \Upsilon_0$ holds true. Therefore, the existence of the compact set Ω_0 makes the universal approximation ability of the linear-in-the-parameter approximation valid since all state variables involved are retained in Ω_0 all the time.

Stage 2: At this stage, by seeking a contradiction, we prove that there indeed exist a finite number of switchings. Let us first suppose that there exist an infinite number of switchings. Therefore, there surely exists a sufficiently large t_{σ_s} such that $\gamma_{i,q}\beta_{i,q} \leq (p_i - p_{i,q} + 4)c_{i,q}\eta_i^{\frac{p_{i,q}-1}{p_i+3}}$, $i = 1, \ldots, N, q = 1, \ldots, n_i$, and such that

$$\boldsymbol{h}_i(t) = \boldsymbol{d}_{i,\sigma_s} = [\mathrm{sign}(\psi_{i,1}), \ldots, \mathrm{sign}(\psi_{i,n_i})]^T \tag{45}$$

on $[t_{\sigma_s}, t_{\sigma_s+1})$. Thus, $\varsigma_i = \gamma_{i,q}\beta_{i,q}, q = 1, \ldots, n_i, i = 1, \ldots, N$. It follows from (43) that (22) becomes

$$\dot{V}_{i,n_i} < -\varsigma_i V_{i,n_i} + \Xi_i \ \text{ as } \ \sum\nolimits_{q=1}^{n_i} \Delta_{i,q} = 0, \tag{46}$$

which, combined with (20) and the Gronwall inequality [11], implies that

$$\vartheta_{i,m}^{p_i+3} < \left[(p_i - p_{i,m} + 4)\left(V_{i,n_i}(0) + \Xi_i/\varsigma_i\right)\right]^{\frac{p_i+3}{p_i - p_{i,m} + 4}} \triangleq \Psi_{i,m}$$

$$\left|\widehat{\Theta}_{i,m}\right| < \sqrt{2\gamma_{i,m}\left(V_{i,n_i}(0) + \Xi_i/\varsigma_i\right)} + \Theta_{i,m} \triangleq \bar{\Lambda}_{i,m} \tag{47}$$

holds on $[t_{\sigma_s}, t_{\sigma_s+1})$ for $i = 1, \ldots, N$, $m = 1, \ldots, n_i$. Thus, it follows from (11) that

$$\left|\dot{\widehat{\Theta}}_{i,m}\right| < \gamma_{i,m}\epsilon_{i,m}^{\overline{p}_{i,m}}\Psi_{i,m}\overline{\Gamma}_{i,m} + \gamma_{i,m}\beta_{i,m}\bar{\Lambda}_{i,m} \triangleq \Upsilon_{i,m} \tag{48}$$

holds on $[t_{\sigma_s}, t_{\sigma_s+1})$, where $\overline{\Gamma}_{i,m}$ is the upper bound of $\Gamma_{i,m}^{\overline{p}_{i,m}}$ according to [16, Lemma 2], for $i = 1, \ldots, N$, $m = 1, \ldots, n_i$.

Recalling (20), (25), (46)-(48), we can obtain that

$$\begin{aligned}\dot{\widehat{V}}_{i,n_i}(t) &< -\varsigma_i V_{i,n_i} + \Xi_i + \sum\nolimits_{m=1}^{n_i} \frac{\Theta_{i,m}\dot{\widehat{\Theta}}_{i,m}}{\gamma_{i,m}} \\ &< -\varsigma_i \widehat{V}_{i,n_i} + \Xi_i + \sum\nolimits_{m=1}^{n_i} \frac{\Theta_{i,m}\dot{\widehat{\Theta}}_{i,m}}{\gamma_{i,m}} - \sum\nolimits_{m=1}^{n_i} \frac{\varsigma_i\Theta_{i,m}^2}{2\gamma_{i,m}} \\ &\quad + \sum\nolimits_{m=1}^{n_i} \frac{\varsigma_i\Theta_{i,m}\widehat{\Theta}_{i,m}}{\gamma_{i,m}} < -\varsigma_i\widehat{V}_{i,n_i} + \widehat{\Xi}_i,\end{aligned} \tag{49}$$

where $\widehat{\Xi}_i = \Xi_i + \sum_{m=1}^{n_i} \frac{\Theta_{i,m}\Upsilon_{i,m}}{\gamma_{i,m}} + \sum_{m=1}^{n_i} \frac{\varsigma_i\Theta_{i,m}\bar{\Lambda}_{i,m}}{\gamma_{i,m}}$ is an unknown positive constant. Hence, we have $\widehat{V}_{i,n_i}(t) \leq \frac{\widehat{\Xi}_i}{\varsigma_i}$, on $[t^*, t_{\sigma_s+1})$, where t^* is the first time instant satisfying $\widehat{V}_{i,n_i}(t^*) = \frac{\widehat{\Xi}_i}{\varsigma_i}$. Then, one has that $\dot{\widehat{V}}_{i,n_i} < 0$ holds when $\widehat{V}_{i,n_i} \geq \frac{\widehat{\Xi}_i}{\varsigma_i}$. The fact that $\widehat{V}_{i,n_i}(\cdot)$ strictly decreases on the time interval $[t_{\sigma_s}, t^*)$ implies that no new switching occurs on $[t_{\sigma_s}, t^*)$ and that $t^* < t_{\sigma_s+1}$.

When $t \in \left[t_{\sigma_s}, t^*\right)$, we can guarantee $0 < \theta_i \leq \varsigma_i$ by choosing proper $\gamma_{i,q}$, and $\beta_{i,q}$, $q = 1, \ldots, n_i$, according to $\varsigma_i = \gamma_{i,q}\beta_{i,q}, q = 1, \ldots, n_i, i = 1, \ldots, N$, which implies that $\widehat{V}_{i,n_i}(t) \leq \ell_i(t)$, on $\left[t_{\sigma_s}, t^*\right)$. When $t \in \left[t^*, t_{\sigma_s+1}\right)$, the condition $\ell_{i,\infty}(t) \geq \frac{\widehat{\Xi}_i}{\varsigma_i}$ can be satisfied via a sufficiently large σ_s in view of (29) and (30). Hence, it holds that $\widehat{V}_{i,n_i}(t) \leq \ell_i(t_{\sigma_s}, t)$, $\forall t \in [t^*, t_{\sigma_s+1})$.

To summarize, we have that $\widehat{V}_{i,n_i}(t) < \ell_i(t_{\sigma_s}, t) < \ell_i(t_{\sigma_s}, t) + \kappa_i$, on $[t_{\sigma_s}, t_{\sigma_s+1})$, which means that switching condition (31) can never be satisfied on the time interval $[t_{\sigma_s}, t_{\sigma_s+1})$. This contradicts the assumption made in the beginning of stage 2. Thus, the proof is completed. ■

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